Name:	
Class:	

YEAR 12
ASSESSMENT TEST 1
TERM 4, 2015

MATHEMATICS EXTENSION 2

Time Allowed – 90 Minutes

(Plus 5 minutes Reading Time)

- *All* questions may be attempted
- *All* questions are of equal value
- Department of Education approved calculators and templates are permitted
- In every Question, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate bundles clearly labeled Question 1, Question 2, etc.

Each question must show (in the top right hand corner) your Candidate Number.

Question 1 (15 marks) **Marks**

(a) Find $(-2+3i)\overline{(3-4i)}$ in the form $x+iy$ $(x,y \in R)$ 2

(b) Given that $z_1 = 2-2i$, $z_2 = \sqrt{3}+i$ and $z_3 = a+bi$ $(a,b \in R)$

i. find the modulus of z_3 when $|z_1 z_3| = 16$ 2

ii. find the argument of z_3 when $\arg\left(\frac{z_3}{z_2}\right) = \frac{7\pi}{12}$ 2

iii. Hence find the values of a and b 2

(c) If $\frac{1-ai}{a-3i}$ is a real number, find all possible values of a $(a \in R)$ 3

(d) i. Find all possible values for the square root of $2+2\sqrt{3}i$ in the form $x+iy$ $(x,y \in R)$ 2

ii. Hence or otherwise, solve the equation $z^2 - \sqrt{10}z + 2 - \frac{\sqrt{3}}{2}i = 0$ 2

Question 2 (15 marks) **Marks**

(a) The points P and Q represents the complex numbers $4+2i$ and $1+5i$ respectively. O is the origin and R is a point on an argand diagram such that $OPRQ$ is a parallelogram. Represent this information on an Argand diagram and hence determine the complex number represented by R. 2

(b) The coefficients of the equation $z^4 + pz^3 + qz^2 + rz + s = 0$ are real. Prove that $r^2 + p^2 s = pqr$ if it is known that one root is purely imaginary. 4

(c) i. Find the equation of the curve if $\left|\frac{(z-1)}{(z+1)}\right| = 2$ 3

ii. Describe the curve. 2

(d) Shade the region defined by: $-\frac{\pi}{3} < \arg(z) \le \frac{\pi}{6}$ and $|z-1| > 4$ 4

Question 3 (15 marks) **Marks**

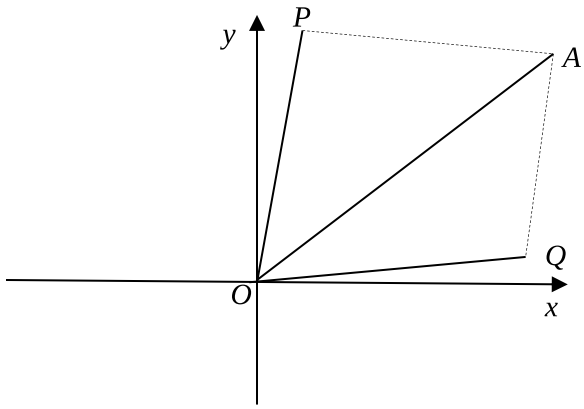
(a) Given that $|z-3| \le 5$, find the greatest and least values of $|z+1|$. 2

(b) i. Use De Moivre's theorem to express $\tan 3\theta$ in terms of $\tan\theta$. 3

ii. Solve the equation $x^3 - 3x^2 - 3x + 1 = 0$ 2

iii. Hence find the exact value of $\tan\left(\frac{5\pi}{12}\right)$ as a surd 3

(c) Points A, P and Q represent the complex numbers a, z_1 and z_2 respectively where z_1 and z_2 are the roots of the equation $z^2 - az + a^2 = 0$



Not to scale

i. Copy the diagram and explain why points O, P, A and Q form a parallelogram. 2

ii. Prove that ΔOPA is equilateral, giving reasons. 3

Continue on next page

Question 4 (15 marks) **Marks**

(a) i. If $\omega^3 = 1$ show that $\omega^2 + \omega + 1 = 0,\ \omega \neq 1$ 1

ii. Prove that:

$$\frac{1}{z-1} + \frac{1}{\omega z-1} + \frac{1}{\omega^2 z-1} = \frac{3}{z^3-1}.\ (z^3 - 1 \neq 0)$$ 3

