Multiple Choice

1. For a certain complex number z, where arg(z)=$\pi/5$, then arg(z7) is

A. $2\pi/5$

B. $3\pi/5$

C. $-7\pi/5$

D. $-3\pi/5$

2. .Consider the complex numbers $2z$, iz, and $2z - iz$, where $z \in \boldsymbol{C}, z \neq 0$.
These three complex numbers are plotted in the Argand plane and together with the origin O they form the vertices of a quadrilateral.
The area of this quadrilateral is

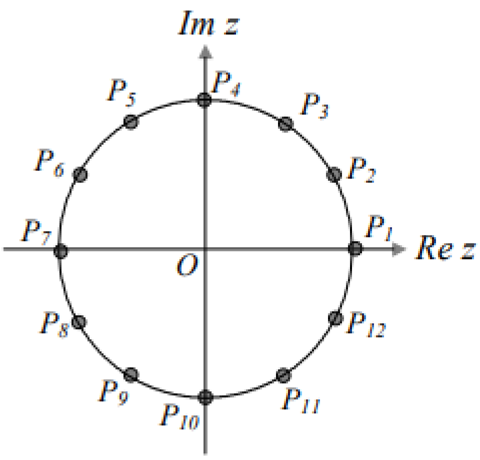
A. $\frac{1}{|2z|}$

B. $|2z|$

C. $|z^2|$

D. $2|z|^2$

3. The points P_1 to P_{12} are twelve equally spaced points around the circumference of a circle.



Point P_3 represents the complex number $z = a + ib$. The complex number $i^{11}\bar{z}$ is represented by point

A. P_5
B. P_8
C. P_9
D. P_{11}

4. It is known that ω is a 4^{th} root of $\frac{\sqrt{7}}{\sqrt{10}} + \frac{\sqrt{3}}{\sqrt{10}} i$. Which of the following is also a root?

A. ω^2

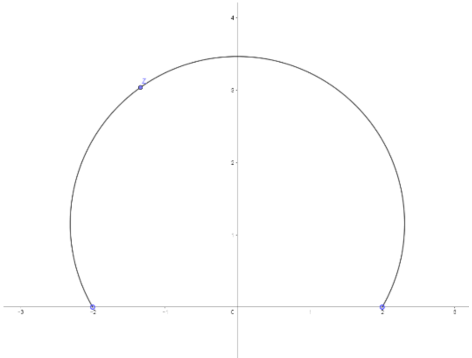
B. $-\omega$

C. $\overline{\omega}$

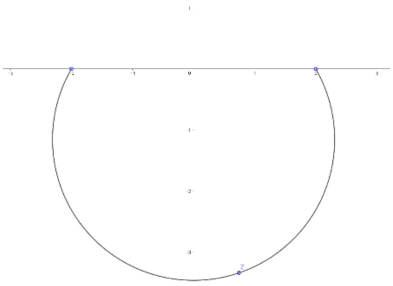
D. $-\overline{\omega}$

5. Which of the following represents the locus of z such that $Arg\left(\frac{z+2}{z-2}\right) = \frac{2\pi}{3}$?

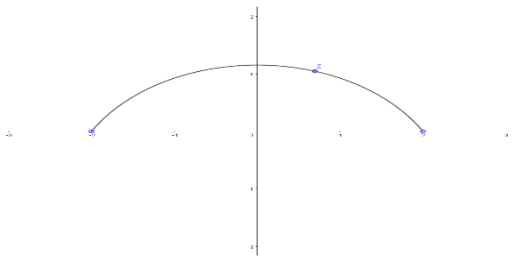
A



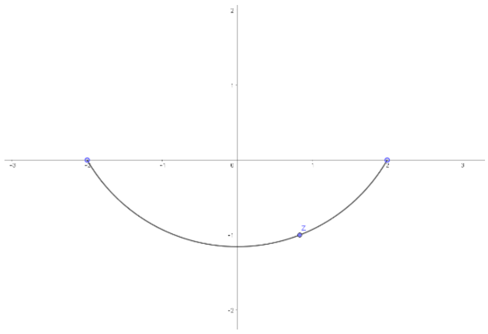
B



C



D



Question 6 (9 marks)

A. Given $z = 2 + 3i$ and $w = 3 + i$, find $\overline{z + iw}$ in the form $a + ib$ 2m

B. Given that $3 + i$ is a root to the equation $z^4 - 4z^3 + 3z^2 - 10z + 50 = 0$, factorise $z^4 - 4z^3 + 3z^2 - 10z + 50$ into:

I. Factors with real coefficients 2m

II. Linear Factors 2m

C. On an Argand Diagram, sketch the locus of z such that $Arg(z + 1 + i) = Arg(z - 2i)$ 3m

Question 7 (Start a New Page) (9 marks)

