Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

Q	Marks
1	/14
2	/16
3	/14
Total	/44

2022

Year 12

Mathematics Extension 2

Task 2
1/4/22

General Instructions:

- Reading time – 5 minutes
- Working time – 60 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations
- Answer all questions in the booklets provided

Total Marks: 44

This question paper must not be removed from the examination room.

This assessment task constitutes 25% of the course.

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In Questions 1 – 3, your responses should include relevant mathematical reasoning and/or calculations.

Question 1: Answer Question 1 in a new booklet. (14 Marks)

(a) (i) Express $1 + i$ and $\sqrt{3} + i$ in **exponential** AND **mod-arg** form. **2**

(ii) Hence, or otherwise, find the values of n such that $\left(\frac{1+i}{\sqrt{3}+i}\right)^n$ is purely imaginary. **2**

(b) Consider the statement "If my pet has 4 legs, then it is a dog." **2**

Write the converse, inverse and contrapositive of this statement.

(c) (i) Find the roots of the equation $z^6 + z^5 + z^4 + z^3 + z^2 + z + 1 = 0$ in mod-arg form. **2**

(ii) Hence, find the exact value of $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$ **2**

(d) (i) Solve $z^3 + 8i = 0$ expressing the roots in the form $re^{i\theta}$ and $a + ib$. **2**

Let ω be a root of $z^3 + 8i = 0$

(ii) Find all possible values of ω^{49} and sketch them on an argand diagram. **2**

Question 2: Answer Question 2 in a new booklet. (16 Marks)

(a) Prove that if p^3 is divisible by 2, then p is divisible by 2. **1**

(b) Prove $\sqrt[3]{2}$ is irrational. **2**

(c) Use the method of proof by contradiction to prove that the cube root of any prime number $p \geq 2$ is irrational. **3**

(d) Prove or disprove that if a and b are rational, then a^b is also rational. **2**

(e) If x and y are positive integers with $x > y$, prove that $6(x+y)^2 - 2(x-y)^2$ is divisible by four. **1**

(f) Prove that $\sqrt[8]{8!} < \sqrt[9]{9!}$ (where, for any integer n, $n!$ is the product of the integers up to and including n, that is, $1 \times 2 \times 3 \times \ \ldots \times n$). **2**

(g) The triangular inequality for complex numbers z and w is shown below:

$$\big||z| - |w|\big| \leq |z - w| \leq |z| + |w|$$

(i) Hence, prove that $\big||z| - |w|\big| \leq |z + w|$ **1**

(ii) It is given that $PQRS$ is a quadrilateral with sides $PQ = 15$, $QR = 19$, $RS = 32$ and $PT = 25$. Find the possible lengths of the diagonal QS. **2**

(h) For a complex number z it is given that $|z - 3| - |z + 1| = 6$.
Find the minimum value of $|z - 1|$. **2**

Question 3: Answer Question 3 in a new booklet. (14 Marks)

(a) Prove that between any two rational numbers, there lies an irrational number. **2**

(b) The formula for the nth term of the Fibonacci sequence, **3**

$$1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

Is given by,

$$a_n = \begin{cases} 1 \text{ for } n = 1 \text{ and } 2 \\ a_{n-2} + a_{n-1} \text{ for } n > 2 \end{cases}$$

Prove by mathematical induction that,

$$a_n = \frac{\left(1+\sqrt{5}\right)^n - \left(1-\sqrt{5}\right)^n}{\sqrt{5} \times 2^n}$$

(c) (i) Given that **2**

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} \ldots$$

show that

$$\frac{\sin\theta}{\theta} = 1 - \frac{\theta^2}{3!} + \frac{\theta^4}{5!} - \frac{\theta^6}{7!} + \frac{\theta^8}{9!} \ldots$$

(ii) Show that the RHS of the result in part (i) can be factorised as: **2**

$$\left(1 - \frac{\theta^2}{\pi^2}\right)\left(1 - \frac{\theta^2}{4\pi^2}\right)\left(1 - \frac{\theta^2}{9\pi^2}\right)\left(1 - \frac{\theta^2}{16\pi^2}\right) \ldots$$