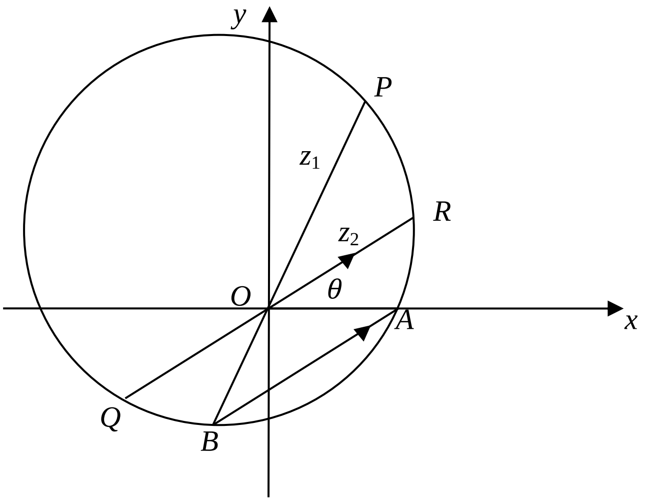
iii. If $z = \cos 2\theta + i\sin 2\theta$ show that $\frac{1}{z-1} = \frac{-\sin\theta - i\cos\theta}{2\sin\theta}$. 2

iv. Hence show that: $3\cot 3\theta = cot\theta + \cot\left(\theta + \frac{\pi}{3}\right) + \cot\left(\theta + \frac{2\pi}{3}\right)$ 3

(You may assume that $\omega = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$ without proof)

(b) On the Argand diagram below, point *P* represents the complex number z_1 and *A* is the point (1,0). *PO* is produced to *B* so that $OB = 1$ unit. A line parallel to *AB* passing through the origin *O* meets the circle through points *P*, *A* and *B* at *Q* and *R*.

Let point *R* represent the complex number z_2 and $\angle ROA = \theta$.



Not to scale

i. Copy the diagram and prove that $arg z_1 = 2 arg z_2$, giving reasons 2

ii. Prove point *R* represents one of the square roots of z_1 4

(You may assume that a cyclic trapezium is isosceles without proof)

End of Exam

MATHEMATICS Extension 2: Question 1

Suggested Solutions	Marks	Marker's Comments
(a) $=(-2+3i)(3+4i)$	1	
$=-6-8i+9i-12$	1	
$=-18+i$	1	
(b)(i) $\lvert z_1\rvert=\sqrt{2^2+2^2}=\sqrt{8}=2\sqrt{2}$		
now $\lvert z_1 . z_3\rvert=16$		
$\lvert z_1\rvert . \lvert z_3\rvert=16$		
$2\sqrt{2} . \lvert z_3\rvert=16$		
$\lvert z_3\rvert=\frac{8}{\sqrt{2}}$	1	
$=4\sqrt{2}$		
(ii) $\arg(z_1)=\pi/6$		
$\arg(z_3)-\arg(z_1)=\frac{7\pi}{12}$	1	
$\arg(z_3)-\pi/6=\frac{7\pi}{12}$	1	
$\arg(z_3)=3\pi/4$		
(iii) $z_3=4\sqrt{2}\left(\cos 3\pi/4+i\sin 3\pi/4\right)$	1	
$=4\sqrt{2}\left(-1/\sqrt{2}+i\frac{1}{\sqrt{2}}\right)$		
$=-4+4i$		
$\therefore a=-4 \quad , b=4$	1	
(c) $\frac{1-ai}{a-3i}\times\frac{a+3i}{a+3i}=\frac{a+3i-a^2i+3a}{a^2+9}$		
$=\frac{4a}{a^2+9}+\frac{i(3-a^2)}{a^2+9}$	1	
For real number, $\frac{3-a^2}{a^2+9}=0$	1	
$3-a^2=0$		
$a^2=3$		
$a=\pm\sqrt{3}$	1	

