Killara HIGH SCHOOL

Student Number

2017

Mathematics Ext 2

HSC Course Assessment Task 3

May 25 2017

General Instructions

- Reading time – 5 minutes
- Working time **1** hour
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- In Questions 6 to 8 show relevant mathematical reasoning and/or calculations
- Answer each question in a separate writing booklet
- This paper must not be removed from the examination room
- A reference sheet is provided at the back of this paper
- Diagrams are NOT to scale

Total Marks – 40

Section I - Pages 2 - 4

5 marks

- Attempt Questions 1 to 5
- Allow about 8 minutes for this section

Section II - Pages 5 to 7

35 marks

- Attempt Questions 6 to 8
- Allow about 52 minutes for this section

Multiple choice	/5
Q6	/16
Q7	/10
Q8	/9
Total	/40

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 20% of the HSC *Course Assessment*

BLANK PAGE

Section I

5 marks
Attempt Questions 1 to 4
Allow about 8 minutes for this section
Use the multiple-choice answer sheet for questions 1 to 4 (Detach from paper).

---

1. The equation $x^3 - y^3 + 3xy + 1 = 0$ defines y implicitly as a function of x. What is the value of $\frac{dy}{dx}$ at the point (-1, 3)?

   (A) $\frac{1}{2}$

   (B) $\frac{2}{5}$

   (C) $\frac{2}{3}$

   (D) $\frac{1}{5}$

2. A directrix of an ellipse has the equation $x = \frac{25}{4}$ and one of its foci has the coordinate (-4, 0). What is the equation of this ellipse?

   (A) $\frac{x^2}{5} + \frac{y^2}{3} = 1$

   (B) $\frac{x^2}{3} + \frac{y^2}{5} = 1$

   (C) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

   (D) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

3. *S(4, 0)* is a focus of the rectangular hyperbola $x^2 - y^2 = k$. Which of the following is the value of *k*.

   (A) $2\sqrt{2}$

   (B) $4\sqrt{2}$

   (C) 8

   (D) 32

4. If the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are inclined to each other at an angle α, then $\tan \alpha$ is equal to:

(A) $\frac{2a}{|a^2-b^2|}$

(B) $-\left|\frac{2ab}{a^2-b^2}\right|$

(C) $\frac{2b}{|a^2-b^2|}$

(D) $\frac{2ab}{|a^2-b^2|}$

5. Which integral is equal to

$$\int_{-a}^{a} f(x)\,dx$$

(A) $\int_{0}^{a} f(x) - f(-x)\,dx$

(B) $\int_{0}^{a} f(x) - f(a-x)\,dx$

(C) $\int_{0}^{a} f(x-a) + f(-x)\,dx$

(D) $\int_{0}^{a} f(x-a) + f(a-x)\,dx$

Section II

35 marks
Attempt Questions 6 to 8
Allow about 52 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 6 to 8, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (16 marks) Use a SEPARATE writing booklet

(a) Find **1**

$$\int \frac{1}{x(lnx)^2}\,dx$$

(b) By completing the square, find **2**

$$\int \frac{1}{x^2+6x+10}\,dx$$

(c) (i) Find real numbers a, b and c such that **2**

$$\frac{3x+1}{(x-1)(x^2+1)} = \frac{a}{x-1} + \frac{bx+c}{x^2+1}$$

(ii) Hence, find $\int \frac{3x+1}{(x-1)(x^2+1)}\,dx$ **2**

(c) Evaluate $\int_1^{\sqrt{3}} \frac{x^2}{\sqrt{4-x^2}}\,dx$ **3**

(d) Find $\int \cos^5 x\,dx$ **3**

(e) Using the substitution $t = \tan\frac{x}{2}$, find $\int \frac{1}{1+\sin x-\cos x}\,dx$ **3**

End of Question 6

Question 7 (10 marks) Use a SEPARATE writing booklet

(a) With respect to the Cartesian axes, the line $x = -1$ is a directrix, and the point (-2, 0) is a focus with eccentricity $\sqrt{2}$.

