Sydney Girls High School

Mathematics Extension 2

YEAR 12

2023 HSC Assessment Task 3

Reading time:	5 minutes
Writing time:	60 minutes
Total marks:	50
Topics:	Complex Numbers II, Vectors and Integration

General Instructions:

- There are four (4) questions which are almost of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write using a black or blue pen.
- White-out or correction tape is NOT to be used.
- A correct answer without working will be awarded a maximum of 1 mark.
- A Mathematics reference sheet is provided.

Student Name:__________________________ **Teacher Name:**____________________

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Question 1 **12 Marks**

a) Find $\int \frac{dx}{\sqrt{8-2x-x^2}}$. **2**

b)

i) Find the real numbers a, b and c such that $\frac{5}{x^2(2-x)} \equiv \frac{ax+b}{x^2} + \frac{c}{(2-x)}$. **2**

ii) Hence, or otherwise, find $\int \frac{20}{x^2(2-x)}\,dx$. **3**

c) Does the point $A(1,0,3)$ lie on the line $\underset{\sim}{r} = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}$? **2**

Justify your answer.

d)

i) Express $\sin\theta$ in complex exponential form. **1**

ii) Hence, show that $\sin^3\theta = \frac{1}{4}\left(3\sin\theta - \sin 3\theta\right)$. **2**

Question 2 (Start writing on a NEW page) **13 Marks**

a)

i) Find $\int \frac{x^2}{1+x^2}\,dx$. **2**

ii) Use integration by parts to evaluate $\int_0^1 2x \tan^{-1} x \, dx$. **2**

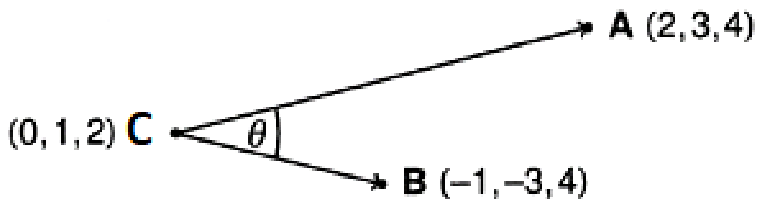
b)

i) Find the values of z for which $z^6 = -64$ where $z = x + iy$. Give your answers in mod-arg form. **2**

ii) Plot the solutions to part i) on an Argand diagram. **2**

iii) The plotted roots are joined to form a polygon. Find the exact area of this polygon. **2**

c) What is the shortest distance between the point **A** and a line passing through points **C** and **B**? **3**



Question 3 (Start writing on a NEW page) **13 Marks**

a) Evaluate $\int_0^2 \sqrt{\frac{8-x}{x}}\, dx.$ **3**

b) A sphere $\mathcal{S}_1$ with centre $\underset{\sim}{c} = 2\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$ passes through $\underset{\sim}{a} = 4\underset{\sim}{i} + 4\underset{\sim}{j} + 4\underset{\sim}{k}$.

i) Find the Cartesian equation of $\mathcal{S}_1$. **2**

ii) A second sphere $\mathcal{S}_2$, has equation $(x-2)^2 + (y-2)^2 + (z-5)^2 = 1$. **3**

Find the equation of the circle on which $\mathcal{S}_1$ and $\mathcal{S}_2$ intersect and state the centre and radius of this circle.

c) Given I_n , where n is an integer, $n \geq 0$:

