SYDNEY BOYS HIGH SCHOOL

NESA Number:

Name:

Maths Class:

2021

YEAR 11
TERM 4
HSC ASSESSMENT
TASK #1

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- **Working time – 60 minutes**
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- Marks may **NOT** be awarded for messy or badly arranged work
- In Questions 6–8, show ALL relevant mathematical reasoning and/or calculations

Total Marks: 38

Section I – 5 marks (pages 2–3)

- Attempt Questions 1–5
- Allow about 8 minutes for this section

Section II – 33 marks (pages 4–6)

- Attempt Questions 6–8
- Allow about 52 minutes for this section

Examiner:

JC

NOT TO BE REMOVED FROM EXAMINATION ROOM

Section I

5 marks
Attempt Questions 1–5
Allow about 8 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–5.

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1 What is the value of $i^{4n-1} + i^{4n+1}$, where $n \in \mathbb{Z}$?

A. $-2i$ B. $2i$ C. 1 D. 0

2 Given that $(x+iy)^{14} = a+ib$, where $x, y, a, b \in \mathbb{R}$.

Which of the following is equal to $(y-ix)^{14}$?

A. $-a-ib$ B. $b-ia$

C. $-b+ia$ D. $-a+ib$

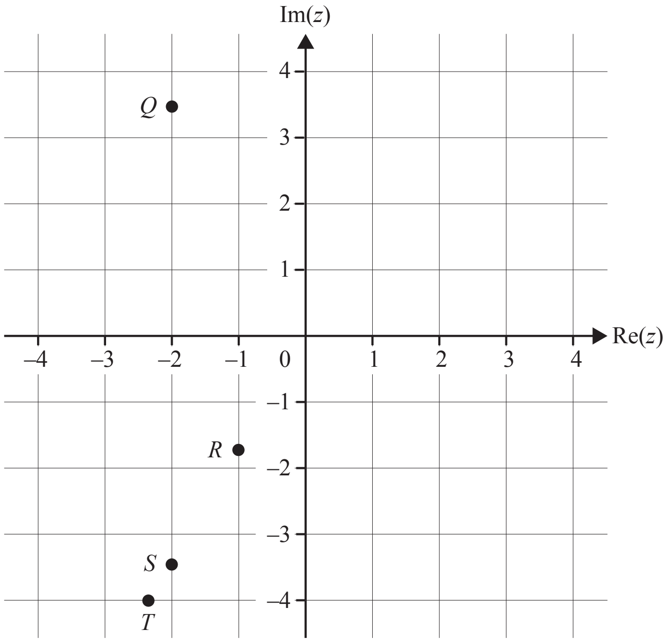
3 Given the complex number $z = a+bi$, where $a, b \in \mathbb{R}$, $a \neq 0$.

Which of the following expression is equivalent to $\dfrac{4z\overline{z}}{(z+\overline{z})^2}$?

A. $4(\operatorname{Re} z \times \operatorname{Im} z)$ B. $1+\left(\dfrac{\operatorname{Im} z}{\operatorname{Re} z}\right)^2$

C. $4\left[(\operatorname{Re} z)^2 + (\operatorname{Im} z)^2\right]$ D. $4\left[1+(\operatorname{Re} z + \operatorname{Im} z)^2\right]$

4 On the Argand diagram below, which point represents the complex number $4e^{-\frac{2\pi i}{3}}$?



A. Q

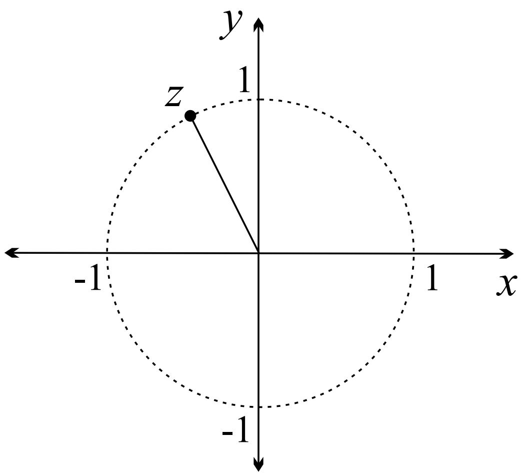
B. R

C. S

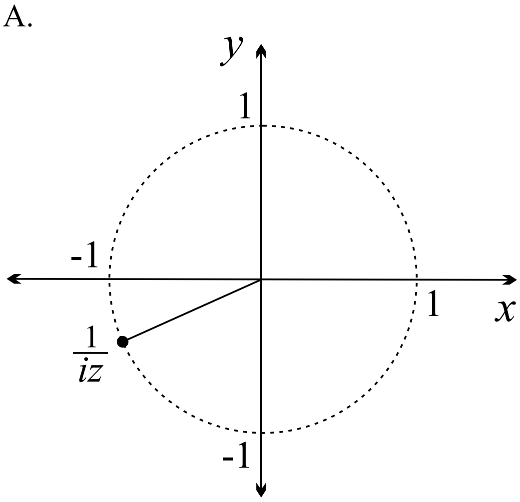
D. T

5 The position of the complex number z is shown on the Argand diagram below.

Which of the following diagrams best represents the position of the complex number $\dfrac{1}{iz}$?