i) Derive from first principles the equation of the conic. **2**

ii) Given that the equation of the tangent to the curve at any point $P(x_1, y_1)$ on it is $xx_1 - yy_1 = 2$. **2**

If the tangent cuts the x-axis at (X, 0) and the y-axis at (0, Y), prove that the locus of the point $R(X, Y)$ is $2Y^2 - 2X^2 = X^2Y^2$

(b)

Consider the hyperbolas $H_1 : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $H_2 : \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$.

i) <u>Derive</u> the equations of the asymptotes (standard formulae are not to be used). What do you notice? **2**

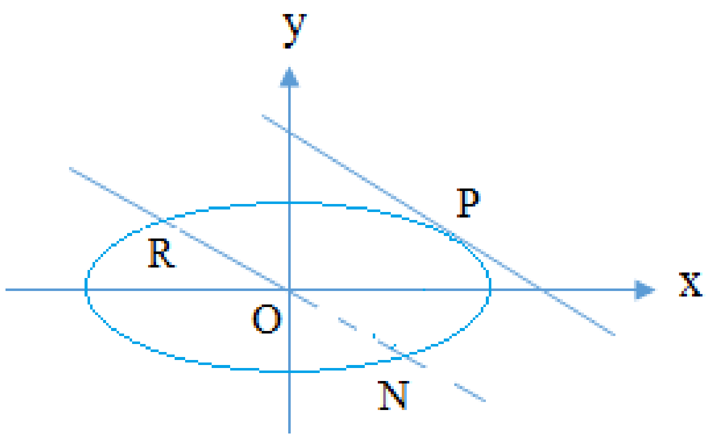
ii) $P(a\sec\theta, b\tan\theta)$ is a point on H_1. Derive the equation of the tangent at P. **2**

iii) The tangent at P to H_1 intersects H_2 at Q and R. The tangents at Q and R to H_2 intersect at T. **2**
Show that T lies on H_1.

End of Question 7

Question 8 (9 marks) Use a SEPARATE writing booklet

(a) $P(a\cos\theta, b\sin\theta)$ is a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. A line through the origin and parallel to the tangent at P, meets the ellipse at R.



i) Find the equation of OR. **2**

ii) Prove that the area of $\Delta\, OPR$ is $\frac{1}{2}ab$ for all position of P. **3**

(b) Let

$$I_n = \int_0^1 x^n\sqrt{1-x^3}\ dx, \quad for\ n \geq 2.$$

(i) Show that $\quad I_n = \frac{2n-4}{2n+5} \times I_{n-3}, \quad for\ n \geq 5.$ **3**

(ii) Hence find I_8 **1**

End of Examination

Student Number

Killara HIGH SCHOOL

Preliminary / HSC Mathematics

Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 10. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

Start Here →

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○

2017-Task3 Solution EXT 2. Marking Scheme.

Q1 **B**

$$3x^2 - 3y^2\frac{dy}{dx} + 3y + 3x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{3x^2+3y}{3y^2-3x}$$

$$= \frac{3+9}{27+3} = \frac{2}{5}$$

Q2 C

$\frac{a}{e} = \frac{25}{4}$, ae = 4 → a = 5 and e = $\frac{4}{5}$

(A) $b^2 = 25(1-\left(\frac{4}{5}\right)^2)$ → b = 3 Equation is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Q3 C

