
CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL

2018 Half-Yearly Examination

FORM VI

MATHEMATICS EXTENSION 2

Thursday 22nd February 2018

General Instructions

- Writing time — 1 hour 30 minutes
- Write using black pen.
- Board-approved calculators and templates may be used.

Total — 55 Marks

- All questions may be attempted.

Section I — 7 Marks

- Questions 1 – 7 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II — 48 Marks

- Questions 8–11 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your candidate number on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your candidate number on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Eight.

Checklist

- SGS booklets — 4 per boy
- Multiple choice answer sheet
- Reference Sheet
- Candidature — 74 boys

Examiner

LYL

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

Which expression is equivalent to $(1-i)^3$?

(A) $2-2i$

(B) $-2-2i$

(C) $2+2i$

(D) $-2+2i$

QUESTION TWO

Given the complex numbers $z = 3i$ and $w = 1 - i\sqrt{5}$, what is the value of $\dfrac{z}{w}$?

(A) $-\frac{\sqrt{5}}{2} + \frac{i}{2}$

(B) $-\frac{3\sqrt{5}}{2} + \frac{3i}{2}$

(C) $-\frac{\sqrt{5}}{6} + \frac{i}{6}$

(D) $-\frac{\sqrt{5}}{2} - \frac{i}{2}$

QUESTION THREE

What is $-\sqrt{3} + i$ expressed in modulus–argument form?

(A) $\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(B) $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(C) $\sqrt{2}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

(D) $2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

QUESTION FOUR

What is the general solution for the equation $\sin 2x = \sin \frac{\pi}{4}$?

(A) $x = \frac{\pi}{4} + 2n\pi$ or $x = \frac{3\pi}{4} + 2n\pi$ where n is an integer.

(B) $x = \frac{\pi}{8} + n\pi$ or $x = -\frac{\pi}{8} + n\pi$ where n is an integer.

(C) $x = \frac{\pi}{8} + n\pi$ or $x = \frac{3\pi}{8} + n\pi$ where n is an integer.

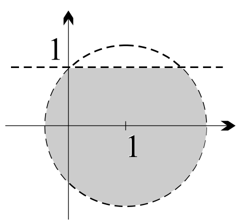
(D) $x = \frac{\pi}{2} + 4n\pi$ or $x = \frac{3\pi}{2} + 4n\pi$ where n is an integer.

QUESTION FIVE

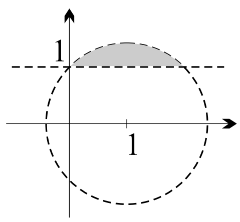
Which Argand diagram best represents the locus of z such that

$|z - 1| \leq \sqrt{2}$ and $\mathrm{Im}(z + 4 + 3i) > 4$?

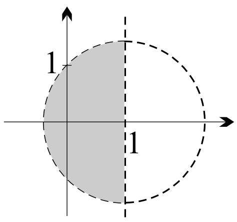
(A)



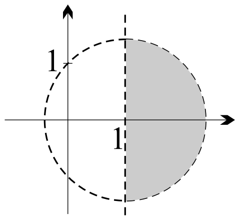
(B)



(C)



(D)



QUESTION SIX

Let $z = x + iy$ and z, $\overline{z}$, $-z$, $i\overline{z}$ and iz be represented by the points P, Q, R, S and T on the Argand diagram. Which geometric statement is incorrect?

(A) The points Q and P are reflections of each other in the real axis.

(B) The points R and P are rotations of each other by π about the origin.

(C) The point S is a reflection of P in the line $y = -x$.

(D) The point T is the result of rotating P by $\frac{\pi}{2}$ anticlockwise about the origin.

Examination continues overleaf . . .

QUESTION SEVEN

Which integral is equivalent to $\displaystyle\int \sqrt{\tfrac{x+1}{x+3}}\, dx$?

(A) $\displaystyle\int \frac{\sqrt{x^2-4x+3}}{x+3}\, dx$

(B) $\displaystyle\int \sqrt{\frac{x^2+2x-3}{x^2-9}}\, dx$

(C) $\displaystyle\int \frac{x+1}{\sqrt{x^2-2x-1}}\, dx$

(D) $\displaystyle\int \frac{x+1}{\sqrt{x^2+4x+3}}\, dx$

End of Section I

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION EIGHT (12 marks) Use a separate writing booklet. **Marks**

(a) Let $z = 4 - 5i$ and $w = -4 + i$. Find:

(i) $|z - w|$ **1**

(ii) $\arg(z + w)$ **1**

(b) Let $z = x + iy$. Prove that:

(i) $\text{Re}(z + \overline{z}) = 2x$ **1**

(ii) $z\overline{z} = |z|^2$ **1**

(c) Differentiate $x^2 - y^3 + xy^2 = 3$ with respect to x and hence find $\dfrac{dy}{dx}$. **2**

(d) Find $\displaystyle\int \frac{6x + 4}{x^2 + 16}\, dx$. **2**

(e) (i) Find the two square roots of $-21 - 20i$. **2**

(ii) Hence, or otherwise, solve $z^2 - (4 + i)z + 9 + 7i = 0$. **2**

Examination continues overleaf ...