2/2

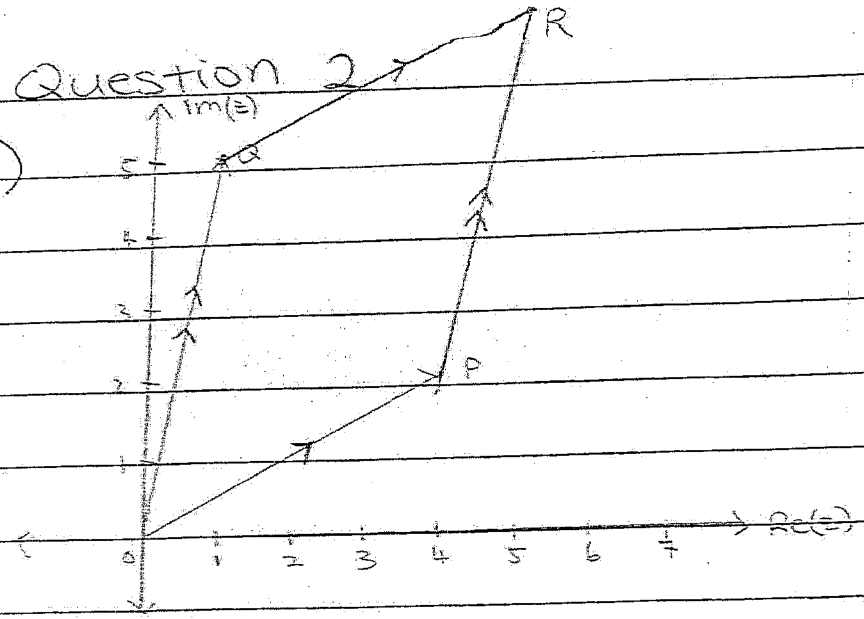
MATHEMATICS Extension 2: Question ...1 cont.

Suggested Solutions	Marks	Marker's Comments
(d)(i) let $2+2\sqrt{3}i = (x+iy)^2$ where $x, y \in R$		
$2+2\sqrt{3}i = x^2-y^2+2xyi$		
equating coefficients		
$2 = x^2-y^2$ ......(1)		
$\sqrt{3} = xy$ ......(2)		
from (2) $y = \sqrt{3}/x$ sub. into (1)		
$2 = x^2 - \frac{3}{x^2}$		
$2x^2 = x^4-3$		
$0 = x^4-2x^2-3$		
$(x^2-3)(x^2+1)=0$		
$x^2=3$ or $x^2=-1$		
$x=\pm\sqrt{3}$ only as $x\in R$	1	
when $x=\sqrt{3}$ $y=1$		
when $x=-\sqrt{3}$, $y=-1$		
$\sqrt{2+2\sqrt{3}} = (\sqrt{3}+i)$ or $(-\sqrt{3}-i)$	1	
(ii) $z^2-\sqrt{10}z+2-\frac{\sqrt{3}}{2}i=0$		
$z = \dfrac{\sqrt{10}\pm\sqrt{10-4(2-\frac{\sqrt{3}}{2}i)}}{2}$		
$= \dfrac{\sqrt{10}\pm\sqrt{10-8+2\sqrt{3}i}}{2}$		
$= \dfrac{\sqrt{10}\pm\sqrt{2+2\sqrt{3}i}}{2}$	1	
$= \dfrac{\sqrt{10}\pm(\sqrt{3}+i)}{2}$ (from (i))		
$= \dfrac{\sqrt{10}+\sqrt{3}+i}{2}$ or $\dfrac{\sqrt{10}-\sqrt{3}-i}{2}$	1	

Student Number

Question 2

a)



1

$R = (4+2i) + (1+5i)$

$= 5 + 7i$ 1

b) $z^4 + pz^3 + qz^2 + rz + s = 0$

let the purely imaginary root be ki

$\therefore -ki$ is also a root (since coefficients are real)

$\therefore$ the roots are $ki, -ki, \alpha, \beta$ — 1

$\alpha + \beta + ki - ki = -p$

$\therefore \alpha + \beta = -p$ — (1)

$\alpha\beta + \alpha ki - \alpha ki + \beta ki - \beta ki + k^2 = q$

$\alpha\beta + k^2 = q$ — (2)

$\alpha\beta ki - \alpha\beta ki + \beta k^2 + \alpha k^2 = -r$

$k^2(\alpha+\beta) = -r$ — (3)

$\alpha\beta k^2 = s$ — (4)