A.



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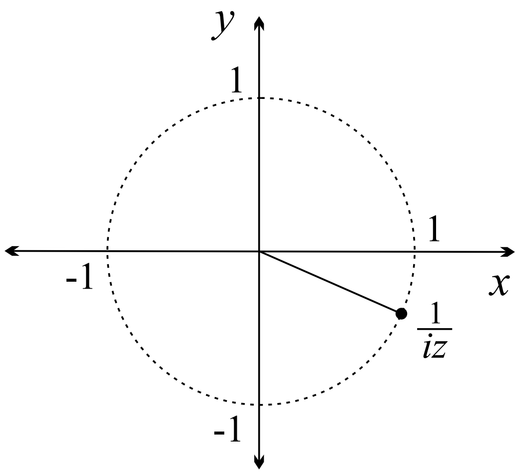
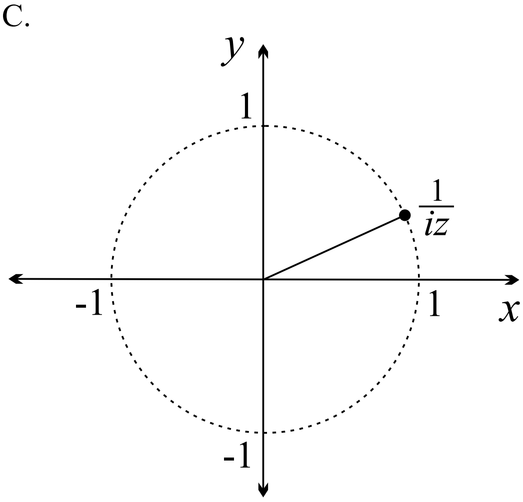
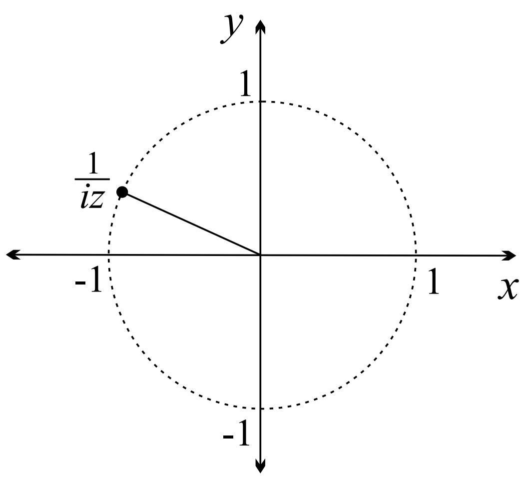


C.



D.



Section II

33 marks
Attempt Questions 6–8
Allow about 52 minutes for this section

In Questions 6–8 your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (11 marks) Start a NEW Answer Booklet

(a) Let $z = \dfrac{2-3i}{1+i}$.

(i) Show that $\overline{z} = -\dfrac{1}{2} + \dfrac{5}{2}i$. **2**

(ii) Find the exact value of $\left| z \right|$. **1**

(b) Let z, w, and u be complex numbers where $w = (1+i)\overline{z}$, $\arg w = \dfrac{\pi}{3}$, $\left| w \right| = 2$, **4**

and $u = \dfrac{z}{2-2i}$.

Express u in modulus-argument form, in exact form.

(c) Consider the complex numbers $\theta = 2\left(\cos\dfrac{\pi}{5} + i\sin\dfrac{\pi}{5}\right)$ and $\phi = 8\left(\cos\dfrac{2k\pi}{5} - i\sin\dfrac{2k\pi}{5}\right)$,

where $k \in \mathbb{Z}^{+}$.

