Name: ______________________

Student Number: ______________________

2025 **YEAR 12 MATHEMATICS EXTENSION 2**
TASK 1

Mathematics

General Instructions

- Reading time – 10 minutes
- Working time – 90 minutes
- Write using black or blue pen
- Diagrams are not to scale
- NESA Approved calculators may be used
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 55

Section I – 5 marks (page 1–2)

- Attempt Questions 1–5
- Allow about 10 minutes for this section

Section II – 45 marks (pages 3–7)

- Attempt Questions 6–8
- Allow about 1 hour and 20 minutes for this section

Two-sided A4 handwritten sheet 5 marks

Section I

5 marks
Attempt Questions 1–5
Each question in this section is worth 1 mark
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for questions 1-5.

Question 1

Which of the following is equivalent to $-3-3\sqrt{3}i$?

A. $6e^{-\frac{\pi}{3}i}$

B. $6e^{-\frac{2\pi}{3}i}$

C. $6e^{\frac{\pi}{3}i}$

D. $6e^{\frac{2\pi}{3}i}$

Question 2

The sum of the roots of $z^3-5z^2+11z-7=0$, where $z\in\mathbb{C}$, is

A. 5

B. $5i$

C. $2+i$

D. $2-i$

Question 3

Given $z=1-i$ which expression is equal to z^5?

A. $z^5=4\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)$

B. $z^5=4\sqrt{2}\left(\cos\left(\frac{-\pi}{4}\right)+i\sin\left(\frac{-\pi}{4}\right)\right)$

C. $z^5=4\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right)$

D. $z^5=4\sqrt{2}\left(\cos\left(\frac{-3\pi}{4}\right)+i\sin\left(\frac{-3\pi}{4}\right)\right)$

Question 4

For what values of n is $(1+\sqrt{3}i)^n$ purely real?

A. Multiples of 2

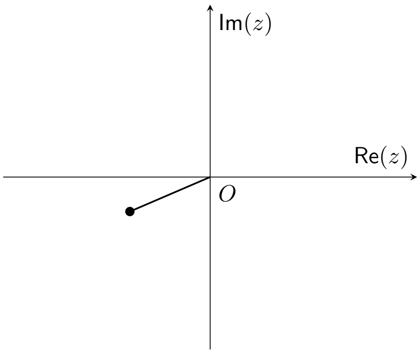
B. Multiples of 3

C. Multiples of 4

D. Multiples of 6

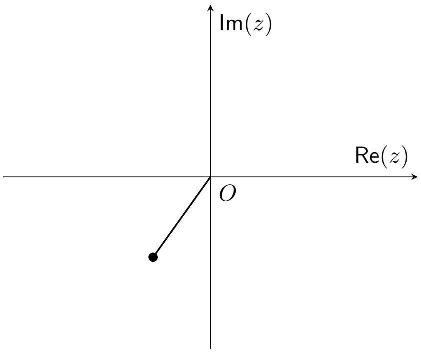
Question 5

The Argand diagram shows the complex number $e^{i\theta}$.

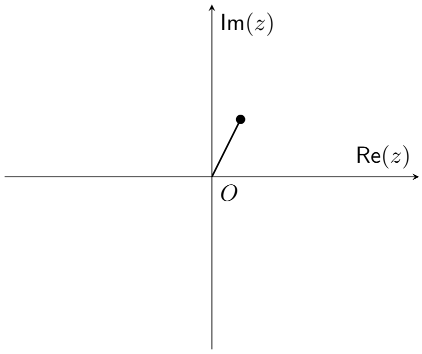


Which of the following could be the complex number $ie^{-i\theta}$?

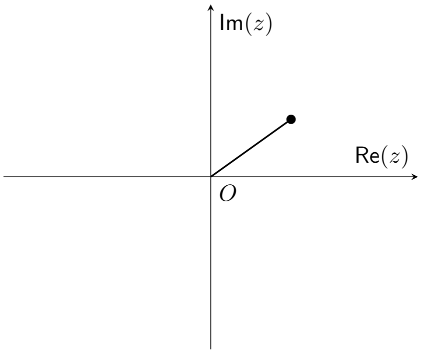
A.



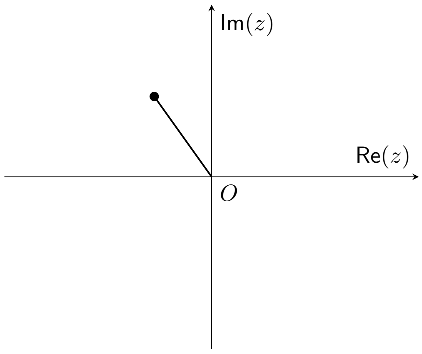
C.



B.



D.



