Northern Beaches Secondary College
Manly Campus

2023

HSC Assessment 2

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- For questions in Section II, show relevant mathematical reasoning and calculations

Total Marks: 56

Section I – 8 marks (pages 1–3)

- Attempt Questions 1–8
- Allow about 15 minutes for this section
- Answer on Multiple Choice Answer Sheet provided

Section II – 48 marks (pages 4–8)

- Attempt Questions 9–11
- Allow about 75 minutes for this section
- Answer each question in a separate booklet

Section I (8 marks)

Attempt Questions 1 – 8

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for questions 1 – 8.

Q1. Let ω be a non-real cube root of 1.

The value of $(1+\omega)^3$ is:

A. -1

B. 0

C. 1

D. 2

Q2. What is the negation of the following statement?
"If Mike stays up late, then he sleeps in until midday."

A. If Mike does not stay up late then he sleeps in until midday.

B. If Mike stays up late then he may not sleep in until midday.

C. Mike stays up late and he does not sleep in until midday.

D. Mike does not stay up late and he sleeps in until midday.

Q3. Let $z = a + ib$ where $a, b \in \mathbb{R}$ and $a \neq 0,\ b \neq 0$.

If $z + \frac{1}{z} \in \mathbb{R}$, which of the following must be true?

A. $Arg(z) = \frac{\pi}{4}$

B. $a = -b$

C. $a = b$

D. $|z| = 1$

Q4. Assume that x and y are negative real numbers with $x > y$.

Which of the following statements might be false?

A. $\frac{1}{x - y} < 0$

B. $\frac{x}{y} - \frac{y}{x} < 0$

C. $x + y > 2y$

D. $2x > 3y$

Q5. Which of the following statements is **false**?

A. $\forall\, x \in \mathbb{R}, x^2 \geq 0$

B. $\forall\, n \in \mathbb{Z}, \exists\, m \in \mathbb{Z},\ m = n + 5$

C. $\exists\, k \in \mathbb{R}, \forall\, x \in \mathbb{R}, kx = x$

D. $\exists\, m \in \mathbb{Z}, \forall\, n \in \mathbb{Z},\ m = n + 5$

Q6. Given that $z = \dfrac{1+\sqrt{3}\,i}{1+i}$, the modulus and argument of the complex number z^5 are:

A. $r = 2\sqrt{2}\ ,\ \theta = \dfrac{5\pi}{6}$

B. $r = 4\sqrt{2}\ ,\ \theta = \dfrac{5\pi}{12}$

C. $r = 4\sqrt{2}\ ,\ \theta = \dfrac{7\pi}{12}$

D. $r = 2\sqrt{2}\ ,\ \theta = -\dfrac{\pi}{12}$

Q7. Given that $(1+i)^n = \alpha i$ where α is a non-zero real constant, then $(1+i)^{2n+2}$ simplifies to:

A. α^4

B. $2\alpha^2 i$

C. $1 + \alpha^2 i$

D. $-2\alpha^2 i$

Q8. On an Argand diagram, a set of points lies on a circle of radius 3, centred at the origin. Which of the following defines this circle?

A. $\{z \in \mathbb{C} : z\bar{z} = 3\}$

B. $\{z \in \mathbb{C} : z^2 = 9\}$

C. $\{z \in \mathbb{C} : (z+\bar{z})^2 - (z-\bar{z})^2 = 36\}$

D. $\{z \in \mathbb{C} : Re(z^2) + Im(z^2) = 9\}$

Section II (48 marks)

Attempt Questions 9 – 11

Allow about 75 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 9 Use a separate writing booklet. **16 marks**

a. Let 1, ω, ω^2 be the cube roots of 1 and let ω be the non-real root with lowest positive argument.

i. Show that $1 + \omega + \omega^2 = 0$ **1**

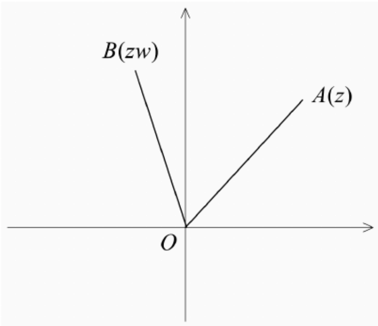
ii. Hence evaluate $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^5)(1-\omega^7)(1-\omega^8)$ **3**

b.

i. Graph the locus of z given that $|z - 2i| = 1$ **2**

ii. Hence or otherwise, find the maximum value of $Arg(z)$ **2**

c. As shown on the Argand diagram below, the complex numbers z and zw are represented by the points A and B respectively.