(iii) Hence, evaluate **1**

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \ldots$$

(d) (i) Show that $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$ **1**

(ii) Solve for $x \in \mathbb{C}$: **3**

$$\sin x = 2$$

End of Examination

2021/2022 Ext 2 Task 2 – Solutions and Feedback

Question 1

Q	Solutions	Marking Scheme	Feedback
(ai)	$1 + i = \sqrt{2}cis\left(\frac{\pi}{4}\right) = \sqrt{2}e^{\frac{i\pi}{4}}$ $\sqrt{3} + i = 2cis\left(\frac{\pi}{6}\right) = 2e^{\frac{i\pi}{6}}$	1 mark for exponential form 1 mark for mod-arg form	Well done.
(ii)	$\left(\frac{1+i}{\sqrt{3}+i}\right)^n = \left(\frac{\sqrt{2}e^{\frac{i\pi}{4}}}{2e^{\frac{i\pi}{6}}}\right)^n$ $= \left(\frac{1}{\sqrt{2}}e^{\frac{i\pi}{12}}\right)^n$ $= 2^{-\frac{n}{2}}e^{\frac{ni\pi}{12}}$ For purely imaginary $\cos\left(\frac{n\pi}{12}\right) = 0$ $\frac{n\pi}{12} = \frac{\pi}{2} + k\pi$ for some integer k $n = 12k + 6, k = 0, \pm 1, \pm 2, etc$	1 mark for simplifying $\left(\frac{1+i}{\sqrt{3}+i}\right)^n$ 1 mark for finding all possible solutions	1st mark was well achieved. Many had issue with general solutions to $\cos\theta = 0$ Some ignored negative values of k which was not justified.
(b)	"If my pet has 4 legs, then it is a dog." Converse: "If my pet is a dog, then it has 4 legs." Contrapositive: "If my pet is not a dog, then it doesn't have 4 legs." Inverse: "If my pet doesn't have 4 legs, then it is not a dog."	1 mark for one statement correct 2 marks for all three correct	Largely well done, some confusion on the converse.
(ci)	$z^6 + z^5 + z^4 + z^3 + z^2 + z + 1 = \frac{z^7 - 1}{z - 1}$ Hence the roots of $z^6 + z^5 + z^4 + z^3 + z^2 + z + 1 = 0$ are the same as $z^7 - 1 = 0$ excepting $z = 1$ $\therefore$The roots are $cis\left(\pm\frac{2\pi}{7}\right)$, $cis(\pm\frac{4\pi}{7})$ and $cis(\pm\frac{6\pi}{7})$	1 mark for recognising connection between the polynomial and $z^7 - 1$, or equivalent working. 1 mark for giving roots	Best responses recognised the polynomial as $\frac{z^7-1}{z-1}$ Some students attempted to divide both sides by z^3 with mixed results. This method is valid, although slower. There was no need to do long forms of working to find the seventh roots of unity, this should be easily recognised. Also, please be careful with giving your responses in principle argument only.

<table>
<tr>
<td>(ii)</td>
<td>By sum of roots
$$cis\left(\frac{2\pi}{7}\right)+cis\left(\frac{-2\pi}{7}\right)+cis\left(\frac{4\pi}{7}\right)+cis\left(\frac{-4\pi}{7}\right)+cis\left(\frac{6\pi}{7}\right)+cis\left(\frac{-6\pi}{7}\right)=-1$$
$$2\cos\left(\frac{2\pi}{7}\right)+2\cos\left(\frac{4\pi}{7}\right)+2\cos\left(\frac{6\pi}{7}\right)=-1$$
$$\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}=-\frac{1}{2}$$</td>
<td>1 mark for using sum of roots