2

sub. (1) into (3) $-k^2 p = -r$

$k^2 p = r$

$k^2 = \frac{r}{p}$ — (5)

sub. (4) into (2) $\frac{s}{k^2} + k^2 = q$ — (6)

sub. (5) into (6) $\frac{sp}{r} + \frac{r}{p} = q$

Student Number

$sp^2 + r^2 = qpr$ 1

$\therefore r^2 + p^2 s = pqr$

c) i) $\left|\frac{(z-1)}{(z+1)}\right| = 2$

let $z = x+iy$

$\left|\frac{(x+iy-1)}{(x+iy+1)}\right| = 2$ 1

$\frac{\sqrt{(x-1)^2+y^2}}{\sqrt{(x+1)^2+y^2}} = 2$ 1

$(x-1)^2+y^2 = 4(x+1)^2+4y^2$

$x^2-2x+1+y^2 = 4x^2+8x+4+4y^2$ 1

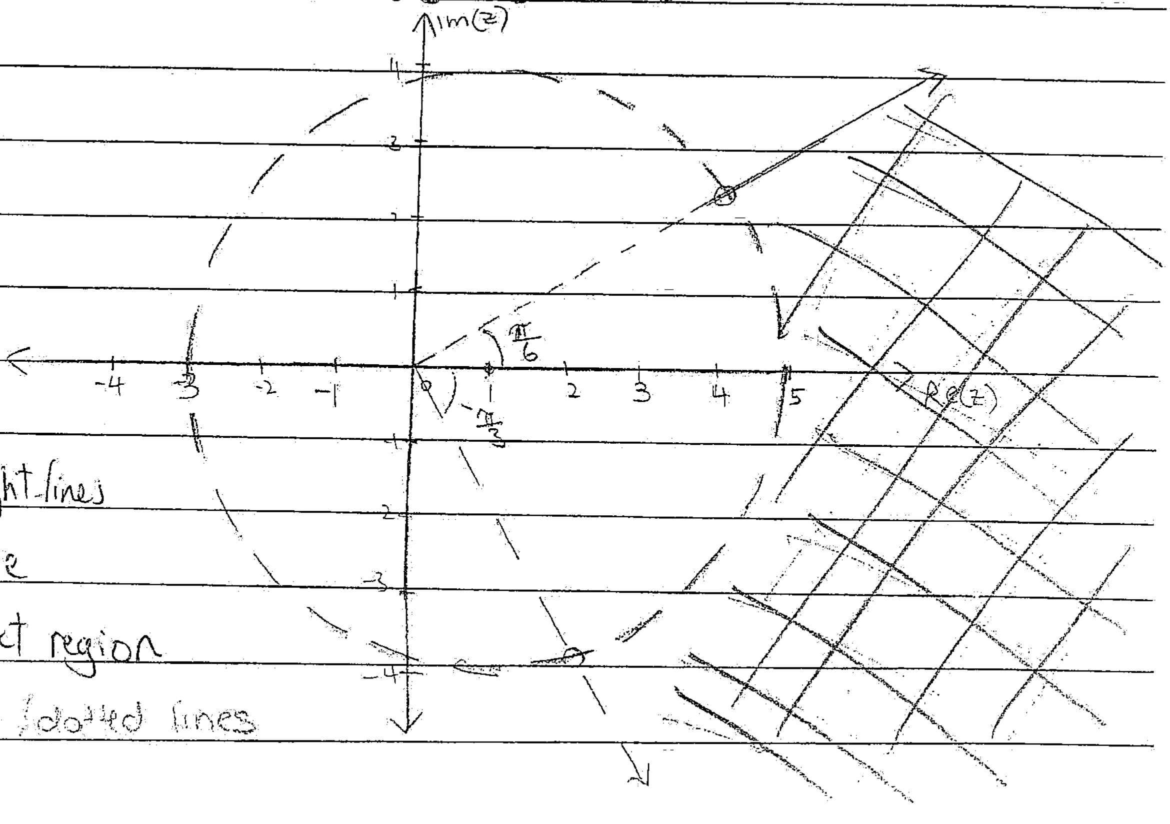
$3x^2+10x+3y^2+3=0$

ii) $x^2+\frac{10}{3}x+y^2=-1$

$\left(x+\frac{5}{3}\right)^2+y^2=\frac{16}{9}$ 1

$\therefore$ locus is a circle with centre $\left(-\frac{5}{3},0\right)$ and radius $\frac{4}{3}$ units. 1

d)