Given $z = re^{i\theta}$ and $w = e^{i\frac{\pi}{3}}$, where $r > 0$,

i. Explain why OAB is an equilateral triangle. **2**

ii. Write the complex number $z - zw$ in exponential form in terms of r and θ. **2**

d. Prove that for any integer $n > 1$, $\log_n(n+1)$ is irrational. **4**

Question 10 Use a separate writing booklet. **16 marks**

a. Convert $4\sqrt{3} - 4i$ into exponential form. **2**

b.

i. Find the square roots of $-8 - 6i$ **2**

ii. Hence or otherwise, solve $z^2 - (1 - i)z + (2 + i) = 0$ **2**

c. A three digit numeral N has a, b and c as its digits when writing from left to right respectively.

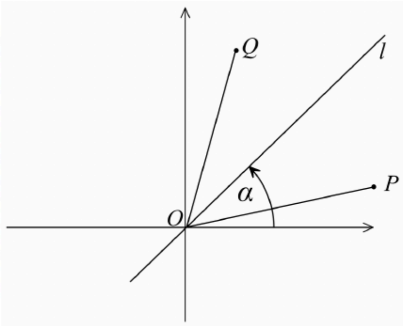
For example, if $N = 735$, $a = 7$, $b = 3$ and $c = 5$.

Show that if $a + c = b$ then N is divisible by 11. **3**

d. In a sequence of numbers, $T_1 = 7$ and $T_n = 2T_{n-1} - 1$ for integers $n \geq 2$.

Use mathematical induction to prove that $T_n = 3(2^n) + 1$ for integers $n \geq 1$. **3**

e. Let l be the line in the complex plane that passes through the origin and makes an angle α with the positive real axis, where $0 < \alpha < \frac{\pi}{2}$.



The point P represents the complex number z_1, where $0 < \arg(z_1) < \alpha$.

The point P is reflected in the line l to produce the point Q, which represents the complex number z_2. Hence $|z_1| = |z_2|$.

i. Explain why $\arg(z_1) + \arg(z_2) = 2\alpha$ **2**

ii. Deduce that $z_1 z_2 = |z_1|^2(\cos 2\alpha + i\sin 2\alpha)$ **2**

Question 11 Use a separate writing booklet. **16 marks**

a. Sketch the region in the complex plane which is defined by the intersection of the regions:

$0 < Arg(z - 1 - i) \leq \frac{\pi}{3}$ and $|z - 1 - i| < \sqrt{2}$ **3**

b. Use contraposition to prove the following proposition, where $m, n \in \mathbb{Z}^+$:

"If $m^3 + n^3$ is odd, then only one of m, n is odd." **3**

c.

i. Given that $x^2 + y^2 \geq 2xy$ for all $x, y \in \mathbb{R}$ show that:

$x^2 + y^2 + z^2 \geq xy + xz + yz$ for all $x, y, z \in \mathbb{R}$ **2**

ii. Hence show that:

$3(x^4 + y^4 + z^4) \geq (x^2 + y^2 + z^2)^2$ for all $x, y, z \in \mathbb{R}$ **3**

d.

i. Show that $(1 + i\tan\theta)^n + (1 - i\tan\theta)^n = \dfrac{2\cos n\theta}{\cos^n \theta}$ for $\cos\theta \neq 0$ and $n \in \mathbb{Z}^+$

2

ii. By using the result in part (i) and by letting $z = i\tan\theta$, show that the roots of

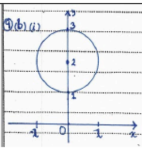
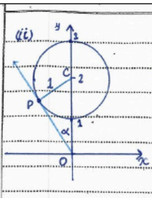
$(1 + z)^2 + (1 - z)^2 = 0$ are $z = \pm i \tan \frac{\pi}{4}$