$$I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x \, dx$$

i) Show that $I_n + I_{n-2} = \frac{1}{n-1}$ for $n \geq 2$. **3**

ii) Evaluate I_1 and hence determine the value of I_5. **2**

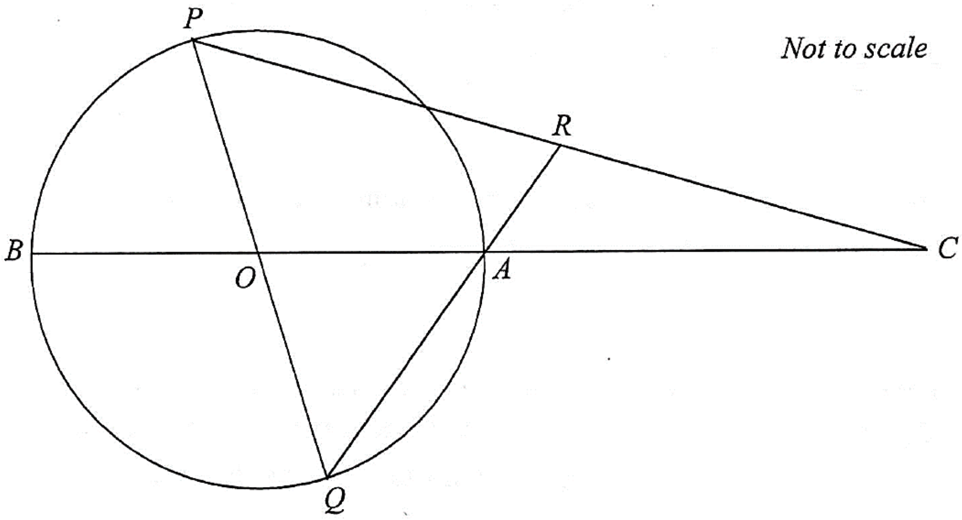
Question 4 (Start writing on a NEW page) **12 Marks**

a) Use the following definite integral property **3**

$$\int_{-a}^{a} f(x)dx = \int_{0}^{a} \{f(x) + f(-x)\}dx$$

to evaluate: $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{e^x \sin^2 3x}{1+e^x}dx.$

b) **4**



In the diagram, PQ and AB are diameters of a circle with centre O.

BA is produced to C so that $BA = AC$. QA produced meets PC at R.

Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OP} = \underset{\sim}{p}$.

Given $\overrightarrow{PR} = \lambda\overrightarrow{PC}$ and $\overrightarrow{QR} = \mu\overrightarrow{QA}$ for some scalars λ and μ, show that R is the midpoint of PC.

Question 4 continues on page 7

c) Consider the equation $z^5 = 1$.

i) Write down, in polar form, the five roots of $z^5 = 1$. **1**

ii) Show that, for $z \neq 1$, **2**

$$\frac{z^5 - 1}{z - 1} = \left(z^2 - 2z\cos\left(\frac{2\pi}{5}\right) + 1\right)\left(z^2 - 2z\cos\left(\frac{4\pi}{5}\right) + 1\right).$$

iii) Deduce that $\cos\left(\frac{2\pi}{5}\right)$ and $\cos\left(\frac{4\pi}{5}\right)$ are roots of the equation $4x^2 + 2x - 1 = 0$. **2**

End of Task

Q1

a) $\displaystyle\int \frac{dx}{\sqrt{8-2x-x^2}}$

$= \displaystyle\int \frac{dx}{\sqrt{9-(x+1)^2}}$ ✓

$= \sin^{-1}\left(\frac{x+1}{3}\right) + c$ ✓

b) i) $\displaystyle\frac{5}{x^2(2-x)} = \frac{ax+b}{x^2} + \frac{c}{2-x}$

$5 = (ax+b)(2-x) + cx^2$ ✓

Coeff of x^2 : $c - a = 0$

coeff of x : $2a - b = 0$

Constant : $2b = 5 \longrightarrow b = \frac{5}{2}$

$2a = b \rightarrow a = \frac{b}{2} = 5/4$ ✓

$c = a \longrightarrow c = 5/4$

ii) $\displaystyle\int \frac{20}{x^2(2-x)}dx = 4\int \frac{5}{x^2(2-x)}dx$

$= 4\left[\displaystyle\int \frac{\frac{5}{4}x + \frac{5}{2}}{x^2}dx + \int \frac{5/4}{(2-x)}dx\right]$ ✓

$= \displaystyle\int \frac{5}{x}dx + \int 10x^{-2}dx + \int \frac{5dx}{(2-x)}$ ✓

$= 5\ln|x| - \frac{10}{x} - 5\ln|2-x| + C$ ✓

Q1 c) If $A(1,0,3)$ lies on

$\underset{\sim}{r} = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}$ then

$\begin{pmatrix} 1+2 \\ 0-3 \\ 3-5 \end{pmatrix} = t\begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}$ must be true for some t. ✓

$\begin{pmatrix} 3 \\ -3 \\ -2 \end{pmatrix} = t\begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}$ (*)

There is No values of t satisfy (*) ✓

$\therefore$ $A(1,0,3)$ does not lie on $\underset{\sim}{r}$.

d) i) $e^{i\theta} = \cos\theta + i\sin\theta$ ①

$e^{-i\theta} = \cos(-\theta) + i\sin(-\theta)$

$e^{-i\theta} = \cos\theta - i\sin(\theta)$ ②

① − ② : $e^{i\theta} - e^{-i\theta} = 2i\sin\theta$

$\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$ ✓

Note: express $\sin\theta$ in the exponential form:

$\therefore$ $\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$

Not $\sin\theta$ in terms of $\cos\theta$.