(i) Find the minimum value of k for $\theta\phi$ to be an integer. **3**

(ii) Write down the value of $\theta\phi$ for the value of k found in part (i). **1**

Question 7 (11 marks) Start a NEW Answer Booklet

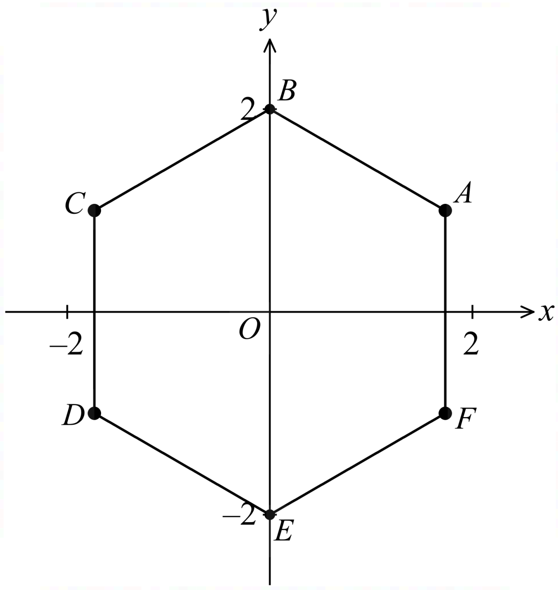
(a) (i) Let $\left(a+ib\right)^2 = -5+12i$, where $a, b \in \mathbb{R}$. **2**

Write down the square roots of $-5+12i$ in the form $a+ib$.

(ii) Hence solve $2z^2 - \left(6+i\right)z + 5 = 0$ for z. **2**

Express your answer in the form $a+ib$.

(b) The diagram below shows the regular hexagon *ABCDEF* drawn on an Argand diagram with vertex *B* at the point (0, 2).
O is the centre of the hexagon.



(i) Find the complex numbers represented by the points *A* and *D*. **2**
Leave in the form $x+iy$.

(ii) The hexagon is rotated anticlockwise about the origin through an angle of $\dfrac{\pi}{4}$. **1**

Express in modulus-argument form the complex number represented by the new position of *F*.

(c) Consider the complex numbers $z = 1+2i$ and $w = 2+ai$, where $a \in \mathbb{R}$.
Find a when

(i) $|\,w\,| = 2|\,z\,|$. **2**

(ii) $\mathrm{Re}\left(zw\right) = 2\,\mathrm{Im}\left(zw\right)$. **2**

Turn Over for Question 8

Question 8 (11 marks) Start a NEW Answer Booklet

(a) Let $z_1 = (1-i)^m$ and $z_2 = \left(1+i\sqrt{3}\right)^n$, where $m, n \in \mathbb{Z}^+$. **4**

Find the smallest positive integers m and n such that $z_1 = z_2$.

(b) Let u and v be non-zero complex numbers. **2**

If $u\overline{v} + \overline{u}v = 0$, what is the relationship between $\arg u$ and $\arg v$? Justify.

(c) (i) Prove that $(1+i\tan\theta)^{2n} + (1-i\tan\theta)^{2n} = 2\sec^{2n}\theta\cos 2n\theta$, $n \in \mathbb{Z}$. **2**

(ii) Show that $\cos^2\dfrac{\pi}{8} = \dfrac{\sqrt{2}+1}{2\sqrt{2}}$. **1**

(iii) Using parts (i) and (ii), show that $\mathrm{Re}\left(1+i\tan\dfrac{\pi}{8}\right)^8 = 64\left(12\sqrt{2}-17\right)$. **2**

End of paper

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Mathematics Extension 2

Sample Solutions

Quick MC Answers

1	D
2	A
3	B
4	C
5	D

NOTE: Before putting in an appeal re-marking, first consider that the mark is not linked to the amount of ink you have used.

Just because you have shown 'working' does not justify that your solution is worth any marks.

If you have used whiteout or pencil, you can not appeal.

1 What is the value of $i^{4n-1} + i^{4n+1}$, where $n \in \mathbb{Z}$?

A. $-2i$ B. $2i$ C. 1 D. 0

$i^4 = 1; i^{-1} = -i$

2 Given that $(x+iy)^{14} = a+ib$, where $x, y, a, b \in \mathbb{R}$.

Which of the following is equal to $(y-ix)^{14}$?