A. Solve the equation $z^2 = |z|^2 - 4$ 3m

A. i) Show that $3 - i\sqrt{3}$ is $2\sqrt{3}\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$. 1m

ii) Find $\left(3 - i\sqrt{3}\right)^6$ and express your solution in the form $x + iy$ where $x, y \in \mathbb{R}$. 2m

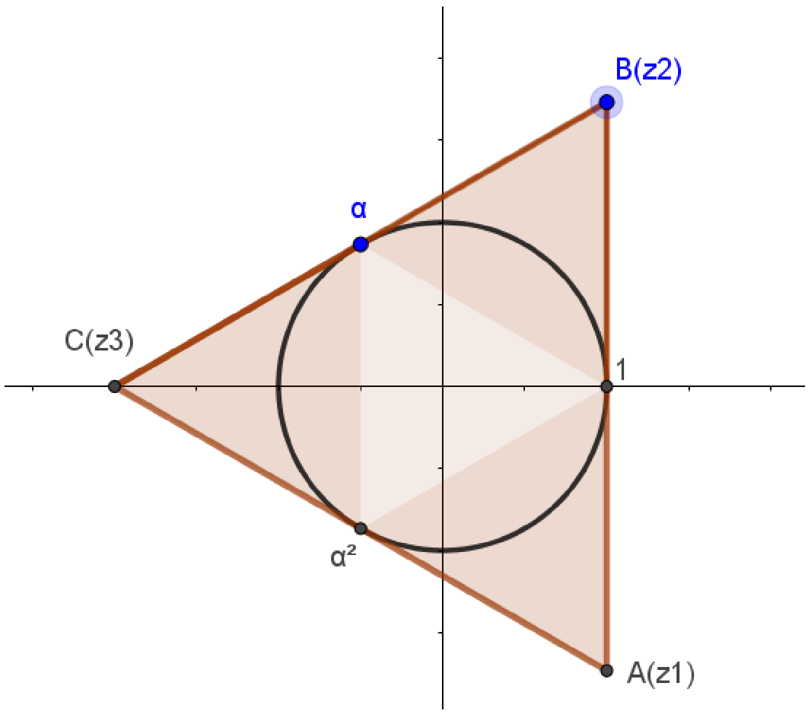
iii) Find the integer values of n for which $\left(3 - i\sqrt{3}\right)^n$ is real. 1m

iv) Find the integer values of n for which $\left(3 - i\sqrt{3}\right)^n = ai$, where a is a real number. 1m

B Express $e^{\frac{i\pi}{3}}$ in Cartesian form. 1m

Question 8 (Start a New Page) (8 marks)

Given that 1, α and α^2 are three complex numbers on the Argand diagram, where α is a non-real cube root of unity.



I. Show that $1 + \alpha + \alpha^2 = 0$. 1m

ii ABC is an equilateral triangle, z_1, z_2 and z_3 are the complex numbers corresponding to A, B and C respectively. Write down z_1, z_2 and z_3 in terms of α. 3m

iii. Show that $z_2 + \alpha z_3 + \alpha^2 z_1 = 0$. 1m

iv Let $\omega_1 = \frac{1}{3}(1 + \alpha + z_2)$, $\omega_2 = \frac{1}{3}(\alpha + \alpha^2 + z_3)$ and $\omega_3 = \frac{1}{3}(1 + \alpha^2 + z_1)$ 3m

be three complex numbers, prove that ω_1, ω_2 and ω_3 are the cube roots of -1.

Question 9 (Start a New Page) (10 marks)

Let $v = 2 + 2\sqrt{3}i$ and $w = 2\sqrt{2} + 2\sqrt{2}i$.

i) If $z = \frac{v}{w}$ give z in modulus-argument form. 3m

ii) On an Argand diagram, plot v, w and z. 2m

iii) Using your results from part (iii), show that $\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$. 2m

iv) Given that $z = \frac{v}{w}$ is a solution of the equation

$$z^3 + \frac{1}{2}(\sqrt{2} - 3\sqrt{6})z^2 + 3z - \sqrt{6} + \sqrt{2} = 0,$$

find the other two solutions of the equation, expressing your answers in Cartesian form. 3m

Question 10 (Start a New Page) (11 marks)

A Given that $z = 2^{\frac{1}{3}}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$,

i) Show that $z^3 + \frac{1}{z^3} = \frac{-5}{2\sqrt{2}} - \frac{3}{2\sqrt{2}}\mathrm{i}$ 3m

ii) Find the real numbers a and b if $z^3 + \frac{1}{z^3} = e^a . e^{ib}$ 3m

B Given $|z_1| = |z_2| = 2$, and $\arg(z_1) = \frac{\pi}{5}$ and $\arg(z_2) = \frac{3\pi}{5}$,

Show that $(z_1 - z_2) = 4\sin\left(\frac{\pi}{5}\right)cis\left(\frac{-\pi}{10}\right)$ 5m

END of PAPER

MATHEMATICS Extension 2: Question MC...

