Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

Q	Marks
MC	/2
3	/15
4	/15
5	/13
Total	/45

2020

Year 12

Mathematics Extension 2

Task 3

28/05/2020

General Instructions:

- Reading time – 5 minutes
- Working time – 60 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations

Total Marks: 45

Section I – 2 marks

- Allow about 3 minutes for this section

Section II – 43 marks

- Allow about 57 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 25% of the course.

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Section I

2 marks
Attempt Questions 1-2
Allow about 3 minutes for this section
Use the multiple-choice sheet for Questions 1–2 (Detach from paper)

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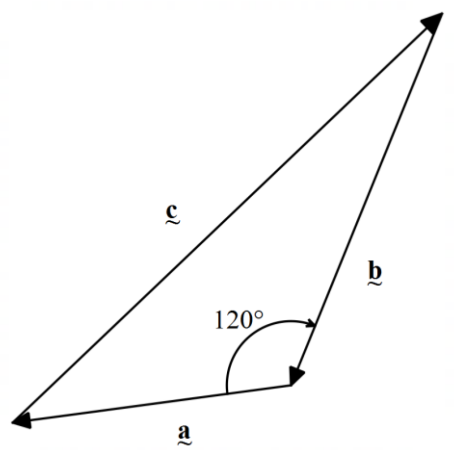
1 The diagram below shows vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$.



From the diagram it follows that:

(A) $\underset{\sim}{a} \cdot \underset{\sim}{b} = \dfrac{|\underset{\sim}{c}|^2 - |\underset{\sim}{a}|^2 - |\underset{\sim}{b}|^2}{2}$

(B) $\underset{\sim}{a} \cdot \underset{\sim}{b} = \dfrac{|\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - |\underset{\sim}{c}|^2}{2}$

(C) $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{c}|^2 - |\underset{\sim}{a}|^2 - |\underset{\sim}{b}|^2$

(D) $\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - |\underset{\sim}{c}|^2$

Section I continues on next page

2 $\int \sin^3\theta\cos^2\theta\,d\theta =$

(A) $\dfrac{\sin^4\theta\cos^3\theta}{12} + C$

(B) $\dfrac{1}{5}\cos^5\theta - \dfrac{1}{3}\cos^3\theta + C$

(C) $\dfrac{1}{3}\cos^3\theta - \dfrac{1}{5}\cos^5\theta + C$

(D) $-\dfrac{\sin^4\theta\cos^3\theta}{12} + C$

End of Section I

Section II

43 marks
Attempt Questions 3-5
Allow about 57 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 3-5, your response should include relevant mathematical reasoning and/or calculations.

Question 3 (15 marks)

(a) Let A, B and C be the points (3, 1, -2), (1, -4, 6) and (2, 0, -4) respectively.

(i) Find $\angle ABC$ to the nearest minute. **2**

(ii) Hence, or otherwise, find the area of the triangle ΔABC to 3 d.p. **1**

(b) Resolve the vector $2\underset{\sim}{i} - 7\underset{\sim}{j} + \underset{\sim}{k}$ into two vector components, one which is parallel to the vector $\underset{\sim}{i} - 4\underset{\sim}{j} - 2\underset{\sim}{k}$ and one which is perpendicular to it. **3**

(c) (i) Show that **2**

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} [f(x) + f(-x)]\, dx$$

(ii) Hence, find **3**

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\sin x}\, dx$$

(d) (i) Express as sum of partial fractions: **2**

$$\frac{7x^2 - 5x + 4}{(x-1)(x^2+1)}$$

(ii) Hence find **2**

$$\int \frac{7x^2 - 5x + 4}{(x-1)(x^2+1)}\, dx$$

End of Question 3

Question 4 (15 marks)

(a) (i) Show that the lines $\underset{\sim}{r}_1 = \begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix} + \lambda_1 \begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$ and $\underset{\sim}{r}_2 = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$ are not skew. **2**

(ii) Find the vector equation of the line that is perpendicular to both $\underset{\sim}{r}_1$ and $\underset{\sim}{r}_2$ that goes through the point of intersection of the two lines. **3**

(b) Evaluate **3**

$$\int_0^{\frac{1}{4}} \frac{x^2 dx}{\sqrt{1-4x^2}}$$

(c) Consider a sphere with equation $\left|\underset{\sim}{r}\right| = 5$ and an identical sphere that has been shifted by the vector $2\underset{\sim}{j}$. Find the circle of intersection of these two spheres in Cartesian form **and** in the form $\underset{\sim}{v} = \underset{\sim}{c} + \underset{\sim}{a}\cos\theta + \underset{\sim}{b}\sin\theta$. **3**