$\boxed{Q1}$

d(ii) from i) $\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$

$$\sin^3\theta = \left[\frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)\right]^3$$

$$= \frac{-1}{8i}\left[e^{i3\theta} - 3e^{i\theta} + 3e^{-i\theta} - e^{-i3\theta}\right] \checkmark$$

$$= -\frac{1}{8i}\left[e^{i3\theta} - e^{-i3\theta} - 3\left(e^{i\theta} - e^{-i\theta}\right)\right]$$

$$= -\frac{1}{8i}\left[2i\sin 3\theta - 3\left(2i\sin\theta\right)\right]$$

$$= -\frac{1}{4}\sin 3\theta + \frac{3}{4}\sin\theta$$

$$= \frac{1}{4}\left(3\sin\theta - \sin 3\theta\right) \checkmark$$

To get full marks, students must use the result from part (i) as the question stated « hence, show that: »

Question 2

2 (a) (i) $\int \frac{x^2+1-1}{x^2+1}\,dx = \int\left(\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\right)dx$ ✓

$$= \int\left(1 - \frac{1}{x^2+1}\right)dx$$

$$= x - \tan^{-1}x + C \checkmark$$

2 (ii) $\int_0^1 \underbrace{2x}_{v'}\,\underbrace{\tan^{-1}x}_{u}\,dx$

$v = x^2$

$v' = 2x$

$u = \tan^{-1}x$

$u' = \frac{1}{x^2+1}$

$$= [uv]_0^1 - \int_0^1 u'v\,dx \checkmark$$

$$= \left[x^2\tan^{-1}x\right]_0^1 - \int_0^1 \frac{x^2}{x^2+1}\,dx$$

$$= 1^2\tan^{-1}1 - \left[x - \tan^{-1}x\right]_0^1 \quad \text{(using (i))}$$

$$= \frac{\pi}{4} - \left(1 - \frac{\pi}{4}\right) \checkmark$$

$$= \frac{\pi}{2} - 1$$

2

(b) (i) Let $z = r\,\mathrm{cis}\,\theta$

$\therefore (r\,\mathrm{cis}\,\theta)^6 = -64$

$r^6\,\mathrm{cis}\,6\theta = 64\,\mathrm{cis}(\pi + 2\pi k)$ ✓ (de Moivre)

Equate moduli and arguments:

$r^6 = 64$ and For $k = -2, -1, 0, 1, 2, 3$:

$r = \sqrt[6]{64}$ $\qquad 6\theta = \pi + 2\pi k$

$r = 2$ $\qquad \theta = \dfrac{\pi + 2\pi k}{6}$

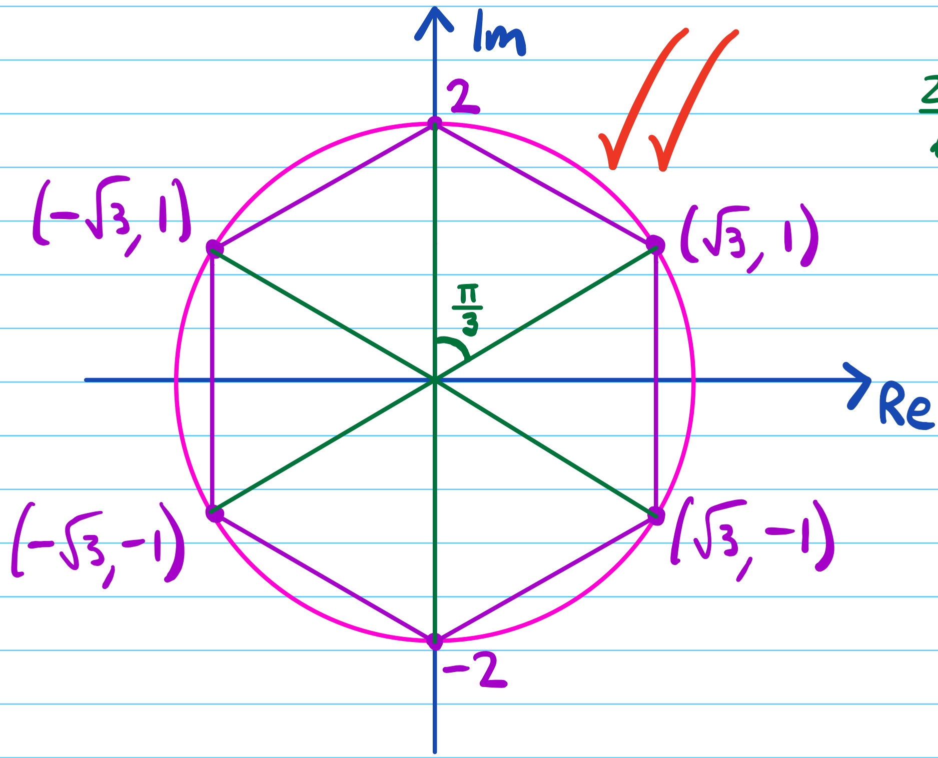
$\theta = -\dfrac{\pi}{2}, -\dfrac{\pi}{6}, \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}$

