Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

	Centre Number
	Student Number

2018

Year 12 Extension 2 Mathematics task 1

Thursday 13th December 2018

General Instructions

- Reading time – 5 minutes
- Working time – 40 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations

Total Marks: 25

Section I – 17 marks

- Allow about 12 minutes for this section

Section II – 13 marks

- Allow about 13 minutes for this section

Section 1 Complex Numbers	Question 1	/7
	Question 2	/5
Section 2 Polynomial	Question 1	/5
	Question 2	/5
	Question 3	/3
Total		/25

Section 1 (12 marks)

Write each question in a separate booklet. Marks

Question 1

(i) Write $\frac{1+i\sqrt{3}}{1+i}$ in the form $x + iy$, where x and y are real. 2

(ii) Write $\frac{1+i\sqrt{3}}{1+i}$ in modulus-argument form. 2

(iii) Hence find the exact value of $cos\, \frac{\pi}{12}$. 1

(iv) Hence calculate $\left(\frac{1+i\sqrt{3}}{1+i}\right)^{12}$. 2

Question 2

a) On an Argand diagram shade the region containing all points representing complex numbers, z, such that

$$1 \leq |z+1| \leq 2 \quad \text{and} \quad Re(z+1) + Im(z) > 0$$ 3

b) Find the range of possible values of arg z? 2

Section 2 (13 marks)

Write each question in a separate booklet. Marks

Question 1

(i) Show that $(-1 + i\sqrt{3}\ )$ is a zero of $3x^3 + 5x^2 + 10x - 4 = 0$. 3

(ii) Hence, find the other roots. 2

Question 2

(i) The polynomial $P(x) \equiv 3x^4 - 5x^3 - x^2 + 6x + 5$ is divided by $(x^2 + x - 2)$.
Find the remainder. 3

(ii) If $A(x) \equiv 3x^4 - 5x^3 - x^2 + ax + b$ is divisible by $(x^2 + x - 2)$, 2

find the values of a and b.

Question 3

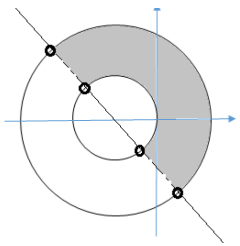
On an Argand diagram, the points A, B and C represent the complex numbers *(1+2i), (5+2i)* and *(3+7i)* respectively. D is the mid-point of AB.

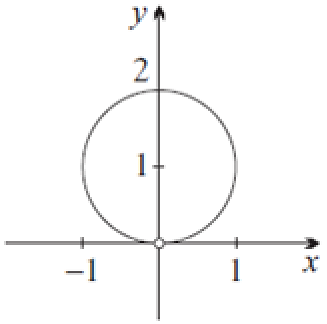
(i) Use vectors to prove that AB is perpendicular to CD. 2

(ii) Hence, find the length of CD. 1

END OF ASSESSMENT

Validation test-solution

<u>Question 1</u> (i) $\frac{(1+i\sqrt{3})(1-i)}{(1+i)(1-i)} = \frac{1-i+i\sqrt{3}+\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2} + \frac{(\sqrt{3}-1)i}{2}$ (1)	2 marks	
(ii) $1+\sqrt{3}i = 2cis\frac{\pi}{3}, \quad 1+i = \sqrt{2}cis\frac{\pi}{4}$ $\rightarrow \frac{1+i\sqrt{3}}{1+i} = \sqrt{2}cis(\frac{\pi}{3}-\frac{\pi}{4})$ $= \sqrt{2}cis\frac{\pi}{12} \quad = \frac{1+\sqrt{3}}{2} + \frac{(\sqrt{3}-1)i}{2}$ from (1)	2 marrks	
(iii) $\therefore \sqrt{2}\cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2} \quad \rightarrow \quad \cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2\sqrt{2}}$		
(iv) $\left(\frac{1+i\sqrt{3}}{1+i}\right)^{12} = \left(\sqrt{2}cis\left(\frac{\pi}{3}-\frac{\pi}{4}\right)\right)^{12}$ $= 2^6 cis\pi = -64$	1 mark	
<u>Question 2</u> a)  $Re(z+1)+Im(z)>0$ is the region $x+1+y>0$	3 marks: 1 for each correct boundary and region. Must have open circle.	
b) Intercepts between $(x+1)^2+y^2=1$ and $y=-(x+1)$ $(-y)^2+y^2=1 \quad \rightarrow \quad y=\pm\frac{1}{\sqrt{2}} \quad and \;\; x=\frac{-1-\sqrt{2}}{\sqrt{2}}, \frac{1-\sqrt{2}}{\sqrt{2}}$ $\therefore$ min. $arg = \tan^{-1}\frac{\frac{1}{\sqrt{2}}}{\frac{-1-\sqrt{2}}{\sqrt{2}}} = -22..5^o = -\frac{7\pi}{8}$ max. $arg = \tan^{-1}\frac{-\frac{1}{\sqrt{2}}}{\frac{1-\sqrt{2}}{\sqrt{2}}} = 67.5^o = -\frac{5\pi}{8}$ $\therefore -\frac{5\pi}{8} < \arg z < \frac{7\pi}{8}$ (2nd region) (3rd region)	2 marks working and correct answers	

Question 3 (i) $w = 1 - \dfrac{2i}{z}$ $= 1 - \dfrac{2i\,\overline{z}}{\lvert z\rvert^2}$ $= \left(1 - \dfrac{2y}{x^2+y^2}\right) - \dfrac{2ix}{x^2+y^2}$ (ii) w is undefined when $z = 0$ (iii) Since w is pure imaginary, $\mathrm{Re}(w) = 0$ so $x^2 + y^2 - 2y = 0$ or $x^2 + (y-1)^2 = 1$  Thus the locus is the unit circle with centre $z = i$, omitting the origin.	2 marks 1 mark 2 marks	