$e = \sqrt{2}$ $S(\sqrt{2k}, 0)$ ∴ $\sqrt{2k} = 4$ → $k = 8$

$a = \sqrt{k}$

Q4 **D**

Equations of asymptotes $y = \pm\frac{b}{a}x$

$$\tan\alpha = \left|\frac{\frac{b}{a} - -\frac{b}{a}}{1+\frac{b}{a}\times-\frac{b}{a}}\right|$$

$$= \left|\frac{\frac{2b}{a}}{1-\frac{b^2}{a^2}}\right| = \left|\frac{2ab}{a^2-b^2}\right|$$

$$= \frac{2ab}{|a^2-b^2|} \quad a \,\&\, b > 0$$

Q5 **D**

$$\int_{-a}^{a} f(x)dx = \int_0^a f(x)dx + \int_{-a}^{0} f(x)dx$$

$\int_0^a f(x-a)dx$ let u = x – a then du = dx

$= \int_{-a}^{0} f(u)du$ x = 0, u = -a and x = -a, u = 0

$= \int_{-a}^{0} f(x)\,dx$

$\int_0^a f(a-x)\,dx$ let u = a - x then du = -dx

$= -\int_a^0 f(u)du$ x = 0, u = a and x = a, u = 0

$= \int_0^a f(x)\,dx$

removed

6a

$$\int \frac{1}{x(\ln x)^2}\,dx$$

let $u = \ln x$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\therefore \int \frac{1}{x} \cdot \frac{1}{(\ln x)^2}\,dx$$

$$= \int \frac{du}{\cancel{dx}} \cdot \frac{1}{u^2} \cdot \cancel{dx}$$

$$= \int \frac{du}{\cancel{dx}} \cdot \frac{1}{u^2} \cdot \cancel{dx}$$

$$= \int u^{-2}\,du$$

$$= \frac{u^{-1}}{-1} = -u^{-1} = \frac{-1}{u}$$

$$= \frac{-1}{\ln x}$$

very well done
although some made the error

$$\int \frac{1}{u^2}\,du = \frac{1}{u^3} + c \quad \times$$ wrong!

b $\int \frac{1}{x^2+6x+10}\,dx$ $= \int \frac{1}{(x^2+6x+9)+1}\,dx$ $= \int \frac{1}{(x+3)^2+1}\,dx$ [1 mark] $= \tan^{-1}(x+3) + C$ [1 mark]	very well done	

c i $\dfrac{3x+1}{(x-1)(x^2+1)} = \dfrac{a}{x-1} + \dfrac{bx+c}{x^2+1}$

with RHS $= \dfrac{a(x^2+1) + (x-1)(bx+c)}{(x-1)(x^2+1)}$

$\therefore 3x+1 = ax^2 + a + bx^2 - bx + cx - c$

$ax^2 + bx^2 = 0x^2$ (equating coefficients of x^2)

$\therefore a + b = 0$ ①

$-bx + cx = 3x$ (equating coefficients of x)

$\therefore -b + c = 3$ ②

$a - c = 1$ ③ (equating constants)

from ① $a = -b$

sub into ② $-b + c = 3 \Rightarrow a + c = 3$ ④

③ + ④ $a + c = 3+$

$a - c = 1$

$2a = 4 \Rightarrow \underline{a = 2}$

very well done although some minor algebraic errors

From ① $a+b=0 \Rightarrow 2+b=0 \Rightarrow \underline{b=-2}$ From ③ $a-c=1 \Rightarrow 2-c=1 \Rightarrow -c=-1 \therefore \underline{c=1}$ $\therefore \dfrac{3x+1}{(x-1)(x^2+1)} = \dfrac{2}{x-1} + \dfrac{-2x+1}{x^2+1}$ [2 marks a,b,c] [1 mark 1 wrong] ii $\therefore \displaystyle\int \frac{3x+1}{(x-1)(x^2+1)}\,dx = \int \frac{2}{x-1} + \frac{-2x+1}{x^2+1}\,dx$ $= \displaystyle\int \frac{2}{x-1}\,dx + \int \frac{-2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$ $= 2\ln(x-1) - \ln(x^2+1) + \tan^{-1}x + c$ [2 marks all correct] [-1 mark for each error]	well done although some sign issues	

d Evaluate $\int_1^{\sqrt{3}} \frac{x^2}{\sqrt{4-x^2}}\,dx$

let $x = 2\sin\theta$.