(d) Use the substitution $t = \tan\left(\frac{x}{2}\right)$ to show that **4**

$$\int_0^{\frac{\pi}{3}} \sec x \, dx = \ln\left(2+\sqrt{3}\right)$$

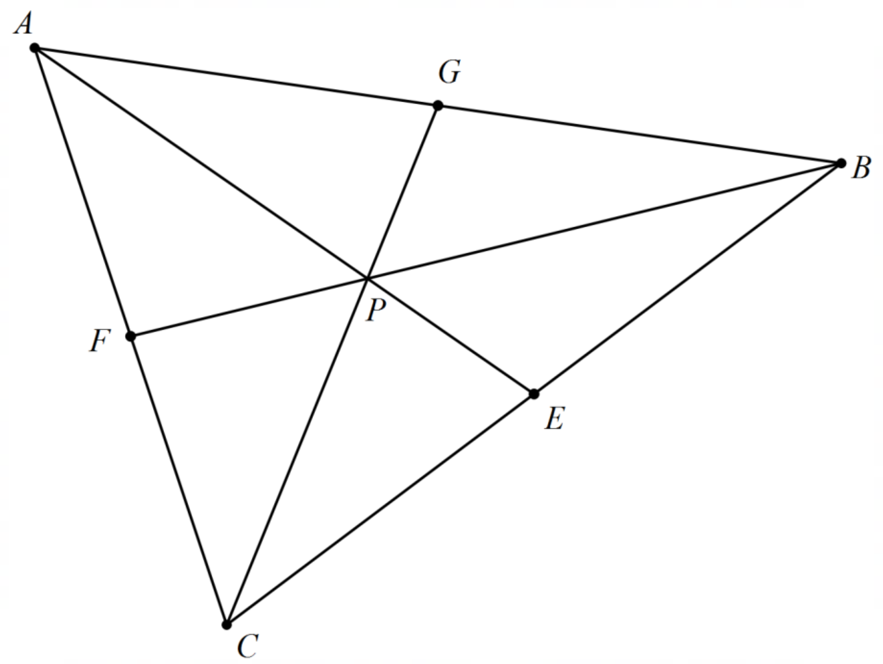
End of Question 4

Question 5 (13 marks)

(a) (i) If point P divides the line segment AB internally in the ratio of $m{:}\,n$, prove that: **1**

$$\overrightarrow{OP} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m+n}$$

(ii) AE, BF and CG are the medians of triangle ABC, where E, F and G are the midpoints of the sides of the triangle. The medians intersect at point P, which is known as the centroid. Using the result from part (i) or otherwise, use vectors to prove that P divides each of the medians in the ratio 2:1. **3**



Question 5 continues on next page

(b) Let:

$$I_n = \int_0^a (a^2 - x^2)^n \, dx, \quad \text{where } n \text{ is an integer and } n \geq 0.$$

(i) Show that **3**

$$I_n = \frac{2a^2 n}{2n+1} \; I_{n-1} \quad \text{for } n \geq 1$$

(ii) Hence, or otherwise, evaluate **1**

$$\int_0^2 (4 - x^2)^3 dx$$

(c) (i) Sketch $y = e^x \sin x$ for the domain $0 \leq x \leq 2\pi$, clearly indicating the x- intercepts **1**

(ii) Let $I = \int_{(k-1)\pi}^{k\pi} e^x \sin x \; dx$, where k is an integer. Show that: **3**

$$2I = (-1)^{k-1} \, e^{k\pi} (1 + e^{-\pi}).$$

(iii) Hence, find the area bounded by $y = e^x \sin x$, the x-axis and the lines $x = 0$ and $x = 2\pi$ **1**

End of Section II and end of test

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Mathematics Extension 2

Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 2. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

