Student Name: ____________________

Maths class: ____________________

James Ruse Agricultural High School

2021 **YEAR 12 Task 3 Examination**

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour 20 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 6–10, show relevant mathematical reasoning and/ or calculations

Total marks: 55

Section I – 5 marks (pages 2–3)

- Attempt Questions 1–5
- Allow about 7 minutes for this section

Section II – 50 marks (pages 4–7)

- Attempt Questions 6–10
- Allow about 1 hour and 13 minutes for this section

Section I

5 marks
Attempt Questions 1–5
Allow about 7 minutes for this section

Use the multiple-choice answer sheet for Questions 1–5.

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1. If $\underset{\sim}{u} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{v} = 3\underset{\sim}{i} - 6\underset{\sim}{j} + 2\underset{\sim}{k}$, the vector projection of $\underset{\sim}{v}$ in the direction of $\underset{\sim}{u}$ is:

(A) $\frac{20}{49}(3\underset{\sim}{i} - 6\underset{\sim}{j} + 2\underset{\sim}{k})$

(B) $\frac{20}{3}(2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k})$

(C) $\frac{20}{7}(3\underset{\sim}{i} - 6\underset{\sim}{j} + 2\underset{\sim}{k})$

(D) $\frac{20}{9}(2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k})$

2. The position vector of a particle at time t is given by $\underset{\sim}{r}(t) = \left(\sqrt{t-2}\right)\underset{\sim}{i} + (2t)\underset{\sim}{j}$ for $t \geq 2$. The Cartesian equation of the path of the particle is

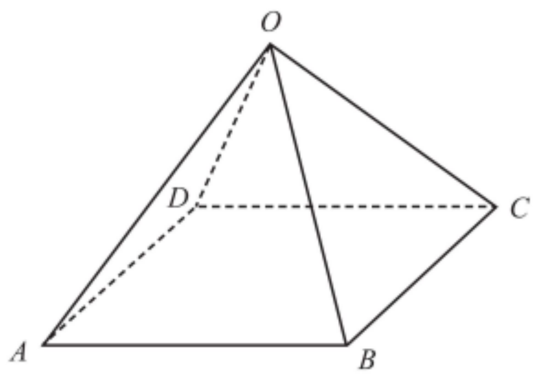
(A) $y = 2x^2 + 4, x \geq 2$

(B) $y = 2x^2 + 2, x \geq 2$

(C) $y = 2x^2 + 4, x \geq 0$

(D) $y = 2x^2 + 2, x \geq 0$

3. Let $OABCD$ be a right square pyramid where $\underset{\sim}{a} = \overrightarrow{OA}, \underset{\sim}{b} = \overrightarrow{OB}, \underset{\sim}{c} = \overrightarrow{OC}, \underset{\sim}{d} = \overrightarrow{OD}$.



An equation correctly relating these vectors is

(A) $\underset{\sim}{a} + \underset{\sim}{c} = \underset{\sim}{b} + \underset{\sim}{d}$

(B) $(\underset{\sim}{a} - \underset{\sim}{c}) \cdot (\underset{\sim}{d} - \underset{\sim}{c}) = 0$

(C) $\underset{\sim}{a} + \underset{\sim}{d} = \underset{\sim}{b} + \underset{\sim}{c}$

(D) $(\underset{\sim}{a} - \underset{\sim}{d}) \cdot (\underset{\sim}{c} - \underset{\sim}{b}) = 0$

4. Suppose a and b are both positive real numbers with $a > b$. If m is any real number, how many of the following statements are always true?

$$\frac{a}{b} < \frac{a+m}{b+m}$$

$$\frac{a}{b} \geq \frac{a+m}{b+m}$$

$$\frac{a+m}{b+m} \leq 1$$

(A) 0

(B) 1

(C) 2

(D) 3

5. Let a and b be positive real numbers with $a < b$. Given that

$$Q = \frac{2}{\frac{1}{a}+\frac{1}{b}}$$

$$M = \sqrt{ab}$$

$$N = \frac{a+b}{2}$$

$$R = \sqrt{\frac{a^2+b^2}{2}}$$

which of the following statements is true?

(A) $Q < N < M < R$

(B) $Q < M < N < R$

(C) $M < N < Q < R$

(D) $Q < M < R < N$

Section II begins on the next page

Section II

50 marks
Attempt Questions 6-10
Allow about 1 hour and 13 minutes for this section

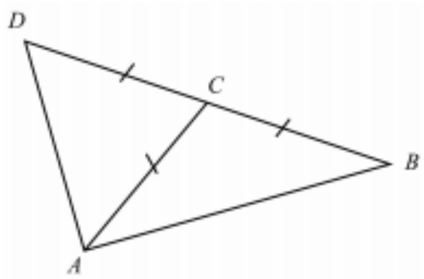
Start each question on a new page. Extra paper is available.

In Questions 6-10, your responses should include relevant mathematical reasoning and/ or calculations.

Question 6 (10 marks) **Start a new page**.

(a) Two spheres $(x-4)^2+(y-1)^2+z^2=27$ and $(x-13)^2+(y-10)^2+(z+9)^2=a$ are externally tangent to each other. What is the value of a? **2**

(b) In triangle $ABD, AC = CD = CB$. **3**



Let $\overrightarrow{AC} = \underset{\sim}{u}$ and $\overrightarrow{BC} = \underset{\sim}{v}$. Use vector methods to prove that $\angle BAD$ is a right angle.

(c) Let α and β be the two roots of the quadratic $p(x) = x^2 + ax + b$, where a and b are both real numbers, and $|a| + |\ b| < 1$.

(i) Prove that $|\alpha + \beta| < 1 - |\alpha\beta|$ **2**

(ii) Given $|\alpha + \beta| \geq |\alpha| - |\beta|$ is true for any real numbers α and β, prove that $|\ \alpha| < 1$ and $|\ \beta|<1$ **3**

Please turn over

Question 7 (11 marks) **Start a new page**.

(a) A line $\mathcal{L}_1$ passes through the points $A(2,1,-1)$ and $B(5,-5,-4)$. Another line $\mathcal{L}_2$ has equation $\underset{\sim}{r} = \begin{pmatrix} 5 \\ 5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$.