$\therefore z = 2\,\mathrm{cis}\left(\pm\dfrac{\pi}{2}\right), 2\,\mathrm{cis}\left(\pm\dfrac{\pi}{6}\right), 2\,\mathrm{cis}\,\dfrac{5\pi}{6}, 2\,\mathrm{cis}\,\dfrac{7\pi}{6}$

$= \pm 2i, 2\left(\dfrac{\sqrt{3}}{2} \pm \dfrac{i}{2}\right), 2\left(-\dfrac{\sqrt{3}}{2} \pm \dfrac{i}{2}\right)$ ✓

$= \pm 2i, \sqrt{3} \pm i, -\sqrt{3} \pm i$

2 (ii)

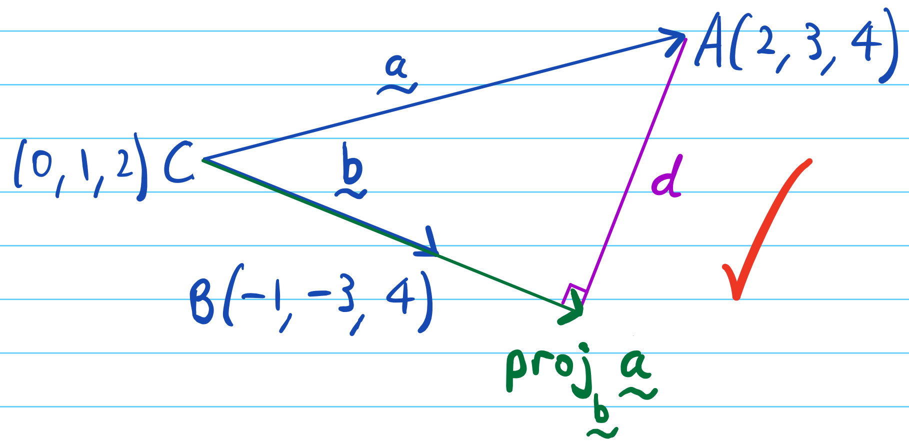


$\frac{2\pi}{6} = \frac{\pi}{3}$

2 (iii) Construct radii as shown above. (green)
This creates six congruent equilateral triangles. The exact area of the hexagon is therefore

$$6 \times \text{Area}\,\triangle = 6 \times \tfrac{1}{2} ab \sin C$$
$$= 6 \times \tfrac{1}{2}(2)(2) \sin \tfrac{\pi}{3}$$
$$= 12\left(\tfrac{\sqrt{3}}{2}\right)$$
$$= 6\sqrt{3} \text{ square units}$$

3 (c)



