Student Name: ______________________________

Teacher: ______________________________

2025	Hurlstone Agricultural High School **Year 12 Assessment Task 2**

Mathematics Extension 2

Examiners	• **Mr D Potaczala** • **Mr G Rawson** • **Mr G Huxley**
General Instructions	• Reading time – 5 minutes • Working time – 1½ hours • Write using black or blue pen • NESA-approved calculators may be used • A Reference sheet is provided for your use • In Questions 9 to 12, show relevant mathematical reasoning and/or calculations
Total marks: 56	**Section I – 8 marks (pages 2–5)** • Attempt Questions 1–8 • Allow about 12 minutes for this section **Section II – 48 marks (pages 6–10)** • Attempt Questions 9–12 • Allow about 78 minutes for this section • For each question use a separate answer booklet

Section I

8 marks
Attempt Questions 1 to 8
Allow about 12 minutes for this section

Use the multiple-choice answer sheet for Questions 1 to 8

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1. Let $z = 5 - i$ and $w = 2 + 3i$.

   What is the value of $2z + \overline{w}$?

   A. $12 + i$

   B. $12 + 2i$

   C. $12 - 4i$

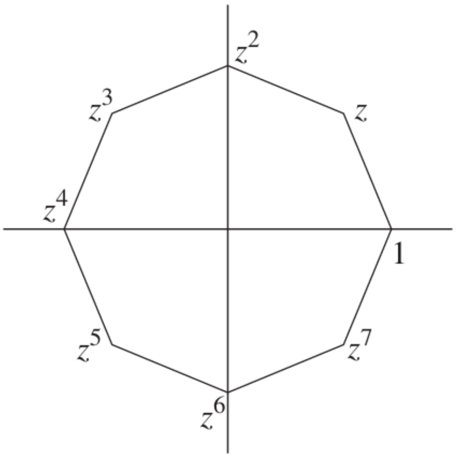
   D. $12 - 5i$

2. It is given that $z = 2 + i$ is a root of $z^3 + az^2 - bz + 5 = 0$, where a and b are real numbers. What is the value of a?

   A. -5 **B.** -3

   C. 3 **D.** 5

3. The complex number z is chosen so that $1, z, z^2, \ldots\ldots, z^7$ form the vertices of the regular polygon shown.



Which polynomial equation has all of these complex numbers as roots?

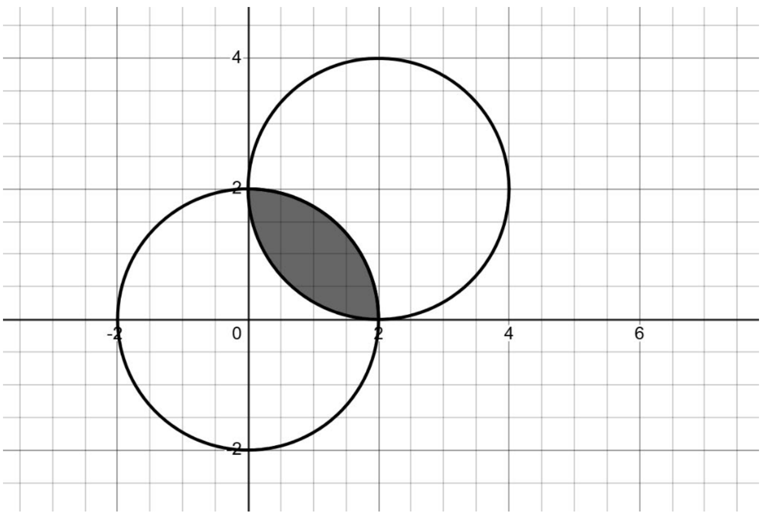
A. $x^7 - 1 = 0$

B. $x^8 - 1 = 0$

C. $x^7 + 1 = 0$

D. $x^8 + 1 = 0$

4. Which of the following inequalities are represented by the shaded area in this Argand diagram?



A. $|z| \leq 4$ and $|z - 2 + 2i| \leq 4$

B. $|z| \leq 2$ and $|z - 2 + 2i| \leq 2$

C. $|z| \leq 4$ and $|z - 2 - 2i| \leq 4$

D. $|z| \leq 2$ and $|z - 2 - 2i| \leq 2$

5. What is the contrapositive of the following statement?

If you're happy and you know it, then you will clap your hands.

A. If you don't clap your hands, then you're happy and you know it.

B. If you clap your hands, then you're either not happy or you don't know it.

C. If you don't clap your hands, then you're not happy and you don't know it.

D. If you don't clap your hands, then you're not happy or you don't know it.

6. Which of the following is the negation of the statement:

"$\forall p \in P$, p is of the form $4m+1 \Rightarrow p$ can be written as a sum of two squares"?