Suggested Solutions	Marks	Marker's Comments

1. D

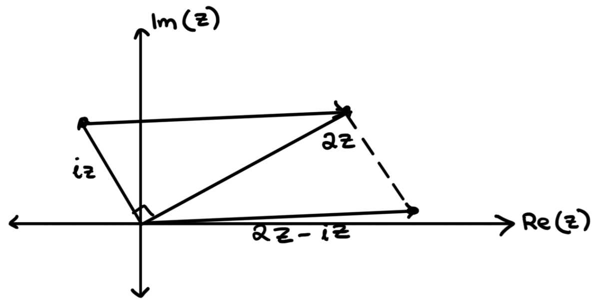
$$\arg(z) = \frac{\pi}{5}$$

$$\arg(z^7) = \frac{7\pi}{5} \quad \text{(De moivre's theorem)}$$

$$\operatorname{Arg}(z^7) = \frac{7\pi}{5} - 2\pi$$

$$= \frac{-3\pi}{5}$$

2. D



$$\text{Area} = 2 \times \frac{1}{2} \times |iz| \times |2z|$$

$$= |z| \times 2 \times |z|$$

$$= 2|z|^2$$

3. B

$$z = P_3$$

$$\bar{z} = P_{11}$$

$$i^4 \bar{z} = P_{11}$$

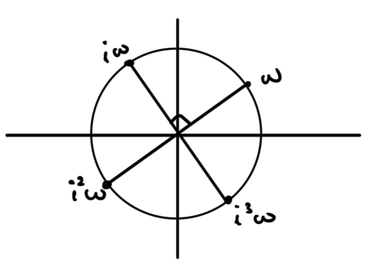
$$i^8 \bar{z} = P_{11}$$

$$i^9 \bar{z} = P_2$$

$$i^{10} \bar{z} = P_5$$

$$i^{11} \bar{z} = P_8$$

4. B



4th roots are equally spaced, $\frac{\pi}{2}$ apart.

$\therefore i^2\omega = -\omega$.

5. D

MATHEMATICS Extension 2: Question 7

Suggested Solutions	Marks	Marker's Comments
i) $1, \alpha$ and α^2 are the roots of the equation $z^3 - 1 = 0$	1	Alternatively, some students solved for α and α^2, and proved it by actually doing the sum.
$\therefore 1 + \alpha + \alpha^2 = \frac{-b}{a}$ (Sum of roots) ← necessary!		
$= \frac{-0}{1}$		
$= 0$	1	
or		
Since α is a root to $z^3 - 1 = 0$		
then $\alpha^3 - 1 = 0$	1	
$(\alpha - 1)(\alpha^2 + \alpha + 1) = 0$		
Since α is non-real, $\alpha \neq 1$ ← necessary!		
$\therefore \alpha^2 + \alpha + 1 = 0$	1	
ii) $z_1 = \alpha^2 - \alpha + 1$	1	Note: $1 + \sqrt{3}i$ is not considered to be "in terms of α".
$z_2 = -\alpha^2 + \alpha + 1$	1	
$z_3 = \alpha^2 + \alpha - 1$	1	
iii) $(-\alpha^2 + \alpha + 1) + \alpha(\alpha^2 + \alpha - 1) + \alpha^2(\alpha^2 - \alpha + 1)$		
$= -\alpha^2 + \alpha + 1 + \alpha^3 + \alpha^2 - \alpha + \alpha^4 - \alpha^3 + \alpha^2$		
$= \alpha^4 + \alpha^2 + 1$		
$= \alpha^2 + \alpha + 1$ (Since $\alpha^3 = 1$)		
$= 0$ (proven from part i)	1	

MATHEMATICS Extension 2: Question……..