$\frac{dx}{d\theta} = 2\cos\theta$

terminals $\sqrt{3} = 2\sin\theta$, $\sin\theta = \frac{\sqrt{3}}{2}$, $\therefore \theta = \frac{\pi}{3}$; $1 = 2\sin\theta$, $\sin\theta = \frac{1}{2}$, $\therefore \theta = \frac{\pi}{6}$

$\therefore \int_1^{\sqrt{3}} \frac{x^2}{\sqrt{4-x^2}}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(2\sin\theta)^2}{\sqrt{4-(2\sin\theta)^2}} \cdot 2\cos\theta \cdot d\theta$ [1 mark]

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4\sin^2\theta}{\sqrt{4-4\sin^2\theta}} \cdot 2\cos\theta\, d\theta$

using $\sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = |\cos\theta| = \cos\theta$ in 1st quad

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4\sin^2\theta}{\cancel{2\cos\theta}} \cdot \cancel{2\cos\theta}\, d\theta$

using $\cos 2\theta = 1 - 2\sin^2\theta$, $2\sin^2\theta = 1 - \cos 2\theta$

$= 2\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 - \cos 2\theta\, d\theta$ [1 mark]

Many skipped steps and lost signs / constants

$$= 2\left[\theta - \frac{\sin 2\theta}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \left[2\theta - \sin 2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) - \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

$$= \frac{\pi}{3}$$ [1 mark]

e $\int \cos^5 x \, dx$

$$= \int \cos x . \cos^4 x \, dx$$

$$= \int \cos x . \cos^2 x . \cos^2 x \, dx$$ using $\cos^2 x = 1 - \sin^2 x$

$$= \int \cos x . (1-\sin^2 x)(1-\sin^2 x) \, dx$$

$$= \int \cos x \left(1 - 2\sin^2 x + \sin^4 x\right) dx$$ [1 mark]

$$= \int \cos x - 2\sin^2 x . \cos x + \sin^4 x \cos x \, dx$$

$$= \int \cos x \, dx - \int 2\sin^2 x \cos x \, dx + \int \sin^4 x \cos x \, dx$$ [1 mark]

$$= \sin x - \left(\begin{array}{l}\text{let } u = \sin x \\ \frac{du}{dx} = \cos x\end{array}\right) + \left(\begin{array}{l}\text{let } u = \sin x \\ \frac{du}{dx} = \cos x\end{array}\right)$$

$$= \sin x - 2\int u^2 \, du + \int u^4 \, du$$

$$= \sin x - \frac{2\sin^3 x}{3} + \frac{\sin^5 x}{5} + C$$ [1 mark]

Generally well done
→ standard question
learn the process

f $\quad t = \tan\left(\frac{x}{2}\right)$

$$\int \frac{1}{1+\sin x-\cos x}\,dx$$

$\sin x = \frac{2t}{1+t^2}$

$$= \int \frac{1}{1+\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}}\cdot\left(\frac{2}{1+t^2}\right).dt$$

$\cos x = \frac{1-t^2}{1+t^2}$

$$= \int \frac{1}{\frac{1+t^2}{1+t^2}+\frac{2t}{1+t^2}-\frac{(1-t^2)}{1+t^2}}\cdot\frac{2}{1+t^2}.dt$$

$$= \int \frac{1}{\frac{2t+2t^2}{1+t^2}}\cdot\frac{2}{1+t^2}\,dt$$

$$= \int \frac{\cancel{(1+t^2)}}{\cancel{2}t(1+t)}\cdot\frac{\cancel{2}}{\cancel{(1+t^2)}}.dt$$