3

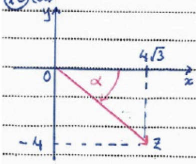
End of paper

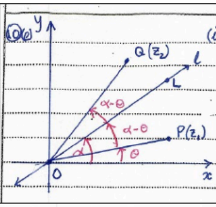
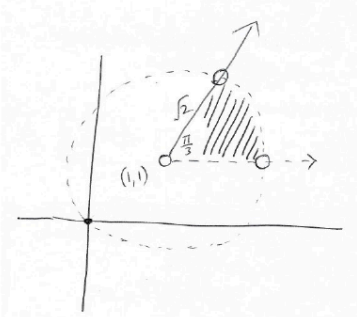
Mathematics Extension 2 Assessment 2		
Q	Solution	Marks
1	$(1+\omega)^3 = 1 + 3\omega + 3\omega^2 + \omega^3$ $= 1 + 3(\omega + \omega^2) + 1$ $= 1 + 3(-1) + 1$ $= -1$	A 35 out of 43 students answered correctly
2	The negation of a statement of the form $P \Rightarrow Q$ is "P and not Q" (ie $P \wedge \sim Q$) $\therefore$ "Mike stays up late" and "he does not sleep in until midday"	C **19 out of 43 students answered correctly**
3	$z + \frac{1}{z} = z + \frac{1}{z} \cdot \frac{\bar{z}}{\bar{z}} = z + \frac{\bar{z}}{z\bar{z}} = z + \frac{\bar{z}}{\|z\|^2}$ $z + \frac{1}{z} = z + \bar{z} = 2\operatorname{Re}(z) \in \mathbb{R}$ if $\|z\|^2 = 1$	D 26 out of 43 students answered correctly
4	$x > y \Rightarrow x - y > 0 \Rightarrow \frac{1}{x-y} > 0$	A 25 out of 43 students answered correctly
5	There exists an integer m, for all integers n, such that $m = n + s$ (That is, for any integer n, add s to obtain a fixed integer m) $\therefore$ D	D 27 out of 43 students answered correctly
6	$z = \frac{1+\sqrt{3}i}{1+i} = \frac{u}{v}$ $\|u\| = \sqrt{(1)^2 + (\sqrt{3})^2} = 2$ $\quad$ $\|v\| = \sqrt{1^2 + 1^2} = \sqrt{2}$ $\operatorname{Arg} u = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$ $\quad$ $\operatorname{Arg} v = \tan^{-1} 1 = \frac{\pi}{4}$ $u = 2 \operatorname{cis} \frac{\pi}{3}$ $\quad$ $v = \sqrt{2} \operatorname{cis} \frac{\pi}{4}$ $z = \frac{u}{v} = \frac{2 \operatorname{cis} \frac{\pi}{3}}{\sqrt{2} \operatorname{cis} \frac{\pi}{4}} = \sqrt{2} \operatorname{cis} \frac{\pi}{12}$ $z^5 = \left\{\sqrt{2} \operatorname{cis} \frac{\pi}{12}\right\}^5 = 4\sqrt{2} \operatorname{cis} \frac{5\pi}{12}$	B 33 out of 43 students answered correctly

7	$(1+i)^{2n+2} = (1+i)^{2n} \cdot (1+i)^2 = \left[(1+i)^n\right]^2 (1+2i+i^2)$ $= (\alpha i)^2 \cdot (2i) = \alpha^2 i^2 \cdot 2i$ $= -2\alpha^2 i$	D 26 out of 43 students answered correctly
8	$(z+\bar{z})^2 - (z-\bar{z})^2 = 36$ $\cancel{z^2} + 2z\bar{z} + \cancel{\bar{z}^2} - \{\cancel{z^2} - 2z\bar{z} + \cancel{\bar{z}^2}\} = 36$ $4z\bar{z} = 36 \Rightarrow z\bar{z} = 9 = \lvert z\rvert^2 = 9$ $\therefore \lvert z\rvert = 3$ C	C **13 out of 43 students answered correctly**