Suggested Solutions	Marks	Marker's Comments
iv) $w_1 = \frac{1}{3}(1+\alpha+(-\alpha^2+\alpha+1))$ $= \frac{1}{3}(2+2\alpha-\alpha^2)$ $= \frac{1}{3}[2(1+\alpha+\alpha^2)-3\alpha^2]$ $= \frac{1}{3}(-3\alpha^2)$ $= -\alpha^2$ $w_1^3 = -\alpha^6$ $= -(1)(1)$ $= -1$ $w_2 = \frac{1}{3}(\alpha+\alpha^2+(\alpha^2+\alpha-1))$ $= \frac{1}{3}(2\alpha^2+2\alpha-1)$ $= \frac{1}{3}[2(\alpha^2+\alpha+1)-3]$ $= \frac{1}{3}(-3)$ $= -1$ $w_2^3 = (-1)^3$ $= -1$ $w_3 = \frac{1}{3}(1+\alpha^2+\alpha^2-\alpha+1)$ $= \frac{1}{3}(2\alpha^2-\alpha+2)$ $= \frac{1}{3}[2(\alpha^2+\alpha+1)-3\alpha]$ $= \frac{1}{3}(-3\alpha)$ $= -\alpha$ $w_3^3 = (-\alpha)^3$ $= -\alpha^3$ $= -1$ $\therefore w_1, w_2$ and w_3 are the cube roots of -1. (Since w_1, w_2 and w_3 are distinct points)		1 for solving one of w_1, w_2 or w_3 in simplified form 1 for solving the other 2. 1 Proving that the cube of all 3 is -1.

MATHEMATICS Extension 2: Question……..

Suggested Solutions	Marks	Marker's Comments
iv) For those who wanted to prove that w_1, w_2 and w_3 are roots to the polynomial $z^3+1=0$		
1 Proving $w_1+w_2+w_3=0$ AND $w_1w_2w_3=-1$		
1 Attempt to prove that $w_1w_2+w_1w_3+w_2w_3=0$ and making significant progress.		
1 Doing the first 2 steps successfully and finishing the rest of the proof with sound argument.		

MATHEMATICS Extension 2: Question……..

Suggested Solutions	Marks	Marker's Comments
Accepted answers: ii)		

z_1	z_2	z_3
$\alpha^2 - \alpha + 1$	$-\alpha^2 + \alpha + 1$	$\alpha^2 + \alpha - 1$
$\alpha^2(1+\sqrt{3}i)$		$\alpha(1+\sqrt{3}i)$
	$\alpha(1-\sqrt{3}i)$	$\alpha^2(1-\sqrt{3}i)$
-2α	$-2\alpha^2$	
$2\alpha^{-\frac{1}{2}}$	$2\alpha^{\frac{1}{2}}$	$2\alpha^{3/2}$
$2\alpha^2 e^{\frac{i\pi}{3}}$		$2\alpha e^{\frac{i\pi}{3}}$
	$2\alpha e^{-\frac{i\pi}{3}}$	$2\alpha^2 e^{-\frac{i\pi}{3}}$

Question 9

ⓐ (i) Show that $3 - i\sqrt{3}$ is $2\sqrt{3}\left(\cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)\right)$

$$3 - i\sqrt{3} = 2\sqrt{3}\left(\frac{\sqrt{3}}{2} + i\left(\frac{-1}{2}\right)\right)$$
$$= 2\sqrt{3}\left(\cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)\right)$$

(ii) Find the integer values of n for which $(3 - i\sqrt{3})^n = ai$, where a is a real number.

$$(3 - i\sqrt{3})^n = ai$$
$$\therefore \left[2\sqrt{3}\left(\cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)\right)\right]^n = ai$$
$$= (2\sqrt{3})^n\left[\cos\left(-\frac{n\pi}{6}\right) + i\sin\left(-\frac{n\pi}{6}\right)\right] = ai \quad \text{by de Moivre Theorem}$$
$$\therefore \cos\frac{n\pi}{6} = 0 \quad \therefore \frac{n\pi}{6} = (2k+1)\frac{\pi}{2}, \ k \text{ is an integer}$$
$$n = 3(2k+1)$$
$$= 6k + 3, \ k \in \mathbb{Z} \qquad ? \ \underline{-6k-3}$$

ⓑ Given that $z = 2^{1/3}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$

(i) Show that $z^3 + \frac{1}{z^3} = \frac{-5}{2\sqrt{2}} - \frac{3}{2\sqrt{2}}i$

$$z^3 = 2\left(\cos\frac{3\pi}{4} - i\sin\frac{3\pi}{4}\right) \quad \text{by de Moivre theorem}$$
$$\frac{1}{z^3} = \frac{1}{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \quad \text{by de Moivre theorem}$$
$$z^3 + \frac{1}{z^3} = \frac{5}{2}\cos\frac{3\pi}{4} - \frac{3}{2}\sin\frac{3\pi}{4}i$$
$$= \frac{5}{2}\left(\frac{-\sqrt{2}}{2}\right) - \frac{3}{2}\left(\frac{\sqrt{2}}{2}\right)i$$
$$= \frac{-5\sqrt{2}}{4} - \frac{3\sqrt{2}i}{4}$$
$$= \frac{-5}{2\sqrt{2}} - \frac{3}{2\sqrt{2}}i$$