1 mark for correct working to arrive at result</td>
<td>Mostly well done. Some minor errors in working. ECF allowed.</td>
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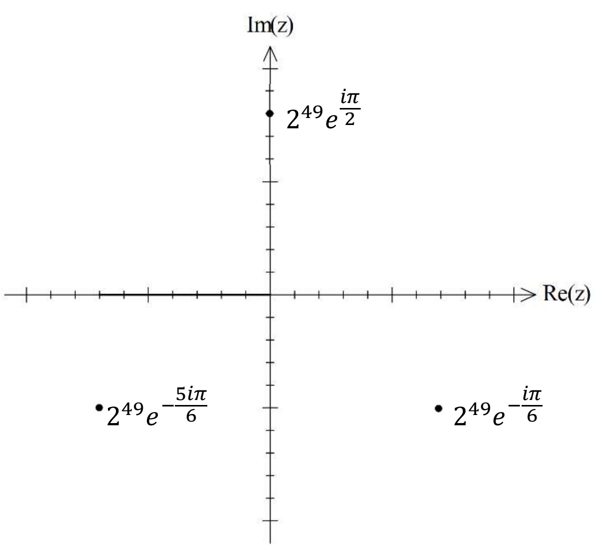
<tr>
<td>(di)</td>
<td>$$z^3=-8i=8e^{-\frac{i\pi}{2}}=r^3e^{3i\theta}$$
$$r=2$$
$$3\theta=-\frac{\pi}{2}+2k\pi$$
$$\theta=\frac{4k-1}{6}\pi$$
∴Roots are $z=2e^{-\frac{i\pi}{6}},2e^{\frac{i\pi}{2}}$ and $2e^{-\frac{5i\pi}{6}}$
$z=\sqrt{3}-i,2i$ and $-\sqrt{3}-i$</td>
<td>1 mark for modulus

1 mark for arguments</td>
<td>Too many took modulus as -2. Modulus must be $+ve$.

Many failed to recognise that $-8i$ has an argument of $-\frac{\pi}{2}$ and hence, $3\theta=-\frac{\pi}{2}+2k\pi$. These students may wish to review the geometry of complex roots (i.e. go look at the spirals)</td>
</tr>
<tr>
<td>(ii)</td>
<td>$$\omega^{48}=(\omega^3)^{16}$$
$$=(-8i)^{16}$$
$$=2^{48}$$
$$\omega^{49}=\omega^{48}\times\omega$$
$$=2^{48}\times\omega$$
$$=2^{49}e^{-\frac{i\pi}{6}},2^{49}e^{\frac{i\pi}{2}}\text{ or }2^{49}e^{-\frac{5i\pi}{6}}$$

</td>
<td>1 mark for sketching the points of an equilateral triangle (must label angles)

1 mark for recognising and appropriately labelling the enlargement factor of 2^{48}</td>
<td>Many seemed to be intimidated by this question and some did not attempt it.

Students should recognise this geometrically as an enlargement of the three roots from part (i) and hence, all that was required was a simple diagram of three evenly spaced points.</td>
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Question 2

Q	Solutions	Marking Scheme	Feedback
(a)	Consider the contrapositive. Assume p isn't divisible by 2, that is, p is odd. So $p = 2k + 1, k \in \mathbb{Z}$ $p^3 = (2k+1)^3$ $= 8k^3 + 3(4k^2) + 3(2k) + 1$ $= 2(4k^3 + 6k^2 + 3h) + 1$ $= 2q + 1, q \in \mathbb{Z}$ $\therefore p^2 + 2$ (p^3 is odd) By contrapositive, if p^3 is divisible by 2, then p is divisible by 2.	1 mark for valid proof with working	Largely well done, however too many students misspoke and called the contrapositive a contradiction. This will cost marks.
(b)	Assume $\sqrt[3]{2}$ is rational. That is, $\sqrt[3]{2} = \frac{p}{q}$ for some $p, q \in \mathbb{Z}, q \neq 0$ and p and q are co-prime. $2 = \frac{p^3}{q^3}$ $2q^3 = p^3$ $\therefore p^3$ is even $\therefore p$ is even, i.e. $p = 2k, k \in \mathbb{Z}$ So, $2q^3 = (2k)^3$ $= 8k^3$ $q^3 = 4k^3$ $\therefore q^3$ is even $\therefore q$ is even This is a contradiction, as p and q are co-prime and both even. $\therefore \sqrt[3]{2}$ is irrational	1 mark for valid proof showing that p is even 1 mark for valid proof showing contradiction and concluding that $\sqrt[3]{2}$ is irrational.	Mostly well done, however too many students were limited in their explanations, and did not adequately justify their conclusion for p even. Too many forgot to list conditions on p and q being coprime and especially q != 0
(c)	Assume $\sqrt[3]{p}$ is rational, p is a prime number and $p \geq 2$ $\therefore \sqrt[3]{p} = \frac{m}{n}$ where $m, n \in \mathbb{Z}$ and co-prime, $n \neq 0$ $p = \frac{m^3}{n^3}$ $n^3 p = m^3$ $\therefore \frac{m^3}{p}$ and hence $\frac{m}{p}$ and so $m = pk, k \in \mathbb{Z}$ so, $n^3 p = (pk)^3$ $n^3 p = p^3 k^3$ $n^3 = p^2 k^3$ $n^3 = p(pk^3)$ $\therefore \frac{n^3}{p}$ and hence, $\frac{n}{p}$ $\therefore m$ and n divisible by p and so, not co-prime. This is a contradiction and hence $\sqrt[3]{p}$ is irrational.	1 mark for assumption of $\sqrt[3]{p}$ and stating the values of m and n 1 mark for working through to conclude p divides n 1 mark for valid proof showing contradiction and	This was not well done, and many students struggled with this generalisation. Too many overused the primality of p inappropriately and thus broke the logical flow of their argument.