9ai	(a) From $z^3=1$, $z^3-1=0$ $\therefore$ roots are $1, \omega, \omega^2$ (i) Sum of roots $= -\frac{b}{a} = 0$ $\Rightarrow 1+\omega+\omega^2=0$ OR $\omega^3=1 \Rightarrow \omega^3-1=0$ $\Rightarrow (\omega-1)(\omega^2+\omega+1)=0$ $\omega\neq 1 \Rightarrow 1+\omega+\omega^2=0$ • **Solutions using vectors and/or Argand diagrams accepted if logically sound** • **Using sum of GS assumes result is TRUE** • **Most common error was lack of rejection of (w-1) solution**	**1 Mark:** Correct solution
9aii	(ii) $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^5)(1-\omega^7)(1-\omega^8)$ $=(1-\omega)(1-\omega^2)(1-\omega)(1-\omega^2)(1-\omega)(1-\omega^2)$ $=(1-\omega)^3(1-\omega^2)^3$ $=[(1-\omega)(1-\omega^2)]^3$ $=[1-\omega-\omega^2+\omega^3]^3$ $=[1+\omega^3-(\omega+\omega^2)]^3=[1+1-(-1)]^3=3^3$ $=27$ $\omega^4=\omega^3.\omega=\omega$ $\omega^5=\omega^3.\omega^2=\omega^2$ $\omega^7=\omega^3.\omega^3.\omega=\omega$ $\omega^8=\omega^3.\omega^3.\omega^2=\omega^2$ $1+\omega+\omega^2=0$ $\Rightarrow \omega+\omega^2=-1$ • **Poorly done for easy Q**	**3 Marks:** Correct solution **2 Marks:** Converts powers 4, 5, 7 and 8 of w to equivalent powers 1, 2 and 3 **AND** progresses to an equivalent version of $(1-w)^3(1-w^2)^3$ **1 Mark:** Converts at least two of the powers 4, 5, 7 and 8 of w to equivalent powers 1, 2 and 3.
9bi	9(b)(i)  $\lvert z-2i\rvert=1$ Circle centre (0, 2) and radius 1 unit • **Poor quality diagrams (should be at least 1/3 page)**	**2 Marks:** Correct answer **1 Mark:** Circle centre (0,0), radius 1 unit OR Circle centre (0, -2i), radius 1 unit OR Any circle with correct centre OR correct radius
9bii	4 (ii)  The maximum value of Arg z occurs at point P where OP is a tangent to the circle and OP ⊥ CP  $\sin\alpha = 1/2 \Rightarrow \alpha=\frac{\pi}{6}$ $\therefore$ maximum value of Arg z is $\frac{\pi}{2}+\frac{\pi}{6}=\frac{2\pi}{3}$ 2 • **Note tangent at left most extremity will be vertical**	**2 Marks:** Correct answer **1 Mark:** Finds sinα OR draws a correct diagram

9ci	⑨(c) $\lvert w\rvert = \lvert e^{i\pi/3}\rvert = 1$. $\quad \text{Arg}\,\omega = \pi/3$ $\lvert zw\rvert = \lvert z\rvert.\lvert w\rvert = \lvert z\rvert.1 = r$ $\therefore \lvert \vec{OA}\rvert = \lvert \vec{OB}\rvert \Rightarrow \Delta AOB$ is isosceles $\text{Arg}(zw) = \text{Arg}\,z + \text{Arg}\,w = \text{Arg}\,z + \pi/3$ $\therefore \angle AOB = \pi/3$ $\therefore \Delta AOB$ is equilateral (isos Δ with apex $\angle = \pi/3$) • **Varied quality of attempts with those presenting logical, sequential relevant statements obtaining full marks**	**2 Marks:** Correct answer **1 Mark:** Nominates one correct feature such as equal sides, correct Arg *w* etc
9cii	$z - zw = z(1-w) = re^{i\theta}(1 - e^{i\pi/3})$ $= re^{i\theta}(1 - \{\cos \pi/3 + i\sin \pi/3\})$ $= re^{i\theta}(1 - \tfrac{1}{2} - \tfrac{\sqrt{3}}{2} i) = re^{i\theta}(\tfrac{1}{2} - \tfrac{\sqrt{3}}{2} i)$ $= re^{i\theta}.\, \overline{w} = re^{i\theta}.\, e^{-i\pi/3}$ $z - zw = re^{i(\theta - \pi/3)}$ • **Only small portion of candidates were able to obtain second mark**	**2 Marks:** Correct answer **1 Mark:** Obtains an expression equivalent to an unsimplified exponential form of $z - zw$
9d	⑨(d) Assume $\log_n(n+1)$ is rational $\therefore \log_n(n+1) = \frac{p}{q}$ where $p, q \in \mathbb{Z}$ and have no common factor and $q \neq 0$ $\therefore n+1 = n^{p/q}$ $\Rightarrow (n+1)^q = n^p$ Case 1 Suppose n is even $\therefore n^p$ is even But $n+1$ is odd $\therefore (n+1)^q$ is odd $\therefore$ odd = even $\therefore$ a contradiction Case 2 Suppose n is odd $\therefore n^p$ is odd But $n+1$ is even $\therefore (n+1)^q$ is even $\therefore$ even = odd $\therefore$ a contradiction $\therefore \log_n(n+1)$ is irrational • **Induction proofs not as successful as proof by contradiction** • **Omission of cases most common deficiency** • **Note:** $n^{\frac{p}{p}}$ **is NOT** $n^{p\,p} - n^{q\,q}$	**4 Marks:** Correct answer **4 Marks:** Correct answer **3 Marks:** Identifies two cases and proves one case and attempts to prove the other. **2 Marks:** Identifies and proves one case **1 Mark:** Obtains $(n+1)^q = n^p$ OR $n + 1 = n^{p/q}$