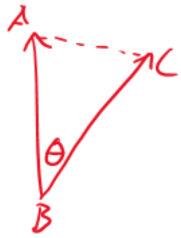
correct

A ~~●~~ B ~~●~~ C ○ D ○

Start Here → **1.** A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

KHS 2020 Ext 2 Task 3 Marking Scheme

1.	A	Tail to tail for dot product
2.	B	
Question 3		
ai)	$\vec{BA} = \underset{\sim}{a} - \underset{\sim}{b}$ $= \begin{pmatrix}3\\1\\-2\end{pmatrix} - \begin{pmatrix}1\\-4\\6\end{pmatrix}$ $= \begin{pmatrix}2\\5\\-8\end{pmatrix}$ $\lvert\vec{BA}\rvert = \sqrt{2^2+5^2+(-8)^2}$ $= \sqrt{93}$  $\vec{BC} = \underset{\sim}{c} - \underset{\sim}{b}$ $= \begin{pmatrix}2\\0\\-4\end{pmatrix} - \begin{pmatrix}1\\-4\\6\end{pmatrix}$ $= \begin{pmatrix}1\\4\\-10\end{pmatrix}$ $\lvert\vec{BC}\rvert = \sqrt{1^2+4^2+(-10)^2}$ $= \sqrt{117}$ 1 mark for $\vec{BA}$ and $\vec{BC}$ $\vec{BA} \cdot \vec{BC} = \lvert\vec{BA}\rvert\lvert\vec{BC}\rvert\cos\theta$ $= 2\times1+5\times4+(-8)\times(-10)$ $= 102$ $\cos\theta = \dfrac{\vec{BA}\cdot\vec{BC}}{\lvert\vec{BA}\rvert\lvert\vec{BC}\rvert}$ $= \dfrac{102}{\sqrt{93}\sqrt{117}}$ $\theta = \cos^{-1}\left(\dfrac{102}{\sqrt{93}\sqrt{117}}\right)$ $= 12^\circ 5'$ 1 mark for θ	A lot of marks lost for confusing $\overrightarrow{AB}$ with $\overrightarrow{BA}$ A lot of marks lost for rounding.
ii)	$A = \frac{1}{2}ab\sin C$ $= \frac{1}{2}\lvert\vec{BA}\rvert\lvert\vec{BC}\rvert\sin\theta$ $= \frac{1}{2}\sqrt{93}\cdot\sqrt{117}\cdot\sin 12^\circ 5'$ $= 10.920\ u^2$ 1 mark for answer	Use exact answer from part (i). Learn to use ANS and STO functions on calculator.
b)	Let $\underset{\sim}{a} = \begin{pmatrix}2\\-7\\1\end{pmatrix}$, $\underset{\sim}{b} = \begin{pmatrix}1\\-4\\-2\end{pmatrix}$ $\underset{\sim}{a}\cdot\underset{\sim}{b} = 2+28-2$ $= 28$ $\lvert\underset{\sim}{b}\rvert^2 = 1^2+(-4)^2+(-2)^2$ $= 21$ $\underset{\sim}{v} = \mathrm{Proj}_{\underset{\sim}{b}}\underset{\sim}{a} = \dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\lvert\underset{\sim}{b}\rvert^2}\underset{\sim}{b}$ 1 mark for applying projection to find λ (or other appropriate method) $= \dfrac{28}{21}\underset{\sim}{b}$ $= \dfrac{4}{3}\begin{pmatrix}1\\-4\\-2\end{pmatrix}$ $= \begin{pmatrix}\frac{4}{3}\\-\frac{16}{3}\\-\frac{8}{3}\end{pmatrix}$ 1 mark Parallel vector for $\underset{\sim}{v}$	Very few realised that vector projection could be used for a parallel vector. Many defaulted to dot product for perpendicular vectors unecessarily.