(i) Find $\overrightarrow{AB}$. **1**

(ii) Write the vector equation of $\mathcal{L}_1$. **1**

(iii) Find the acute angle between $\mathcal{L}_1$ and $\mathcal{L}_2$. **2**

(iv) $\mathcal{L}_1$ and $\mathcal{L}_2$ intersect at C. Find $\overrightarrow{OC}$. **3**

(b) Let α_i be an angle such that $0 < \alpha_i < \frac{\pi}{4}$ and $0 \leq \sum_{r=1}^{n} \alpha_r \leq \frac{\pi}{4}$ for all positive integers i. Prove by mathematical induction that for all positive integer $n \geq 2$, **4**

$$\tan \sum_{r=1}^{n} \alpha_r \geq \sum_{r=1}^{n} \tan \alpha_r$$

Question 8 (10 marks) **Start a new page**.

(a) In this question, you are given the Arithmetic-Geometric Mean Inequality, that for real values of x and y

$$x^2 + y^2 \geq 2xy$$

Let a, b and c be positive real numbers such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$.

(i) Prove that $1 - \frac{1}{a} \geq \frac{2}{\sqrt{bc}}$ **1**

(ii) Prove that $(a - 1)(b - 1)(c - 1) \geq 8$ **2**

(b) Find the two points on the line $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ whose distance from point $A(1,1,-1)$ is $3\sqrt{5}$ units. **4**

Question 8 continues on the next page

(c) In the following diagram, P is the midpoint of AB and Q is the midpoint of CD. **3**

If $\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD}$, show that $\overrightarrow{PQ} = \frac{3}{4}\overrightarrow{AD}$.

Question 9 (9 marks) **Start a new page**.

(a) (i) Prove that **1**

$$\sqrt{n+1} - 1 = \sqrt{n+2} - 1 - \frac{1}{\sqrt{n+1} + \sqrt{n+2}}$$

(ii) Hence, use mathematical induction to prove that for all integers $n \geq 2$ **3**

$$2\left(\sqrt{n+1} - 1\right) < 1 + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}}$$

(iii) Is the following inequality true for all positive integers N? Give reasons for your answer. **1**

$$\sum_{k=1}^{N} \frac{1}{\sqrt{k}} < 10^{10}$$

Question 9 continues on the next page

(b) (i) Given that $\sin x > \frac{2x}{\pi}$ for $0 < x < \frac{\pi}{2}$, explain why **2**

$$\int_0^{\frac{\pi}{2}} e^{-\sin x}\,dx < \int_0^{\frac{\pi}{2}} e^{-\frac{2x}{\pi}}\,dx$$

(ii) Hence, given that **2**

$$\int_{\frac{\pi}{2}}^{\pi} e^{-\sin x}\,dx = \int_0^{\frac{\pi}{2}} e^{-\sin x}\,dx$$

show that

$$\int_0^{\pi} e^{-\sin x}\,dx < \frac{\pi}{e}(e-1)$$

Question 10 (10 marks) **Start a new page**.

(a) A sequence is defined by $x_n = -x_{n-1} + 3 \cdot 2^n$ and $x_1 = 1$ for $n \geq 2$. Another sequence is defined by $y_n = x_{2n} - 3$. Prove by mathematical induction that y_n is divisible by 8 for $n \geq 1$. **4**

(b) (i) Let p, q and r be positive real numbers. Prove that **2**

$$(p+q+r)\left(\frac{1}{p}+\frac{1}{q}+\frac{1}{r}\right) \geq 9$$

(ii) Hence, given that a, b and c are three sides of a triangle, prove that **4**

$$\frac{a}{b+c-a}+\frac{b}{c+a-b}+\frac{c}{a+b-c} \geq 3$$

END OF EXAMINATION

(Q1) $\underset{\sim}{u} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ $\quad\quad$ $\underset{\sim}{v} = 3\underset{\sim}{i} - 6\underset{\sim}{j} + 2\underset{\sim}{k}$

$$\text{proj}_{\underset{\sim}{u}}(\underset{\sim}{v}) = \left(\frac{\underset{\sim}{v}\cdot\underset{\sim}{u}}{|\underset{\sim}{u}|^2}\right)\underset{\sim}{u}$$

$$= \left(\frac{6+12+2}{4+4+1}\right)\underset{\sim}{u}$$

$$= \frac{20}{9}\underset{\sim}{u}$$

$$= \frac{20}{9}\left(2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right)$$

D

(Q2) $\underset{\sim}{r}(t) = (\sqrt{t-2})\underset{\sim}{i} + 2t\,\underset{\sim}{j}$ $\quad\quad$ $t \geq 2$

$x = \sqrt{t-2} \geq 0$ $\quad\quad$ $y = 2t$ $\quad$ ②

$x^2 = t-2$

$x^2 + 2 = t$ $\quad$ ①

① -> ②: $\quad y = 2x^2 + 4 \;, \; x \geq 0$

C

(Q3) A

(Q4) $\frac{a}{b} < \frac{a+m}{b+m}$ $\quad$ is false (choose $m=0$)

$\frac{a}{b} \geq \frac{a+m}{b+m}$ $\quad$ is false (choose $m<0$)

$\frac{a+m}{b+m} \leq 1$ $\quad$ is false (choose $m>0$)

A

(Q5) This can be determined by subbing in a value of a & b

e.g. $a=5$, $b=3$

$\Rightarrow Q < M < N < R$ (B) ✓

<u>Interesting Note:</u>

This question is about the "generalised mean" of exponent p.

$$M_p = \left(\frac{a^p + b^p}{2}\right)^{1/p}$$

$p=-1$: $Q = \dfrac{2}{\frac{1}{a}+\frac{1}{b}}$ (Harmonic Mean)

$p \to 0$: $M = \sqrt{ab}$ (Geometric Mean)

$p=1$: $N = \dfrac{a+b}{2}$ (Arithmetic Mean)

$p=2$: $R = \sqrt{\dfrac{a^2+b^2}{2}}$ (Quadratic Mean / Root Square Mean)