10a	(10)(a)  Let $z=4\sqrt{3}-4i$ $\lvert z\rvert=r=\sqrt{(4\sqrt{3})^2+(-4)^2}=8$ $\alpha=\tan^{-1}\left(\frac{4}{4\sqrt{3}}\right)=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}$ $\therefore \operatorname{Arg} z=\frac{-\pi}{6}$ $\Rightarrow 4\sqrt{3}-4i=8e^{-i\left(\frac{\pi}{6}\right)}$	**2 Marks:** Correct answer **1 Mark:** Finds $\lvert z\rvert$ or Arg(z)
10bi	(10)(b) Let $z=x+iy$ Then $z^2=-8-6i \Rightarrow (x+iy)^2=-8-6i$ $x^2-y^2+2xyi=-8-6i$ $x^2-y^2=-8$ $\quad 2xy=-6$ $y=\frac{-3}{x}$ $x^2-\left(\frac{-3}{x}\right)^2=-8$ $x^2-\frac{9}{x^2}=-8$ $\Rightarrow x^4+8x^2-9=0$ $(x^2+9)(x^2-1)=0$ $x^2\neq-9$ $\quad x^2=1$ $\Rightarrow x=\pm1,\ y=\mp3$ $\therefore x+iy=1-3i$ or $x+iy=-1+3i$	**2 Marks:** Correct answer **1 Mark:** Obtains $x^2-y^2=-8$ and $2xy=-6$
10bii	$z^2-(1-i)z+(2+i)=0$ $\Rightarrow z=\frac{(1-i)\pm\sqrt{(1-i)^2-4(2+i)}}{2}$ $=\frac{(1-i)\pm\sqrt{-8-6i}}{2}$ $z_1=\frac{1-i+1-3i}{2}=\frac{2-4i}{2}$ $\quad z_2=\frac{1-i-(1-3i)}{2}=\frac{2i}{2}$ $z_1=1-2i$ $\quad z_2=i$	**2 Marks:** Correct answer **1 Mark:** Obtains a correct quadratic formula expression for z
10c	(10)(c) Let $N=100a+10b+c$ Then $b=a+c \Rightarrow N=100a+10(a+c)+c$ $=100a+10a+10c+c$ $=110a+11c$ $=11\{10a+c\}$ $=11M$ where $M=10a+c\in\mathbb{Z}^+$ $\therefore$ N is divisible by 11	**3 Marks:** Correct answer **2 Marks:** Recognises $b=a+c$ gives a correct initial expression for N in terms of a and c. **1 Mark:** Writes an expression equivalent to N=100a + 10b +c