$t = \tan\left(\frac{x}{2}\right)$

$\tan^{-1}t = \tan^{-1}\left(\tan\left(\frac{x}{2}\right)\right)$

$\tan^{-1}t = \frac{x}{2}$

$\therefore x = 2\tan^{-1}t$

$\frac{dx}{dt} = 2.\frac{1}{1+t^2}$

$\therefore dx = \frac{2}{1+t^2}.dt$

Generally very well done but those who skipped steps often made mistakes

$$= \int \frac{1}{t(1+t)} \cdot dt \quad \text{[1 mark]} \Longrightarrow \left(\frac{1}{t(1+t)} = \frac{a}{t} + \frac{b}{1+t} \right.$$

$$a(1+t) + bt = 1$$

$$a + at + bt = 1$$

$$a = 1$$

$$at + bt = 0t$$

$$\therefore a + b = 0$$

$$\therefore b = -1$$

$$= \int \frac{1}{t}\,dt + \int \frac{-1}{1+t}\,dt \quad \text{[1 mark]}$$

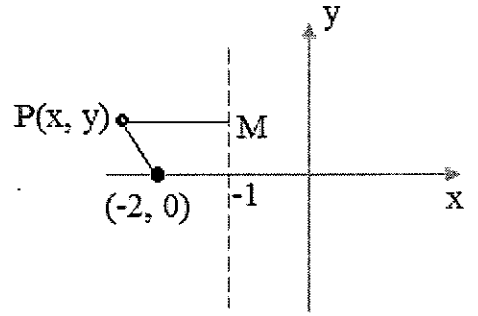
$$= \ln(t) - \ln(1+t) + C$$

$$= \ln\left[\tan\left(\frac{x}{2}\right)\right] - \ln\left[1 + \tan\left(\frac{x}{2}\right)\right] + C$$

[1 mark]

Question 7

(a)
(i)



$\frac{PS}{PM} = \sqrt{2}$

$PS^2 = 2PM^2$

$(x+2)^2 + y^2 = 2(-1-x)^2$
$x^2 + 4x + 4 + y^2 = 2x^2 + 4x + 2$

$x^2 - y^2 = 2$

is the rectangular hyperbola.

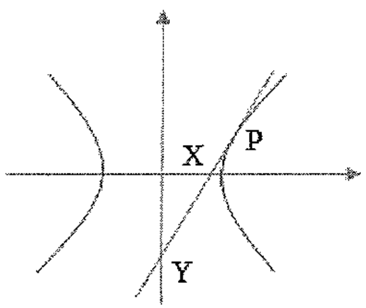
1-relationship between PS & PM

1 - correct equation

(student must derive the result from 1st principle to obtain the mark)

Many students used the formulae ~~for~~ for $S(ae, 0)$ and $x = \frac{a}{e}$ to arrive at the equation.

(ii)



Sub (X,0) into $xx_1 - yy_1 = 2$ $\Rightarrow Xx_1 = 2$

$x_1 = \frac{2}{X}$ **(1)**

Sub (0, Y) into $xx_1 - yy_1 = 2$ $\Rightarrow Yy_1 = 2$

$y_1 = \frac{2}{Y}$ **(2)**

1- coordinates of x_1 and y_1.

Well done!

Since (x_1, y_1) lies on $x^2 - y^2 = 2$
Sub (1) & 2) into the equation

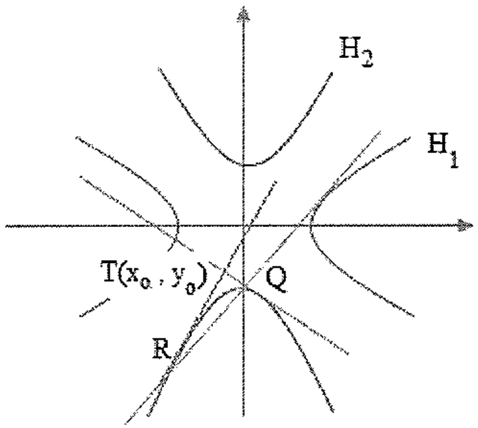
$\Rightarrow \frac{4}{X^2} - \frac{4}{Y^2} = 2$

$4X^2 - 4Y^2 = 2X^2Y^2$

$\therefore \quad 2X^2 - 2Y^2 = X^2Y^2$

1 - correct locus equation.