Section II

45 marks
Attempt Questions 6 –8
Allow about 1hrs and 20 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (15 marks) **Marks**

(a) Consider the complex numbers $w = 1 + 2i$ and $z = 3 - i$. Find in the form $x + iy$,

(i) $\overline{w}z$, **2**

(ii) $\overline{\left(\frac{10}{z}\right)}$. **2**

(b) Factorise $z^4 - z^2 - 20$ over:

(i) the real field, **1**

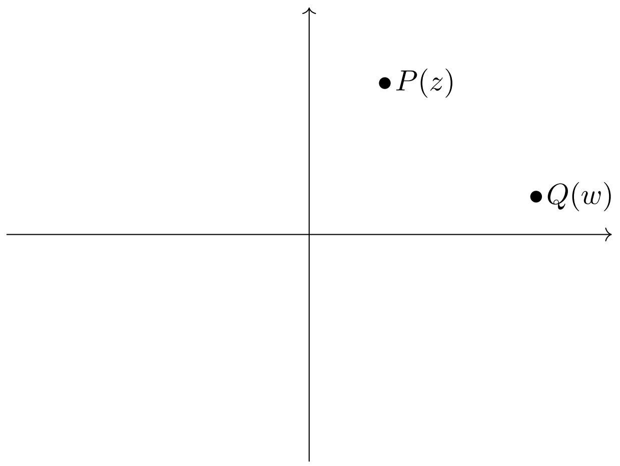
(ii) the complex field. **1**

(c) (i) Find the square roots of $-8 + 6i$ in Cartesian form, **2**

(ii) Hence solve $z^2 - (7 - i)z + (14 - 5i) = 0$. **2**

(d) Find a quadratic equation in the form $az^2 + bz + c = 0$, where $a, b, c \in \mathbb{R}$ that has the complex root $\sqrt{3} + 5i$. **2**

(e) The points P and Q on the Argand diagram represent the complex numbers z and w respectively.



On the **separate answer sheet** provided for Question 6(e) mark on it the following points:

(i) the point R representing $\overline{z}$, **1**

(ii) the point S representing $z + w$, **1**

(iii) the point T representing $z - w$. **1**

End of Question 6

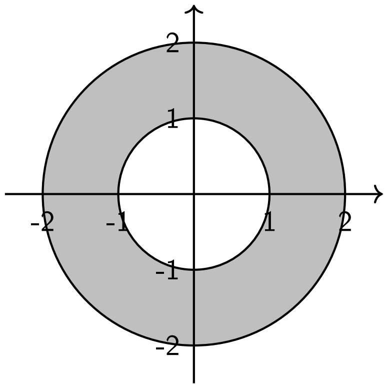
Question 7 (15 marks) **Marks**

(a) Let $\omega = -\sqrt{3} + i$.

(i) Write ω in modulus-argument form, **2**

(ii) Hence find ω^9 in Cartesian form. **1**

(b) The diagram below shows the shaded region in the Argand diagram. Find the inequality which describes the shaded area. **1**



(c) If p, q and r are the roots of $z^3 + (1-5i)z^2 + (-5+2i)z + 15 - 3i = 0$, find:

(i) $\overline{p} + \overline{q} + \overline{r}$, **2**

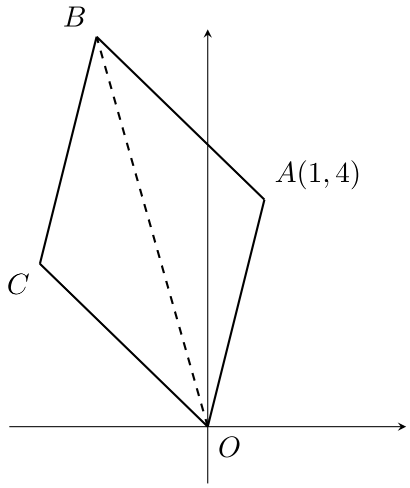
(ii) $\overline{p} \times \overline{q} \times \overline{r}$. **2**

(d) Find the complex roots of $z^6 = -1$ and express them in exponential form. **2**

(e) Sketch the graph of **2**

$$\text{Arg}\left[\frac{z-(2+i)}{z-(1+2i)}\right] = \pi$$

(f) $OABC$ is a rhombus in the Argand diagram, where O is the origin and point B is in the second quadrant. The point A is $(1, 4)$ and the angle AOB is $\frac{\pi}{6}$. **3**



Find the complex numbers represented by the points B and C in Cartesian form.