A. $\forall p \in P$, p is of the form $4m+1$ and p can not be written as a sum of two squares.

B. $\exists p \in P$, p is not of the form $4m+1$ and p can be written as a sum of two squares.

C. $\forall p \in P$, p is not of the form $4m+1$ or p can not be written as a sum of two squares.

D. $\exists p \in P$, p is of the form $4m+1$ and p can not be written as a sum of two squares.

7. Given $P(z)$ is a polynomial function with real coefficients, which of the following is true?

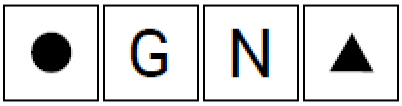
A. $\forall \alpha \in \mathbb{C}, P(\alpha)=0 \Rightarrow P(\overline{\alpha})=0$

B. $P(\alpha)=0 \Rightarrow P(\overline{\alpha})=0$ iff $\alpha \neq 0$

C. $P(\alpha)=0 \Rightarrow P(\overline{\alpha})=0$ iff $\alpha \in \mathbb{R}$

D. $P(\alpha)=0 \Rightarrow P(\overline{\alpha})=0$ iff $\alpha \notin \mathbb{R}$

8. Four cards each have a letter on one face and a shape on the other. They are placed on a table as follows:



You are given the rule "If N is on a card, then a circle is on the other side."

Which cards need to be turned over to check if this rule holds?

A. N and G.

B. G and the triangle.

C. The circle and N.

D. N and the triangle.

~ END OF SECTION I ~

Section II

48 marks
Attempt Questions 9 to 12
Allow about 78 minutes for this section

Answer each question in a new answer booklet.

All necessary working should be shown in every question.

Question 9 (12 marks) **Marks**

(a) Express $\dfrac{2\sqrt{5}+i}{\sqrt{5}-i}$ in the form $x+iy$, where x and y are real. **2**

(b) (i) Let $z=-2\sqrt{3}-2i$. Express z in exponential form. **2**

(ii) Find a positive integer n such that z^n is purely imaginary. **2**

(c) Factorise $z^2+4iz+5$. **2**

(d) Let $z=\cos\theta+i\sin\theta$.

(i) By considering the real part of z^4, show that $\cos 4\theta$ is

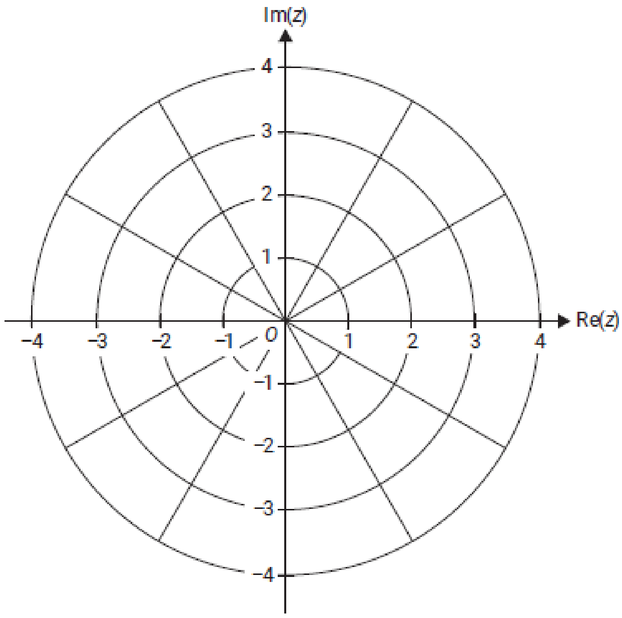
$$\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta.$$ **2**

(ii) Hence, or otherwise, find an expression for $\cos 4\theta$ involving only powers of $\cos\theta$. **2**

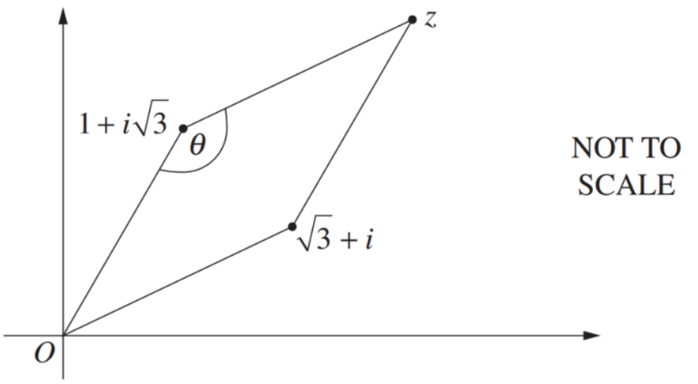
~ End of question 9 ~

Question 10 (12 marks) **Marks**

(a) Sketch the ray given by $Arg\left(z-\left(\sqrt{3}-i\right)\right)=\frac{5\pi}{6}$ on the Argand diagram given in your answer booklet. A copy of the diagram is below. **2**