(b) (i) For the asymptotes of H_1 $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow b^2x^2-a^2y^2=a^2b^2 \quad (\div x^2)$ $\Rightarrow \quad b^2-\frac{a^2y^2}{x^2}=\frac{a^2b^2}{x^2}$ As $x\to\infty$, $\quad \frac{a^2b^2}{x^2}\to 0$ $\therefore \quad \frac{a^2y^2}{x^2}=b^2 \quad \Rightarrow \quad y=\pm\frac{b}{a}x$ You could also do $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow$ $\left\lvert\frac{x}{a}+\frac{y}{b}\right\rvert\left\lvert\frac{x}{a}-\frac{y}{b}\right\rvert=1.$ $\left\lvert\frac{b}{a}x\mp y\right\rvert=\frac{1}{\left\lvert\frac{x}{a}\pm\frac{y}{b}\right\rvert}$ $x\to\infty,\ y\to\infty,\ \pm\frac{bx}{a}\to y.$ $y\to\pm\frac{b}{a}x.$ Similarly the asymptotes of H_2 are $y=\mp\frac{b}{a}x$ Both H_1 & H_2 have the same asymptotes. (ii) $\frac{2x}{a^2}-\frac{2y}{b^2}\frac{dy}{dx}=0 \quad \Rightarrow \quad \frac{dy}{dx}=\frac{b^2x}{a^2y}$ At $(a\sec\theta, b\tan\theta) \quad m=\frac{b\sec\theta}{a\tan\theta}$ Equation of the tangent to H_1 is $y-b\tan\theta=\frac{b\sec\theta}{a\tan\theta}(x-a\sec\theta)$ $\Rightarrow \quad \frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=\sec^2\theta-\tan^2\theta$ $=1 \quad$ (1)	1 – dividing by x^2 2 – correct equation of asymptotes For both H_1 & H_2 with conclusion 1 – finding the gradient 1 – correct equation of tangent	Most students got 1 mark here for making the correct conclusion. Well done.

(iii)  **Not drawn to scale** The equation of the tangent to H_1, $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$, is also the equation of the chord of contact RQ from $T(x_0, y_0)$. The equation of the chord of contact from the point $T(x_0, y_0)$ is $\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1$ (2) $\overrightarrow{(1)} \equiv (2)$ $\frac{\sec\theta}{a} = \frac{x_0}{a^2} \Rightarrow x_0 = a\sec\theta$ $\frac{\tan\theta}{b} = \frac{y_0}{b^2} \Rightarrow y_0 = b\tan\theta$ $\therefore (x_0, y_0)$ lies on H_1.	1 – explanation 1 – showing (x_0, y_0) lies on H_1	Except one students no one got this right. It was important to know right from the beginning RQ is a chord of contact.

Question 8

8 i $I_n = \int_0^1 x^n \sqrt{1-x^3}\, dx$ $= \frac{1}{3}\int_0^1 x^{n-2} \cdot 3x^2 \sqrt{1-x^3}\, dx$ using $\int u\, dv = u.v - \int v\, du$ $= \left[-\frac{1}{3} x^{n-2} \sqrt{(1-x^3)^3}\right]_0^1 + \frac{1}{3} \times \frac{2}{3} \times (n-2) \int_0^1 x^{n-3}(1-x^3)\sqrt{1-x^3}\, dx$ [1 mark] $I_n = \frac{2(n-2)}{9}\int_0^1 x^{n-3}\sqrt{1-x^3}\, dx - \frac{2(n-2)}{9}\int_0^1 x^n . \sqrt{1-x^3}\, dx$ [1 mark] $I_n\left(1 + \frac{2(n-2)}{9}\right) = \frac{2(n-2)}{9} . I_{n-3}$ $I_n\left(\frac{2n+5}{9}\right) = \left(\frac{2n-4}{9}\right) . I_{n-3}$ $\therefore I_n = \frac{2n-4}{2n+5} . I_{n-3}$ [1 mark]	2 marks : All correct. 1 mark: Applying integration by parts and significant progress to arrive at the reduction formula.	