End of Question 7

Question 8 (15 marks) **Marks**

(a) The complex number z moves such that $\text{Im}\left[\dfrac{1}{\overline{z}-i}\right] = 1$. Find the Cartesian equation that describes the locus of z. **3**

(b) Sketch the intersection of the two regions below: **3**

$$|z + 1 - i| < \sqrt{2}$$

$$-\pi \leq \text{ Arg}(z + 1 - i) \leq \frac{3\pi}{4}$$

in the complex number plane.

(c) Given $|z| < \dfrac{1}{3}$, show that $|(1 + i)z^3 + iz| < \dfrac{4}{9}$. **3**

(d) (i) Use the binomial theorem to expand $(\cos(\theta) + i \sin(\theta))^5$, **1**

(ii) Use de Moivre's theorem and your results from part (i) to prove that **1**

$$\sin(5\theta) = \sin^5(\theta) - 10\cos^2(\theta)\sin^3(\theta) + 5\cos^4(\theta)\sin(\theta),$$

(iii) Hence or otherwise prove, **2**

$$\tan(5\theta) = \frac{5\tan(\theta) - 10\tan^3(\theta) + \tan^5(\theta)}{1 - 10\tan^2(\theta) + 5\tan^4(\theta)},$$

(iv) Hence show that $x^4 - 10x^2 + 5 = 0$ has roots $x = \pm\tan\frac{\pi}{5}$ and $x = \pm\tan\frac{2\pi}{5}$. **2**

End of Question 8

End of Task

Name ____________________ Teacher ____________________

2025 HSC Mathematics Extension 2
Task 1 Examination

Section I – Multiple Choice

Allow about 10 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ⊗ B ● C ○ D ○

If you have changed your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word **correct** and drawing an arrow as follows:

A ○ B ⊗ C ○ D ○

correct

1. A ○ B ● C ○ D ○
2. A ● B ○ C ○ D ○
3. A ○ B ○ C ● D ○
4. A ○ B ● C ○ D ○
5. A ● B ○ C ○ D ○

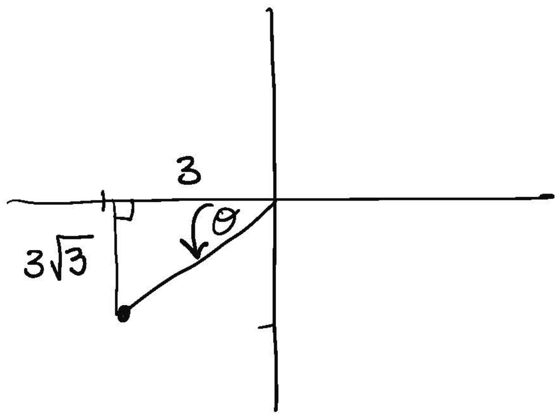
1. $-3 - 3\sqrt{3}\, i$

let $z = -3 - 3\sqrt{3}\, i$

$$|z| = \sqrt{(-3)^2 + (3\sqrt{3})^2}$$

$$= \sqrt{36}$$

$$= 6$$



$$\arg(z) = -\pi + \tan^{-1}\left(\frac{3\sqrt{3}}{3}\right)$$

$$= -\pi + \frac{\pi}{3}$$

$$= -\frac{2\pi}{3}$$

$$\therefore \quad -3 - 3\sqrt{3}\, i = 6e^{-\frac{2\pi}{3} i}$$

B

2. $z^3 - 5z^2 + 11z - 7 = 0$

let α, β and γ be the roots

let $\alpha = a + bi$

$\beta = a - bi$

$\alpha + \beta + \gamma = \frac{-b}{a}$

$2a + \gamma = +5$

note. 3rd root, as it must be real.

$\therefore$ (A) is the answer.

test $z = 1$

$1 - 5 + 11 - 7 = 0$

$-12 + 12 = 0$

$\therefore \gamma = 1$

$2a + 1 = 5$

$2a = 4$

$a = 2$

$\therefore$ the other roots $2+i$, $2-i$

sum of roots.

$1 + 2 + i + 2 - i = 5$.