(b) On the Argand Diagram, the complex numbers $0, 1+i\sqrt{3}, \sqrt{3}+i$ and z form a rhombus.



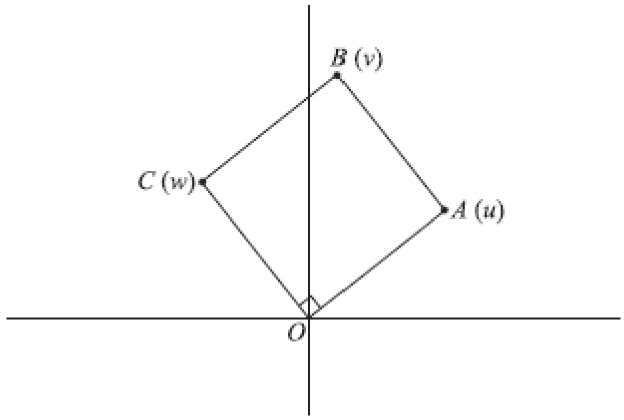
(i) Find z in the form $a+ib$, where a and b are real numbers. **1**

(ii) An interior angle, θ, of the rhombus is marked on the diagram.

Show reasoning and find the size of θ. **2**

~ *Question 10 continues on the next page*~

(c) The points *A, B* and *C* on the Argand Diagram represent the complex numbers *u, v* and *w* respectively. The points *O, A. B* and *C* form a square as shown on the diagram.



It is given that $u = 7 + 4i$

(i) Find the value of *v*. **2**

(ii) Calculate $\arg\left(\frac{w}{v}\right)$. **1**

(d) Let the complex numbers 0, *u* and *v* form the vertices of an equilateral triangle in the Argand Diagram, where $Arg(v) > Arg(u) > 0$.

(i) By considering the number *v* as a transformation of the number *u* on the diagram, write an expression for *v* in terms of *u.* **1**

(ii) Hence, or otherwise, show that $u^3 + v^3 = 0$. **1**

(iii) Show that $u^2 + v^2$ must be equal to $-uv$. **2**

~ End of question 10 ~

Question 11 (12 marks) **Marks**

(a) Consider the statement: "If $a^2 - 2a + 7$ is even then a is odd."

(i) Write the contrapositive of this statement. **1**

(ii) Use proof by contrapositive to prove that the statement is true. **2**

(b) Consider the statement:

$$(2n)! \geq 2^n (n!)^2 \text{ for all positive integers } n.$$

Prove that if the statement is true when $n = k$, then it will be true when $n = k + 1$. **3**

(c) Suppose that a, b, c and d are positive integers, and c is not a perfect square.

(i) Given that $\dfrac{a}{b+\sqrt{c}} + \dfrac{d}{\sqrt{c}}$ is rational, prove that $b^2 d = c(a+d)$. **3**

(ii) Hence, prove that $\dfrac{a}{1+\sqrt{c}} + \dfrac{d}{\sqrt{c}}$ is not rational. **3**

~ End of question 11 ~

Question 12 (12 marks) **Marks**

(a) (i) For real numbers $a, b \geq 0$, prove that $\dfrac{a+b}{2} \geq \sqrt{ab}$ **2**

(ii) Hence or otherwise, find the minimum value of the expression $4x + \dfrac{9}{x}$ when $x > 0$. **2**

(b) Use mathematical induction to prove that:

$2^n + (-1)^{n+1}$ is divisible by 3 for all positive integers *n*. **3**

(c) By considering the reciprocals of the left-hand and right-hand sides, or otherwise, prove the following without using a calculator or decimal approximations.

$$\sqrt{6} - \sqrt{5} < \sqrt{5} - 2$$ **2**

(d) Use mathematical induction to prove that $7^n + 15^n$ is divisible by 11, where ***n* is a positive odd integer.** **3**

~ End of question 12 ~

~ END OF SECTION II ~

~ END OF PAPER ~

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

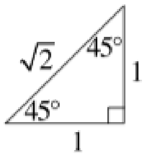
$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