		concluding that $\sqrt[3]{p}$ is irrational.	
(d)	If $a, b \in \mathbb{Z}$, then $a^b \in \mathbb{Z}$ Consider, $a, b \in \mathbb{Q}$ with $a = 2$ and $b = \frac{1}{3}$ $a^b = 2^{\frac{1}{3}} = \sqrt[3]{2}$ which is irrational by 2(b). $\therefore$ the statement is disproved.	2 marks for identifying a suitable counterexample and concluding that the statement is disproved.	Too many students proved the statement, where in fact the statement was untrue. The format of the question should have been a hint for students to find a counter example.
(e)	$6(x+y)^2 - 2(x-y)^2$ $= 6(x^2 + 2xy + y^2) - 2(x^2 - 2xy + y^2)$ $= 6x^2 + 12xy + 6y^2 - 2x^2 + 4xy - 2y^2$ $= 4x^2 + 16xy + 4y^2$ $= 4(x^2 + 4xy + y^2)$ $= 4m$ where $m \in \mathbb{Z}$ $\therefore 6(x+y)^2 - 2(x-y)^2$ is divisible by 4.	1 mark for valid proof with working	Very well done.
(f)	Required to prove: $\sqrt[8]{8!} < \sqrt[9]{9!}$ $(8!)^9 < (9!)^8$ $(8!)^8 8! < (9 \cdot 8!)^8$ $(8!)^8 8! < (8!)^8 9^8$ $8! \le 9^8$ Each term on the left is less than 9, so their product is less than eight 9's on the RHS. So, the inequality holds	1 mark for showing $8! \le 9^8$ 1 mark for stating that inequlity holds	Many students were able to achieve 1 of 2 marks here by trying to manipulate the expression with powers. Most struggled to express the final conclusion.
(gi)	Taking the left hand inequality: $\big\vert \vert z\vert - \vert w\vert \big\vert \le \vert z - w\vert$ And letting $w = -w$ $\big\vert \vert z\vert - \vert -w\vert \big\vert \le \vert z + w\vert$ $\big\vert \vert z\vert - \vert w\vert \big\vert \le \vert z + w\vert$	1 mark for valid proof with working	Very poorly done for a very simple proof. Showed a lack of familiarity what should be a fundamental result. Some students ignored the "Hence" and attempted to prove the result without the given inequality. This was leniently awarded, however, it may not be in future.
(ii)	For ΔQRS: $13 \le QS \le 51$ For ΔPQS: $10 \le QS \le 40$ Therefore, to satisfy both inequalities, $13 \le QS \le 51$ QS must be between 13 and 51 units.	2 marks for recognising QS between 13 and 51 not inclusive. 1 mark for any appropriate application of triangular inequality to find a bound.	Poorly attempted. Shows a lack of understanding of the geometry of the triangular inequality. "Side of a triangle is less than the sum and more than the difference of the other two sides."