A. $-a-ib$ B. $b-ia$

C. $-b+ia$ D. $-a+ib$

$(y-ix)^{14} = [-i(x+iy)]^{14} = i^{3\times4+2}(a+ib) = -(a+ib)$

3 Given the complex number $z = a+bi$, where $a, b \in \mathbb{R}$, $a \neq 0$.

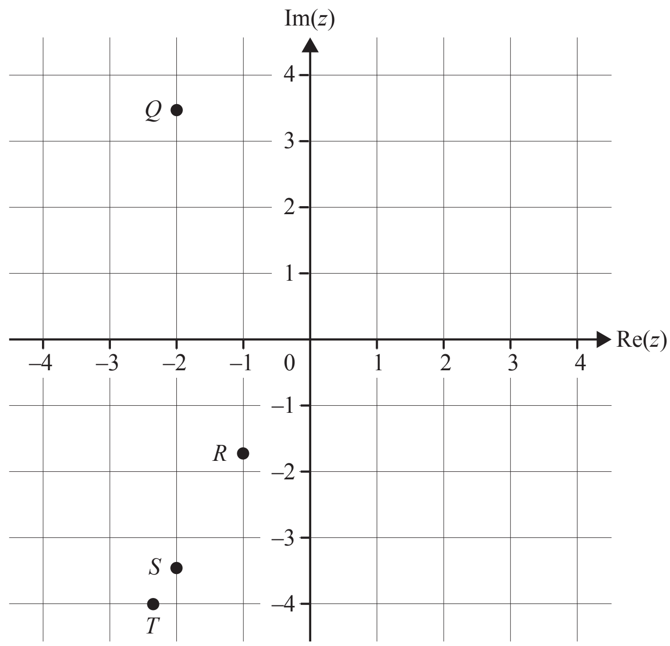
Which of the following expression is equivalent to $\dfrac{4z\bar{z}}{(z+\bar{z})^2}$?

A. $4(\operatorname{Re} z \times \operatorname{Im} z)$ B. $1+\left(\dfrac{\operatorname{Im} z}{\operatorname{Re} z}\right)^2$

C. $4\left[(\operatorname{Re} z)^2 + (\operatorname{Im} z)^2\right]$ D. $4\left[1+(\operatorname{Re} z + \operatorname{Im} z)^2\right]$

$$\frac{4z\bar{z}}{(z+\bar{z})^2} = \frac{4(x^2+y^2)}{(2x)^2} = 1+\frac{y^2}{x^2}$$

4 On the Argand diagram below, which point represents the complex number $4e^{-\frac{2\pi i}{3}}$?



A. Q

B. R

C. S

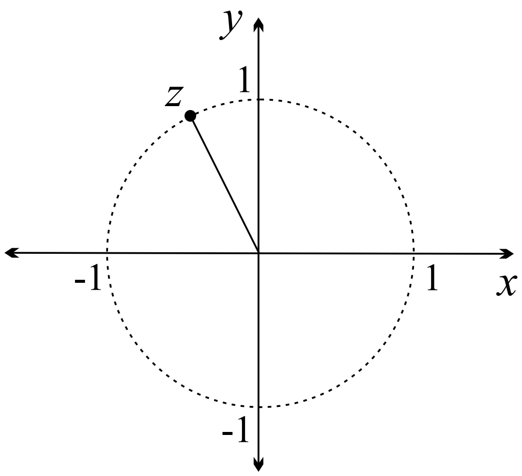
D. T

$e^{-\frac{2\pi i}{3}}$ is in 3rd quad i.e. R, S, or T

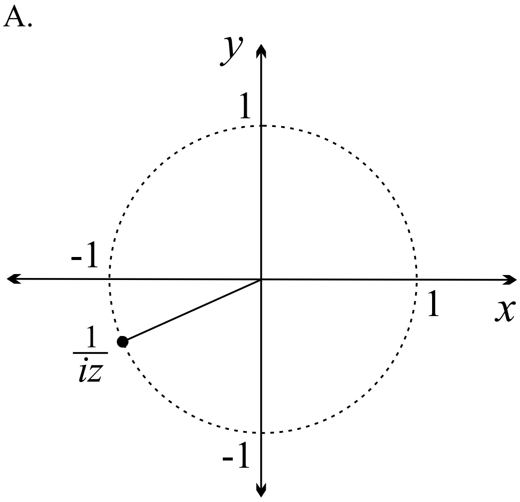
$\cos\left(-\dfrac{2\pi}{3}\right) = -\dfrac{1}{2}$

5 The position of the complex number z is shown on the Argand diagram below.

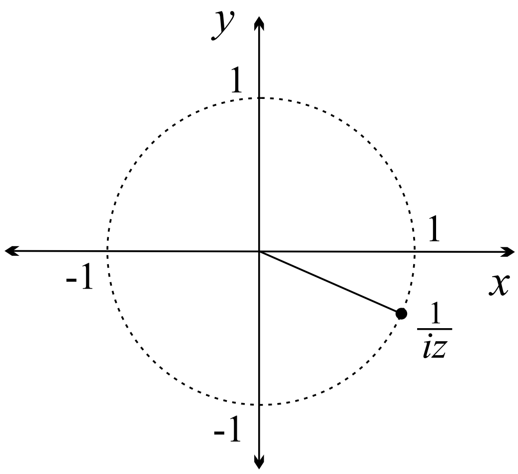
Which of the following diagrams best represents the position of the complex number $\dfrac{1}{iz}$?