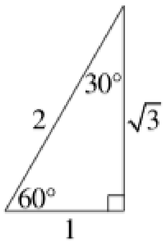
Trigonometric Functions

$\sin A = \dfrac{\text{opp}}{\text{hyp}}, \quad \cos A = \dfrac{\text{adj}}{\text{hyp}}, \quad \tan A = \dfrac{\text{opp}}{\text{adj}}$



$A = \dfrac{1}{2}ab\sin C$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$



$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \dfrac{1}{2}r^2\theta$

Trigonometric identities

$\sec A = \dfrac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \dfrac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \dfrac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1+t^2}$

$\cos A = \dfrac{1-t^2}{1+t^2}$

$\tan A = \dfrac{2t}{1-t^2}$

$\cos A \cos B = \dfrac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A \sin B = \dfrac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A \cos B = \dfrac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A \sin B = \dfrac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \dfrac{1}{2}(1 - \cos 2nx)$

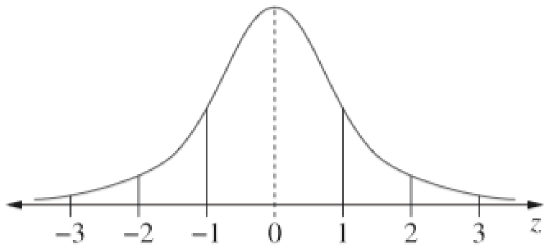
$\cos^2 nx = \dfrac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \dfrac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \le x) = \displaystyle\int_a^x f(x)\,dx$

$P(a < X < b) = \displaystyle\int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$

$X \sim \operatorname{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \dbinom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left| x\underset{\sim}{i} + y\underset{\sim}{j} \right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$

Student Name: ______________________________

Teacher: ______________________________

2025

Hurlstone Agricultural High School
Year 12 Mathematics Extension 2
Assessment Task 2

Section I – Multiple Choice Answer Sheet

Allow about 12 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

Start Here

1. A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

5. A ○ B ○ C ○ D ○

6. A ○ B ○ C ○ D ○

7. A ○ B ○ C ○ D ○

8. A ○ B ○ C ○ D ○

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Ext 2 2025 AT2 Multiple Choice Answers and Solutions

1. $2z+\overline{w}=2(5-i)+2-3i$ **D**

$=12-5i$

2. $\alpha\beta\gamma=-\frac{d}{a}$

$(2+i)(2-i)\gamma=-5$

$5\gamma=-5$

$\gamma=-1$

Now, $\sum\alpha=-\frac{b}{a}$ **B**

$(2+i)+(2-i)+(-1)=-a$

$a=-3$

3. There are 8 roots indicated including $z^0=1$. Selecting the complex root with the smallest positive argument, z, we can multiply its argument by 8 to get to the real number 1. Hence $z^8=1$ and so $z^8-1=0$.

B

4. The shaded area is the intersection of the interior of both circles. They both have radius =2, so the final difference to determine is the correct subtraction of the number represented by the centre of the circle point (2,2).

D

5. Contrapositive of $A\Rightarrow B$ is $\neg B\Rightarrow\neg A\ldots$

Note:

A: you're happy _and_ you know it

$\neg A$: you're ***not*** happy **_or_** you ***don't*** know it

B: you clap your hands

$\neg B$: you don't clap your hands **D**

6. it's just D **D**

7. The first statement is true, referring to the complex roots and their conjugates. The other statements may be true on some occasions, but it is the *iff* in each statement that renders them untrue.

A

8. Considering each card in turn:
The reverse of the circle card will not help us because there is the possibility that it is on the reverse of another letter.
The statement does not affect cards with a G as letter.
If N is on one side there has to be a circle on the other side, which we will need to confirm.
For a triangle on one side, then there can not be an N on the other side.

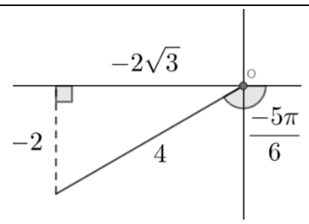
So option D: Turning the N to ensure the statement (If N then circle) and turning the triangle to ensure the contrapositive is also true. (If not a circle, then not an N).