MATHEMATICS Extension 2: Question 6

Suggested Solutions	Marks	Marker's Comments
a) Distance between 2 centres is:		
$\sqrt{(13-4)^2+(10-1)^2+(-9-0)^2}$		
$= \sqrt{81+81+81}$		
$= \sqrt{243}$	1	or if you found the vector from 1 centre to the other
$= 9\sqrt{3}$		
$\therefore \sqrt{27}+\sqrt{a} = 9\sqrt{3}$		
$3\sqrt{3}+\sqrt{a} = 9\sqrt{3}$		
$\sqrt{a} = 6\sqrt{3}$		
$= \sqrt{108}$		
$\therefore a = 108$	1	
b) $\overrightarrow{CD} = \overrightarrow{BC} = \underset{\sim}{v}$ (equal in magnitude and direction)		
$\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD}$		
$= \underset{\sim}{u} + \underset{\sim}{v}$		
$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB}$	1	
$= \underset{\sim}{u} - \underset{\sim}{v}$		
$\overrightarrow{AB} \cdot \overrightarrow{AD} = (\underset{\sim}{u}-\underset{\sim}{v})\cdot(\underset{\sim}{u}+\underset{\sim}{v})$		
$= \underset{\sim}{u}\cdot\underset{\sim}{u} - \underset{\sim}{v}\cdot\underset{\sim}{v}$		
$= \lvert\underset{\sim}{u}\rvert^2 - \lvert\underset{\sim}{v}\rvert^2$		must give a reason why
$= 0$ (Since $\lvert\underset{\sim}{u}\rvert = \lvert\underset{\sim}{v}\rvert$)	1	$\lvert\underset{\sim}{u}\rvert^2 - \lvert\underset{\sim}{v}\rvert^2 = 0$
$\therefore \overrightarrow{AD} \perp \overrightarrow{AB}$ ($\overrightarrow{AD}\cdot\overrightarrow{AB} = 0$)		Must make a
$\therefore \angle BAD = 90^\circ$	1	conclusion

MATHEMATICS Extension 2: Question……..		
Suggested Solutions	**Marks**	**Marker's Comments**
c) i $\alpha+\beta=-a$ (sum of roots)		
$\alpha\beta=b$ (product of roots)	1	
$\lvert\alpha+\beta\rvert=\lvert a\rvert$ and $\lvert\alpha\beta\rvert=\lvert b\rvert$ * Absolutely necessary!		
$\lvert a\rvert+\lvert b\rvert<1$		
$\downarrow$		
$\lvert\alpha+\beta\rvert+\lvert\alpha\beta\rvert<1$	1	
$\lvert\alpha+\beta\rvert<1-\lvert\alpha\beta\rvert$		
ii $\lvert\alpha+\beta\rvert<1-\lvert\alpha\beta\rvert$ (from i)		
$\therefore \lvert\alpha+\beta\rvert\geq\lvert\alpha\rvert-\lvert\beta\rvert$		
$\Downarrow$		
$1-\lvert\alpha\beta\rvert>\lvert\alpha\rvert-\lvert\beta\rvert$	1	
$\therefore 1-\lvert\alpha\rvert+\lvert\beta\rvert-\lvert\alpha\beta\rvert>0$		
$(1+\lvert\beta\rvert)(1-\lvert\alpha\rvert)>0$		
Since $(1+\lvert\beta\rvert)>0$, then $(1-\lvert\alpha\rvert)>0$, $\therefore \lvert\alpha\rvert<1$	1	
Similarly, $1-\lvert\alpha\beta\rvert>\lvert\beta\rvert-\lvert\alpha\rvert$ (general identity)		Note: Do NOT square both sides of an inequality, you will receive 1/3 MAX if you did so.
$1-\lvert\beta\rvert+\lvert\alpha\rvert+\lvert\alpha\beta\rvert>0$		
$(1+\lvert\alpha\rvert)(1-\lvert\beta\rvert)>0$		
Since $1+\lvert\alpha\rvert>0$, then $1-\lvert\beta\rvert>0$		
$\therefore \lvert\beta\rvert<1$		1

Question 7 (11 marks) **Start a new page.**

(a) A line $\mathscr{L}_1$ passes through the points $A(2,1,-1)$ and $B(5,-5,-4)$. Another line $\mathscr{L}_2$ has equation $\underset{\sim}{r} = \begin{pmatrix} 5 \\ 5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$.

(i) Find $\overrightarrow{AB}$. 1

(ii) Write the vector equation of $\mathscr{L}_1$. 1

(iii) Find the acute angle between $\mathscr{L}_1$ and $\mathscr{L}_2$. 2

(iv) $\mathscr{L}_1$ and $\mathscr{L}_2$ intersect at C. Find $\overrightarrow{OC}$. 3

(b) Let α_i be an angle such that $0 < \alpha_i < \frac{\pi}{4}$ and $0 \leq \sum_{r=1}^{n} \alpha_r \leq \frac{\pi}{4}$ for all positive integers i.

Prove by mathematical induction that for all positive integer $n \geq 2$, 4

$$\tan \sum_{r=1}^{n} \alpha_r \geq \sum_{r=1}^{n} \tan \alpha_r$$

(a) A line $\mathscr{L}_1$ passes through the points $A(2, 1, -1)$ and $B(5, -5, -4)$. Another line $\mathscr{L}_2$ has equation $\underset{\sim}{r} = \begin{pmatrix} 5 \\ 5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$.

(i) Find $\overrightarrow{AB}$. 1

ai, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$$= \begin{pmatrix} 5-2 \\ -5-1 \\ -4--1 \end{pmatrix}$$

$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ -6 \\ -3 \end{pmatrix}$$

(a) A line $\mathscr{L}_1$ passes through the points $A(2,1,-1)$ and $B(5,-5,-4)$.

(ii) Write the vector equation of $\mathscr{L}_1$. 1

ii) $L_1: \underset{\sim}{r} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -6 \\ -3 \end{pmatrix}$, $\mu \in \mathbb{R}$ ✓

(a) A line $\mathscr{L}_1$ passes through the points $A(2,1,-1)$ and $B(5,-5,-4)$. Another line $\mathscr{L}_2$ has equation $\underset{\sim}{r} = \begin{pmatrix}5\\5\\1\end{pmatrix} + \lambda\begin{pmatrix}-3\\4\\2\end{pmatrix}$.