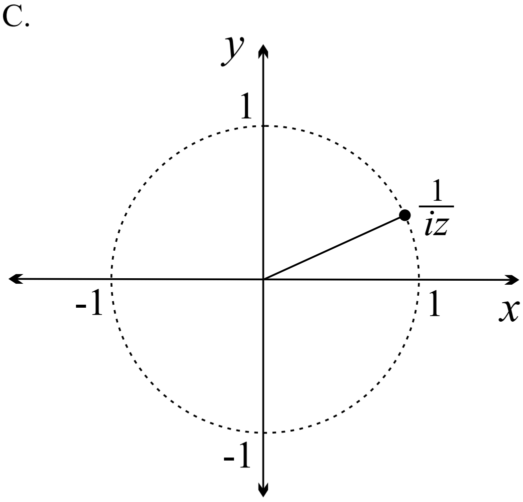
A.



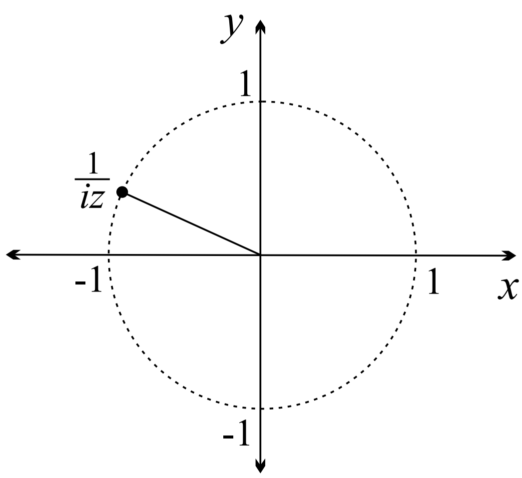
B.



C.



D.



As $|z| = 1$ then $\dfrac{1}{iz} = -i \times \overline{z}$ i.e. a clockwise rotation of 90° from the conjugate of z

Question 6

a) i) $z = \dfrac{2-3i}{1+i} \times \dfrac{1-i}{1-i}$

$= \dfrac{2-2i-3i-3}{1^2+1^2}$

$= -\dfrac{1}{2} - \dfrac{5i}{2}$

$\therefore \bar{z} = -\dfrac{1}{2} + \dfrac{5}{2}i$

OR

$\bar{z} = \overline{\left(\dfrac{2-3i}{1+i}\right)}$

$= \dfrac{\overline{2-3i}}{\overline{1+i}}$

$= \dfrac{2+3i}{1-i} \times \dfrac{1+i}{1+i}$

$= \dfrac{2+2i+3i-3}{1^2+1^2}$

$= -\dfrac{1}{2} + \dfrac{5i}{2}$

ii) $|z| = |\bar{z}| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{5}{2}\right)^2}$

$= \sqrt{\dfrac{26}{4}}$

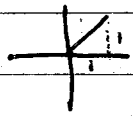
$= \dfrac{\sqrt{26}}{2}$

COMMENT:

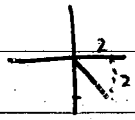
Part (i) is a show that so more working is required.

b) $w = 2e^{\frac{\pi}{3}i}$

$1+i = \sqrt{2}e^{\frac{\pi}{4}i}$



$2-2i = 2\sqrt{2}e^{-\frac{\pi}{4}i}$



$w = (1+i)\bar{z}$

$2e^{\frac{\pi}{3}i} = \sqrt{2}e^{\frac{\pi}{4}i}\bar{z}$

$\therefore \bar{z} = \sqrt{2}e^{\frac{\pi}{12}i}$

$z = \sqrt{2}e^{-\frac{\pi}{12}i}$

$$u = \frac{z}{2-2i}$$

$$= \frac{\sqrt{2}e^{-\frac{\pi}{12}i}}{2\sqrt{2}e^{-\frac{\pi}{4}i}}$$

$$= \frac{1}{2}e^{\frac{\pi}{6}i}$$

$$\therefore u = \frac{1}{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

COMMENT:

Multiplying and dividing complex numbers is best done using the modulus and argument.
Also we are asked to express in modulus-argument form.
Why use cartesian form?