	$\underset{\sim}{w} = \underset{\sim}{a} - \underset{\sim}{v}$ $= \begin{pmatrix} 2 \\ -7 \\ 1 \end{pmatrix} - \begin{pmatrix} \frac{4}{3} \\ -\frac{16}{3} \\ -\frac{8}{3} \end{pmatrix}$ $= \begin{pmatrix} \frac{2}{3} \\ -\frac{5}{3} \\ \frac{11}{3} \end{pmatrix}$ 1 mark using subtraction for $\underset{\sim}{w}$		
ci)	$\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(x)\,dx + \int_{-a}^{0} f(x)\,dx$ Changing the variable of the second integral; $Let\ x = -u$ $dx = -du$ $When\ x = -a,\ u = a$ $When\ x = 0,\ u = 0$ ❶ $\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(x)\,dx + \int_{a}^{0} f(-u) \times -1\,du$ $= \int_{0}^{a} f(x)\,dx - \int_{a}^{0} f(-u)\,du$ $= \int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(-u)\,du$ ❶ $= \int_{0}^{a} f(x)\,dx + \int_{0}^{a} f(-x)\,dx$ $= \int_{0}^{a} f(x) + f(-x)\,dx$	1 mark: correct change of variables and limits of integration Correct proof	Well done but a few students struggled to complete the solution due to a poor choice of substitution. Some students simply decided to fudge the answer by putting negative at various places with no reasoning or working.
ii)	$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx = \int_{0}^{\frac{\pi}{4}} \frac{1}{1+\sin x} + \frac{1}{1+\sin(-x)}\,dx$ ❶ $= \int_{0}^{\frac{\pi}{4}} \frac{1}{1+\sin x} + \frac{1}{1-\sin x}\,dx$ $= \int_{0}^{\frac{\pi}{4}} \frac{1-\sin x+1+\sin x}{1-\sin^2 x}\,dx$ $= 2\int_{0}^{\frac{\pi}{4}} \frac{1}{\cos^2 x}\,dx$ $= 2\int_{0}^{\frac{\pi}{4}} \sec^2 x\,dx$ ❶ $= 2[\tan x]_{0}^{\frac{\pi}{4}}$ $= 2[1-0]$ $= 2$ ❶	3 mark: applies the result from (a) and evaluates correctly 2 mark: applies the result from (a) and minor error in working 1 mark: applies the result from (a)	Must show the application of result from (i) $= \int_{0}^{\frac{\pi}{4}} \frac{1}{1+\sin x} + \frac{1}{1-\sin x}\,dx$ This cannot be the first step. Most students successfully got to $2\int_{0}^{\frac{\pi}{4}} \frac{1}{\cos^2 x}\,dx$ but then some chose to use the t-results to proceed from there. This resulted in a much longer solution with many students making errors. Students are reminded that 2-unit results are still relevant in the Extension 2 exam. Lack of understanding of basic integration was evident in many cases.

<table>
<tr>
<td></td>
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<td>Unfortunate observations:

1. students not putting ‘dx’ in the integral notation.
2. Not recognising $1-\sin^2 x$ as $\cos^2 x$
3. Not knowing $\frac{1}{\cos^2 x} = \sec^2 x$
4. Above all $\int \frac{1}{\cos^2 x}\,dx = \int \frac{2}{1+cos2x}\,dx = \frac{2}{x+\frac{sin2x}{2}}$

As Ext 2 students, we expect you to have understanding of the fundamentals. The above examples do not demonstrate this.</td>
</tr>
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<td>di)</td>
<td>$\frac{7x^2-5x+4}{(x-1)(x^2+1)} \equiv \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$
$\therefore 7x^2-5x+4 = A(x^2+1)+(Bx+C)(x-1)$
When $x=i$,   When $x=1$,
$-3-5i=(Bi+C)(i-1)$ $6=2A$
$-3-5i=-B-C+(-B+C)i$ $A=3$ ❶
Equating real and imaginary parts
$B+C=3$...①
$B-C=5$...②
①+② $2B=8$
$B=4$
①−② $2C=-2$
$C=-1$

$\therefore \frac{7x^2-5x+4}{(x-1)(x^2+1)} \equiv \frac{3}{x-1} + \frac{4x-1}{x^2+1}$ ❶</td>
<td>1 mark: Expresses in the rational form and correctly evaluates one of the constants

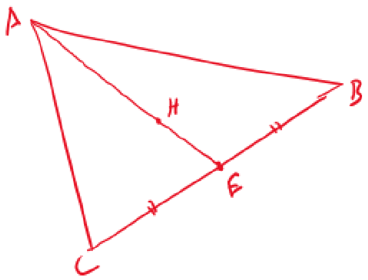
1 mark: correctly evaluates all the constants and gives the correct partial fraction form</td>
<td>Generally well done, except minor error in calculations.</td>
</tr>
<tr>
<td>ii)</td>
<td>$\int \frac{7x^2-5x+4}{(x-1)(x^2+1)}dx$
$=\int \frac{3}{x-1} + \frac{4x-1}{x^2+1}dx$
$=3\int \frac{1}{x-1}\,dx + 2\int \frac{2x}{x^2+1}\,dx - \int \frac{1}{x^2+1}\,dx$ ❶
$=3\ln\lvert x-1\rvert + 2\ln\lvert x^2+1\rvert - \tan^{-1}x + c$ ❶</td>
<td>2 marks: correct integration