(iii) Find the acute angle between $\mathscr{L}_1$ and $\mathscr{L}_2$. 2

iii, let θ be the angle between L_1 & L_2

$\therefore \cos\theta = \left| \dfrac{\underset{\sim}{d_1} \cdot \underset{\sim}{d_2}}{|\underset{\sim}{d_1}||\underset{\sim}{d_2}|} \right|$, where $\underset{\sim}{d_1}$ & $\underset{\sim}{d_2}$ are direction vectors of L_1 & L_2 respectively.

(note the abs. value to ensure the acute $\angle$ is found)

$= \left| \dfrac{\begin{pmatrix}3\\-6\\-3\end{pmatrix} \cdot \begin{pmatrix}-3\\4\\2\end{pmatrix}}{\left|\begin{pmatrix}3\\-6\\-3\end{pmatrix}\right| \left|\begin{pmatrix}-3\\4\\2\end{pmatrix}\right|} \right|$ ✓

$= \left| \dfrac{-9-24-6}{\sqrt{54}\sqrt{29}} \right|$

$\cos\theta = \dfrac{39}{3\sqrt{174}}$

$\therefore \theta = 9°46''$ (nearest min)

$= 9.76°$ (2 d.p.) ✓

(a) A line $\mathscr{L}_1$ passes through the points $A(2, 1, -1)$ and $B(5, -5, -4)$. Another line $\mathscr{L}_2$ has equation $\underset{\sim}{r} = \begin{pmatrix} 5 \\ 5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$.

(iv) $\mathscr{L}_1$ and $\mathscr{L}_2$ intersect at C. Find $\overrightarrow{OC}$. 3

iv, $L_1: \underset{\sim}{r} = \begin{pmatrix} 2+3\mu \\ 1-6\mu \\ -1-3\mu \end{pmatrix}$ $L_2: \underset{\sim}{r} = \begin{pmatrix} 5-3\lambda \\ 5+4\lambda \\ 1+2\lambda \end{pmatrix}$

$$\begin{pmatrix} 2+3\mu \\ 1-6\mu \\ -1-3\mu \end{pmatrix} = \begin{pmatrix} 5-3\lambda \\ 5+4\lambda \\ 1+2\lambda \end{pmatrix}$$

$2+3\mu = 5-3\lambda \quad (1)$

$1-6\mu = 5+4\lambda \quad (2)$

$-1-3\mu = 1+2\lambda \quad (3)$

$(1)+(3): \quad 1 = 6-\lambda \Rightarrow \underline{\underline{\lambda = 5}}$

sub into (3): $-1-3\mu = 1+10$

$-3\mu = 12$

$\underline{\underline{\mu = -4}}$

check for consistent system by subbing $\lambda = 5$ & $\mu = -4$ into ②:

$$LHS = 1 - 6 \times -4$$
$$= 25$$

$$RHS = 5 + 4 \times 5$$
$$= 25$$

$\therefore \lambda = 5, \mu = -4$ is consistent

sub into L_1: $\underset{\sim}{r} = \begin{pmatrix} 2 + 3(-4) \\ 1 - 6(-4) \\ -1 - 3(-4) \end{pmatrix}$

$$= \begin{pmatrix} -10 \\ 25 \\ 11 \end{pmatrix}$$

(b) **4**

Let α_i be an angle such that $0 < \alpha_i < \frac{\pi}{4}$ and $0 \le \sum_{r=1}^{n} \alpha_r \le \frac{\pi}{4}$ for all positive integers i.

Prove by mathematical induction that for all positive integer $n \ge 2$,

$$\tan \sum_{r=1}^{n} \alpha_r \ge \sum_{r=1}^{n} \tan \alpha_r$$

b, Base case: $n = 2 \rightarrow LHS = \tan(\alpha_1 + \alpha_2)$

$$= \frac{\tan(\alpha_1) + \tan(\alpha_2)}{1 - \tan(\alpha_1)\tan(\alpha_2)}$$

since $\alpha_i \in (0, \frac{\pi}{4}) \Rightarrow 0 < \tan(\alpha_i) < 1$

$\Rightarrow 0 < \tan(\alpha_1)\tan(\alpha_2) < 1$

$\Rightarrow 0 < 1 - \tan(\alpha_1)\tan(\alpha_2) < 1$ ✓

$\Rightarrow \dfrac{1}{1 - \tan(\alpha_1)\tan(\alpha_2)} > 1$

$$\frac{\tan(\alpha_1)+\tan(\alpha_2)}{1-\tan(\alpha_1)\tan(\alpha_2)} > \tan(\alpha_1) + \tan(\alpha_2)$$

$$\therefore \quad LHS = \frac{\tan(\alpha_1)+\tan(\alpha_2)}{1-\tan(\alpha_1)\tan(\alpha_2)}$$

$$> \tan(\alpha_1)+\tan(\alpha_2)$$

$$= RHS$$

$\therefore$ true for $n=2$.

<u>Inductive Assumption:</u> assume $n=k$ is true

i.e. assume $\tan\left(\sum_{r=1}^{k} \alpha_r\right) \geq \sum_{r=1}^{k} \tan(\alpha_r)$ where $0 < \alpha_i < \frac{\pi}{4}$

<u>RTP:</u> $n=k+1$ is true

i.e. $\tan\left(\sum_{r=1}^{k+1} \alpha_r\right) \geq \sum_{r=1}^{k+1} \tan(\alpha_r)$

$$LHS = \tan\left(\sum_{r=1}^{k+1} \alpha_r\right)$$

$$= \tan\left(\alpha_{k+1} + \sum_{r=1}^{k} \alpha_r\right) \geq \tan\left(\sum_{r=1}^{k} \alpha_r\right) + \tan\alpha_{k+1}$$ Using the initial case (n=2)

$$\geq \sum_{r=1}^{k} \tan\alpha_r + \tan\alpha_{k+1},$$ By Assumption

$$= \sum_{r=1}^{k+1} \tan\alpha_r$$

The alternative approach which more efficient in the next slides

Inductive Assumption: assume $n=k$ is true

i.e. assume $\tan\left(\sum_{r=1}^{k}\alpha_r\right) \geq \sum_{r=1}^{k}\tan(\alpha_r)$ where $0<\alpha_i<\frac{\pi}{4}$