(h)	$6 = \lvert 6 \rvert = \big\lvert \lvert z-3 \rvert - \lvert z+1 \rvert \big\rvert$ $\leq \lvert z-3+z+1 \rvert$ $= \lvert 2z-2 \rvert$ $= 2\lvert z-1 \rvert$ i.e. $2\lvert z-1 \rvert \geq 6$ and hence $\lvert z-1 \rvert \geq 3$ and the minimum value of $\lvert z-1 \rvert$ is 3.	1 mark for applying triangular inequality or equivalent working. 1 mark for achieving result, with sufficient working.	Not a single student recognised the connection between this question and the triangular inequality proven in question (gi). Considering this question geometrically, it concerns the difference between the lengths of two vectors. This should make you think about diagonals of a triangle and hence, the triangular inequality.

Question 3

Q	Solutions	Marking Scheme	Feedback
(a)	Let a, b be rational with $b > a$ We know, $0 < \frac{1}{\sqrt{2}} < 1$ $0 < \frac{1}{\sqrt{2}}(b-a) < (b-a) \qquad (b-a>0)$ $a < a + \frac{1}{\sqrt{2}}(b-a) < b$ So, $c = a + \frac{1}{\sqrt{2}}(b-a)$ lies between a and b and is irrational as $\frac{1}{\sqrt{2}}$ is irrational.	1 mark for identifying an irrational number bounded in some way. 1 mark for rearranging a stated bounded irrational to generalise an irrational strictly between two given rationals.	This question was not well done, with many students struggling to formulate an initial approach to the question. Students should work on expressing the requirements in mathematical language with perhaps a specific case first to work towards a generalised case.
(b)	We are given $a_1 = 1$ and $a_2 = 1$ Assume true for a_k and a_{k+1} RTP: $a_{k+2} = \frac{(1+\sqrt{5})^{k+2} - (1-\sqrt{5})^{k+2}}{\sqrt{5}\times 2^{k+2}}$ We are given, $a_{k+2} = a_k + a_{k+1}$ $= \frac{(1+\sqrt{5})^{k} - (1-\sqrt{5})^{k}}{\sqrt{5}\times 2^{k}} + \frac{(1+\sqrt{5})^{k+1} - (1-\sqrt{5})^{k+1}}{\sqrt{5}\times 2^{k+1}}$ By assumption $= \frac{2^2(1+\sqrt{5})^{k} - 2^2(1-\sqrt{5})^{k}}{\sqrt{5}\times 2^{k+2}} + \frac{2(1+\sqrt{5})^{k+1} - 2(1-\sqrt{5})^{k+1}}{\sqrt{5}\times 2^{k+2}}$ $= \frac{4(1+\sqrt{5})^{k} - 4(1-\sqrt{5})^{k} + 2(1+\sqrt{5})(1+\sqrt{5})^{k} - 2(1-\sqrt{5})(1-\sqrt{5})^{k}}{\sqrt{5}\times 2^{k+2}}$ $= \frac{\left([4+2(1+\sqrt{5})](1+\sqrt{5})^{k} - [4+2(1-\sqrt{5})](1-\sqrt{5})^{k}\right)}{\sqrt{5}\times 2^{k+2}}$ $= \frac{(6+2\sqrt{5})(1+\sqrt{5})^{k} - (6-2\sqrt{5})(1-\sqrt{5})^{k}}{\sqrt{5}\times 2^{k+2}}$ $= \frac{(1+\sqrt{5})^{k+2} - (1-\sqrt{5})^{k+2}}{\sqrt{5}\times 2^{k+2}}$ As required. Hence, as a_1 and a_2 are true, and a_{k+2} is true if a_k and a_{k+1} are true, then the statement is true by mathematical induction.	1 mark for initial cases confirmed 1 mark for the recurrence relationship 1 mark for conclusion	Question was challenging, it seems many students made only a limited attempt, possibly due to time constraints. Many students used recurrence relationship but had only assumed one previous statement correct, not both previous statements correct.
ci)	$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} \ldots$ $e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} \ldots$ $= 1 + i\theta - \frac{\theta^2}{2} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} \ldots$	1 mark for substituting $x = i\theta$	Few attempted this question, but those who did mostly achieved the result.