1 Straight lines

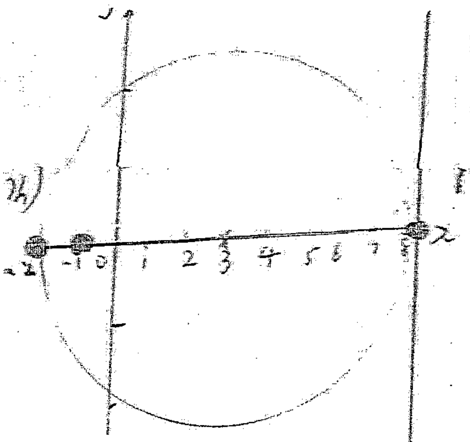
1 circle

1 correct region

1 solid / dotted lines

a) greatest $|z+1| = 9$ 1m } with justification

least $|z+1| = 2$ 1m



b i) $(\cos\theta + i\sin\theta)^3$ (De Moivre's Th) 1m must state D.M.Th

$= \cos 3\theta + i\sin 3\theta$

$\operatorname{cis} 3\theta = (\operatorname{cis}\theta)^3$

$= c^3 + i3c^2s - 3cs^2 - is^3$

key

$c = \cos\theta$

$s = \sin\theta$

$t = \tan\theta$

Real part $\cos 3\theta = c^3 - 3cs^2$

Imaginary part $\sin 3\theta = 3c^2s - s^3$

$\tan 3\theta = \dfrac{\sin 3\theta}{\cos 3\theta} = \dfrac{3c^2s - s^3}{c^3 - 3cs^2} \begin{matrix}\div c^3 \\ \div c^3\end{matrix}$ 1m

$= \dfrac{3t - t^3}{1 - 3t^2}$ 1m

$\therefore \tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$ #

ii) Let $x = \tan\theta$

$\tan 3\theta = \dfrac{3x - x^3}{1 - 3x^2}$

when $\tan 3\theta = 1$ $\quad 3x - x^3 = 1 - 3x^2$

$x^3 - 3x^2 - 3x + 1 = 0$

$x^3 + 1 - 3x(x+1) = 0$

$(x+1)(x^2 - x + 1) - 3x(x+1) = 0$ 1m

$(x+1)(x^2 - x + 1 - 3x) = 0$

$(x+1)(x^2 - 4x + 1) = 0$

$\therefore x = -1$ or $x = \dfrac{4 \pm \sqrt{16 - 4}}{2} = \dfrac{4 \pm 2\sqrt{3}}{2}$

$\therefore \underline{x = -1 \text{ or } 2 \pm \sqrt{3}}$ 1m

Alternatively,

$\tan 3\theta = \frac{3x - x^3}{1 - 3x^2}$

when $\tan 3\theta = 1$, $x^3 - 3x^2 - 3x + 1 = 0$ | 1m

$\tan 3\theta = 1$ when $3\theta = \frac{\pi}{4} + k\pi$ $k \in J$

$\theta = \frac{\pi}{12} + \frac{k\pi}{3}$

$k = 0$, $x = \tan \frac{\pi}{12}$

$k = 1$, $x = \tan \frac{5\pi}{12}$

$k = 2$, $x = \tan \frac{9\pi}{12} = \tan \frac{3\pi}{4}$

$\therefore$ roots of $x^3 - 3x^2 - 3x + 1 = 0$ are

$\tan \frac{\pi}{12}$, $\tan \frac{5\pi}{12}$, $\tan \frac{3\pi}{4}$ $(= -1)$ | 1m for the 3 roots

$\tan \frac{\pi}{12} = \tan\left(\frac{\pi}{2} - \tan \frac{5\pi}{12}\right) = \cot \frac{5\pi}{12} = \frac{1}{\tan \frac{5\pi}{12}}$

$\therefore$ sum of Rts $= -\frac{b}{a} = 3$

$\tan \frac{\pi}{12} + \tan \frac{5\pi}{12} + \tan \frac{3\pi}{4} = 3$

$\frac{1}{\tan \frac{5\pi}{12}} + \tan \frac{5\pi}{12} - 1 = 3$

$\left(\tan \frac{5\pi}{12}\right)^2 - 4 \tan\left(\frac{5\pi}{12}\right) + 1 = 0$ | 1m for the quadratic eq.