RTP: $n=k+1$ is true

i.e. $\tan\left(\sum_{r=1}^{k+1}\alpha_r\right) \geq \sum_{r=1}^{k+1}\tan(\alpha_r)$

$$\begin{aligned}
LHS &= \tan\left(\sum_{r=1}^{k+1}\alpha_r\right) \\
&= \tan\left(\alpha_{k+1} + \sum_{r=1}^{k}\alpha_r\right) \\
&= \frac{\tan(\alpha_{k+1}) + \tan\left(\sum_{r=1}^{k}\alpha_r\right)}{1 - \tan(\alpha_{k+1})\tan\left(\sum_{r=1}^{k}\alpha_r\right)}
\end{aligned}$$

now, since $0 < \alpha_{k+1} < \frac{\pi}{4}$ & $0 \le \sum_{r=1}^{k} \alpha_r \le \frac{\pi}{4}$

$$\Rightarrow 0 < \tan(\alpha_{k+1}) < 1 \quad \& \quad 0 \le \tan\left(\sum_{r=1}^{k} \alpha_r\right) \le 1$$

$$\Rightarrow 0 < \tan(\alpha_{k+1}) \tan\left(\sum_{r=1}^{k} \alpha_r\right) < 1$$

$$\Rightarrow 0 < 1 - \tan(\alpha_{k+1}) \tan\left(\sum_{r=1}^{k} \alpha_r\right) < 1$$

$$\Rightarrow \frac{1}{1 - \tan(\alpha_{k+1}) \tan\left(\sum_{r=1}^{k} \alpha_r\right)} > 1$$

$$\therefore \text{LHS} = \frac{\tan(\alpha_{k+1}) + \tan\left(\sum_{r=1}^{k} \alpha_r\right)}{1 - \tan(\alpha_{k+1})\tan\left(\sum_{r=1}^{k} \alpha_r\right)}$$

$$> \tan(\alpha_{k+1}) + \tan\left(\sum_{r=1}^{k} \alpha_r\right)$$

$$> \tan(\alpha_{k+1}) + \sum_{r=1}^{k} \tan(\alpha_r) \qquad \text{, by assumption}$$

$$= \sum_{r=1}^{k+1} \tan(\alpha_r)$$

$$= \text{RHS}$$

$\therefore$ true for $n = k+1$, if $n = k$ is true

Therefore True n>=2 by Mathematical induction.

Question 8a)

Saturday, 5 June 2021 9:01 pm

(a) Let a, b and c be positive real numbers such that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$.

(i) Prove that $1-\frac{1}{a} \geq \frac{2}{\sqrt{bc}}$ **1**

(ii) Prove that $(a-1)(b-1)(c-1) \geq 8$ **2**

Solution:

a) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$

(i) $(\sqrt{x}-\sqrt{y})^2 \geq 0$

$x+y-2\sqrt{xy} \geq 0$

$x+y \geq 2\sqrt{xy}$

$1-\frac{1}{a}=\frac{1}{b}+\frac{1}{c}$

$\frac{1}{b}+\frac{1}{c} \geq 2\sqrt{\frac{1}{b}\times\frac{1}{c}}$ AM-GM inequality

$\frac{1}{b}+\frac{1}{c} \geq \frac{2}{\sqrt{bc}}$ 1

$\therefore 1-\frac{1}{a} \geq \frac{2}{\sqrt{bc}}$

This questions was well done by most students
It was sufficient to say that you used the AM-GM inequality to arrive at the second line.

(ii) $1-\frac{1}{a} \geq \frac{2}{\sqrt{bc}} \Rightarrow a-1 \geq \frac{2a}{\sqrt{bc}}$ 1

1 mark was awarded for rearranging the inequality from part (i)

Similarly,

$1-\frac{1}{b} \geq \frac{2}{\sqrt{ac}}$ $\qquad b-1 \geq \frac{2b}{\sqrt{ac}}$

and $1-\frac{1}{c} \geq \frac{2}{\sqrt{ab}}$ $\qquad c-1 \geq \frac{2c}{\sqrt{ab}}$

$(a-1)(b-1)(c-1) \geq \frac{2a}{\sqrt{bc}} \times \frac{2b}{\sqrt{ac}} \times \frac{2c}{\sqrt{ab}}$ 1

$= \frac{8abc}{\sqrt{a^2b^2c^2}}$

$= \frac{8abc}{abc}$

$= 8$

1 mark was awarded for using a-1 to arrive at the given inequality showing all steps.
Each step must be shown to earn the mark.
Students who merely wrote the first line and then 8 were not awarded the mark.

Question 8b)

Saturday, 5 June 2021 9:01 pm

(b) Find the two points on the line $\underset{\sim}{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\1\end{pmatrix}$ whose distance from point $A(1,1,-1)$ is $3\sqrt{5}$ units. **4**

Solution:

b) Let (x,y,z) be the point on $\underset{\sim}{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\1\end{pmatrix}$ whose distance from point $A(1,1,-1)$ is $3\sqrt{5}$ u.

$$\left|\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix} - \begin{pmatrix}1\\1\\-1\end{pmatrix}\right| = 3\sqrt{5}$$

But (x,y,z) lies on the line

$$\therefore \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\1\end{pmatrix}$$

$$\therefore \left|\begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\1\end{pmatrix} - \begin{pmatrix}1\\1\\-1\end{pmatrix}\right| = 3\sqrt{5} \quad (1)$$

1 mark was awarded for writing an expression for the distance involving λ and equating it to $3\sqrt{5}$.
Many students were not able to interpret the question well enough to arrive at this step and were unable to earn any further marks.

$$\left|\begin{pmatrix}0\\1\\4\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\1\end{pmatrix}\right| = 3\sqrt{5}$$

$$\left|\begin{matrix}2\lambda\\1+3\lambda\\4+\lambda\end{matrix}\right| = 3\sqrt{5}$$

$$(2\lambda)^2 + (1+3\lambda)^2 + (4+\lambda)^2 = 45$$

$$4\lambda^2 + 1 + 6\lambda + 9\lambda^2 + 16 + 8\lambda + \lambda^2 = 45$$

$$14\lambda^2 + 14\lambda - 28 = 0$$

$$\lambda^2 + \lambda - 2 = 0$$

$$(\lambda+2)(\lambda-1) = 0$$

$$\lambda = -2 \text{ or } 1 \quad (1)$$

1 mark was awarded for solving a quadratic expression in λ to find two values of λ

$\lambda = 1$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \begin{pmatrix}2\\3\\1\end{pmatrix} = \begin{pmatrix}3\\5\\4\end{pmatrix}$$

$\lambda = -2$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix} - 2\begin{pmatrix}2\\3\\1\end{pmatrix} = \begin{pmatrix}-3\\-4\\1\end{pmatrix} \quad (1)$$

1 mark was awarded for arriving at the two correct position vectors by substituting in the values of λ

$\therefore$ The points are $(3,5,4)$ and $(-3,-4,1)$ (1)

The final mark was awarded only if students wrote the correct form for the two POINTS
Many students did not read the question carefully enough to realise they needed points.
If students used ordered triple notation throughout their working, they had to state clearly that their answers now represented points.