3. $z = 1 - i$

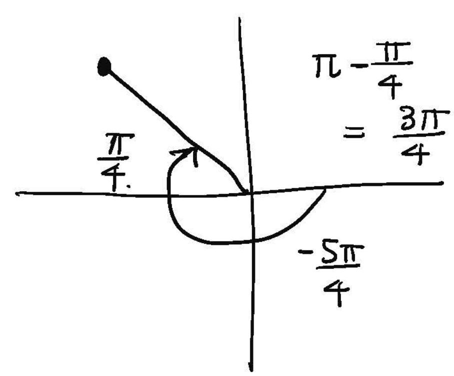
$|z| = \sqrt{2}$

$\arg(z) = \frac{-\pi}{4}$

$z = \sqrt{2} \operatorname{cis} \frac{-\pi}{4}$

$z^5 = \left(\sqrt{2} \operatorname{cis} -\frac{\pi}{4}\right)^5$

$z^5 = \left(\sqrt{2}^5 \operatorname{cis} -\frac{5\pi}{4}\right)$

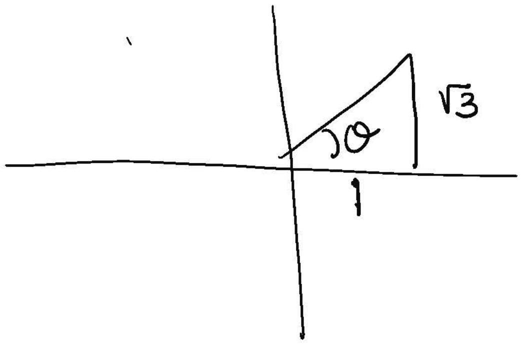


from the diagram

$z^5 = 4\sqrt{2} \operatorname{cis} \frac{3\pi}{4}$

(C)

Q4

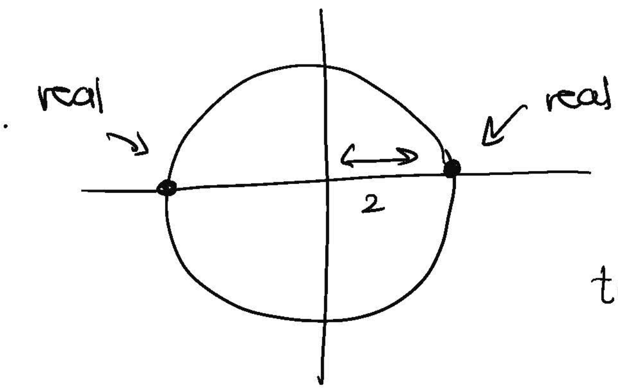


$(1+\sqrt{3}i)^n$

$|z| = 2$

$\arg(z) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \frac{\pi}{3}$



$(1+\sqrt{3}i)^n = (2\operatorname{cis}\frac{\pi}{3})^n$

$= 2^n \operatorname{cis}\frac{n\pi}{3}$

to be real

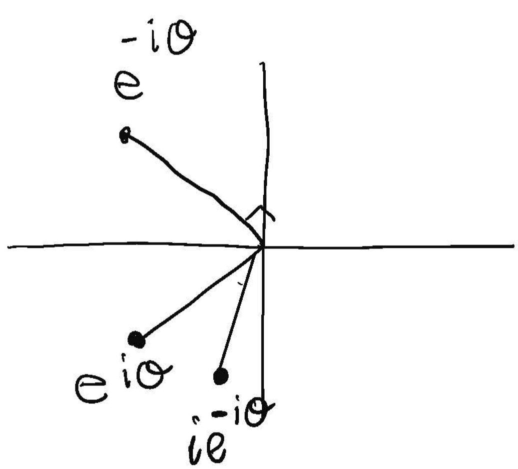
$\frac{n\cancel{\pi}}{3} = k\cancel{\pi}$ where $k \in \mathbb{Z}$

$n = 3k$

ie its a multiple of 3.

B

Q5



A

Question 6

a) $w = 1 + 2i$

1 mark correct conjugate

(i) $\bar{w}z = (1-2i)(3-i)$

$= 3 - i - 6i - 2$

$= 1 - 7i$

1 mark for correct solution.

(ii) $\overline{\left(\frac{10}{z}\right)} = ?$

$\left(\frac{10}{z}\right) = \frac{10}{3-i} \times \frac{3+i}{3+i}$

1 mark realising the denominator correctly

$= \frac{30 + 10i}{9 + 1}$

$= \frac{\cancel{10}(3+i)}{\cancel{10}}$

$= 3 + i$

$\overline{\left(\frac{10}{z}\right)} = 3 - i$

1 mark correct solution

Question 6.

(b) $z^4 - z^2 - 20 = (z^2 - 5)(z^2 + 4)$ rational field

real field $= (z - \sqrt{5})(z + \sqrt{5})(z^2 + 4)$

complex field $= (z - \sqrt{5})(z + \sqrt{5})(z - 2i)(z + 2i)$

1 mark for each correct solution

(c) (i) $-8 + 6i$

$\frac{6}{2} = 3 <$ 3, 1 / 1, 3

$1^2 - 3^2 = -8$

$\therefore$ the square roots are $\pm(1 + 3i)$ 2 mark.