QUESTION NINE (12 marks) Use a separate writing booklet. **Marks**

(a) Determine $\displaystyle\int \sin^4 x \cos^3 x \, dx$. **2**

(b) Use mathematical induction to prove that **3**

$$n! \geq 2^{n-1}$$

for all positive integers n.
[NOTE: by definition $n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$]

(c) (i) Find real numbers A, B and C such that **2**

$$\frac{9x^2}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}.$$

(ii) Hence find $\displaystyle\int \frac{9x^2}{(x-1)^2(x+2)} \, dx$. **1**

(d) Using integration by parts, find $\displaystyle\int xe^{2x} \, dx$. **2**

(e) In an Argand diagram, the points P and Q represent the complex numbers w and $\frac{1}{w}$ respectively. The interval PQ is produced to R such that $QR = 2PQ$. Find the complex number represented by the point R. Leave your answer in terms of w. **2**

Examination continues next page ...

QUESTION TEN (12 marks) Use a separate writing booklet. **Marks**

(a) Find $\displaystyle\int \frac{1}{x(\log_e x)^3}\,dx$. **1**

(b) (i) Show that the derivative of $\sec\theta$ is $\sec\theta\tan\theta$. **1**

(ii) Use the substitution $x = \sec\theta - 1$ to find the exact value of **3**

$$\int_{\sqrt{2}-1}^{1} \frac{2}{(x+1)\sqrt{x^2+2x}}\,dx.$$

(c) Let $I_n = \displaystyle\int_1^e \frac{(\ln x)^n}{x^2}\,dx$ for $n \geq 0$.

(i) Show that $I_n = -\frac{1}{e} + nI_{n-1}$ for $n \geq 1$. **2**

(ii) Hence find I_2. **2**

(d) The complex number z satisfies $|z - 3i| \leq 4$.

(i) Find the least possible value of $|z - 5 + 2i|$. **2**

(ii) For what value of z is the least possible value attained? **1**

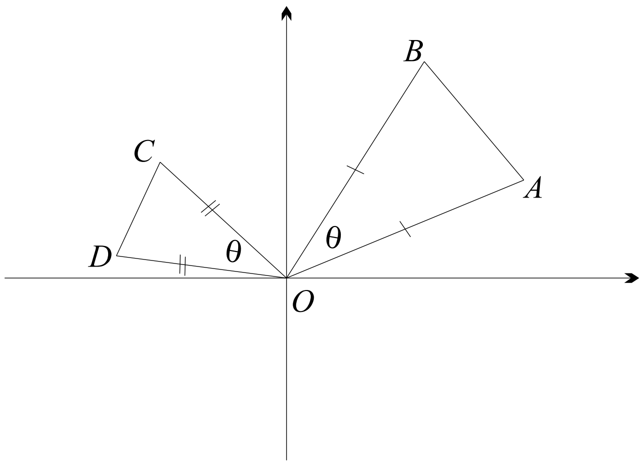
Examination continues overleaf ...

QUESTION ELEVEN (12 marks) Use a separate writing booklet. **Marks**

(a) Sketch the locus of the points such that $\left|z^2 - \overline{z}^2\right| < 4$. **2**

(b) Use the substitution $t = \tan\frac{x}{2}$ to determine $\displaystyle\int \frac{1}{4 + 3\sin x}dx$. **3**

(c)



The Argand diagram above shows the points O, A, B, C and D. These points represent the complex numbers, 0, a, b, c and d respectively. Triangles OAB and OCD are similar and isosceles with $\angle AOB = \angle COD = \theta$ where θ is acute.

Let $\omega = \cos\theta + i\sin\theta$.

(i) Explain why $d = \omega c$. **1**

(ii) Find the complex number a in terms of b. **1**

(iii) Using complex numbers, show that the lengths AC and BD are equal. **2**

(d) Find $\displaystyle\int xe^x \sin x\, dx$. **3**

End of Section II

END OF EXAMINATION

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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ◯ B ◯ C ◯ D ◯