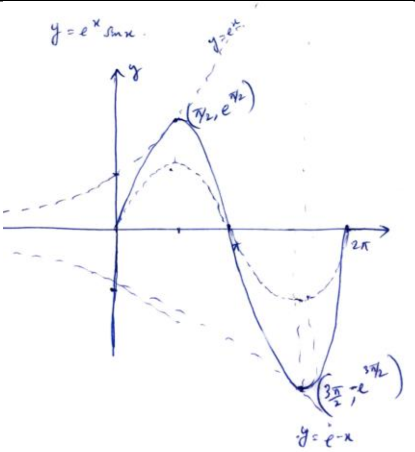
1 mark: Minor error</td>
<td>Done well</td>
</tr>
</table>

Question 4		
ai)	Point of intersection occurs where: $\underset{\sim}{r_1} = \underset{\sim}{r_2}$ $\begin{pmatrix}-1\\-3\\2\end{pmatrix} + \lambda_1\begin{pmatrix}-3\\2\\2\end{pmatrix} = \begin{pmatrix}4\\-1\\0\end{pmatrix} + \lambda_2\begin{pmatrix}2\\-4\\-2\end{pmatrix}$ $\begin{pmatrix}-1\\-3\\2\end{pmatrix} - \begin{pmatrix}4\\-1\\0\end{pmatrix} = \lambda_2\begin{pmatrix}2\\-4\\-2\end{pmatrix} - \lambda_1\begin{pmatrix}-3\\2\\2\end{pmatrix}$ $\begin{pmatrix}-5\\-2\\2\end{pmatrix} = \begin{pmatrix}2\lambda_2+3\lambda_1\\-4\lambda_2-2\lambda_1\\-2\lambda_2-2\lambda_1\end{pmatrix} \Longrightarrow$ $2\lambda_2+3\lambda_1=-5$ (1), $-4\lambda_2-2\lambda_1=-2$ (2), $-2\lambda_2-2\lambda_1=2$ (3) (3) - (2) $2\lambda_2 = 4$ $\boxed{\lambda_2 = 2}$ (4) Sub (4) in (1) $4 + 3\lambda_1 = -5$ $\lambda_1 = -\frac{9}{3}$ $\boxed{\lambda = -3}$ 1 mark for $\lambda_1 = -3$ and $\lambda = 2$ For $\underset{\sim}{r_1}$: $\begin{pmatrix}-1\\-3\\2\end{pmatrix} - 3\begin{pmatrix}-3\\2\\2\end{pmatrix} = \begin{pmatrix}8\\-9\\-4\end{pmatrix}$ For $\underset{\sim}{r_2}$: $\begin{pmatrix}4\\-1\\0\end{pmatrix} + 2\begin{pmatrix}2\\-4\\-2\end{pmatrix} = \begin{pmatrix}8\\-9\\-4\end{pmatrix}$ $\therefore$ $(8,-9,-4)$ is the point of intersection of $\underset{\sim}{r_1}$ and $\underset{\sim}{r_2}$ $\therefore$ The lines are not skew 1 mark for finding POI OR showing that λ values were consistent for $\underset{\sim}{r_1} = \underset{\sim}{r_2}$	Fairly well done. A bit of time was lost with long algebraic methods showing the lines as not parallel. If the lines are not skew, this implies a single POI, which in turn implies not parallel lines. Otherwise, simply show direction vectors are not scalar multiples. Some confused scalar multiple direction vectors with scalar multiple vector line equations.
ii)	Let $\underset{\sim}{d} = \begin{pmatrix}a\\b\\c\end{pmatrix}$ be the direction vector of the line $\begin{pmatrix}a\\b\\c\end{pmatrix}\cdot\begin{pmatrix}-3\\2\\2\end{pmatrix} = 0$ and $\begin{pmatrix}a\\b\\c\end{pmatrix}\cdot\begin{pmatrix}2\\-4\\-2\end{pmatrix} = 0$ 1 mark for applying dot product $-3a+2b+2c=0$ (1) $\quad 2a-4b-2c=0$ (2) (1) + (2) $-a-2b=0$ $-a=2b$ (3) Sub (3) in (1) $-3a-a+2c=0$ $2c=4a$ $c=2a$ $\therefore$ Let $a=2 \Rightarrow \underset{\sim}{d} = \begin{pmatrix}2\\-1\\4\end{pmatrix}$ 1 mark for valid direction vector $\therefore$ The line $\underset{\sim}{r_3} = \begin{pmatrix}8\\-9\\-4\end{pmatrix} + \lambda_3\begin{pmatrix}2\\-1\\4\end{pmatrix}$ 1 mark for equivalent vector equation	Many started to apply dot product but did not follow through. Some found the proportions of a,b and c but did not know what to do with them.