Alt

$$2^{-\frac{1}{3}} z = \cos\frac{\pi}{4} - i\sin\frac{\pi}{4}$$
$$\frac{2^{1/3}}{z} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$$

$\therefore$ $\frac{z^3}{2} = \cos\frac{3\pi}{4} - i\sin\frac{3\pi}{4}$, $\frac{2}{z^3} = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}$ By de Moivre theorem

$$z^3 = 2\left(\cos\frac{3\pi}{4} - i\sin\frac{3\pi}{4}\right) \text{ and } \frac{1}{z^3} = \frac{1}{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

(ii) $$z^3 + \frac{1}{z^3} = e^a \cdot e^{ib} = e^a(\cos b + i\sin b)$$

$$e^a\cos b = \frac{-5}{2\sqrt{2}} \text{ and } e^a\sin b = \frac{-3}{2\sqrt{2}}$$

$$e^{2a} = \frac{25+9}{8} = \frac{34}{8} = \frac{17}{4} \therefore e^a = \frac{\sqrt{17}}{2} \therefore a = \ln\left(\frac{\sqrt{17}}{2}\right)$$

$$a = \frac{1}{2}\ln(17) - \ln 2$$

$$\cos b = \frac{-5}{\sqrt{34}}, \quad \sin b = \frac{-3}{\sqrt{34}}$$

$$b = \tan^{-1}\left(\frac{3}{5}\right) - \pi$$

? $\tan^{-1}\left(\frac{3}{5}\right)$

? $\pi + \tan^{-1}\left(\frac{3}{5}\right)$

? $-\pi + \tan^{-1}\left(\frac{3}{5}\right) + 2k\pi,\ k\in\mathbb{Z}$

(a) (i) 1 for correct — 0 for wrong

(ii) 1 for correct — 0 for wrong.

$6k+3,\ k\in\mathbb{Z}$

or $-6k-3,\ k\in\mathbb{Z}$

(b) (i) 1 for z^3

1 for $\frac{1}{z^3}$

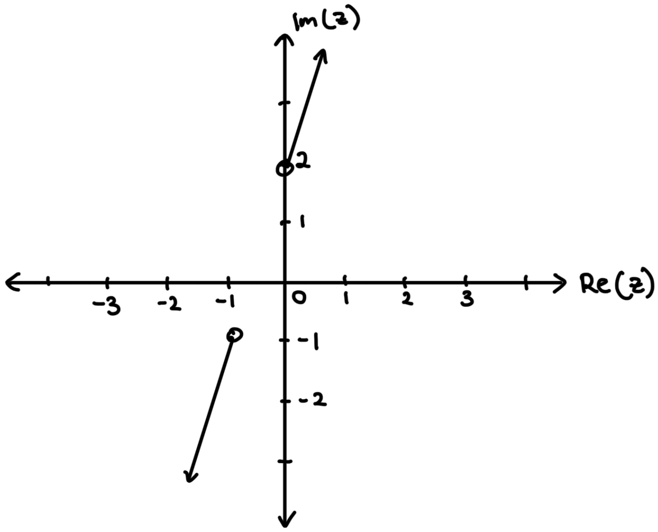
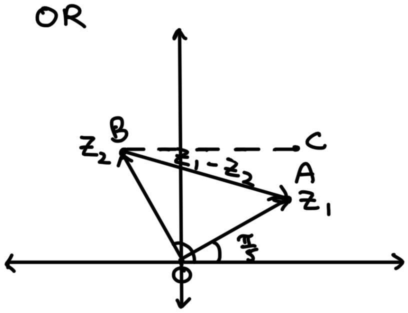
1 for final correct answer