$\tan \frac{5\pi}{12} = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$ | 1m for $2 \pm \sqrt{3}$

Since $\tan \frac{5\pi}{12} > \tan \frac{\pi}{12} > 0$

$\therefore$ $\tan \frac{5\pi}{12} = 2 + \sqrt{3}$, $\tan \frac{\pi}{12} = 2 - \sqrt{3}$ | 1m for $\tan \frac{5\pi}{12} = 2 + \sqrt{3}$ with justification

Rts of $x^3 - 3x^2 - 3x + 1 = 0$ can also be stated as $2 \pm \sqrt{3}$, -1 #

c i) Given $z^2 - az + a = 0$

$\therefore$ Sum of Rts $z_1 + z_2 = a$

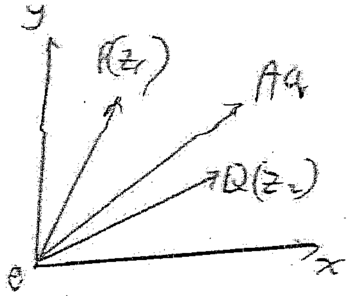
$\vec{OP} + \vec{PA} = \vec{OA}$

$z_1 + \vec{PA} = a$

$\vec{PA} = z_2$ ie $\vec{PA} = \vec{OQ}$

$\therefore PA \parallel OQ$, $|PA| = |OQ|$

$\therefore OPAQ$ is a parm (1 pair of opposite sides parallel and equal)



c ii) $z^2 - az + a^2 = 0$

$$z = \frac{a \pm \sqrt{a^2 - 4a^2}}{2} = \frac{a \pm i\sqrt{3}a}{2}$$

$Im(z_1) > Im(z_2) \therefore z_1 = a\left(\frac{1+\sqrt{3}i}{2}\right) = a \operatorname{cis}\frac{\pi}{3}$

$z_2 = a\left(\frac{1-\sqrt{3}i}{2}\right) = a \operatorname{cis}\left(-\frac{\pi}{3}\right)$

$|z_1| = |z_2| = |PA| = |a|$

$\therefore OPA$ is an equilateral triangles

(3 sides equal in a triangle)

Alternative method for $|z_1| = |z_2| = |a|$

c ii

$z_1 \times z_2 = a^2$ product of Rt } 1m

$\therefore |z_1 \times z_2| = |a^2| = |a| \times |a|$

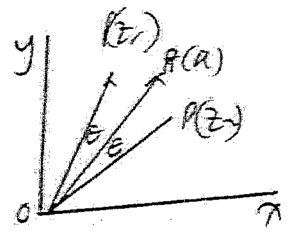
$\arg(z_1) + \arg(z_2) = 2\arg(a)$

$\therefore^* \arg(z_1) - \arg(a) = \arg(a) - \arg(z_2)$

$\therefore OPAQ$ is a rhombus (parm with (1m) diagonal bisect vertex angle)

$\therefore |z_1| = |z_2|$ (All sides of rhombus are equal) }

$\therefore |z_1|^2 = |a|^2 \therefore |z_1| = |z_2| = |a|$



1m sum of Rts (...) $z_1 + z_2 = a$

must state $\vec{OP} + \vec{PA} = \vec{OA}$ and prove $\vec{PA} = z_2$

1m for proving parm

v. poorly done

* do not accept as parm is formed when 2 vectors are added

c(ii)

1m $\begin{cases} z_1 = a\left(\frac{1+\sqrt{3}i}{2}\right) \\ z_2 = a\left(\frac{1-\sqrt{3}i}{2}\right) \end{cases}$

1m for proving $|z_1| = |z_2| = |a|$

1m prove equilateral triangle

Note students can't get any of the first 2 marks won't get the last mark

* Students wrote $\arg(a) = \frac{\arg(z_1) + \arg(z_2)}{2}$

1m is not clear enough to prove the vertex angle is bisected.

Generally part (c) is very badly done.