b)	$I=\int_0^{\frac{1}{4}}\frac{x^2dx}{\sqrt{1-4x^2}}$ $Let\ 2x=\sin\theta$ $x=\frac{1}{2}\sin\theta$ $dx=\frac{1}{2}\cos\theta\ d\theta$ $When\ x=\frac{1}{4},\ \theta=\frac{\pi}{6}$ $When\ x=0,\ \theta=0.$ ❶ $I=\int_0^{\frac{\pi}{6}}\frac{\frac{1}{4}\sin^2\theta}{\sqrt{1-\sin^2\theta}}\times\frac{1}{2}\cos\theta\ d\theta$ $=\frac{1}{8}\int_0^{\frac{\pi}{6}}\sin^2\theta\ d\theta$ $=\frac{1}{16}\int_0^{\frac{\pi}{6}}1-\cos 2\theta\ d\theta$ $=\frac{1}{16}\left[\theta-\frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{6}}$ ❶ $=\frac{1}{16}\left[\frac{\pi}{6}-\frac{1}{2}\sin\frac{\pi}{3}-\left(0-\frac{1}{2}\sin 0\right)\right]$ $=\frac{1}{16}\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)$ $=\frac{1}{32}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$ ❶	3 marks: correct answer from correct working 2 marks: chooses an appropriate substitution, expresses in an easier integrable form and attempts to integrate and evaluate 1 mark: chooses an appropriate substitution, attempts to express in an easier integrable form and attempts to integrate and evaluate	Most students got 2 marks for this question, with some error in the evaluation. Students who used the substitution were more successful than the ones who used separation of the numerator.
	Method 2 $I = -\frac{1}{4}\int_0^{\frac{1}{4}}\frac{-4x^2}{\sqrt{1-4x^2}}\,dx$ $= -\frac{1}{4}\int_0^{1/4}\frac{1-4x^2-1}{\sqrt{1-4x^2}}\,dx$ $= -\frac{1}{4}\int_0^{1/4}\sqrt{1-4x^2}\,dx + \frac{1}{4}\int\frac{1}{\sqrt{1-4x^2}}\,dx$ $-\frac{1}{4}\left[\sqrt{1-4x^2}\cdot x\right]_0^{1/4} + \frac{1}{4}\int\frac{x\cdot x\cdot -8x}{2\sqrt{1-4x^2}}\,dx$ $+\frac{1}{8}\int\frac{1}{\sqrt{\frac{1}{4}-x^2}}\,dx.$ $= -\frac{1}{4}\left[\frac{1}{4}\cdot\frac{\sqrt{3}}{2} - I + \frac{1}{8}\left[\sin^{-1}2x\right]_0^{1/4}\right.$ $2I = \frac{-\sqrt{3}}{32} + \frac{1}{8}\times\frac{\pi}{6}.$ $I = \frac{\pi}{96} - \frac{\sqrt{3}}{64}.$	All correct – 3 marks Separates the variables, applies integration by parts and attempts to evaluate correctly – 2 mark Separates the variables and does integration by parts – 1 mark	
c)	$\lvert \underset{\sim}{r}\rvert = 5$ $\quad\lvert \underset{\sim}{r}-2\underset{\sim}{j}\rvert = 5$ $\lvert \underset{\sim}{r}\rvert = \lvert \underset{\sim}{r}-2\underset{\sim}{j}\rvert$		Very poorly done. Students need to be more familiar with vector, Cartesian and parametric