The shortest distance between a point and a line is the perpendicular distance, so we first find the projection of $\overrightarrow{CA}$ onto $\overrightarrow{CB}$:

$$\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a} = \frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\underset{\sim}{b}$$

$$= \frac{\begin{pmatrix}2\\2\\2\end{pmatrix}\cdot\begin{pmatrix}-1\\-4\\2\end{pmatrix}}{\begin{pmatrix}-1\\-4\\2\end{pmatrix}\cdot\begin{pmatrix}-1\\-4\\2\end{pmatrix}}\begin{pmatrix}-1\\-4\\2\end{pmatrix}$$

$$= \frac{-2-8+4}{1+16+4}\begin{pmatrix}-1\\-4\\2\end{pmatrix}$$

$$= -\frac{2}{7}\begin{pmatrix}-1\\-4\\2\end{pmatrix}$$

Note:

$$\underset{\sim}{a} = \begin{pmatrix}2\\3\\4\end{pmatrix} - \begin{pmatrix}0\\1\\2\end{pmatrix} = \begin{pmatrix}2\\2\\2\end{pmatrix}$$

$$\underset{\sim}{b} = \begin{pmatrix}-1\\-3\\4\end{pmatrix} - \begin{pmatrix}0\\1\\2\end{pmatrix} = \begin{pmatrix}-1\\-4\\2\end{pmatrix}$$

$$\therefore d = \left|\underset{\sim}{a} - \text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}\right|$$

$$= \left|\begin{pmatrix}2\\2\\2\end{pmatrix} - -\frac{2}{7}\begin{pmatrix}-1\\-4\\2\end{pmatrix}\right|$$

$$= \frac{6\sqrt{14}}{7} \text{ units}$$

Question 3 (Start writing on a NEW page) **13 Marks**

a) Evaluate $\int_0^2 \sqrt{\frac{8-x}{x}}\, dx$. **3**

$$= \int_0^2 \frac{\sqrt{8-x}}{\sqrt{x}} \times \frac{\sqrt{8-x}}{\sqrt{8-x}}$$ "rationalise the numerator"

$$= \int_0^2 \frac{8-x}{\sqrt{8x-x^2}}$$ ✓

$$= \int_0^2 \frac{4-x}{\sqrt{8x-x^2}} + \frac{4}{\sqrt{8x-x^2}}$$

$$= \int_0^2 (4-x)(8x-x^2)^{-\frac{1}{2}}\,dx + \int_0^2 \frac{4}{\sqrt{16-(x-4)^2}}\,dx$$

$$= \int_0^2 \frac{1}{2}(8-2x)(8x-x^2)^{-\frac{1}{2}}\,dx + \int_0^2 \frac{4}{\sqrt{16-(x-4)^2}}\,dx$$

$$= \left[\sqrt{8x-x^2}\right]_0^2 + \left[4\sin^{-1}\left(\frac{x-4}{4}\right)\right]_0^2$$ ✓

$$= \sqrt{12} + 4\left(-\frac{\pi}{6}+\frac{\pi}{2}\right)$$

$$= 2\sqrt{3} + \frac{4\pi}{3}$$ ✓ 3

Comment:

- This question was marked generously.
- Half the students didn't realise that they needed to 'rationalise the numerator' as the first step.
- Refer to the SGHS booklet examples for revision.

b) A sphere S_1 with centre $\underset{\sim}{c} = 2\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$ passes through $\underset{\sim}{a} = 4\underset{\sim}{i} + 4\underset{\sim}{j} + 4\underset{\sim}{k}$.

i) Find the Cartesian equation of S_1. **2**

ii) A second sphere S_2, has equation $(x-2)^2 + (y-2)^2 + (z-5)^2 = 1$. **3**

Find the equation of the circle on which S_1 and S_2 intersect and state the centre and radius of this circle.

i) The radius of the sphere is given by:

$$r = |\underset{\sim}{a} - \underset{\sim}{c}|$$
$$= \sqrt{2^2+2^2+2^2}$$
$$= \sqrt{12} \checkmark$$

$\therefore$ The cartesian equation of S_1:

$$(x-2)^2 + (y-2)^2 + (z-2)^2 = 12 \checkmark$$

ii) Solving simultaneously:

$$(x-2)^2 + (y-2)^2 + (z-2)^2 = 12$$
$$(x-2)^2 + (y-2)^2 + (z-5)^2 = 1$$

subtracting equation S_2 from S_1

$$(z-2)^2 - (z-5)^2 = 12-1 \checkmark$$

one mark for process of solving simultaneous equations.

$$z^2 - 4z + 4 - (z^2 - 10z + 25) = 11$$
$$z^2 - 4z + 4 - z^2 + 10z - 25 = 11$$
$$6z - 21 = 11$$
$$6z = 32$$
$$z = \frac{32}{6} = \frac{16}{3}.$$

Substitute $z = \frac{16}{3}$ into S_2:

$$(x-2)^2 + (y-2)^2 + \left(\frac{16}{3} - 2\right)^2 = 12$$

one mark for eqn of circle.

$$\therefore \quad (x-2)^2 + (y-2)^2 = \frac{8}{9} \checkmark$$ which is the equation of the circle.

