

Mr Feng
Mrs Kerr
Ms Lau

Name:

Student Number:

Teacher:.....



Pymble Ladies' College

Year 12

HSC ASSESSMENT TASK 1

**Term 4
2021**

Time Allowed: 50 minutes

Mathematics Extension 2

General Instructions

- Write your answers in the Answer Booklet.
- Write using black or blue pen.
- Erasable pens are **not** to be used.
- Approved calculators may be used.
- Diagrams are not drawn to scale.
- All necessary working should be shown in questions **6 - 8**.
- Marks may be deducted for careless or untidy work.
- A reference sheet is provided.

Mark	/30
Rank	/
Highest Mark	/30

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Section I

Multiple Choice.

Each question worth 1 mark in this section

- 1 What is the value of $\text{Im}(z)$ if $z = 4 - i\sqrt{3}$?
- (A) $-i\sqrt{3}$
(B) $-\sqrt{3}$
(C) $\sqrt{3}$
(D) $i\sqrt{3}$
- 2 What is the conjugate of the complex number $\sqrt{2}i - 2$?
- (A) $\sqrt{2}i + 2$
(B) $\sqrt{2} + 2i$
(C) $\sqrt{2} - 2i$
(D) $-\sqrt{2}i - 2$
- 3 What is the argument of ω given that $\omega = -4 + 3i$?
- (A) $\tan^{-1}\frac{3}{4}$
(B) $-\tan^{-1}\frac{3}{4}$
(C) $\tan^{-1}\frac{3}{4} + \frac{\pi}{2}$
(D) $\pi - \tan^{-1}\frac{3}{4}$

- 4 The complex numbers $p = 5 - 2i$ and $q = -8 - 7i$ are represented by the points P and Q on the Argand diagram. Which of the following complex numbers is represented by the vector $\overrightarrow{PQ}$?
- (A) $-13 - 5i$
(B) $13 + 5i$
(C) $13 - 5i$
(D) $-13 + 5i$
- 5 Which of the following equations has the roots $1 + 2i$, $1 - 2i$ and -3 ?
- (A) $z^3 + z^2 + z + 15 = 0$
(B) $z^3 - z^2 + z + 15 = 0$
(C) $z^3 + z^2 - z + 15 = 0$
(D) $z^3 - z^2 - z - 15 = 0$

End of Section I

Section II (25 marks)

Question 6 (8 marks)

- (a) Given the complex numbers $z = \sqrt{3} - i$ and $\omega = -2 + 2i$.
- (i) Write z in modulus and argument form. **1**
- (ii) Hence, simplify $\frac{z^2}{\omega^4}$. Leave your answer in the form $a + ib$, where a and b are real. **3**
- (b) (i) Find the two square roots of the complex number $-5 + 12i$. **2**
- (ii) Hence, or otherwise, solve the equation $z^2 + (2 + i)z + (2 - 2i) = 0$. **2**

Question 7 (8 marks)

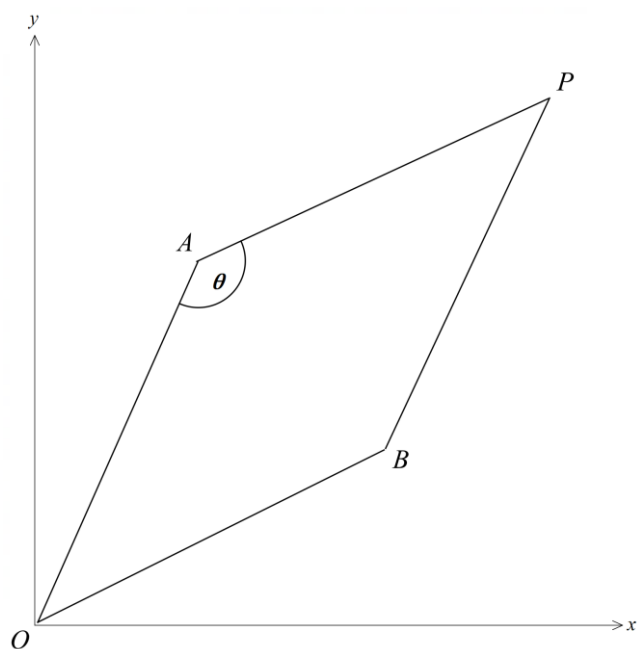
(a) Find the value of $\frac{e^{i\frac{\pi}{6}} - e^{-i\frac{\pi}{6}}}{3i}$. **2**

(b) Consider the equation $x^3 - 8x^2 + 25x - 26 = 0$.

(i) Show that $3 - 2i$ is a root of the equation. **1**

(ii) Find the other roots of the equation. **2**

(c) On the Argand diagram, the points O , A , B and P represent the complex numbers 0 , $1 + \sqrt{3}i$, $\sqrt{3} + i$ and z respectively. $OAPB$ forms a rhombus.