(ii) 1 for equating real and imaginary parts

1 for correct a

1 for correct b

MATHEMATICS Extension 2: Question 10

Suggested Solutions	Marks	Marker's Comments
a) $\text{Arg}(z+1+i) = \text{Arg}(z-2i)$ 		1 correct points 1 open circles 1 correct lines
b) $\lvert z_1\rvert = \lvert z_2\rvert = 2$, $\arg(z_1) = \frac{\pi}{5}$ and $\arg(z_2) = \frac{3\pi}{5}$ $z_1 = 2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$ $z_2 = 2\left(\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}\right)$	1	
$z_1 - z_2 = 2\left(\cos\frac{\pi}{5} - \cos\frac{3\pi}{5} + i\left(\sin\frac{\pi}{5} - \sin\frac{3\pi}{5}\right)\right)$ $= 2\left[\cos\left(\frac{2\pi}{5} - \frac{\pi}{5}\right) - \cos\left(\frac{2\pi}{5} + \frac{\pi}{5}\right) + i\left(\sin\left(\frac{2\pi}{5} + \left(-\frac{\pi}{5}\right)\right) - \sin\left(\frac{2\pi}{5} - \left(-\frac{\pi}{5}\right)\right)\right)\right]$ $= 2\left[2\sin\left(\frac{2\pi}{5}\right)\sin\left(\frac{\pi}{5}\right) + 2i\cos\left(\frac{2\pi}{5}\right)\sin\left(-\frac{\pi}{5}\right)\right]$	1	using product to sum on the formula sheet.
$= 4\left(\sin\frac{2\pi}{5}\sin\frac{\pi}{5} - i\cos\frac{2\pi}{5}\sin\frac{\pi}{5}\right)$ as $\sin(-x) = -\sin x$ $= 4\sin\frac{\pi}{5}\left(\sin\frac{2\pi}{5} - i\cos\frac{2\pi}{5}\right)$	1	must also show that $\sin x$ is an odd function
$= 4\sin\frac{\pi}{5}\left(\cos\left(\frac{\pi}{2} - \frac{2\pi}{5}\right) - i\sin\left(\frac{\pi}{2} - \frac{2\pi}{5}\right)\right)$ $= 4\sin\frac{\pi}{5}\left(\cos\frac{\pi}{10} - i\sin\frac{\pi}{10}\right)$ $= 4\sin\frac{\pi}{5}\left(\cos\left(-\frac{\pi}{10}\right) - i\sin\left(-\frac{\pi}{10}\right)\right)$	1	show working for complementary angles
OR  $\angle AOB = \frac{3\pi}{5} - \frac{\pi}{5}$ $= \frac{2\pi}{5}$ $OB = OA = 2$ $\angle OBA = \angle OAB$ (equal angles are opposite equal sides in triangle)	1	correct diagram

MATHEMATICS Extension 2: Question..10...

Suggested Solutions	Marks	Marker's Comments
$\angle OBA = \dfrac{\pi - \frac{2\pi}{5}}{2}$ (angle sum of a triangle)		
$= \frac{3\pi}{10}$		
$\angle CBO = \pi - \frac{3\pi}{5}$ (co-interior angles, parallel lines)	1	for working out angle
$= \frac{2\pi}{5}$		
$\angle CBA = \frac{2\pi}{5} - \frac{3\pi}{10}$		
$= \frac{\pi}{10}$		
$\therefore \arg(z_1 - z_2) = -\frac{\pi}{10}$	1	recognising arg is negative
$\lvert z_1 - z_2\rvert = \sqrt{\lvert z_1\rvert^2 + \lvert z_2\rvert^2 - 2\lvert z_1\rvert\lvert z_2\rvert \cos\angle AOB}$		
$= \sqrt{4 + 4 - 8\cos\frac{2\pi}{5}}$		
$= \sqrt{8 - 8\cos\frac{2\pi}{5}}$		
$= \sqrt{8 - 8(1 - 2\sin^2\frac{\pi}{5})}$		
$= \sqrt{16\sin^2\frac{\pi}{5}}$		
$= 4\sin\frac{\pi}{5}$	1	correct modulus plus conclusion.
$\therefore z_1 - z_2 = 4\sin\frac{\pi}{5}\left(\cos\left(-\frac{\pi}{10}\right) + i\sin\left(-\frac{\pi}{10}\right)\right)$		

Question 6

Sunday, 12 December 2021 10:11 pm

a) $z=2+3i$, $w=3+i$

$z+iw = 2+3i + i(3+i)$

$= 2+3i+3i-1$

$= 1+6i$ 1m for $z+iw$

$\overline{z+iw} = 1-6i$ 1m for $\overline{z+iw}$

Generally well done

b) $e^{i\pi/3} = \cos\pi/3 + i\sin\pi/3$

$= \frac{1}{2} + \frac{\sqrt{3}}{2}i$

1m for Cartesian form
0m for mod-arg form

Generally well done.
Students must read question carefully and give answers in specified form.

c) $z = x+iy$

$z^2 = |z|^2 - 4$

$(x+iy)^2 = \left(\sqrt{x^2+y^2}\right)^2 - 4$

$x^2+2xiy-y^2 = x^2+y^2-4$ 1m equation in terms of x and y in expanded form

$x^2-y^2 = x^2+y^2-4$

$-2y^2 = -4$

$y^2 = 2$

$y = \pm\sqrt{2}$ 1m for y

$\therefore z = \pm\sqrt{2}i$ 1m for z

Answers must be given as z values

d) Since coefficients are real $3-i$ is also a root by the conjugate root theorem

$(z-(3+i))(z-(3-i))(z^2+az+b) = z^4-4z^3+3z^2-10z+50$

$(z^2-6z+10)(z^2+az+b) = z^4-4z^3+3z^2-10z+50$

$-6+a = -4$

$\therefore a = 2$

$10b = 50$

$b = 5$

Students should practise factorisation by inspection.