c) i) $\theta = 2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$

$$\phi = 8\left(\cos\frac{2k\pi}{5} - i\sin\frac{2k\pi}{5}\right)$$

$$= 8\left(\cos\left(-\frac{2k\pi}{5}\right) + i\sin\left(-\frac{2k\pi}{5}\right)\right)$$

$$\theta\phi = 16\left(\cos\left(\frac{\pi}{5} + -\frac{2k\pi}{5}\right) + i\sin\left(\frac{\pi}{5} + -\frac{2k\pi}{5}\right)\right)$$

$$= 16\left(\cos\frac{\pi}{5}(1-2k) + i\sin\frac{\pi}{5}(1-2k)\right)$$

Note: For $\theta\phi$ to be an integer it first must be real.
ie $Im(\theta\phi) = 0$

And, if $\theta\phi$ is real it will be an integer since
If $\sin\theta = 0$ then $\cos\theta = \pm 1$.

$$\sin\frac{\pi}{5}(1-2k) = 0$$

$$\frac{\pi}{5}(1-2k) = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$$

$$1-2k = 0, \pm 5, \pm 10, \pm 15, \ldots$$

$$-2k = -1, -6, 4, -11, 9, -16, -14, \ldots$$

$$k = \frac{1}{2}, 3, -2, \frac{11}{2}, -\frac{9}{2}, 8, -7, \ldots$$

$k = 3$ is the minimum value, since $k \in \mathbb{Z}^+$

ii) when $k = 3$

$$\theta\phi = 16\left(\cos\frac{\pi}{5}(1-2(3)) + i\cancel{\sin\frac{\pi}{5}(1-2(3))}^{\,0}\right)$$

$$= 16(-1)$$

$$= -16$$

COMMENT:

$\phi = 8\left(\cos\frac{2k\pi}{5} - i\sin\frac{2k\pi}{5}\right)$ is <u>not</u> in mod-arg form.

$\phi = 8\left(\cos\left(-\frac{2k\pi}{5}\right) + i\sin\left(-\frac{2k\pi}{5}\right)\right)$ is in mod-arg form

The most common mistake was to forget that $k \in \mathbb{Z}^+$ (positive integers)

which $k = \frac{1}{2}$ is <u>not</u>.

Marks could still be gained in part (ii) as long as $\theta\phi$ was an integer.

Question 7

(a)(i) $(a+ib)^2 = -5+12i$

$a^2 - b^2 + 2abi = -5 + 12i$

$\therefore$ real: $a^2 - b^2 = -5 \quad (1)$
imaginary: $2ab = 12 \quad (2)$

also $a^2 + b^2 = 13 \quad (3)$

$(1)+(3)$: $2a^2 = 8$
$a^2 = 4$
$\therefore a = \pm 2$

$(3)-(1)$: $2b^2 = 18$
$b^2 = 9$
$\therefore b = \pm 3$

$\therefore$ The square roots are $2+3i$ and $-2-3i$
$[= \pm(2+3i)]$

[2]

(ii) $2z^2 - (6+i)z + 5 = 0$

$$\therefore z = \frac{-{-(6+i)} \pm \sqrt{[-(6+i)]^2 - 4 \times 2 \times 5}}{2 \times 2}$$

$$= \frac{6+i \pm \sqrt{36 + 12i - 1 - 40}}{4}$$

$$= \frac{6+i \pm \sqrt{-5+12i}}{4}$$

$$= \frac{6+i \pm (2+3i)}{4}$$

$$= \frac{8+4i}{4} \quad \text{or} \quad \frac{4-2i}{4}$$

$$= 2+i \quad \text{or} \quad 1 - \frac{1}{2}i$$

[2]

Comments:

This question was well done.

A few students made small numerical incorrect calculations.

(b)(i) $\angle AOB = \frac{\pi}{3}$, $OB = OA$
$= 2$

B is represented by $2i$

$\therefore A: \quad b = ae^{i\pi/3}$
$\therefore a = be^{-i\pi/3}$
$= 2ie^{-i\pi/3}$
$= 2i\left(\frac{-1}{2} - \frac{\sqrt{3}}{2}i\right)$
$= \sqrt{3} + i$

By symmetry $d = -a$
$= -(\sqrt{3} + i)$
$= -\sqrt{3} - i$

$\therefore A: \sqrt{3} + i$ and $D: -\sqrt{3} - i$ [2]

OR

$B = 2\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$
$= 2i$

Interior angles of a hexagon $= \frac{\pi}{3}$

$\therefore A = 2 \text{ cis } \theta$, $D = 2 \text{ cis } \alpha$

$A = B \times \text{cis}\left(-\frac{\pi}{3}\right)$
$= 2 \text{ cis}\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$
$= 2 \text{ cis}\left(\frac{\pi}{6}\right)$

$D = B \times \text{cis}\left(\frac{2\pi}{3}\right)$
$= 2 \text{ cis}\left(\frac{\pi}{2} + \frac{2\pi}{3}\right)$
$= 2 \text{ cis}\left(\frac{7\pi}{6}\right)$

$= 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$ $\qquad$ $= 2 \text{ cis } \left(\frac{5\pi}{6}\right)$

$= \sqrt{3} + i$ $\qquad$ $= 2\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$

$= -\sqrt{3} - i$ [2]

(ii) By symmetry, $F = -\overline{D}$

$= \sqrt{3} - i \ \left(= 2e^{-i\pi/6}\right)$

When F is rotated anticlockwise by $\frac{\pi}{4}$.