(i) Find z in the form of $a + ib$, where a and b are real numbers. **1**

(ii) An interior angle of the rhombus is marked as θ on the diagram. Find the value of θ . **2**

Question 8 (9 marks)

Given that ω is a non-real seventh root of -1 with the smallest positive argument:

- (i) Find ω in polar form. 1
- (ii) Show that $1 - \omega + \omega^2 - \omega^3 + \omega^4 - \omega^5 + \omega^6 = 0$. 1
- (iii) If $z = \cos \theta + i \sin \theta$, show that $z^n + z^{-n} = 2 \cos n\theta$. 1
- (iv) Hence, or otherwise, find the exact value of $\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{3\pi}{7}$. 2
- (v) Show that $\alpha = \omega - \omega^2 - \omega^4$ and $\beta = \omega^3 + \omega^5 - \omega^6$ are the roots of the equation $z^2 - z + 2 = 0$. 2
- (vi) Hence, find the exact value of $\sin \frac{\pi}{7} - \sin \frac{2\pi}{7} - \sin \frac{3\pi}{7}$. 2

End of Task

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NSW Education Standards Authority

HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced Mathematics Extension 1 Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

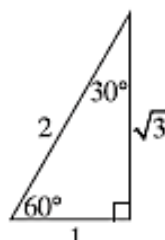
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1 + t^2}$$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2} (1 - \cos 2nx)$$

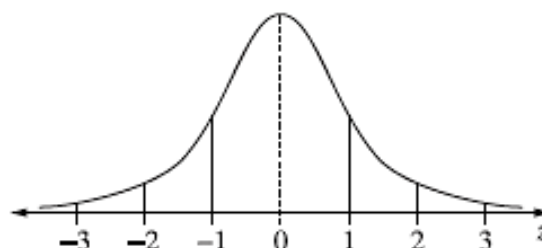
$$\cos^2 nx = \frac{1}{2} (1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between -1 and 1
- approximately 95% of scores have z-scores between -2 and 2
- approximately 99.7% of scores have z-scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

Binomial distribution

$$P(X = r) = {}^n C_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= {}^n C_x p^x (1 - p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function

Derivative

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

Integral Calculus

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \left\{ f(a) + f(b) + 2[f(x_1) + \dots + f(x_{n-1})] \right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \dots + \binom{n}{r}x^{n-r}a^r + \dots + a^n$$

Vectors

$$|\underline{u}| = |x\underline{i} + y\underline{j}| = \sqrt{x^2 + y^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1x_2 + y_1y_2,$$

$$\text{where } \underline{u} = x_1\underline{i} + y_1\underline{j}$$

$$\text{and } \underline{v} = x_2\underline{i} + y_2\underline{j}$$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

Complex Numbers

$$\begin{aligned} z &= a + ib = r(\cos \theta + i \sin \theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos \theta + i \sin \theta)]^n &= r^n(\cos n\theta + i \sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

Multiple Choice Answer Sheet
Mathematics Extension 2 2021

Name:.....

Teacher's Name:.....

Question 1

- (A) ☐ (B) ☒ (C) ☐ (D) ☐

Question 2

- (A) ☐ (B) ☐ (C) ☐ (D) ☒

Question 3

- (A) ☐ (B) ☐ (C) ☐ (D) ☒

Question 4

- (A) ☒ (B) ☐ (C) ☐ (D) ☐

Question 5

- (A) ☐ (B) ☐ (C) ☒ (D) ☐

SECTION II
25 marks

Name:

Teacher's Name:

Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the end of each part in this Answer Booklet.

Question 6 (8 marks)

Part (a)

Marks

(i)

1

$$|z| = \sqrt{(\sqrt{3})^2 + (-1)^2}$$

$$= 2$$

$$\tan \theta = \frac{-1}{\sqrt{3}}$$

$$\theta = -\frac{\pi}{6}$$



1 mark.
Correct solution with indication of finding the argument.

$$\therefore z = 2 \operatorname{cis}\left(-\frac{\pi}{6}\right)$$

(ii)

3

$$|w| = \sqrt{(-2)^2 + (2)^2}$$

$$= 2\sqrt{2}$$

$$\tan \alpha = \frac{2}{-2} = -1$$

$$\alpha = -\frac{\pi}{4}$$



1 mark.
Find w in mod-arg form.