2015 Extension 2 Term 4 Task Q 4

Part (a)

(i)

$\omega^3 = 1$

$\omega^3 - 1 = 0$

$(\omega - 1)(\omega^2 + \omega + 1) = 0$

$\omega \neq 1 \Rightarrow \quad \omega^2 + \omega + 1 = 0$

(1 mark)

(ii)

LHS:

$\frac{1}{z-1} + \frac{1}{\omega z-1} + \frac{1}{\omega^2 z-1}$

$= \frac{1}{z-1} + \frac{\omega^2 z-1+\omega z-1}{(\omega z-1)(\omega^2 z-1)}$

(1 mark)

$= \frac{1}{z-1} + \frac{z(\omega^2+\omega)-2}{\omega^3 z^2 - z(\omega^2+\omega)+1}$

Now: $\omega^2 + \omega = -1, \ \omega^3 = 1 \ \Rightarrow$

$= \frac{1}{z-1} + \frac{z(-1)-2}{z^2+z+1}$

(1 mark)

$= \frac{z^2+z+1-z^2-z+2}{z^3-1}$

$= \frac{3}{z^3-1}$

= RHS

(1 mark)

Comments:

If the student added the fractions and began to simplify, one mark was awarded. Two marks were only awarded if the student correctly used the properties of the cube roots of zero to further simplify the expression.

(iii)

LHS, using given information:

$\frac{1}{z-1} = \frac{1}{\cos 2\theta + i \sin}$

$= \frac{1}{1 - 2 \text{si} \ ^2 \theta + 2i \sin\theta \text{ co}}$

$= \frac{1}{-2\sin^2\theta + 2i \sin\theta \cos}$

$$= \frac{1}{2\sin\theta(-\sin\theta + i\cos\theta)} \times \frac{-\sin\theta - i\cos\theta}{-\sin\theta - i\cos\theta}$$

$$= \frac{\sin\theta - i\cos\theta}{2\sin\theta(\sin^2\theta + \cos^2\theta)}$$

$$= \frac{-\sin\theta - i\cos\theta}{2\sin\theta}$$

As required.

(1 mark)

Comments:

Some students straightaway multiplied by the expression by $\frac{\cos 2\theta - 1 + i s}{\cos 2\theta - 1 + i s}$ and proceeded to simplify. The first mark, in all cases, was awarded when this operation was successfully completed.

(iv)

We need to use the results in part (ii) to prove this result. We'll express each fraction as a trigonometric function, add, then simplify.

(1) Consider z^3 and $\frac{3}{z^3-1}$:

By de Moivre's theorem:

$$z^3 = (\cos 2\theta + i\sin 2\theta)^3$$

$$= \cos 6\theta + i\sin 6\theta$$

Note, using (iii): $z = \cos\theta + i\sin\theta \Rightarrow \frac{1}{z-1} = \frac{-\sin\theta - i\cos\theta}{2\sin\theta}$

$$\therefore \frac{3}{z^3-1} = \frac{-3(\sin 3\theta + i\cos 3\ )}{2\sin 3}$$

$$= -\frac{3\sin 3}{2\sin 3} + i\left(-\frac{3\cos 3}{2\sin 3}\right)$$

(2) Consider ωz and $\frac{1}{\omega z-1}$:

$$\omega z = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)(\cos 2\theta + i\sin 2\theta)$$

Expanding the above and using the trig identities for $\cos(a+b)$ and $\sin(a+b)$:

$$\omega z = \cos\left(\frac{2\pi}{3} + 2\theta\right) + i\sin\left(\frac{2\pi}{3} + 2\theta\right)$$

Therefore, using (iii) again:

$$\frac{1}{\omega z-1} = -\frac{\sin\left(\theta+\frac{\pi}{3}\right) + i\cos\left(\theta+\frac{\pi}{3}\right)}{2\sin\left(\theta+\frac{\pi}{3}\right)}$$

$$= -\frac{\sin\left(\theta+\frac{\pi}{3}\right)}{2\sin\left(\theta+\frac{\pi}{3}\right)} + i\left(-\frac{\cos\left(\theta+\frac{\pi}{3}\right)}{2\sin\left(\theta+\frac{\pi}{3}\right)}\right)$$