The circle has centre $= \left(2, 2, \frac{16}{3}\right) \checkmark$ and radius $r = \frac{\sqrt{8}}{3}$.

one mark for centre.

Comment:

* Many students lost marks because they made errors when solving simultaneous equations.
* Students needed to show a 2-D equation for the circle and centre $= (x, y, z) = \left(2, 2, \frac{16}{3}\right)$.

c) Given I_n, where n is an integer, $n \geq 0$:

$$I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x \, dx$$

i) Show that $I_n + I_{n-2} = \frac{1}{n-1}$ for $n \geq 2$. **3**

ii) Evaluate I_1 and hence determine the value of I_5. **2**

i) $I_n = \int_{\pi/4}^{\pi/2} \cot^n x \, dx$

$= \int_{\pi/4}^{\pi/2} \cot^2 x \cot^{n-2} x \, dx$

$= \int_{\pi/4}^{\pi/2} (\operatorname{cosec}^2 x - 1) \cot^{n-2} x \, dx$

$= \int_{\pi/4}^{\pi/2} \operatorname{cosec}^2 x \cot^{n-2} x \, dx - \int_{\pi/4}^{\frac{\pi}{2}} \cot^{n-2} x \, dx$

$= \left[-\frac{\cot^{n-1} x}{n-1} \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - I_{n-2}$

$\therefore I_n + I_{n-2} = \frac{\cot^{n-1} \frac{\pi}{4}}{n-1} - \cancel{\frac{\cot^{n-1} \pi/2}{n-1}}^{\,0}$

$\therefore I_n + I_{n-2} = \frac{1}{n-1}$ for $n \geq 2$.

Carefully shown last step.

Comment: This was the simplest approach to answer this question. Refer to SGHS booklet for revision.

ii) $I_1 = \int_{\pi/4}^{\pi/2} \cot x \, dx$

$= \int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} \, dx$

$= \left[\ln|\sin x| \right]_{\pi/4}^{\pi/2}$

$= \ln\left|\sin \frac{\pi}{2}\right| - \ln\left|\sin \frac{\pi}{4}\right|$

$= \ln 1 - \ln \frac{1}{\sqrt{2}}$

$= \frac{1}{2} \ln 2$. (or $\ln\sqrt{2}$)

$I_5 + I_3 = \frac{1}{4}$ $(n=5)$

$I_5 = \frac{1}{4} - I_3$

$= \frac{1}{4} - \left(\frac{1}{2} - I_1 \right)$ $(n=3)$

$= -\frac{1}{4} + I_1$

$= -\frac{1}{4} + \frac{1}{2} \ln 2$

* Many students made errors evaluating I_1 and hence I_5
* Carry errors were awarded 1 mark.

Question 4

(a) $$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{e^x \sin^2 3x}{1+e^x}\,dx = \int_0^{\frac{\pi}{3}} \frac{e^x \sin^2 3x}{1+e^x} + \frac{e^{-x}\sin^2(-3x)}{1+e^{-x}}\,dx$$ ✓

$$= \int_0^{\frac{\pi}{3}} \frac{e^x \sin^2 3x}{1+e^x} + \frac{\sin^2 3x}{e^x+1}\,dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{\sin^2 3x\,(e^x+1)}{1+e^x}\,dx$$

$$= \int_0^{\frac{\pi}{3}} \sin^2 3x\,dx$$ ✓

$$= \frac{1}{2}\int_0^{\frac{\pi}{3}} 1-\cos 6x\,dx$$

$$= \frac{1}{2}\left[x - \frac{1}{6}\sin 6x\right]_0^{\frac{\pi}{3}}$$

$$= \frac{1}{2}\left(\frac{\pi}{3} - \cancel{\frac{1}{6}\sin 2\pi} - 0\right)$$

$$= \frac{\pi}{6}$$ ✓

Comment

Most students were successful in applying the given property.