Question 8c)

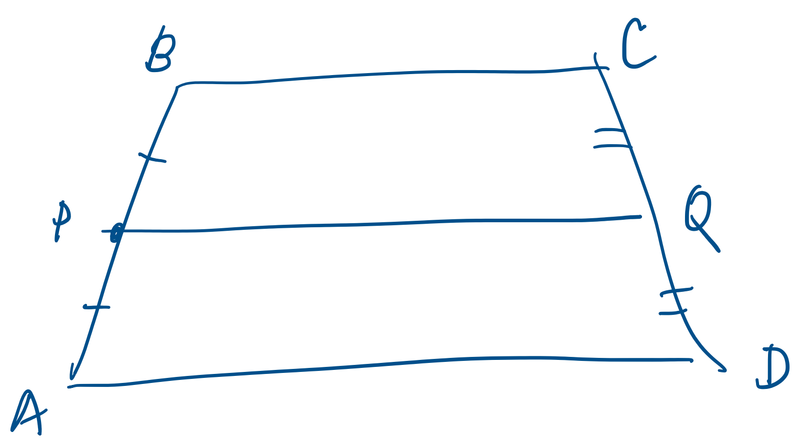
Saturday, 5 June 2021 9:01 pm

(c) In the following diagram, P is the midpoint of AB and Q is the midpoint of CD. **3**

If $\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD}$, show that $\overrightarrow{PQ} = \frac{3}{4}\overrightarrow{AD}$.

Solution:

c)



$$\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD}$$

$$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BC} + \overrightarrow{CQ}$$
$$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD} + \frac{1}{2}\overrightarrow{CD}$$

$$\Rightarrow \overrightarrow{PQ} - \frac{1}{2}\overrightarrow{AD} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{CD}) \quad 1$$

1 mark was awarded for writing vector PQ in terms of vectors
- **AD, AB and CD.**

OR
- **AB, BC and CD**

$$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$$
$$= \overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD} + \overrightarrow{CD}$$

$$\Rightarrow \frac{1}{2}\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{CD} \quad 1$$
$$\frac{1}{2}\overrightarrow{AD} = 2\left(\overrightarrow{PQ} - \frac{1}{2}\overrightarrow{AD}\right)$$
$$= 2\overrightarrow{PQ} - \overrightarrow{AD}$$
$$\frac{3}{2}\overrightarrow{AD} = 2\overrightarrow{PQ}$$
$$\overrightarrow{PQ} = \frac{3}{4}\overrightarrow{AD} \quad 1$$

1 mark was awarded for using the expression found for vector PQ to write an expression for vector AD in terms of two other vectors
- **PQ and BC**

OR
- **AB and CD**

The final mark was awarded for correct working to arrive at the required result.

Students who did not use correct vector notation earned a maximum of 2 marks

Students who wrote expressions involving the zero vector but wrote only 0 earned a maximum of 2 marks

Year 12 Extension 2: Q9		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
(a) (i) $\text{RHS} = \sqrt{n+2} - 1 - \dfrac{1}{\sqrt{n+1}+\sqrt{n+2}}$ $= \sqrt{n+2} - 1 - \dfrac{1}{\sqrt{n+1}+\sqrt{n+2}} \cdot \dfrac{\sqrt{n+2}-\sqrt{n+1}}{\sqrt{n+2}-\sqrt{n+1}}$ $= \sqrt{n+2} - 1 - \dfrac{\sqrt{n+2}-\sqrt{n+1}}{(n+2)-(n+1)}$ $= \sqrt{n+2} - 1 - \left(\sqrt{n+2}-\sqrt{n+1}\right)$ $= \sqrt{n+1} - 1$ $= \text{LHS}$ The left-hand side could have been worked, too, starting with $\sqrt{n+1} - 1 = \sqrt{n+1} - \sqrt{n+2} + \sqrt{n+2} - 1$ $= \left(\sqrt{n+1}-\sqrt{n+2}\right) \times \dfrac{\sqrt{n+1}+\sqrt{n+2}}{\sqrt{n+1}+\sqrt{n+2}} + \sqrt{n+2} - 1$ $= -\dfrac{1}{\sqrt{n+1}+\sqrt{n+2}} + \sqrt{n+2} - 1$ $= \text{RHS}$	1	Setting out was lacking in a lot of cases. Some started with the equality and showed…equality. You can't start by assuming that thing you're trying to prove true as being true. Also, if you want to take a pause in a derivation and start something else that's related, tell the person to whom you're communicating. E.g. "Now…" often signifies you wish to start something that you think will help and you'll patch it back in later. Don't write as a stream of consciousness. Everything you write down is meant to be an argument, so it therefore has to be coherent.

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(ii)

Base case:

$$\text{LHS} = 2\left(\sqrt{3} - 1\right)$$

$$\text{RHS} = 1 + \frac{1}{\sqrt{2}}$$

Assume for contradiction that

$$2\sqrt{3} - 2 \geq 1 + \frac{\sqrt{2}}{2}$$

Then

$$4\sqrt{3} - 6 \geq \sqrt{2}$$

So, squaring,

$$16 \cdot 3 - 48\sqrt{3} + 36 \geq 2$$

$$41 \geq 24\sqrt{3}$$

So, squaring again,

$$1681 \geq 1728$$

which is false.

Hence, the negation of the original assumption is true; that is,

$$2\left(\sqrt{3} - 1\right) < 1 + \frac{1}{\sqrt{2}}$$

Let $S(n)$ be

$$2\left(\sqrt{n+1} - 1\right) < \sum_{k=1}^{n} \frac{1}{\sqrt{k}}$$

for some $2 \leq n \in \mathbb{N}$.