(ii) $z^2 - (7-i)z + (14 - 5i) = 0$

$$z = \frac{+(7-i) \pm \sqrt{(7-i)^2 - 4 \times 1 \times (14-5i)}}{2}$$ $-i \times -i$

$$= \frac{7 - i \pm \sqrt{49 - 14i - 1 - 56 + 20i}}{2}$$

$$= \frac{7 - i \pm \sqrt{-8 + 6i}}{2}$$

$\therefore z = \frac{7 - i + 1 + 3i}{2}$ or $\frac{7 - i - 1 - 3i}{2}$

$= \frac{8 + 2i}{2}$ 1 mark

$= 4 + i$

$= \frac{6 - 4i}{2}$ 1 mark.

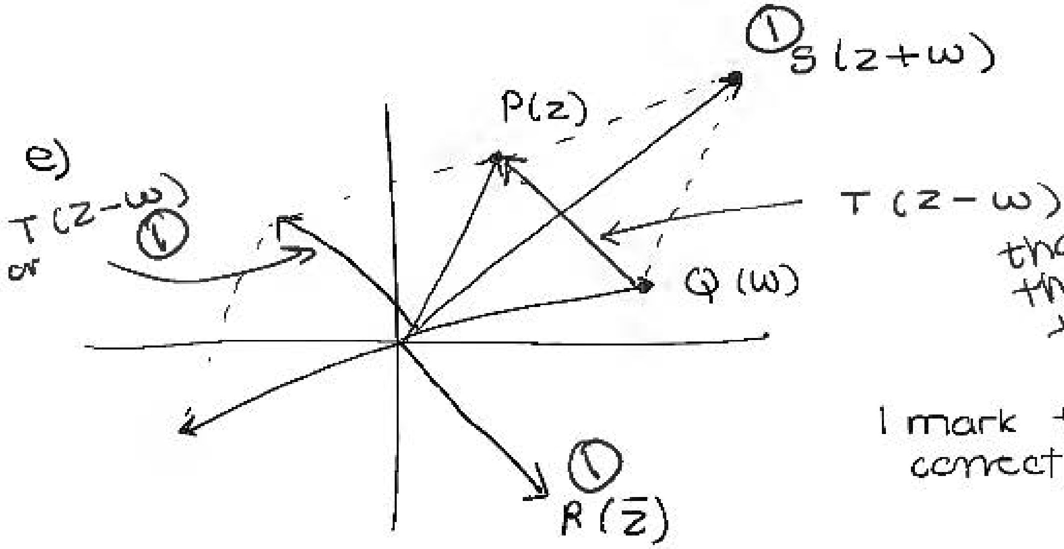
$= 3 - 2i$ 1 mark

d) $(z - (\sqrt{3} + 5i))(z - (\sqrt{3} - 5i)) = 0$

$z^2 - z(\sqrt{3} - 5i) - z(\sqrt{3} + 5i) + (\sqrt{3} + 5i)(\sqrt{3} - 5i) = 0$

$z^2 - 2\sqrt{3}.z + (3 + 25) = 0$

$z^2 - 2\sqrt{3}z + 28 = 0$ 1 mark for correct solution e)



thats the vector
the point
the position
is the vector

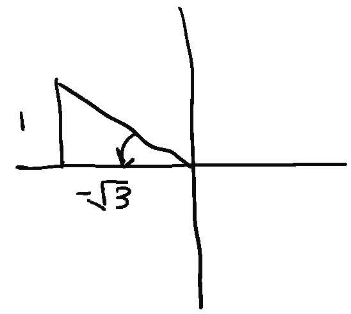
1 mark for each correct solution

Q7. Simplify

a) (i) $\omega = -\sqrt{3} + i$

$$|\omega| = \sqrt{(-\sqrt{3})^2 + (1)^2}$$

$$= 2$$



$$\arg(\omega) = \pi - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \pi - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}$$

$\therefore \ \omega = 2 \operatorname{cis} \frac{5\pi}{6}$ 2 mark

(ii)

$$\omega^9 = \left(2 \operatorname{cis} \frac{5\pi}{6}\right)^9$$

$$= 2^9 \operatorname{cis}\left(\frac{5\pi}{6} \times 9\right)$$

$$= 512 \operatorname{cis}\left(\frac{15\pi}{2}\right)$$

$$= 512 \operatorname{cis}\left(\frac{3\pi}{2}\right)$$

$$= 512 \left[\cos\frac{3\pi}{2} \quad i\sin\frac{3\pi}{2}\right]$$

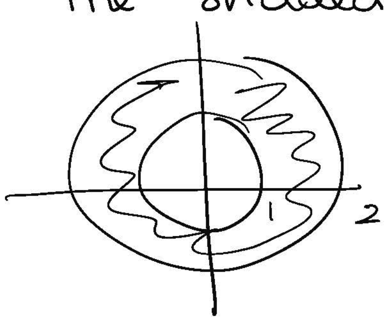
$$= 512 \left[0 - i\right]$$

$$= -512i$$

1 mark

(b) Write the inequality that best describes the shaded area

1 mark



$1 \leq |z| \leq 2$

Q7(c) $z^3 + (1-5i)z^3 + (-5+2i)z + 15 - 3i = 0$

i) $\bar{p} + \bar{q} + \bar{r} = \overline{p+q+r}$ 1 mark

$= \overline{\left(\frac{-b}{a}\right)}$

$= \overline{-\left(\frac{1-5i}{1}\right)}$

$= \overline{-1+5i}$

$= -1 - 5i$ 1 mark

ii) $\bar{p} \times \bar{q} \times \bar{r} = \overline{p \times q \times r}$ 1 mark