	$x^2+y^2+z^2 = x^2+(y-2)^2+z^2$ $y^2 = y^2-4y+4$ $4y=4$ $y=1 \quad (1)$ $x^2+y^2+z^2=5^2$ $x^2+z^2=24$ 1 mark for $x^2+z^2=24$ Centre at $(0,1,0)$ $\underset{\sim}{c} = \begin{pmatrix}0\\1\\0\end{pmatrix}$ 1 mark for $y=1$ or $\underset{\sim}{c}=\begin{pmatrix}0\\1\\0\end{pmatrix}$ or centre at $(0,1,0)$ $\lvert \underset{\sim}{a}\rvert = \lvert \underset{\sim}{b}\rvert = r$ and $\underset{\sim}{a}\cdot\underset{\sim}{b}=0$ $=2\sqrt{6}$ Choose $\underset{\sim}{a} = \begin{pmatrix}2\sqrt{6}\\0\\0\end{pmatrix}$ $\underset{\sim}{b} = \begin{pmatrix}0\\0\\2\sqrt{6}\end{pmatrix}$ $\therefore$ Circle: $\underset{\sim}{v} = \begin{pmatrix}0\\1\\0\end{pmatrix}+\begin{pmatrix}2\sqrt{6}\\0\\0\end{pmatrix}\cos\theta+\begin{pmatrix}0\\0\\2\sqrt{6}\end{pmatrix}\sin\theta$ 1 mark for appropriate $\underset{\sim}{a}$ and $\underset{\sim}{b}$		equations. Many did not seem to understand the concept of the question and did not realise that $y=1$ referred to the plane of intersection. Many did not even attempt to find the centre.
d)	$I = \int_0^{\frac{\pi}{3}} secx\, dx$ $t = \tan\frac{x}{2}$ $dt = \frac{1}{2}\sec^2\frac{x}{2}$ $= \frac{1}{2}(1+\tan^2\frac{x}{2})\, dx$ $dx = \frac{2}{1+t^2}dt$ $\sec x = \frac{1}{cosx} = \frac{1+t^2}{1-t^2}$ When $x=0, t=0$ $x = = \frac{\pi}{3},\ t=\frac{1}{\sqrt{3}}$ $I = \int_0^{\frac{1}{\sqrt{3}}}\frac{1+t^2}{1-t^2}\,\frac{2}{1+t^2}dt$ $= \int_0^{\frac{1}{\sqrt{3}}}\frac{2dt}{1-t^2}dt$ $= \int_0^{\frac{1}{\sqrt{3}}}\frac{1}{1-t}+\frac{1}{1+t}dt$ $= \left[\ln\left(\frac{1-t}{1+t}\right)\right]_0^{\frac{1}{\sqrt{3}}}$ $= \ln\left(\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}\right) - ln1$ $=\ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ Rationalise, $=\ln\left(\frac{3+2\sqrt{3}+1}{3-1}\right)$ $=\ln(2+\sqrt{3})$	1 mark: proves the correct expression for dx 1 mark: converts the integrand in the t form 1 mark: expresses as partial fractions 1 mark: evaluates and proves the result	This question was mostly well done. It was answered using a range of methods. Some student's used inefficient approaches that involved making two substitutions. Experience through greater practice will develop your understanding of the best substitution to make.

Question 5			
ai)	a) $\vec{AP}:\vec{PB} = m:n$ $\therefore n\vec{AP} = m\vec{PB}$ $n(\vec{OP}-\vec{OA}) = m(\vec{OB}-\vec{OP})$ $n\vec{OP} - n\vec{OA} = m\vec{OB} - m\vec{OP}$ $(m+n)\vec{OP} = m\vec{OB}+n\vec{OA}$ $\vec{OP} = \frac{m\vec{OB}+n\vec{OA}}{m+n}$	1 mark	Poorly done for what should be a routine proof.
ii)	Let H be the point that divides AE in the ratio 2:1 i.e. AH:HE = 2:1 $\vec{OH} = \frac{2\vec{OE}+\vec{OA}}{3}$ (from part (a)) $= \frac{2\underset{\sim}{e}+\underset{\sim}{a}}{3}$ Now $\vec{OE} = \frac{\vec{OB}+\vec{OC}}{2}$ $\underset{\sim}{e} = \frac{\underset{\sim}{b}+\underset{\sim}{c}}{2}$ $\therefore \vec{OH} = \frac{2\frac{(\underset{\sim}{b}+\underset{\sim}{c})}{2}+\underset{\sim}{a}}{3}$ $\underset{\sim}{h} = \frac{\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}}{3}$  Similarly, if R and S are the points that divide BF and CG respectively in the ratio of 2:1, it can be shown that $\underset{\sim}{r} = \frac{\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}}{3}$ and $\underset{\sim}{s} = \frac{\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}}{3}$ $\therefore$ H, R and S are the same point which is also the intersection of the medians at P. Hence, P divides all 3 medians in the ratio 2:1.	1 mark for successfully applying part (a) 1 mark for applying midpoint 1 mark for completed proof	Many did not attempt this question.
bi)	$I_n = \int_0^a (a^2-x^2)^n dx$ $=\int_0^a (a^2-x^2)^n .1dx$ $= [(a^2-x^2)^n .x]_0^a - \int_0^a n(a^2-x^2)^{n-1}.-2x.x\,dx$ $= 0 + 2n\int_0^a (a^2-x^2)^{n-1}x^2\,dx$ $= 2n\int_0^a (a^2-x^2)^{n-1}(a^2-(a^2-x^2))dx$ $= 2na^2\int_0^a (a^2-x^2)^{n-1}dx - 2n\int_0^a (a^2-x^2)^n dx$ $I_n = 2na^2 I_{n-1} - 2n\,I_n$ $(2n+1)I_n = 2na^2 I_{n-1}$ $I_n = \frac{2na^2}{2n+1}I_{n-1}$	Performs integration by part 2 mark: Manipulates and separates the terms and gives final result correctly 1 mark: minor error	Separation of variables must be shown.
ii)	$I_3 = \int_0^2 (4-x^2)^3 dx$ $I_0 = \int_0^2 (4-x^2)^0 dx$	1 mark: Applies the result from (i) and evaluates	Many errors