$$w = 2\sqrt{2} \operatorname{cis} \frac{3\pi}{4}$$

$$z^2 = (2 \operatorname{cis}(-\frac{\pi}{6}))^2$$

$$= 4 \operatorname{cis}(-\frac{\pi}{3})$$

$$w^4 = (2\sqrt{2} \operatorname{cis} \frac{3\pi}{4})^4$$

$$= 64 \operatorname{cis} 3\pi$$

$$= 64 \operatorname{cis} \pi$$

$$\frac{z^2}{w^4} = \frac{4}{64} \operatorname{cis}(-\frac{\pi}{3} - \pi)$$

$$= \frac{1}{16} \operatorname{cis}(-\frac{4\pi}{3})$$

$$= \frac{1}{16} \operatorname{cis} \frac{2\pi}{3}$$

(De Moivre's Theorem). Apply De Moivre's Theorem

1 mark

Correct solutions

$$= \frac{1}{16} (\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$$

$$= -\frac{1}{32} + \frac{\sqrt{3}}{32} i$$

Question 6 (continued)

Part (b)

Marks

2

(i)

$$\begin{aligned}\sqrt{-5+12i} &= \sqrt{4+12i-9} \\ &= \sqrt{2^2+2(3i)(2)+(3i)^2} \\ &= \sqrt{(2+3i)^2} \\ &= \pm(2+3i)\end{aligned}$$

$$\begin{array}{c} 2 \\ \nearrow \\ 6i \quad \searrow \quad 3i \end{array}$$

1 mark.

Separate $-5+12i$ as an expansion of a perfect square.

or
Generate the simultaneous equations.

1 mark.

Correct answers.

(ii)

2

$$\begin{aligned}\Delta &= (2+i)^2 - 4(2-2i) \\ &= 4+4i+(-1)-8+8i \\ &= -5+12i\end{aligned}$$

1 mark.

Apply quadratic formula correctly.

$$z = \frac{-(2+i) \pm \sqrt{-5+12i}}{2}$$

1 mark.

Correct answers.

$$z = \frac{-2-i+2+3i}{2} \quad \text{or} \quad z = \frac{-2-i-2-3i}{2} \quad (\text{part (i)}).$$

$$\therefore z = i \quad \text{or} \quad z = -2-2i$$

Name:

Question 7 (8 marks)

Teacher's Name:

Part (a)

Marks

2

$$\begin{aligned}
 \text{(i)} \quad \frac{e^{i\frac{\pi}{6}} - e^{-i\frac{\pi}{6}}}{2i} &= \frac{\cos\frac{\pi}{6} + i\sin\frac{\pi}{6} - (\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6}))}{2i} \\
 &= \frac{2i\sin\frac{\pi}{6}}{2i} \\
 &= \frac{1}{1} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 &= \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} - (\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}) \\
 &= \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} - \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} \\
 &= 2i\sin\frac{\pi}{6}
 \end{aligned}$$

Part (b)

(i)

1

Sub $x=3-2i$ into the equation:

$$\begin{aligned}
 \text{LHS} &= (3-2i)^3 - 8(3-2i)^2 + 15(3-2i) - 26 \\
 &= 27 + 3(3)^2(-2i) + 3(3)(-2i)^2 + (-2i)^3 - 72 + 96i + 32 + 75 - 50i - 26 \\
 &= 27 - 54i - 36 + 8i - 72 + 96i + 32 + 75 - 50i - 26 \\
 &= 0 = \text{RHS} \\
 \therefore x=3-2i \text{ is a root.}
 \end{aligned}$$

(ii)

2

Since all the coefficients are real & $3-2i$ is a root
 $\therefore 3+2i$ is also a root (Conjugate Root Theorem). 1

Let the roots be $3-2i$, $3+2i$ & α .

By the sum of roots:

$$3-2i + (3+2i) + \alpha = 8$$

$$\alpha = 2$$

$\therefore$ The roots of the equation are $3-2i$, $3+2i$, 2 . 1

Question 7 (continued)

Part (c)

Marks

(i)

1

$$z = 1 + \sqrt{3}i + \sqrt{3} + i$$

$$= (1 + \sqrt{3}) + (1 + \sqrt{3})i$$

(ii)

2

$$\arg(1 + \sqrt{3}i) = \frac{\pi}{3} \quad \text{--- } \frac{\pi}{6}$$

$$\arg(\sqrt{3} + i) = \frac{\pi}{6} \quad \text{--- } \frac{\pi}{6}$$

$$\angle AOB = \arg(1 + \sqrt{3}i) - \arg(\sqrt{3} + i)$$

$$\text{The other interior angle of the rhombus } \alpha = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

$$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \quad (\text{co-interior angles in parallel lines})$$

Name:

Question 8 (9 marks)

Teacher's Name:

Marks

(i)

1

$$\begin{aligned}
 z^7 &= -1 \\
 z^7 &= \text{cis}(\pi + 2k\pi) \\
 z &= (\text{cis}(2k\pi)\pi)^{\frac{1}{7}} \\
 &= \text{cis} \frac{2k\pi}{7} \quad \text{De Moivre's theorem} \\
 \text{Let } k=0. \\
 \therefore w &= \text{cis} \frac{\pi}{7}
 \end{aligned}$$