ii

$$I_8 = \frac{2\times8-4}{2\times8+5} \times I_5$$

$$= \frac{12}{21} \times \frac{2\times5-4}{2\times5+5} \cdot I_2$$

$$= \frac{12}{21} \times \frac{6}{15} \cdot I_2$$

$$I_2 = \int_0^1 x^2\sqrt{1-x^3}\,dx \qquad \text{let } u = 1-x^3$$

$$\frac{du}{dx} = -3x^2$$

$$= -\frac{1}{3}\int_0^1 -3x^2\sqrt{1-x^3}\,dx$$

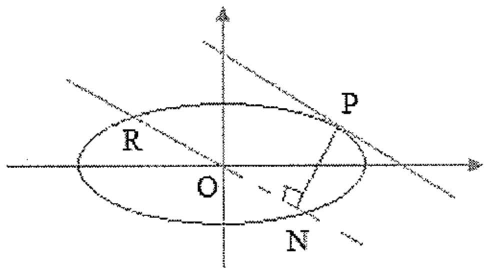
terminals $x=1 \Rightarrow u=0$, $x=0 \Rightarrow u=1$

$$= -\frac{1}{3}\int_1^0 u^{\frac{1}{2}}\frac{du}{dx}\,du$$

$$= -\frac{1}{3}\left[\frac{2u^{3/2}}{3}\right]_1^0 = -\frac{1}{3}\left(0-\frac{2}{3}\right) = \frac{2}{9} \Rightarrow \therefore I_8 = \frac{12}{21}\times\frac{6}{15}\times\frac{2}{9} = \frac{16}{315}$$

1 mark: Correct answer from correct working.

(b)



$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0 \quad \Rightarrow \quad \frac{dy}{dx}=-\frac{b^2x}{a^2y}$$

At $(a\cos\theta, b\sin\theta)$

$$m = \frac{-b^2a\cos\theta}{a^2b\sin\theta} = -\frac{b\cos\theta}{a\sin\theta}$$

This question was very well done.

Equation of OR is $y=-\dfrac{b\cos\theta}{a\sin\theta}x$ **(1)** (ii) Sub **(1)** into $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1 \Rightarrow \dfrac{x^2}{a^2}+\dfrac{b^2\cos^2\theta}{b^2a^2\sin^2\theta}=1 \Rightarrow x=\pm a\sin\theta$ sub into (1) $\Rightarrow y=\pm b\cos\theta$ 1 mark was given for getting these points $OR=\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$ Perpendicular distance PN to the line $\dfrac{b\cos\theta}{a\sin\theta}x+y=0$ is : For trying to find the perpendicular distance 1 mark was awarded. (Significant progress to in Simplifying the expression). $\left\lvert\dfrac{\dfrac{ab\cos^2\theta}{a\sin\theta}+b\sin\theta}{\sqrt{\dfrac{b^2\cos^2\theta}{a^2\sin^2\theta}+1}}\right\rvert=\left\lvert\dfrac{\dfrac{ab\cos^2\theta+ab\sin^2\theta}{a\sin\theta}}{\sqrt{\dfrac{b^2\cos^2\theta+a^2\sin^2\theta}{a^2\sin^2\theta}}}\right\rvert=\dfrac{ab}{\sqrt{a^2\sin^2\theta+b\cos^2\theta}}$ Area $\Delta OPR=\dfrac{1}{2}\times\dfrac{ab}{\sqrt{a^2\sin^2\theta+b\cos^2\theta}}\times\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$ $=\frac{1}{2}ab$ Which is independent of P (for all position of P).	1 – correct gradient of OR 1 – correct equation of OR 1 – correct distance of OR 1- correct perpendicular distance PN 1 – correct area of triangle OPR	