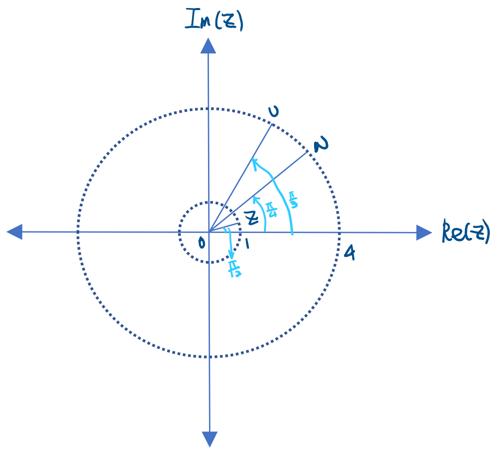
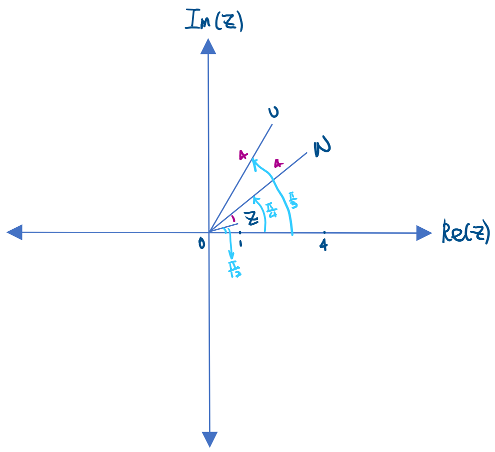
$\therefore z^4-4z^3+3z^2-10z+50 = (z^2-6z+10)(z^2+2z+5)$

1m for stating $3-i$ is also a root
1m for $(z^2-6z+10)$ as a factor
1m for (z^2+2z+5) as a factor and for <u>not</u> finding linear factors

2021 Year 12 Term 1 Mathematics Extension 2 Question 8

Solution	Marks	Marker's Comments
i) $z = \frac{2+2\sqrt{3}i}{2\sqrt{2}+2\sqrt{2}i} = \frac{1+\sqrt{3}i}{\sqrt{2}+\sqrt{2}i}$		
$= \frac{(1+\sqrt{3}i)(1-i)}{\sqrt{2}(1+i)(1-i)}$	1	Multiplying the conjugate of the denominator
$= \frac{1+\sqrt{3}i-i+\sqrt{3}}{\sqrt{2}(1+1)}$		
$= \frac{1+\sqrt{3}}{2\sqrt{2}} + \frac{\sqrt{3}-1}{2\sqrt{2}}i$	1	"$a+bi$" form (Some students write "$\frac{1+\sqrt{3}+(\sqrt{3}-1)i}{2\sqrt{2}}$" (x))
ii) $\lvert v \rvert = \sqrt{2^2+(2\sqrt{3})^2} = 4$		Final answers need to be in the expanded form "$r(\cos\theta + i\sin\theta)$"
$\text{Arg}(v) = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = \frac{\pi}{3}$ (fir quadrant)		
$v = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$		
$\lvert w \rvert = \sqrt{(2\sqrt{2})^2+(2\sqrt{2})^2} = 4$		
$\text{Arg}(w) = \tan^{-1}\left(\frac{2\sqrt{2}}{2\sqrt{2}}\right) = \frac{\pi}{4}$ first quad		
$w = 4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$	1	correct v and w
$z = \frac{v}{w} = \frac{4\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}{4\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)}$		
$= \cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$	1	correct z (Some students use modulus argument form to find the answer, which is really long)

2021 Year 12 Term 1 Mathematics Extension 2 Question 8

Solution	Marks	Marker's Comments
iii)		* The complex numbers can be represented by ① a dot ② an interval ③ an arrow * if one complex number is wrong, deduct one mark.
	1 1	arguments modulus (circle)
	1 1	arguments modulus (labeled) - v and w same length - z is shorter