However, many students had difficulty in going from $\frac{e^{-x}\sin^2(-3x)}{1+e^{-x}}$ to $\frac{\sin^2 3x}{e^x+1}$. $\times \frac{e^x}{e^x}$

$\left\langle \text{Note: } \sin^2(-3x) = \left(\sin(-3x)\right)^2 = \left(-\sin 3x\right)^2 = \sin^2 3x \right\rangle$

(b) $\overrightarrow{PR} = \lambda \overrightarrow{PC}$

$= \lambda(\overrightarrow{OC} - \overrightarrow{OP})$

$= \lambda(3\underset{\sim}{a} - \underset{\sim}{p})$ ✓ Finds $\overrightarrow{PR}$ (or equivalent merit)

Also,

$\overrightarrow{PR} = \overrightarrow{QR} - \overrightarrow{QP}$

$= \mu \overrightarrow{QA} - 2\underset{\sim}{p}$

$= \mu(\underset{\sim}{p} + \underset{\sim}{a}) - 2\underset{\sim}{p}$ ✓ Finds another expression for $\overrightarrow{PR}$ (or another expression for the previously obtained vector)

$\therefore \lambda(3\underset{\sim}{a} - \underset{\sim}{p}) = \mu(\underset{\sim}{p} + \underset{\sim}{a}) - 2\underset{\sim}{p}$

$3\lambda\underset{\sim}{a} - \lambda\underset{\sim}{p} = \mu\underset{\sim}{a} + (\mu - 2)\underset{\sim}{p}$

$$\begin{cases} 3\lambda = \mu \\ -\lambda = \mu - 2 \end{cases}$$

✓

$-\lambda = 3\lambda - 2$

$\lambda = \frac{1}{2}$

$\therefore \overrightarrow{PR} = \lambda \overrightarrow{PC}$

$= \frac{1}{2} \overrightarrow{PC}$

ie R is the midpt of PC.

✓

Comment Poorly done.

(c)

(i) $z^5 = 1$

$= \text{cis}(0 + 2k\pi)$

$= \text{cis}(2k\pi)$

$z = \text{cis}\left(\frac{2k\pi}{5}\right), \quad k = 0, \pm 1, \pm 2$

$= \text{cis}\,0, \ \text{cis}\left(\pm\frac{2\pi}{5}\right), \ \text{cis}\left(\pm\frac{4\pi}{5}\right)$

1

(ii) $z^5 - 1 = (z-1)\left(z - \text{cis}\frac{2\pi}{5}\right)\left(z - \text{cis}\left(-\frac{2\pi}{5}\right)\right)\left(z - \text{cis}\frac{4\pi}{5}\right)\left(z - \text{cis}\left(-\frac{4\pi}{5}\right)\right)$

$= (z-1)\left(z^2 - 2\cos\frac{2\pi}{5}z + 1\right)\left(z^2 - 2\cos\frac{4\pi}{5}z + 1\right)$

$\therefore \frac{z^5 - 1}{z - 1} = \left(z^2 - 2\cos\frac{2\pi}{5}z + 1\right)\left(z^2 - 2\cos\frac{4\pi}{5}z + 1\right)$

(iii) $z^5 - 1 = (z-1)(1 + z + z^2 + z^3 + z^4)$

$\therefore$ From (ii),

$1 + z + z^2 + z^3 + z^4 = \left(z^2 - 2\cos\frac{2\pi}{5}z + 1\right)\left(z^2 - 2\cos\frac{4\pi}{5}z + 1\right)$

Equate coeff of z:

$1 = -2\cos\frac{2\pi}{5} - 2\cos\frac{4\pi}{5}$

$\therefore \cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}$

Equate coeff of z^2:

$$1 = 1 + 4\cos\frac{2\pi}{5}\cos\frac{4\pi}{5} + 1$$

$$\therefore \cos\frac{2\pi}{5}\cos\frac{4\pi}{5} = -\frac{1}{4}$$

$\therefore \cos\frac{2\pi}{5}$ and $\cos\frac{4\pi}{5}$ are roots of

$$x^2 + \frac{1}{2}x - \frac{1}{4} = 0$$

ie. $4x^2 + 2x - 1 = 0$

Comment

Part (iii) was poorly done.

Students were asked to deduce (ie use previous result(s)).

Hence, other methods (such as substitution or using the quadratic formula and exact values for $\cos\frac{2\pi}{5}$ and $\cos\frac{4\pi}{5}$) were not awarded any marks.