$= \overline{\left(\frac{-d}{a}\right)}$

$= \overline{-\left(\frac{15-3i}{1}\right)}$

$= \overline{-15+3i}$

$= -15 - 3i$ 1 mark

d) $z^6 = -1$

Find the first root

let a root be $z = r\,\text{cis}\,\theta$

$\therefore \quad z^6 = (r\,\text{cis}\,\theta)^6 = -1$

$r^6\,\text{cis}\,6\theta = -1$

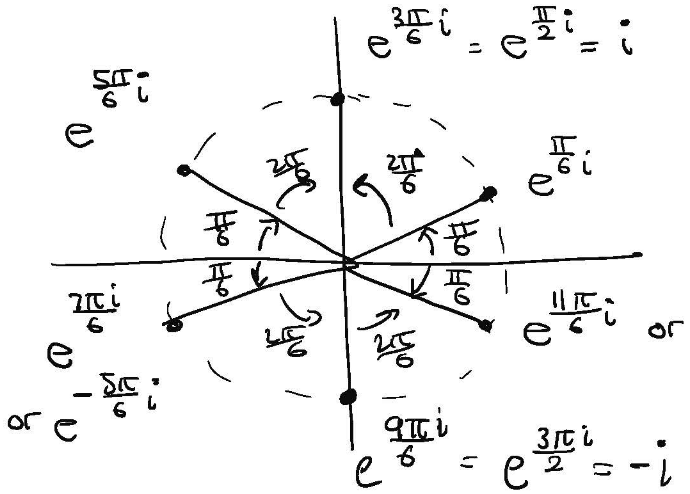
$r^6 = 1 \qquad\qquad \text{cis}\,6\theta = \text{cis}\,\pi$

$\therefore \quad r = 1 \qquad\qquad 6\theta = \pi$

$\theta = \frac{\pi}{6}$

$\therefore$ first root $z = \text{cis}\,\frac{\pi}{6}$

$z = e^{\frac{\pi}{6}i}$ 1 mark.



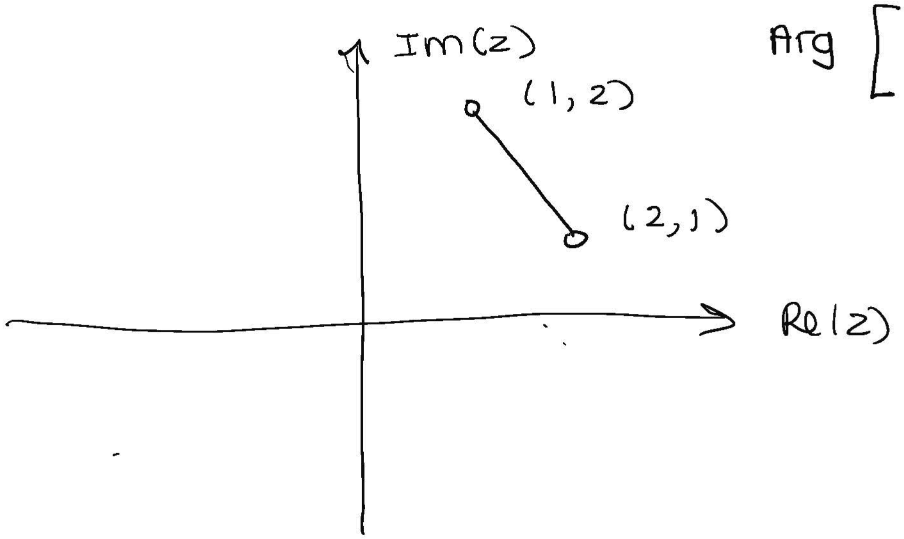
Equal spacing $\frac{2\pi}{6} = \frac{\pi}{3}$

$\therefore$ the roots are

$e^{\frac{\pi}{6}i}, e^{\frac{\pi}{2}i}, e^{\frac{5\pi}{6}i}, e^{\frac{7\pi}{6}i}, e^{\frac{3\pi}{2}i}, e^{\frac{11\pi}{6}i}$

1 mark for correct solution

e) $\text{Arg}\left[\frac{z-(2+i)}{z-(1+2i)}\right] = \pi$



2 marks,

f) $\overrightarrow{OA} = r \operatorname{cis} \theta$ Rhombus so no scaling 1 mark.