10d	(10)(d) For $n=1$, $T_1 = 3(2^1)+1 = 7$ $\therefore$ S(1) is true S(k): Assume $T_k = 3(2^k)+1$ S(k+1): Prove $T_{k+1} = 3(2^{k+1})+1$ $T_{k+1} = 2T_k - 1$ $= 2\{3(2^k)+1\} - 1$ $= 3\{2.2^k\} + 2 - 1$ $= 3(2^{k+1}) + 1$ $\therefore$ $S(k) \Rightarrow S(k+1)$ and the result is proved by mathematical induction	**3 Marks:** Correct answer **2 Marks:** Shows S(1) is true, correctly states S(k) & S(k+1) and attempts the implication proof **1 Mark:** Proves S(1) AND correctly states S(k+1)
10ei	(10)(e)  (i) Let Arg $z_1 = \theta$ $\angle POL = \alpha - \theta$ $\angle QOL = \alpha - \theta$ Arg $z_2 = \alpha + \alpha - \theta$ $= 2\alpha - \theta$ $\therefore$ Arg z_1 + Arg $z_2 = \theta + 2\alpha - \theta$ $= 2\alpha$	**2 Marks:** Correct answer **1 Mark:** Recognises that angle QOL = ά-ɵ
10eii	(ii) $z_1 = \|z_1\| \text{cis}\, \theta$ $\quad z_2 = \|z_2\| \text{cis}(2\alpha - \theta)$ $= \|z_1\| \text{cis}(2\alpha - \theta)$ $\therefore z_1 z_2 = \|z_1\| \text{cis}\,\theta . \|z_1\| \text{cis}(2\alpha - \theta)$ $= \|z_1\|^2 \text{cis}(\theta + 2\alpha - \theta)$ $z_1 z_2 = \|z_1\|^2 \text{cis}\, 2\alpha$ $= \|z_1\|^2 (\cos 2\alpha + i \sin 2\alpha)$	**2 Marks:** Correct answer **1 Mark:** Forms a correct expression for $z_1 z_2$
11a	 $\|z - (1+i)\| < \sqrt{2}$ "distance to $1+i$ is less than $\sqrt{2}$" $0 < \underbrace{\text{Arg}(z - (1+i))}_{} \leq \frac{\pi}{3}$ direction of the vector pointing from $1+i$ to z	**3 Marks:** Correct diagram **2 Marks:** Draws the correct region with some features missing: - open circle at (1, 1), since Arg0 undefined - dotted lines for < and solid lines for ≤ - radius or angle not labelled / inconsistent radius (Note: the circle passes through *O* !) **1 Mark:** Draws the correct circle OR the correct rays ***Marker's comment:*** **Most students obtained 2 marks (correct region with missing features).** **Draw a large diagram (about 1/3 page) so that all information can be seen clearly.**