	Given that $e^{i\theta} = cis\theta$ and taking imaginary components: $$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} \dots$$ $$\frac{\sin\theta}{\theta} = 1 - \frac{\theta^2}{3!} + \frac{\theta^4}{5!} - \frac{\theta^6}{7!} + \frac{\theta^8}{9!} \dots$$	1 mark for achieving the result with sufficient working.	This question required a recognition of substituting $x = i\theta$ to then utilise $e^{i\theta} = cis\theta$. The algebra is quite straightforward and should be attempted as an exercise.
ii)	This is a polynomial with constant of 1 and $\sin\theta = 0$ when $\theta = k\pi$ for $k = 0, \pm 1, \pm 2, etc$. However, $\theta \neq 0$. Hence the RHS can be factorised linearly as: $$\left(1-\frac{\theta}{\pi}\right)\left(1+\frac{\theta}{\pi}\right)\left(1-\frac{\theta}{2\pi}\right)\left(1+\frac{\theta}{2\pi}\right)\left(1-\frac{\theta}{3\pi}\right)\left(1+\frac{\theta}{3\pi}\right)\left(1-\frac{\theta}{4\pi}\right)\left(1+\frac{\theta}{4\pi}\right)\dots$$ And by expanding using difference of two squares: $$\left(1-\frac{\theta^2}{\pi^2}\right)\left(1-\frac{\theta^2}{4\pi^2}\right)\left(1-\frac{\theta^2}{9\pi^2}\right)\left(1-\frac{\theta^2}{16\pi^2}\right)\dots$$	1 mark for explaining the consequences of the constant term being 1 1 mark for using the zeros of $\sin\theta$ to give factors	Difficult marks to achieve. Required a strong grasp of Ext 1 polynomials and trigonometry. Worth spending time wrestling with this result.
iii)	$$1 - \frac{\theta^2}{3!} + \frac{\theta^4}{5!} - \frac{\theta^6}{7!} + \frac{\theta^8}{9!} \dots$$ $$= \left(1-\frac{\theta^2}{\pi^2}\right)\left(1-\frac{\theta^2}{4\pi^2}\right)\left(1-\frac{\theta^2}{9\pi^2}\right)\left(1-\frac{\theta^2}{16\pi^2}\right)\dots$$ Comparing coefficient of θ^2 $$-\frac{\theta^2}{3!} = -\frac{\theta^2}{\pi^2} - \frac{\theta^2}{4\pi^2} - \frac{\theta^2}{9\pi^2} - \frac{\theta^2}{16\pi^2} \dots$$ $$\frac{1}{3!} = \frac{1}{\pi^2} + \frac{1}{4\pi^2} + \frac{1}{9\pi^2} + \frac{1}{16\pi^2} \dots$$ $$1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} \dots = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \dots = \frac{\pi^2}{3!} = \frac{\pi^2}{6}$$	1 mark for result with sufficient working.	This was a difficult mark to achieve. It required a recognition that the coefficients of θ^2 from the expansion are the denominators we are looking for in the expression.
di)	$$e^{ix} - e^{-ix} = cis(x) - cis(-x)$$ $$= 2i\sin x$$ $$\sin x = \frac{e^{ix} - e^{-i}}{2i}$$	1 mark for result with sufficient working.	Well done. Please be careful to show full working when taking $cis(-x) = \cos(-x) + i\sin(-x)$
ii)	$$\sin x = 2$$ $$\frac{e^{ix} - e^{-ix}}{2i} = 2$$ $$e^{ix} - e^{-ix} = 4i$$ Let $e^{ix} = z$ $$z - z^{-1} = 4i$$ $$z^2 - 4iz - 1 = 0$$ $$z = \frac{4i \pm \sqrt{-16+4}}{2}$$	1 mark for finding the quadratic in $z = e^{ix}$ 1 mark for finding the roots of the quadratic	Difficult question to achieve full marks, however, it is still worth attempting for partial marks given enough time in the test.

	$= \frac{4i \pm 2\sqrt{3}i}{2}$ $= (2 \pm \sqrt{3})i$ $e^{ix} = (2 \pm \sqrt{3})i$ $= (2 \pm \sqrt{3})e^{\frac{i\pi}{2}}$ $ix = \ln(2 \pm \sqrt{3}) + i\left(\frac{\pi}{2} + 2k\pi\right), k \in \mathbb{Z}$ $= \ln(2 \pm \sqrt{3}) + i\pi\left(\frac{4k+1}{2}\right)$ $x = \pi\left(\frac{4k+1}{2}\right) - i\ln(2 \pm \sqrt{3})$	1 mark for finding the solutions for x.	