$F' = F \times e^{i\pi/4}$

$= 2e^{-i\pi/6} \times e^{i\pi/4}$

$= 2e^{i\pi/12}$

$\therefore \ F' = 2\left(\cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)\right)$ [1]

Comments:

This question was generally well done.

From part (i) some students made small errors in calculations.

In part (i) and (ii), if students had their answer in the incorrect form, they did not receive full marks.

(c) $z = 1 + 2i$, $w = 2 + ai$

(i) $|w| = 2|z|$

$\therefore |w|^2 = 4|z|^2$

$2^2 + a^2 = 4(1^2 + 2^2)$

$4 + a^2 = 20$

$a^2 = 16$

$\therefore a = \pm 4$ [2]

(ii) $\mathrm{Re}(zw) = 2\,\mathrm{Im}(zw)$

$zw = (1 + 2i)(2 + ai)$

$= 2 + ai + 4i - 2a$

$= 2 - 2a + i(a + 4)$

$\therefore \mathrm{Re}(zw) = 2\,\mathrm{Im}(zw)$

$2 - 2a = 2(a + 4)$

$2 - 2a = 2a + 8$

$4a = -6$

$a = -\frac{3}{2}$ [2]

Comments:

This question was generally well done.
Some students made small errors in calculations.

Question 8

(a) $z_1 = (1-i)^m$

$= \left[\sqrt{2}\left(\cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4}\right)\right]^m$

$= 2^{\frac{m}{2}}\left(\cos\frac{-m\pi}{4} + i\sin\frac{-m\pi}{4}\right)$, by De Moivre's Theorem

$z_2 = (1+\sqrt{3}i)^n$

$= \left[2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^n$

$= 2^n\left(\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}\right)$

Since $z_1 = z_2$, $|z_1| = |z_2|$

$2^{\frac{m}{2}} = 2^n$

$\frac{m}{2} = n$

$m = 2n$

Since $z_1 = z_2$, $\text{Arg}(z_1) = \text{Arg}(z_2)$

$\frac{-m\pi}{4} = \frac{n\pi}{3} + 2k\pi$, where $k \in \mathbb{Z}$

$-3m\pi = 4n\pi + 24k\pi$

$-6 \cdot n = 4n + 24k$, since $m = 2n$

$-10n = 24k$

$n = \frac{-12}{5}k$

Since n and k are both integers, k must be a multiple of 5.
When $k = -5$, $n = 12$, and $m = 24$.

* Poorly done by many students.
Most correctly converted one or both of $(1-i)$ and $(1+\sqrt{3}i)$ to polar form.
Some students did not exponentiate the $\sqrt{2}$ and 2 correctly.
Some students had difficulty converting $(\sqrt{2})^m$ and 2^n to the same base.
Most students attempted to show that $\frac{-m\pi}{4} = \frac{n\pi}{3}$, not recognising that the arguments may differ by a multiple of 2π.

(b) Let $u = r_1 e^{i\theta}$ $\quad v = r_2 e^{i\phi}$
$\bar{u} = r_1 e^{-i\theta}$ $\quad \bar{v} = r_2 e^{-i\phi}$

Note that $\arg(u) = \theta$ and $\arg(v) = \phi$.

$$u\bar{v} + \bar{u}v = 0$$
$$r_1 e^{i\theta} . r_2 e^{-i\phi} + r_1 e^{-i\theta} . r_2 e^{i\phi} = 0$$
$$r_1 r_2 \left(e^{i(\theta-\phi)} + e^{-i(\theta-\phi)}\right) = 0$$
$$2r_1 r_2 . \operatorname{Re}\left(e^{i(\theta-\phi)}\right) = 0$$
$$\cos(\theta - \phi) = 0$$
$$\theta - \phi = (2k+1) . \frac{\pi}{2}, \text{ where } k \in \mathbb{Z}$$
$$\theta = \phi + (2k+1)\frac{\pi}{2}$$
$$\arg(u) = \arg(v) + (2k+1)\frac{\pi}{2}$$