11b	(11)(b) Contrapositive: If neither m nor n is odd, or if both m and n are odd then m^3+n^3 is not odd (i.e even) Case 1 Assume neither m nor n is odd $\therefore m=2p$ and $n=2q$ for $p,q\in\mathbb{Z}^+$ $\therefore m^3+n^3=(2p)^3+(2q)^3$ $=8p^3+8q^3$ $=2\{4p^3+4q^3\}$ $=2k \quad k\in\mathbb{Z}^+$ $\therefore m^3+n^3$ is even Case 2 Assume both m and n are odd $\therefore m=2p+1 \quad n=2q+1 \; ; p,q\in\mathbb{Z}^+$ $m^3+n^3=(2p+1)^3+(2q+1)^3$ $=8p^3+12p^2+6p+1+8q^3+12q^2+6q+1$ $=8p^3+12p^2+6p+8q^3+12q^2+6q+2$ $=2\{4p^3+6p^2+3p+4q^3+6q^2+3q+1\}$ $=2k, \quad k\in\mathbb{Z}^+$ $\therefore m^3+n^3$ is even $\therefore$ result is proved by contraposition	**3 Marks:** Correct answer **2 Marks:** Proves one case OR Identifies two cases and makes relevant substitutions in each case **1 Mark:** Makes a relevant substitution for either Case 1 or Case 2 ***Marker's comment:*** **The correct negation of "only one of *m*, *n* is odd" is "*m* and *n* are both even or both odd". Many students wrote "*m*, *n* are both even", which gave an incorrect contrapositive, which simplified the proof to 2 marks.**
11ci	(11)(c)(i) $x^2+y^2\geq 2xy$ (1) $\Rightarrow x^2+z^2\geq 2xz$ (2) $\Rightarrow y^2+z^2\geq 2yz$ (3) (1)+(2)+(3): $2x^2+2y^2+2z^2\geq 2xy+2xz+2yz$ $\cancel{2}\{x^2+y^2+z^2\}\geq \cancel{2}\{xy+xz+yz\}$ $x^2+y^2+z^2\geq xy+xz+yz$	**2 Marks:** Correct answer **1 Mark:** Identifies inequalities 1 and 2 ***Marker's comment:*** **Mostly done well**
11cii	(ii) $3(x^4+y^4+z^4)-(x^2+y^2+z^2)^2$ $=3x^4+3y^4+3z^4-\{x^4+y^4+z^4+2x^2y^2+2x^2z^2+2y^2z^2\}$ $=2x^4+2y^4+2z^4-2\{x^2y^2+x^2z^2+y^2z^2\}$ $=2\{x^4+y^4+z^4-(x^2y^2+x^2z^2+y^2z^2)\}$ $\geq 2\times 0$ from (i) with $x\to x^2$, $y\to y^2$ and $z\to z^2$ ≥ 0 $\therefore 3(x^4+y^4+z^4)\geq(x^2+y^2+z^2)^2$	**2 Marks:** Correct answer **1 Mark:** Forms a correct difference expression and attempts to simplify OR substitutes x^2 , y^2 , z^2 into part (i) ***Marker's comment:*** **Mostly done well. Examining LHS – RHS was a straightforward method in this question.**
11di	(11)(d)(i) $(1+i\tan\theta)^n+(1-i\tan\theta)^n$ $=\left(1+\dfrac{i\sin\theta}{\cos\theta}\right)^n+\left(1-\dfrac{i\sin\theta}{\cos\theta}\right)^n$ $=\dfrac{(\cos\theta+i\sin\theta)^n}{\cos^n\theta}+\dfrac{(\cos\theta-i\sin\theta)^n}{\cos^n\theta}$ $=\dfrac{\cos n\theta+\cancel{i\sin n\theta}+\cos n\theta-\cancel{i\sin n\theta}}{\cos^n\theta}$ $=\dfrac{2\cos n\theta}{\cos^n\theta}$ <u>Alternative method</u>	**2 Marks:** Correct answer **1 Mark:** Forms a correct expression involving $\cos^n\theta$ ***Marker's comment:*** **Students who converted tan to sin/cos were generally successful. Be sure to show all steps in "show that" questions, particularly shorter questions!**

	Convert $1 + i\tan\theta$ to mod-arg form: $r = \sqrt{1+\tan^2\theta} = \sqrt{\sec^2\theta} = \sec\theta$ $\arg z = \tan^{-1}(\tan\theta) = \theta$ $\Rightarrow \quad 1 + i\tan\theta = \sec\theta \text{ cis } \theta = \frac{\text{cis}\theta}{\cos\theta}$ $\Rightarrow \quad (1+i\tan\theta)^n + (1-i\tan\theta)^n = \frac{(\text{cis}\theta)^n}{\cos^n\theta} + \frac{(\text{cis}(-\theta))^n}{\cos^n\theta}$ etc	
11dii	(ii) $(1+z)^2 + (1-z)^2 = 0$ $z = i\tan\theta \Rightarrow (1+i\tan\theta)^2 + (1-i\tan\theta)^2 = 0$ For $n=2$ $\frac{2\cos 2\theta}{\cos^2\theta} = 0$ $\cos 2\theta = 0$ $\therefore 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$ For $\theta = \frac{\pi}{4}$, $z_1 = i\tan\frac{\pi}{4} = i \times 1 = i$ For $\theta = \frac{3\pi}{4}$, $z_2 = i\tan\frac{3\pi}{4} = i \times -1 = -i$ $\therefore z_{1,2} = \pm i = \pm i\tan\frac{\pi}{4}$	**3 Marks:** Correct answer **2 Marks:** Finds two correct values for Θ **1 Mark:** Forms a correct expression by substituting *n =2* ***Marker's comment:*** **Mostly done well.**