D

Year 12	Mathematics Extension 2	Ass Task 2 2025 HSC
Question 9	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

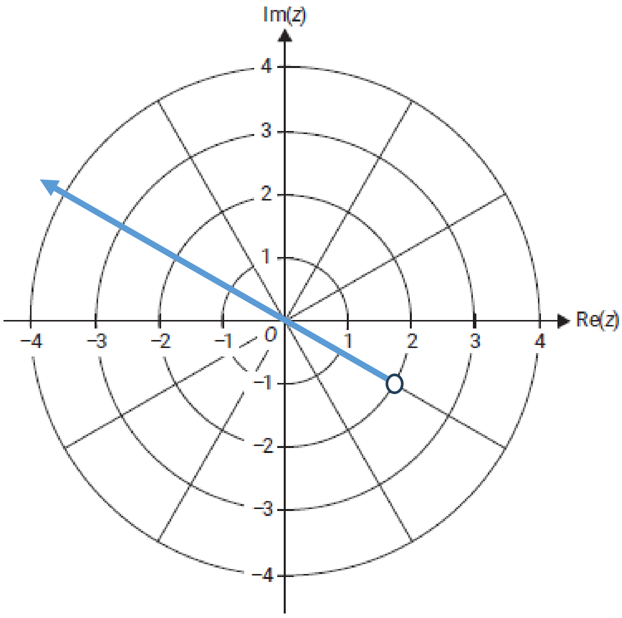
MEX 12-1: understands and uses different representations of numbers and functions to model, prove results and find solutions to problems in a variety of contexts

Question	Solutions	Marking Guidelines
(a)	$\dfrac{2\sqrt{5}+i}{\sqrt{5}-i}=\dfrac{\left(2\sqrt{5}+i\right)\left(\sqrt{5}+i\right)}{5+1}$ $=\dfrac{10-1+3\sqrt{5}i}{6}=\dfrac{3}{2}+\dfrac{\sqrt{5}}{2}i$	<u>2 marks</u>: correct solution $x+yi$ => real and imaginary components need to be separate <u>1 mark</u>: substantially correct solution (writes $\dfrac{2\sqrt{5}+i}{\sqrt{5}-i}\times\dfrac{\sqrt{5}+i}{\sqrt{5}+i}$ or equivalent merit)
(b)(i)	From the diagram, $z=4e^{-\frac{5\pi}{6}i}$ $\left(\text{or } z=4e^{\frac{7\pi}{6}i}\right)$  	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution
(ii)	Now, $z^n=\left(4e^{-\frac{5\pi}{6}i}\right)^n=4^n e^{-\frac{5n\pi}{6}i}$ For purely imaginary, the argument must be $k\pi+\dfrac{\pi}{2}$, for integral k *Or an odd multiple of* $\dfrac{\pi}{2}$, ie $(2k+1)\dfrac{\pi}{2}$. $(2k+1)\dfrac{\pi}{2}=-\dfrac{5\pi n}{6}$ $3(2k+1)=-5n$ $n=-\dfrac{3(2k+1)}{5}$ for n to be +ve integer, $-(2k+1)=5M$ (ie multiple of 5) $k=-3,\,-8,\,\ldots$ $n=3,\,9,\,\ldots$ Smallest positive integer is $n=3$ (but accept any $3+6p\ldots$ p integer)	<u>2 marks</u>: correct solution <u>1 mark</u>: recognising the argument is $-\dfrac{5\pi n}{6}$ and must be equal to $\dfrac{\pi}{2}$ $\pm$ a multiple of π. (or equivalent merit) Note: the most common statement by candidates was for the argument to be $\dfrac{\pi}{2}+2k\pi$, which does not count for imaginary numbers on the -ve y-axis. This generally led to an answer of $n=9$. While this answer is correct in this case, had the question asked for the smallest value of n, very few would have attained full marks