Alt. Method.
$$u\bar{v} + \bar{u}v = 0$$
$$u\bar{v} + \overline{(u\bar{v})} = 0$$
$$2\operatorname{Re}(u\bar{v}) = 0$$
$\therefore u\bar{v}$ is purely imaginary

i.e. $\arg(u\bar{v}) = (2k+1)\frac{\pi}{2}$
$$\arg(u) + \arg(\bar{v}) = (2k+1)\frac{\pi}{2}$$
$$\arg(u) - \arg(v) = (2k+1)\frac{\pi}{2}$$
$$\arg(u) = \arg(v) + (2k+1)\frac{\pi}{2}$$

* Poorly done by many students.
* Most students attempted to use Cartesian form eg $u=x+iy$ etc, but very few made meaningful progress with this method.
* Students who did not clearly express all possible values did not receive full marks.
* Students who arrived at the conclusion that the vectors representing u and v were perpendicular, but who did not explain the relationship between $\arg(u)$ and $\arg(v)$, did not receive full marks.

(c) (i) $$\begin{aligned} LHS &= (1+i\tan\theta)^{2n} + (1-i\tan\theta)^{2n} \\ &= \left(\frac{\cos\theta + i\sin\theta}{\cos\theta}\right)^{2n} + \left(\frac{\cos\theta - i\sin\theta}{\cos\theta}\right)^{2n} \\ &= \sec^{2n}\theta\,(\cos\theta + i\sin\theta)^{2n} + \sec^{2n}\theta\,(\cos(-\theta) + i\sin(-\theta))^{2n} \\ &= \sec^{2n}\theta\,[\cos 2n\theta + i\sin 2n\theta + \cos(-2n\theta) + i\sin(-2n\theta)] \\ &= 2\sec^{2n}\theta.\cos 2n\theta = RHS \end{aligned}$$

* Many students did this question well.
Students who went from $(\cos\theta - i\sin\theta)^{2n}$ to $(\cos 2n\theta - i\sin 2n\theta)$ without an intermediate step did not receive full marks, as De Moivre's Theorem only applies to complex numbers in the form $\cos\theta + i\sin\theta$.

(ii) Consider $\cos 2A = 2\cos^2 A - 1$, letting $A = \frac{\pi}{8}$

$$\begin{aligned} \cos\frac{\pi}{4} &= 2\cos^2\frac{\pi}{8} - 1 \\ 2\cos^2\frac{\pi}{8} &= \cos\frac{\pi}{4} + 1 \\ &= \frac{1}{\sqrt{2}} + 1 \\ \cos^2\frac{\pi}{8} &= \frac{1}{2}\left(\frac{1}{\sqrt{2}} + 1\right) \\ &= \frac{1}{2}\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right) \\ &= \frac{\sqrt{2}+1}{2\sqrt{2}} \text{ as needed.} \end{aligned}$$

* Some students did this well, but many attempted to use the result from (i).

(iii) Let $z = 1 + i\tan\frac{\pi}{8}$

$\bar{z} = 1 - i\tan\frac{\pi}{8}$

$$\left(1+i\tan\frac{\pi}{8}\right)^8 + \left(1-i\tan\frac{\pi}{8}\right)^8 = z^8 + \bar{z}^8$$
$$= z^8 + \overline{(z^8)}$$
$$= 2\operatorname{Re}(z^8)$$
$$= 2\operatorname{Re}\left[\left(1+i\tan\frac{\pi}{8}\right)^8\right]$$

* This can be assumed, as applying it correctly shows it is known.

$$\text{LHS} = \operatorname{Re}\left[\left(1+i\tan\frac{\pi}{8}\right)^8\right]$$

$$= \frac{1}{2}\left[2\sec^8\frac{\pi}{8}\cos\left(\frac{\pi}{8}\times 8\right)\right] \text{, using (i)}$$

$$= \frac{1}{\left(\cos^2\frac{\pi}{8}\right)^4} \times \cos(\pi)$$

$$= \frac{1}{\left(\frac{\sqrt{2}+1}{2\sqrt{2}}\right)^4} \times -1$$

$$= \frac{-(2\sqrt{2})^4}{(\sqrt{2}+1)^4}$$

$$= \frac{-64}{4+8\sqrt{2}+12+4\sqrt{2}+1}$$

$$= \frac{-64}{17+12\sqrt{2}} \times \frac{17-12\sqrt{2}}{17-12\sqrt{2}}$$

$$= \frac{-64(17-12\sqrt{2})}{(17)^2-(12\sqrt{2})^2}$$

$$= 64(12\sqrt{2}-17) = \text{RHS}$$

* Poorly done. Many different arithmetic errors, and students did not know how to use the previous results in proving the equality.