1 r/w

(ii)

1

$$\begin{aligned}
 w^7 &= -1 \\
 (1+w^7) &= 0 \\
 (1+w)(1-w+w^2-w^3+w^4-w^5+w^6) &= 0 \\
 \text{Since } w \text{ is non-real \& } w \neq -1 \\
 \therefore 1-w+w^2-w^3+w^4-w^5+w^6 &= 0
 \end{aligned}$$

1 r/w

(iii)

1

$$\begin{aligned}
 z &= \cos \theta + i \sin \theta \quad z^n = (\cos \theta + i \sin \theta)^n \\
 &= \cos n\theta + i \sin n\theta \quad (\text{De Moivre's theorem}) \\
 \text{Similarly } z^{-n} &= \cos(-n\theta) + i \sin(-n\theta) \\
 &= \cos n\theta - i \sin n\theta \\
 \therefore z^n + z^{-n} &= \cos n\theta + i \sin n\theta + \cos n\theta - i \sin n\theta \\
 &= 2 \cos n\theta
 \end{aligned}$$

1 r/w

Question 8 (continued)

(iv)

2

From part (iii): $w + w^7 = 2\cos\frac{\pi}{7}$ ①

$w^2 + w^6 = 2\cos\frac{2\pi}{7}$ ②

$w^3 + w^5 = 2\cos\frac{3\pi}{7}$ ③

$\therefore ① \times ③ \times ②: 8\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7} = (w + w^7)(w^2 + w^6)(w^3 + w^5)$

$= (w^3 + w^4 + w^1 + w^3)(w^3 + w^5)$

$= w^6 + 1 + w + w^{-4} + w^4 + w^{-2} + 1 + w^{-6}$

$= w^6 + 1 + w^2 - w^{-4}(w^7) + w^4 - w^2(w^7) + 1 - (w^{-6})(w^7)$

since $w^7 = -1$.

$= w^6 + 1 + w^2 - w^3 + w^4 - w^5 + 1 - w$

$= 0 + 1$ (from (i)).

$\therefore \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7} = \frac{1}{8}$

2	Correct
1	Use of fact from (iii) + attempt at expansion of RHS

2

(v)

$\alpha + \beta = w - w^2 - w^4 + w^3 + w^5 - w^6$

$= 1$ (since $1 - w + w^2 - w^3 + w^4 - w^5 + w^6 = 0$.)

$\alpha\beta = (w - w^2 - w^4)(w^3 + w^5 - w^6)$

$= w^4 + w^6 - w^7 - w^5 - w^7 + w^8 - w^7 - w^9 + w^{10}$

$= w^4 + w^6 + 1 - w^5 + 1 + w(w^7) + 1 - w^2(w^7) + w^3(w^7)$

$= w^4 + w^6 + 1 - w^5 + 1 - w + 1 + w^2 - w^3$ (since $w^7 = -1$)

$= 0 + 1 + 1$ (part (ii))

$= 2$

By the sum & product of roots; α & β are roots of the equation

$z^2 - z + 2 = 0$.

2	Correct
1	Sum of roots or product of roots correctly ascertained

Question 8 (continued)

(vi)

2

Now solve $z^2 - z + 2 = 0$

$$z = \frac{1 \pm \sqrt{1 - 4(2)}}{2}$$

$$z = \frac{1 + \sqrt{7}i}{2} \quad \text{or} \quad z = \frac{1 - \sqrt{7}i}{2}$$

$$\alpha = u - w^2 - w^4$$

$$= e^{i\frac{\pi}{7}} - e^{i\frac{2\pi}{7}} - e^{i\frac{4\pi}{7}} \quad (\text{De Moivre's theorem})$$

$$\begin{aligned} \text{Im}(\alpha) &= \sin\frac{\pi}{7} - \sin\frac{2\pi}{7} - \sin\frac{4\pi}{7} \\ &= \sin\frac{\pi}{7} - \sin\frac{2\pi}{7} - \sin\frac{3\pi}{7} \end{aligned}$$

$y = \sin x$ is increasing for $0 < x < \frac{\pi}{2}$.
 $\therefore \sin\frac{\pi}{7} - \sin\frac{2\pi}{7} - \sin\frac{3\pi}{7} < 0$.

$$\begin{aligned} \therefore \sin\frac{\pi}{7} - \sin\frac{2\pi}{7} - \sin\frac{3\pi}{7} &= \text{Im}(\alpha) \\ &= -\frac{\sqrt{7}}{2} \end{aligned}$$

2	correct.
1	part of process achieved.

End of Task

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Blank lined paper with horizontal ruling lines.