$\overrightarrow{OC} = \overrightarrow{OA} \times \operatorname{cis}\left(\frac{\pi}{6}\right) \times \operatorname{cis}\left(\frac{\pi}{6}\right)$

$= (1+4i) \times \left(\cos\left(\frac{\pi}{6}+\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6}+\frac{\pi}{6}\right)i\right)$

$= (1+4i) \times \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$= (1+4i) \times \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$\overrightarrow{OC} = \frac{1}{2} + \frac{\sqrt{3}}{2}i + \frac{4}{2}i - \frac{4\sqrt{3}}{2}$

ie $C = \frac{1}{2} - \frac{4\sqrt{3}}{2} + \left(\frac{4}{2} + \frac{\sqrt{3}}{2}\right)i$

$\therefore \overrightarrow{OC} = \frac{1-4\sqrt{3}}{2} + \left(\frac{4+\sqrt{3}}{2}\right)i$ 1 mark

$\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{OA}$

$= \frac{1-4\sqrt{3}}{2} + \left(\frac{4+\sqrt{3}}{2}\right)i + 1 + 4i$

$\overrightarrow{OB} = \frac{1-4\sqrt{3}}{2} + \left(\frac{4+\sqrt{3}}{2}\right)i + \frac{2+8i}{2}$

ie $B = \frac{3-4\sqrt{3}}{2} + \frac{12+\sqrt{3}}{2}i$ 1 mark.

Q 8

(a) $\operatorname{Im}\left[\dfrac{1}{\bar{z}-i}\right] = 1$

let $z = x + iy$

$\bar{z} = x - iy$

$$\frac{1}{\bar{z}-i} = \frac{1}{x - iy - i}$$

$$= \frac{1}{x - (y+1)i} \times \frac{x + (y+1)i}{x + (y+1)i}$$

$$= \frac{x + (y+1)i}{x^2 + (y+1)^2}$$

1 mark.

$$\longrightarrow \quad = \frac{x}{x^2 + (y+1)^2} + \frac{y+1}{x^2+(y+1)^2}\, i$$

$$\operatorname{Im}\left[\frac{1}{\bar{z}-i}\right] = \frac{y+1}{x^2+(y+1)^2} = 1$$

$$y + \cancel{1} = x^2 + y^2 + 2y + \cancel{1}$$

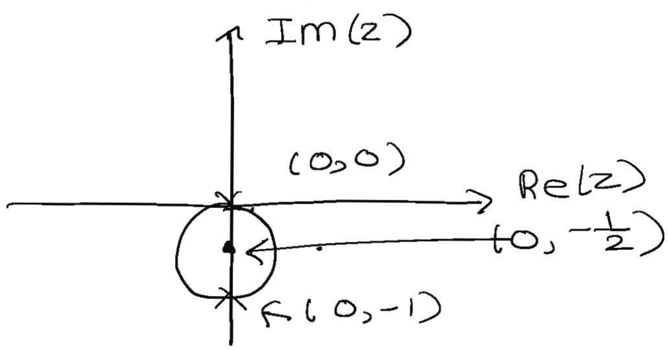
$$0 = x^2 + y^2 + y$$

$$\left(\tfrac{1}{2}\right)^2 = x^2 + y^2 + y + \left(\tfrac{1}{2}\right)^2$$

$$\left(\tfrac{1}{2}\right)^2 = x + \left(y + \tfrac{1}{2}\right)^2$$

$$\tfrac{1}{4} = x + \left(y + \tfrac{1}{2}\right)^2 \quad \longleftarrow \text{1 mark}$$