(c)(i)	*Question 9 continued…* Find zeros by using the quadratic formula: $z=\frac{1}{2}\left(-4i\pm\sqrt{(4i)^2-4.5.1}\right)$ $=\frac{1}{2}\left(-4i\pm\sqrt{-36}\right)$ $=-2i\pm 3i$ The zeros are $-5i$ and i, and therefore $z^2+4iz+5=(z+5i)(z-i)$ Common problems were: • not distinguishing between factors and the solution to a quadratic equation – using the quadratic formula correctly and getting $z=i$ or $z=-5i$, but not stating that the factors were $(z-i)(z+5i)$ • not stating the quadratic formula before using it, eg a number of candidates simply wrote $\frac{-4i\pm\sqrt{-16+20}}{2}$. This incorrect '+20' did not allow the markers to know whether this was due to an incorrect formula being used or to a simple arithmetic error • not realising that the co-efficient of z was $4i$ and not 4 • many candidates wrote $z^2+4iz+5=(z+2i)^2+9$, but had difficulty in converting the + 9 to – $9i^2$ to achieve the next step equal to $(z+2i+3i)(z+2i-3i)$.	<u>2 marks</u>: correct factorisation <u>1 mark</u>: substantially correct solution Attempts to use formula to solve a quadratic equation, or equivalent merit this was extremely common!
(d)(i) **(ii)**	$(\cos\theta+i\sin\theta)^4=\cos^4\theta+4i\cos^3\theta\sin\theta+6i^2\cos^2\theta\sin^2\theta+4i^3\cos\theta\sin^3\theta+i^4\sin^4\theta$ $\cos 4\theta+i\sin 4\theta=\cos^4\theta+4i\cos^3\theta\sin\theta-6\cos^2\theta\sin^2\theta-4i\cos\theta\sin^3\theta+\sin^4\theta$ Equating reals: $\cos 4\theta=\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta$ $\cos 4\theta=\cos^4\theta-6\cos^2\theta\sin^2\theta+\sin^4\theta$ $=\cos^4\theta-6\cos^2\theta\left(1-\cos^2\theta\right)+\left(1-\cos^2\theta\right)^2$ $=\cos^4\theta-6\cos^2\theta+6\cos^4\theta+1-2\cos^2\theta+\cos^4\theta$ $=8\cos^4\theta-8\cos^2\theta+1$	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution <u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution (which means, in this case, <u>demonstrating</u> an understanding of what is required, but not executing effectively due to calculation error)

Question 10
Outcomes Addressed in this Question

MEX12-4 uses the relationship between algebraic and geometric representations of complex numbers and complex number techniques to prove results, model and solve problems

Question	Solutions	Marking Guidelines
(a)	 	**(a) 2 marks**: Correct ray with open circle at point indicated on polar grid. **1 mark:** 1 correct feature (either point of origin or argument of ray)
(b)	(i) $$z = 1 + i\sqrt{3} + \sqrt{3} + i$$ $$= 1 + \sqrt{3} + \left(1 + \sqrt{3}\right)i$$ (ii) From the point representing $\left(1 + i\sqrt{3}\right)$ draw a vertical line down to the real axis, and a horizontal line towards the right side of the diagram. This shows that θ is made up of: $\left(\frac{\pi}{2} - arg\left(1 + i\sqrt{3}\right)\right) + \left(\frac{\pi}{2}\right) + \left(arg\left(\sqrt{3} + i\right)\right)$ $$\therefore \theta = \frac{\pi}{6} + \frac{\pi}{2} + \frac{\pi}{6} = \frac{5\pi}{6}$$	**(b)(i) 1 mark:** Correct response. **(ii) 2 marks:** Correct response with justification, or equivalent merit. **1 mark:** Solution incomplete.
(c)	(i) $$v = u + w = u + iu$$ $$= 7 + 4i + (7 + 4i)i$$ $$= 3 + 11i$$ (ii) $arg\left(\frac{w}{v}\right) = arg(w) - arg(v) = \frac{\pi}{4}$, Due to the diagonal property of square *OABC*.	**(c)(i) 2 marks:** Correct solution with working. **1 mark:** 1 component correct; either correct expression for *w* or correct *v* from incorrect *w*. **(ii) 1 mark:** Correct solution.

(d)	(i) $v = ue^{\frac{\pi}{3}i}$ or $v = u\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)$ (ii) Method 1: $u^3 + v^3 = u^3 + u^3e^{\pi i}$ $= u^3 + (-u^3) = 0$ Method 2: $u^3 + v^3 = u^3 + u^3(cos\pi + isin\pi)$ $= u^3 + (-u^3) = 0$ (iii) From (ii): $u^3 + v^3 = 0$ $(u + v)(u^2 - uv + v^2) = 0$ $\therefore u + v = 0$ or $(u^2 - uv + v^2) = 0$ But $u \neq -v$ because u, v form sides of an equilateral triangle (and from part (i)). So $(u^2 - uv + v^2) = 0$ is the only solution. $\therefore u^2 + v^2 = +uv$ EDIT: The highlighted text above includes the intended solution to the question. A typo in the question gave $u^2 + v^2 = -uv$ The marking scheme has been adjusted accordingly: **2 marks:** Either solution: $u^2 + v^2 = \pm uv$ that discards $u + v = 0$. **1 mark:** Attempts factorisation of sum of 2 cubes, or equivalent merit.	**d(i) 1 mark:** Correct expression. **(ii) 1 mark:** Correct communication of property that makes $v^3 = -u^3$. **(iii) 2 marks:** Correct solution, including the reason why $u + v = 0$ needs to be discarded as a solution. **1 mark:** Factorisation of sum of 2 cubes, without justification of solution.