(3) Consider $\omega^2 z$ and $\frac{1}{\omega^2 z-1}$:

$$\omega^2 z = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^2 (\cos 2\theta + i\sin 2\theta)$$

$$= \left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)(\cos 2\theta + i\sin 2\theta) \text{ (By de Moivre's Theorem)}$$

$$= \cos\left(\frac{4\pi}{3} + 2\theta\right) + i\sin\left(\frac{4\pi}{3} + 2\theta\right)$$

$$\therefore \frac{1}{\omega^2 z - 1} = -\frac{\sin\left(\theta + \frac{2\pi}{3}\right)}{2\sin\left(\theta + \frac{2\pi}{3}\right)} + i\left(-\frac{\cos\left(\theta + \frac{2\pi}{3}\right)}{2\sin\left(\theta + \frac{2\pi}{3}\right)}\right)$$

(2 marks)

(4) Equate imaginary parts:

$$\operatorname{Im}\left(\frac{3}{z^3-1}\right) = \operatorname{Im}\left(\frac{1}{z-1}\right) + \operatorname{Im}\left(\frac{1}{\omega z-1}\right) + \operatorname{Im}\left(\frac{1}{\omega^2 z-1}\right)$$

$$-\frac{3\cos 3}{2\sin 3} = -\frac{\cos\theta}{2\sin\theta} - \frac{\cos\left(\theta + \frac{\pi}{3}\right)}{2\sin\left(\theta + \frac{\pi}{3}\right)} - \frac{\cos\left(\theta + \frac{2\pi}{3}\right)}{2\sin\left(\theta + \frac{2\pi}{3}\right)}$$

$$3\cot 3\theta = \cot\theta + \cot\left(\theta + \frac{\pi}{3}\right) + \cot\left(\theta + \frac{2\pi}{3}\right)$$

As required.

(1 mark)

Comments:

This question was generally poorly answered. If the student constructed trigonometric expressions from the expressions that were in terms of ω and z then they were awarded one mark.

The mark was awarded if they had one or more expression correct, or if they **clearly showed** they could use the previously proved results but made an error in their working. If the student constructed all of the expressions correctly then they were awarded two marks.

Equating the imaginary parts and simplifying earned the other mark.

Many students took shortcuts and, for instance, derived an expression like

$$\frac{1}{z-1} = -\frac{1}{2} - \frac{1}{2} i \cot\theta$$

without clearly explaining what results they used and how they arrived at this expression. Unless the working was clear, the mark was not awarded. Note the question requires students to **show**.

Part (b)

(i)

$\angle ROA = \angle OAB = \theta$ (alternate angles $QR \parallel AB$)

$OA = OB = 1$ (given)

$\therefore \angle OBA = \angle OAB = \theta$ (equal angles opposite equal sides in ΔOAB)

(1 mark)

$\angle POA = \angle OBA + \angle OAB$ (exterior angle of ΔOAB)

$= \theta + \theta$

$= 2\theta$

$\therefore \arg z_1 = 2\arg z_2$

(1 mark)

(ii)

$\angle QOB = \angle POR$ (vertically opposite angles are equal)

$\angle POR = \angle POA - \angle ROA$

$= 2\theta - \theta$

$= \theta$

$\therefore \angle QOB = \angle ROA = \theta$

(1 mark)

In ΔOQB and ΔORA

$OB = OA$ (given)

$\angle QOB = \angle ROA$ (proven above)

$\angle OQB = \angle ORA$ (adjacent angles of isosceles trapezium are equal)

$\therefore \Delta OQB \equiv \Delta ORA$ (AAS)

(1 mark)

$OQ \times OR = OP \times OB$ (products of intercepts of intersecting chords)

(1 mark)

$OQ = OR$ (corresponding sides of congruent triangles OQB and ORA are equal)

$\therefore OQ = OR = |z_2|$

And $|z_2||z_2| = |z_1| \times 1$

$|z_2|^2 = |z_1|$

$\because |z_2| = \sqrt{|z_1|}$ and $\arg z_2 = \frac{1}{2}\arg z_1$

$z_2 = \sqrt{z_1}$

As required.

(1 mark)

Comments:

Stating that $OQ \times OR = OP \times OB$, with a reason, was sufficient to earn one mark. If the student could prove that $\Delta POR \,|||\, \Delta OQB$ by showing they were equiangular, it was possible to equate ratios of sides and complete the proof in this way.

To earn the final mark, the student had to show the relationships between the moduli AND the arguments of z_1 and z_2 before concluding that $z_2 = \sqrt{z_1}$.