Then for $S(n+1)$:

$$\sum_{k=1}^{n+1} \frac{1}{\sqrt{k}} = \sum_{k=1}^{n} \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{n+1}}$$

$$> 2\left(\sqrt{n+1} - 1\right) + \frac{1}{\sqrt{n+1}} \quad \text{(if } S(n) \text{ is true)}$$

$$= 2\left(\sqrt{n+2} - 1 - \frac{1}{\sqrt{n+1} + \sqrt{n+2}}\right) + \frac{1}{\sqrt{n+1}} \quad \text{(by part (i))}$$

$$= 2\left(\sqrt{n+2} - 1\right) + \frac{1}{\sqrt{n+1}} - \frac{2}{\sqrt{n+1} + \sqrt{n+2}}$$

$$> 2\left(\sqrt{n+2} - 1\right) + \frac{1}{\sqrt{n+1}} - \frac{2}{\sqrt{n+1} + \sqrt{n+1}} \quad (*)$$

$$= 2\left(\sqrt{n+2} - 1\right)$$

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A great portion of the cohort calculated a LHS and a RHS, in terms of fractions with surds, and then stated that LHS ≤ RHS.

There's a big problem with this: it's not obvious that that's the case, and this is only guaranteed because of the contrived nature of the problem.

Think: in 'real life', if you were testing something out for induction, would you stop at two numbers with surds and operations defining them, or would you keep going to prove that one number is indeed less than or equal to the other? If you didn't do this, BECAUSE it's the case that you KNOW what the target is, claiming the inequality to be true without further proof or justification isn't enough.

In Extension 2, you work through the topics of **Inequalities** and **Proof**. Use proof by contradiction, say, to get you out of trouble here.

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where $(*)$ follows since sqrt is a monotonic increasing function, so

$$\sqrt{n+2} > \sqrt{n+1}$$

$$\sqrt{n+1} + \sqrt{n+2} > 2\sqrt{n+1}$$

$$\frac{1}{\sqrt{n+1}+\sqrt{n+2}} < \frac{1}{2\sqrt{n+1}}$$

$$\frac{-2}{\sqrt{n+1}+\sqrt{n+2}} > \frac{-2}{2\sqrt{n+1}}$$

$$\frac{-2}{\sqrt{n+1}+\sqrt{n+2}} > \frac{-1}{\sqrt{n+1}}$$

$$\frac{1}{\sqrt{n+1}} + \left(\frac{-2}{\sqrt{n+1}+\sqrt{n+2}}\right) > \frac{1}{\sqrt{n+1}} + \left(-\frac{1}{\sqrt{n+1}}\right) = 0$$

Hence

$$\sum_{k=1}^{n+1} \frac{1}{\sqrt{k}} > 2(\sqrt{n+2} - 1)$$

if the hypothesis is true.

Since $S(2)$ is true, and since $S(n) \rightarrow S(n+1)$ is true for $2 \leq n \in \mathbb{N}$, it follows, by the principle of mathematical induction, that $S(n)$ is true $\forall n \in \mathbb{N}$ and $n \geq 2$.

(iii)

Assume, for all $n \in \mathbb{N}$, that

$$\sum_{k=1}^{n} \frac{1}{\sqrt{k}} < 10^{10}$$

is true. From (ii), we have that for all positive integers $n \geq 2$,

$$2\left(\sqrt{n+1} - 1\right) < \sum_{k=1}^{n} \frac{1}{\sqrt{k}}$$

so we would have for all integers $n \geq 2$,

$$2\left(\sqrt{n+1} - 1\right) < 10^{10}$$

This is a 'spelt out' justification for line $(*)$ **above.** Many people provided sound justification at this point, which was good to see.

About 1/3 did not attempt this question.

Those who did received the mark for sufficient reasoning to justify why the sum had no limiting value.

Let $n = 10^{20} - 1$. Then $n \in \mathbb{N}$ while $$\begin{aligned} 2\left(\sqrt{n+1} - 1\right) &= 2\left(\sqrt{10^{20} - 1 + 1} - 1\right) \\ &= 2(10^{10} - 1) \\ &= 2 \times 10^{10} - 2 \\ &= 10^{10} + (10^{10} - 2) \\ &> 10^{10} \end{aligned}$$ not less than 10^{10}. Hence the statement is false: the sum is **not** less than 10^{10} for all $n \in \mathbb{N}$.	1	
(b) (i) Let $x \in \left(0, \frac{\pi}{2}\right) \subseteq \mathbb{R}$. Over that interval, we are given that $$\sin x > \frac{2x}{\pi}$$ Since the exponential function is monotonic increasing for all $\mathbb{R}$, it follows that $$e^{\sin x} > e^{\frac{2x}{\pi}}$$ giving $$e^{-\sin x} < e^{-\frac{2x}{\pi}}$$ Since this relation is true for all x over the given interval, it follows that $$\int_0^{\pi/2} e^{-\sin x}\ dx < \int_0^{\pi/2} e^{-2x/\pi}\ dx$$	1 1	Some students lost a mark here because they did not justify the leap from $$\sin x > \frac{2x}{\pi}$$ to $$e^{\sin x} > e^{\frac{2x}{\pi}}$$ or similar. If someone said $\alpha(x) < \beta(x)$ and said show that $\int_a^b f(\alpha(x))dx < \int_a^b f(\beta(x))dx$, wouldn't you need to prove something about the behaviour of f first? The $'f(x)'$ here is e^x. Also again, **why** do you need to do this in an exam? Because you **know** what the target is/what hurdle you have to cross before making the conclusion the examiners want. This is the problem – you