Year 12	Mathematics Extension 2	Ass Task 2 2025 HSC
Question 11	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MEX 12-2: chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings

Question	Solutions	Marking Guidelines
(a)(i)	Contrapositive is: "If a is not odd then a^2-2a+7 is not even." (Which is equivalent to: "If a is even then a^2-2a+7 is odd")	1 mark: correct contrapositive statement
(ii)	Let $a=2k$, where k is an integer Then: $a^2-2a+7=(2k)^2-2(2k)+7$ $=4k^2-4k+7$ $=2(2k^2-2k+3)+1$ $=2M+1$ which is odd, as M is an integer $\therefore$ If a is even then a^2-2a+7 is odd And by contraposition, If a^2-2a+7 is even then a is odd	2 marks: correct solution (*qualifying $2k^2$-$2k$+3 as an integer, and 2($2k^2$-$2k$+3)+1 as odd*) 1 mark: substantially correct solution (defines k as an integer, or equivalent merit)
(b)	Assuming the statement true for $n=k$ *ie*, assume $(2k)!\geq 2^k(k!)^2$ is true Prove true for $n=k+1$ ie prove $[2(k+1)]!\geq 2^{k+1}[(k+1)!]^2$ $\text{LHS}=(2(k+1))!$ $=(2k+2)!$ $=(2k+2)(2k+1)(2k)!\geq(2k+2)(2k+1)2^k(k!)^2$ (assumption) $=2(k+1)(2k+1)2^k(k!)^2$ $>2^{k+1}(k+1)(k+1)(k!)^2$ $=2^{k+1}((k+1)!)^2$ $=\text{RHS}$ $\therefore$ if true for $n=k$, then true for $n=k+1$. There were many different methods used to answer this question. Some unusual approaches were attempted. The better responses worked with only one side of the inequality, rather than with both sides at once. A common error was to state $(2k)!=2!k!$. In the better responses, candidates simplified the left hand side, $[2(k+1)]!=(2k+2)(2k+1)(2k)!$, then used the induction hypothesis to simplify. The most common error was to not use the induction hypothesis to allow comparison of both sides of the inequality.	3 marks: correct solution Note: the most common reason for not gaining the full three marks was for an inability to demonstrate the RHS was greater than what was required to be proved, after introducing the induction hypothesis. (ie the bit in orange) 2 marks: correctly writes what is to be proved, ***and*** correctly introduces the assumption and inequality into the proof, or equivalent merit 1 mark: correctly writes what is to be proved, ***or*** correctly introduces the assumption and inequality into the proof, or equivalent merit

Question 11 continued…

(c)(i)

$$\frac{a}{b+\sqrt{c}}+\frac{d}{\sqrt{c}}=\frac{a\sqrt{c}+bd+d\sqrt{c}}{b\sqrt{c}+c}\times\frac{b\sqrt{c}-c}{b\sqrt{c}-c}$$

$$=\frac{abc+b^2d\sqrt{c}+bdc-ac\sqrt{c}-bdc-dc\sqrt{c}}{b^2-c^2}$$

$$=\frac{abc+bdc-bdc+\sqrt{c}\left(b^2d-ac-dc\right)}{b^2-c^2}=\frac{p}{q},\ \left(p,q\text{ integral}\right)$$

For n to be integral,

$b^2d-ac-dc=0$ $\qquad\left(c\text{ not perfect square }\Rightarrow\sqrt{c}\text{ is irrational}\right)$

$$b^2d=ac+dc$$

$$b^2d=a\left(c+d\right)$$

3 marks: correct solution
1. writing the expression as a single fraction ***and*** noting p/q.
2. separating rational and irrational parts of numerator.
3. recognising coefficient of $\sqrt{c}=0$ ***and*** rearranging to show result

2 marks: substantially correct solution (two of the above, or equivalent merit)

1 mark: partially correct solution (one of the above, or equivalent merit)

(ii)

From (i) $\dfrac{a}{b+\sqrt{c}}+\dfrac{d}{\sqrt{c}}$ is rational $\Rightarrow b^2d=c\left(a+d\right)$

Let $b = 1$: $\dfrac{a}{1+\sqrt{c}}+\dfrac{d}{\sqrt{c}}$ is rational $\Rightarrow$ $\qquad d=c\left(a+d\right)$

$$d=ac+cd$$

$$d-cd=ac$$

$$d\left(1-c\right)=ac$$

a, c, d are positive integers. So RHS > 0

c is not a perfect square, so $\qquad c\ge 2$

$$1-c<0\ \therefore \text{LHS}<0$$

this is a contradiction, ie $d\ne c\left(a+d\right)$

hence $\dfrac{a}{1+\sqrt{c}}+\dfrac{d}{\sqrt{c}}$ can *NOT* be rational.