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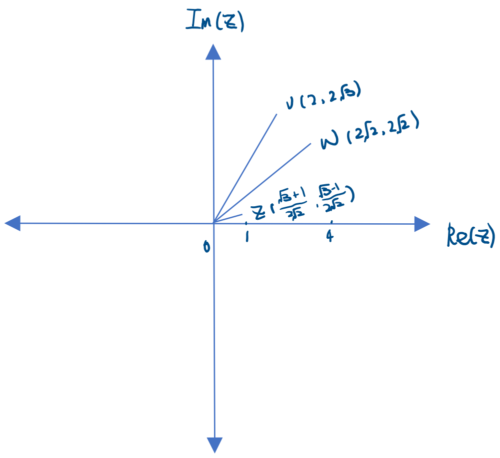
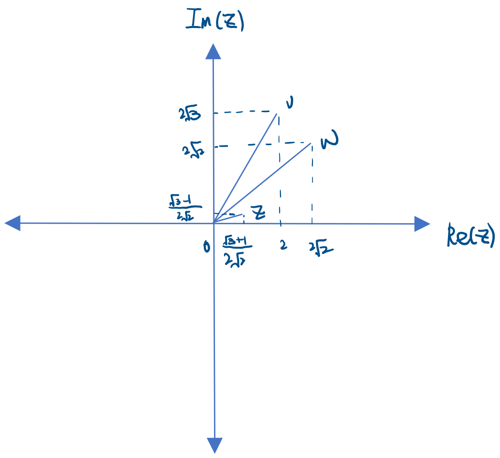
Solution	Marks	Marker's Comments
iii) continue		
	1 1	coordinate pairs v and w same length z is shorter
	1 1	coordinates labelled on the axis v and w same length z is shorter
		Common problems: ① label i — coordinate / y-axis ② no Im, Re ③ v and w different length

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Solution	Marks	Marker's Comments
iv) z is a solution for real coefficient equation		The question asks for answers in cartesian form
$\Rightarrow \bar{z} = \frac{\sqrt{3}+1}{2\sqrt{2}} + \frac{1-\sqrt{3}}{2\sqrt{2}}i$ is also a solution	1	
let the third solution be β.		
Method 1:		
product of roots $= z \times \bar{z} \times \beta = -\frac{d}{a} = \frac{-(-\sqrt{6}+\sqrt{2})}{1} = \sqrt{6}-\sqrt{2}$	1	Some students forget the "$-$" of "$-\frac{d}{a}$"
let $r\text{cis}\theta = r(\cos\theta + i\sin\theta)$		
$\text{cis}(\frac{\pi}{12}) \times \text{cis}(-\frac{\pi}{12}) \times \beta = \sqrt{6}-\sqrt{2}$		
$\text{cis}(0) \times \beta = \sqrt{6}\ \sqrt{2}$		
$1 \times \beta = \sqrt{6}-\sqrt{2}$		
$\beta = \sqrt{6}-\sqrt{2}$	1	
Method 2:		
Sum of roots $= z + \bar{z} + \beta = -\frac{b}{a}$	1	Some students forget "$-$" of "$-\frac{b}{a}$"
$2\text{Re}(z) + \beta = -\frac{1}{2}(\sqrt{2}-3\sqrt{6})$		
$2 \times \frac{\sqrt{6}+\sqrt{2}}{4} + \beta = -\frac{1}{2}(\sqrt{2}-3\sqrt{6})$		A lot of calculation mistake here!!!
$\times 2$ $\sqrt{6}+\sqrt{2}+2\beta = -\sqrt{2}+3\sqrt{6}$ $\times 2$		
$2\beta = -2\sqrt{2}+2\sqrt{6}$		Please don't rush!!!
$\beta = \sqrt{6}-\sqrt{2}$	1	
Method 3:		
Change variable z to x		
$(x-z)(x-\bar{z})$		
$= x^2 - 2\text{Re}(z)x + z\bar{z}$		
$= x^2 - (\frac{\sqrt{2}+\sqrt{6}}{2})x + (\frac{\sqrt{2}+\sqrt{6}}{4})^2 + (\frac{\sqrt{6}-\sqrt{2}}{4})^2$		A lot of calculation mistakes.
$= x^2 - (\frac{\sqrt{2}+\sqrt{6}}{2})x + \frac{2+6+2\sqrt{12}+2+6-2\sqrt{12}}{16}$		
$= x^2 - (\frac{\sqrt{2}+\sqrt{6}}{2})x + 1$	1	
$(x^2 - \frac{\sqrt{2}+\sqrt{6}}{2}x + 1)(x-\beta) = x^3 + \frac{1}{2}(\sqrt{2}-3\sqrt{6})z^2 + 3z - \sqrt{6}+\sqrt{2}$		
By equating the constant, $1 \times (-\beta) = -\sqrt{6}+\sqrt{2}$		
$\beta = \sqrt{6}-\sqrt{2}$	1	

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Solution	Marks	Marker's Comments
iv) continue Not recommended method let the other two solutions be α and β $\alpha+\beta+z=-\frac{1}{2}(\sqrt{2}-3\sqrt{6})$ $\alpha\beta z=-(-\sqrt{6}+\sqrt{2})$ and then solve simultaneous equations to find out α and β, which involves too much calculation.	1 1 1	 Find out α Find out β