(ii) We are given that $\int_0^{\pi/2} e^{-\sin x}\ dx = \int_{\pi/2}^{\pi} e^{-\sin x}\ dx \quad \ldots (1)$		basically have the answer there, so it's your job to explain **why** what's there is true.
Now, $I := \int_0^{\pi} e^{-\sin x}\ dx = \int_0^{\frac{\pi}{2}} e^{-\sin x}\ dx + \int_{\frac{\pi}{2}}^{\pi} e^{-\mathrm{si}}\ \ dx$ $= 2\int_0^{\frac{\pi}{2}} e^{-\mathrm{si}}\ \ dx$		**Part (ii) was handled very well.** First mark for reasonable headway (at least using information from part (i)), second mark for full conclusion.
$< 2\int_0^{\frac{\pi}{2}} e^{-\frac{2x}{\pi}}\, dx \quad (\text{by } (1))$	1	
$= 2\left[-\frac{\pi}{2}e^{-\frac{2x}{\pi}}\right\rvert_0^{\pi/2}$		
Hence $I < -\pi[e^{-1} - e^{0}] = \pi\left(1 - \frac{1}{e}\right) = \frac{\pi}{e}(e-1)$ as required.	1	

MATHEMATICS Extension 2: Question 10		
Suggested Solutions	**Marks**	**Marker's Comments**
a) $y_n = x_{2n} - 3$ $x_n = -x_{n-1} + 3\times 2^n \quad , x_1 = 1$ Prove true for $n=1$ $y_1 = x_2 - 3$ $= -x_1 + 3\times 2^2 - 3$ $= -(1) + 12 - 3$ $= 8$ which is divisible by 8. $\therefore$ true for $n=1$	1	must show all working!
Assume true for $n=k,\ k\in\mathbb{Z}^+$ ie. $y_k = x_{2k} - 3$ $= 8M \quad , M\in\mathbb{Z}$		the integer for M does not have to be positive only.
Prove true for $n=k+1$ $y_{k+1} = x_{2(k+1)} - 3$ $= x_{2k+2} - 3$ $= -x_{2k+1} + 3\times 2^{2k+2} - 3$ $= -(-x_{2k} + 3\times 2^{2k+1}) + 3\times 2^{2k+2} - 3$ $= x_{2k} - 3 - 3\times 2^{2k+1} + 3\times 2^{2k+2}$ $= 8M - 3\times 2^{2k+1} + 3\times 2\times 2^{2k+1}$ (by assumption)	1	correctly using the definition of x_n twice. A lot of students got the power incorrect.
$= 8M + 3\times 2^{2k+1}$ as $k\geq 1$, $2k+1\geq 3$ $= 8M + 3\times 2^3 \times 2^{2k-2}$ $= 8M + 3\times 8\times 2^{2k-2}$ $= 8(M + 3\times 2^{2k-2})$ which is divisible by 8 as $M + 3\times 2^{2k-2}$ is an integer.	1	need to show that $2k+1\geq 3$ and to state that $M+3\times 2^{2k-2}$ is an integer.

MATHEMATICS Extension 2: Question ..10...

Suggested Solutions	Marks	Marker's Comments
If the statement is true for $n=k$ and proven true for $n=k+1$. Since it is true for $n=1$, then it will be true for $n=2,3,4\ldots$ and so on. Therefore the statement is true by the principle of mathematical induction.	1	for setting out. must have: - conclusion - by assumption - $M \in \mathbb{Z}$ - statement for $n=k$.
b) i) $(p+q+r)\left(\frac{1}{p}+\frac{1}{q}+\frac{1}{r}\right) \geq 9$		
$\text{LHS} = 1+\frac{p}{q}+\frac{p}{r}+\frac{q}{p}+1+\frac{q}{r}+\frac{r}{p}+\frac{r}{q}+1$		
$= \frac{p}{q}+\frac{q}{p}+\frac{p}{r}+\frac{r}{p}+\frac{q}{r}+\frac{r}{q}+3$	1	
as $(p-q)^2 \geq 0$ $p^2-2pq+q^2 \geq 0$ $p^2+q^2 \geq 2pq$ $\frac{p^2}{pq}+\frac{q^2}{pq} \geq 2$ as p and q are positive real numbers $\frac{p}{q}+\frac{q}{p} \geq 2$		
Similarly $\frac{p}{r}+\frac{r}{p} \geq 2$ $\frac{q}{r}+\frac{r}{q} \geq 2$		
$\therefore \text{LHS} \geq 2+2+2+3$ $= 9$ $\therefore (p+q+r)\left(\frac{1}{p}+\frac{1}{q}+\frac{1}{r}\right) \geq 9$	1	must show how the 2 was obtained
NOTE: If using AM-GM with three terms, this must be proven first before it can be used. Two terms is in the syllabus so can use without the proof.		

MATHEMATICS Extension 2: Question 10

Suggested Solutions	Marks	Marker's Comments
ii) as a, b and c are sides of a triangle. $a+b>c$ $\therefore a+b-c>0$ similarly, $a+c-b>0$ $b+c-a>0$ as they are all positive real numbers.	1	must show this to be able to use part (i)
from (i) let $p = a+b-c$ $q = b+c-a$ $r = a+c-b$		
$\therefore (a+b-c+b+c-a+a+c-b)\left(\frac{1}{a+b-c}+\frac{1}{b+c-a}+\frac{1}{a+c-b}\right) \geq 9$ $(a+b+c)\left(\frac{1}{a+b-c}+\frac{1}{b+c-a}+\frac{1}{a+c-b}\right) \geq 9$	1	using part (i) correctly
$\frac{a+b+c}{a+b-c}+\frac{a+b+c}{b+c-a}+\frac{a+b+c}{a+c-b} \geq 9$	1 correct expansion	$\left(\frac{a}{b+c-a}+\frac{b}{c+a-b}+\frac{c}{a+b-c}\right)\left(\frac{b+c-a}{a}+\frac{c+a-b}{b}+\frac{a+b-c}{c}\right) \geq 9$ does not lead you far as you will have made a incorrect assumption along the way. if $ab \geq 9$ and $ab \geq c$ does not imply that $9 \geq c$.
$\frac{a+b-c+2c}{a+b-c}+\frac{b+c-a+2a}{b+c-a}+\frac{a+c-b+2b}{a+c-b} \geq 9$ $1+\frac{2c}{a+b-c}+1+\frac{2a}{b+c-a}+1+\frac{2b}{a+c-b} \geq 9$		
$2\left(\frac{c}{a+b-c}+\frac{a}{b+c-a}+\frac{b}{a+c-b}\right) \geq 6$ $\therefore \frac{a}{b+c-a}+\frac{b}{c+a-b}+\frac{c}{a+b-c} \geq 3$	1	