3 marks: correct solution
1. substitutes $b = 1$ to obtain expression to disprove.
2. uses proof by contradiction to show $d \ne c(a + d)$
(or equivalent merit)

2 marks: substantially correct solution
1. substitutes $b = 1$ to obtain expression to disprove.
2. ***attempts*** proof by contradiction to show $d \ne c(a + d)$
(or equivalent merit)

1 mark: partially correct solution
substitutes $b = 1$ to obtain expression to disprove.
or attempts proof by contradiction
(or equivalent merit)

Question 12
Outcomes Addressed in this Question

MEX12-2: chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings.

Question	Solutions	Marking Guidelines
(a)	(i) $\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$ (This is an axiomatic fact.) $\therefore a - 2\sqrt{a}\sqrt{b} + b \geq 0$ $a + b \geq 2\sqrt{ab}$ $\dfrac{a+b}{2} \geq \sqrt{ab}$	**(a)(i) 2 marks:** Correct solution communicated. **1 mark:** Some aspect of the solution not included.
	(ii) For the final equation above, substitute $a = 4x; b = \frac{9}{x}$ So: $\frac{4x+\frac{9}{x}}{2} \geq \sqrt{36}$, but also. There is equality when *a*=*b*. i.e. $4x = \frac{9}{x} \rightarrow x = \frac{3}{2}$ So the minimum value will be $4\left(\frac{3}{2}\right) + \frac{9}{\frac{3}{2}} = 6 + 6 = 12.$	**(ii)2 marks:** Finding the relevant *x* value and the minimum value. **1 mark:** One aspect of the method correct.
(b)	Step 1: Show true for $n = 1$. $LHS = 2^1 + (-1)^2 = 2 + 1 = 3$ $= 3 \times 1$, which is hence divisible by 3. Step 2: Assume true for *n*=*k:* $i.e.\, 2^k + (-1)^{k+1} = 3P$ Where *P* is a positive integer. Prove true when *n*=*k*+1 $2^{k+1} + (-1)^{k+2} = 2 \times 2^k - (-1)^{k+1}$ $= 2 \times 2^k + 2(-1)^{k+1} - 3(-1)^{k+1}$ $= 2(3P) - 3(-1)^{k+1}$ $= 3(2P - (-1)^{k+1})$ $= 3(2P + (-1)^k)$ Which is divisible by 3 since $(2P + (-1)^k)$ is an integer. Therefore, the statement is true for all positive integers *n*.	**(b) 3 marks:** Correct base case; correct integration of the assumption; correct solution with conclusion. **2 marks:** 2 components correct(or equivalent merit). **1 mark:** 1 component correct.
(c)	Reciprocals: $\frac{1}{\sqrt{6}-\sqrt{5}} = \sqrt{6} + \sqrt{5}$ $\frac{1}{\sqrt{5}-2} = \sqrt{5} + 2$ Using the axiom: $\sqrt{6} > 2$ $\sqrt{6} + \sqrt{5} > 2 + \sqrt{5}$ Taking reciprocals reverses the inequality sign. $\dfrac{1}{\sqrt{6}+\sqrt{5}} < \dfrac{1}{\sqrt{5}+2}$ $\therefore \sqrt{6} - \sqrt{5} < \sqrt{5} - 2$	**(c) 2 marks:** Correct calculation of reciprocals and communication of the relevant inequality. **1 mark:** One of the components correct.

(d)	Step 1: Show true for n=1. $$LHS = 7^1 + 15^1 = 22$$ $$= 2 \times 11$$ Which is hence divisible by 11. Step 2: Assume true for n=k where k is odd. Prove true for the next odd number, n=k+2. i.e. Assume $7^k + 15^k = 11M$ for integer M. Then $7^{k+2} + 15^{k+2} = 49 \times 7^k + 225 \times 15^k$ $$= 49(7^k + 15^k) + 176$$ $$= 49(11M) + 16 \times 11$$ $$= 11(49M + 16)$$ Which is a multiple of 11 since $(49M + 16)$ must be an integer. Hence the statement is true for all odd integers.	**(d) 3 marks:** Correct base case; correct integration of the assumption; correct solution with conclusion. **2 marks:** 2 components correct(or equivalent merit). **1 mark:** 1 component correct.