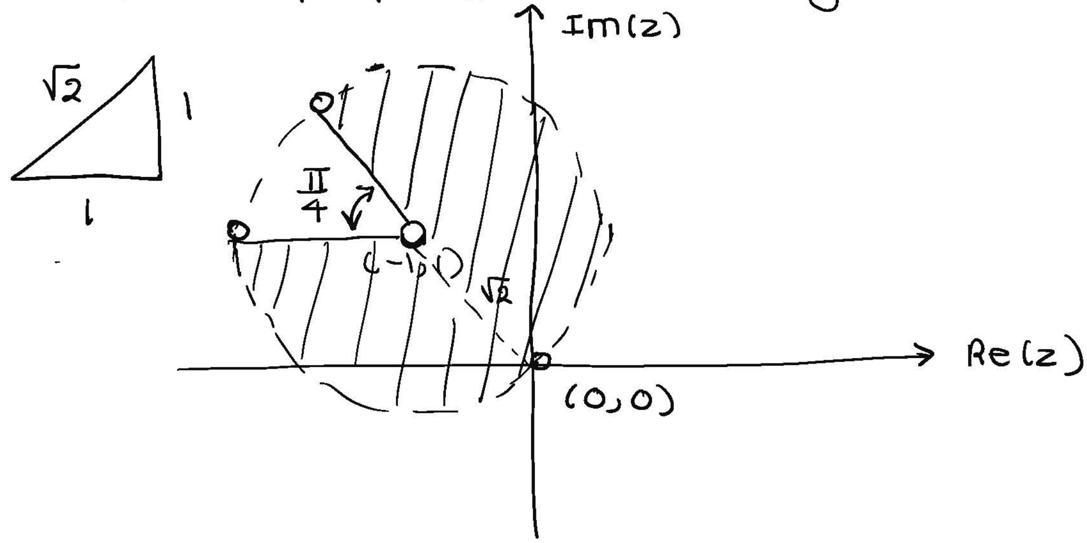
$\therefore$ the locus is a circle at $(0, -\frac{1}{2})$ with a radius $\frac{1}{2}$.



1 mark.

(b) $|z+1-i| < \sqrt{2}$ and $-\pi \leq \arg(z+1-i) \leq \frac{3\pi}{4}$

$|z-(-1+i)| < \sqrt{2}$ and $-\pi \leq \arg(z-(-1+i)) \leq \frac{3\pi}{4}$



correct circle 1 mark.
correct angles 1 mark.

correct solution 1 mark.

c) $|z| < \frac{1}{3}$ show that $|(1+i)z^3 + iz| < \frac{4}{9}$

$$|(1+i)z^3 + iz| \leq |(1+i)z^3| + |iz|$$ Triangle inequality (1 mark)

$$= |(1+i)||z^3| + |i||z|$$

$$= \sqrt{2}\,|z^3| + 1 \times |z|$$

$$< \sqrt{2}\,|(\tfrac{1}{3})|^3 + 1 \times (\tfrac{1}{3})$$ 1 mark

$$= \frac{\sqrt{2}}{27} + \frac{1}{3}$$

$$= \frac{\sqrt{2}}{27} + \frac{9}{27}$$

$$< \frac{3+9}{27}$$

$$= \frac{12}{27}$$

$$= \frac{4}{9}$$ 1 mark.

d) (i) $(\cos\theta + i\sin\theta)^5$

Binomial theorem to expand

$(\cos\theta + i\sin\theta)^5 =$

$\binom{5}{0}\cos^5\theta + \binom{5}{1}\cos^4\theta\sin\theta\, i + \binom{5}{2}\cos^3\theta\sin^2\theta\, i^2$

$+ \binom{5}{3}\cos^2\theta\sin^3\theta\, i^3 + \binom{5}{4}\cos\theta\sin^4\theta\, i^4$

$+ \binom{5}{5}\sin^5\theta\, i^5$

$= \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta$

$- 10\cos^2\theta\sin^3\theta\, i + 5\cos\theta\sin^4\theta$

$+ \sin^5\theta\, i$

$= (\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta)$

$+ i(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$

1 mark

(ii) Using de Moivre's theorem

$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$

equating the imaginary parts

$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$

1 mark

(iii) equating the real parts

$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$

$\tan 5\theta = \dfrac{\sin 5\theta}{\cos 5\theta}$ 1 mark

$= \dfrac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta}$

$$= \frac{5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta}{\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta} \div \frac{\cos^5\theta}{\cos^5\theta}$$

$$= \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$$

$$= \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{5\tan^4\theta - 10\tan^2\theta + 1}$$ 1 mark.

$\square$

(iv) $\tan 5\theta = 0$ when $\sin 5\theta = 0$

$5\theta = k\pi$ where $k \in \mathbb{Z}$

$\theta = \frac{k\pi}{5}$

$\theta = 0, \pm\frac{\pi}{5}, \pm\frac{2\pi}{5}$

Also $5\tan\theta - 10\tan^3\theta + \tan^5\theta = 0$

let $\tan\theta = x$

$\therefore \quad 5x - 10x^3 + x^5 = 0$

$x(x^4 - 10x^2 + 5) = 0$

1 mark $\therefore \quad x = 0 \qquad x^4 - 10x^2 + 5 = 0$

$x = \tan 0$ is the solution to $x = 0$

so the other solutions

$\tan(\pm\frac{\pi}{5}), \tan(\pm\frac{2\pi}{5})$ odd function

1 mark. $= \pm\tan\frac{\pi}{5}, \pm\tan(\frac{2\pi}{5})$ are also solutions