	$= \int_0^2 dx$ $= [x]_0^2 = 2$ $I_3 = \dfrac{2\times 3\times 2^2}{1+2\times 3} I_2$ $= \frac{24}{7} I_2$ $= \frac{24}{7}\times\frac{16}{5} I_1$ $= \frac{24}{7}\times\frac{16}{5}\times\frac{8}{3} I_0$ $= \frac{24}{7}\times\frac{16}{5}\times\frac{8}{3}\times 2$ $= \frac{2048}{35}$		
ci)		1 mark: Shape with x- intercepts and shape correct (may not be in the correct proportion)	
ii)	$I = \int_{(k-1)\pi}^{k\pi} e^x \sin x\, dx$ $= [-e^x cosx]_{(k-1)\pi}^{k\pi} + \int_{(k-1)\pi}^{k\pi} e^x \cos x\, dx$ $= [-e^x cosx]_{(k-1)\pi}^{k\pi} + [e^x sinx]_{(k-1)\pi}^{k\pi} - \int_{(k-1)\pi}^{k\pi} e^x \sin x\, dx$ **1 mark** $I = [e^x(sinx - cosx)]_{(k-1)\pi}^{k\pi} - I$ $2I = [e^x(sinx - cosx)]_{(k-1)\pi}^{k\pi}$ $= [e^{k\pi}(sink\pi - cosk\pi)] - [e^{(k-1)\pi}(\sin(k-1)\pi - \cos(k-1)\pi)]$ for integer k. $sink\pi = 0, \quad \sin(k-1)\pi = 0$ $2I = -e^{k\pi}cosk\pi + e^{(k-1)\pi}\cos(k-1)\pi$ **1 mark** When k is even, $cosk\pi = 1$; When k is odd, $\cos k\pi = -1$ $\therefore cosk\pi = (-1)^k$ Similarly, $\cos(k-1)\pi = (-1)^{k-1}$ $\therefore 2I = (-1)^k(-e^{k\pi}) + (-1)^{k-1}(-e^{(k-1)\pi})$ $= (-1)^{k-1}(e^{k\pi}) + (-1)^{k-1}e^{k\pi}\times e^{-\pi}$ $= (-1)^{k-1}e^{k\pi}(1+e^{-\pi})$ **1 mark**	1 mark: applies integration by part 1 mark: gives the general solution here 1 mark: Uses the odd and even value of k, and proves the result	Students who got to this point, in general received 2 out of 3 Please note how the $\cos k\pi$ changes in value. Must explain this.
iii)	Required area = $\int_0^{\pi} e^x sinx\, dx +$ $\left\lvert \int_{\pi}^{2\pi} e^x sinx\, dx \right\rvert$ $= \frac{1}{2}e^{\pi}(1+e^{-\pi}) + \left\lvert -\frac{1}{2}e^{2\pi}(1+e^{-\pi}) \right\rvert$ 1 mark $\dfrac{e^{\pi}(1+e^{-\pi})(1+e^{\pi})}{2}$ $= \dfrac{(e^{\pi}+1)(e^{\pi}+1)}{2}$ $= \dfrac{(e^{\pi}+1)^2}{2}$	1 mark: correct answer from correct working 1 mark: minor in calculations	Seldom attempted