Question Two

A ◯ B ◯ C ◯ D ◯

Question Three

A ◯ B ◯ C ◯ D ◯

Question Four

A ◯ B ◯ C ◯ D ◯

Question Five

A ◯ B ◯ C ◯ D ◯

Question Six

A ◯ B ◯ C ◯ D ◯

Question Seven

A ◯ B ◯ C ◯ D ◯

HALF-YEARLY EXTENSION 2, 2018

MC

Q1 $(1-i)^3$ Let $z = 1-i$

$$z^2 = (1-i)^2$$
$$= -2i$$
$$z^3 = z^2 \times z$$
$$= -2i(1-i)$$
$$= -2i - 2$$

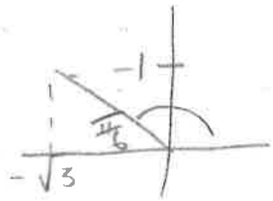
B

Q2 $\dfrac{3i}{1-i\sqrt{5}} \times \dfrac{1+i\sqrt{5}}{1+i\sqrt{5}} = \dfrac{3i - 3\sqrt{5}}{6}$

$$= \frac{i}{2} - \frac{\sqrt{5}}{2}$$

A

Q3 $-\sqrt{3} + i = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$



$\theta = \frac{5\pi}{6}$

$r = 2$

D

Q4 $\sin 2x = \sin\frac{\pi}{4}$

$2x = \frac{\pi}{4} + 2n\pi$ or $2x = \left(\pi - \frac{\pi}{4}\right) + 2n\pi$

$x = \frac{\pi}{8} + n\pi$ $\quad 2x = \frac{3\pi}{4} + 2n\pi$

C $\quad x = \frac{3\pi}{8} + n\pi$

MC

Q 5 $|z-1| \leq \sqrt{2}$

region inside a circle centre (1,0)
radius $\sqrt{2}$
solid boundary

$\text{Im}(z + 4 + 3i) > 4$

$z + 4 + 3i = z - (-4 - 3i)$

let $z = x + iy$

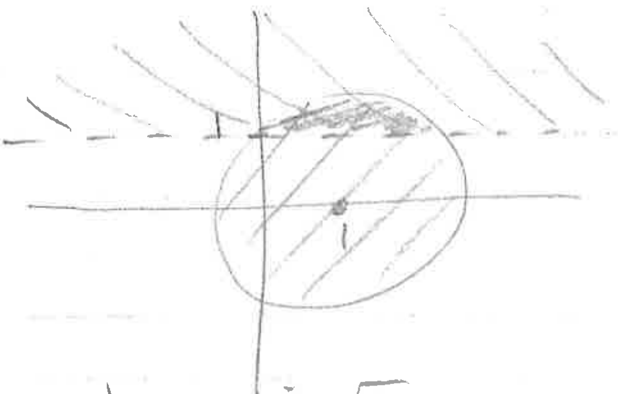
$x + iy + 4 + 3i = x + 4 + i(y+3)$

$\text{Im}(z + 4 + 3i) = y + 3$

$y + 3 > 4$

$y > 1$

Dotted horizontal line $y = 1$
shaded above the line



B

$|z-1| \leq \sqrt{2}$ and $\text{Im}(z + 4 + 3i) > 4$

MC/Q6

The point S is a reflection of P in line $y = x$ NOT $y = -x$

C

Q 7

$$\int \sqrt{\frac{x+1}{x+3}} \times \frac{\sqrt{x+1}}{\sqrt{x+1}} \, dx$$

$$= \int \frac{x+1}{\sqrt{(x+3)(x+1)}} \, dx$$

$$= \int \frac{x+1}{\sqrt{x^2+4x+3}} \, dx$$

D

Section II

Q8 $\quad z = 4 - 5i \qquad w = -4 + i$

a) i) $z - w = 4 - 5i - (-4 + i)$

$= 8 - 6i$

$|z - w|^2 = 8^2 + (-6)^2$

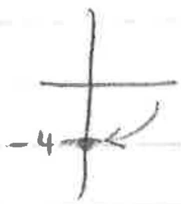
$= 100$

$|z - w| = 10$

ii) $z + w = 4 - 5i + (-4 + i)$

$= -4i$

$\arg(z + w) = -\frac{\pi}{2}$



b) i) Since $z = x + iy \qquad \bar{z} = x - iy$

$z + \bar{z} = x + iy + x - iy$

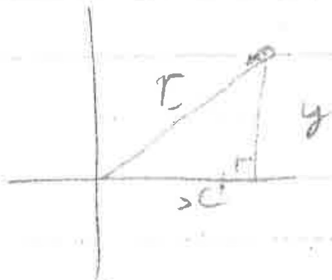
$= 2x$

$\text{Re}(z + \bar{z}) = 2x$ as $\text{Im}(z + \bar{z}) = 0$

ii) $z\bar{z} = (x + iy)(x - iy)$

$= x^2 - i^2y^2$

$= x^2 + y^2$



$|z|^2 = r^2$

$= x^2 + y^2$

$= z\bar{z}$

c) $x^2 - y^3 + xy^2 = 3$

$2x - 3y^2 \frac{dy}{dx} + x\,2y\frac{dy}{dx} + y^2 = 0$

$\frac{dy}{dx}(2xy - 3y^2) = -2x - y^2$

$\frac{dy}{dx} = \frac{-(2x + y^2)}{2xy - 3y^2}$

must have differential 3 terms correctly

Q8 c) $\int \frac{6x+4}{x^2+16}\,dx$

$= \int \frac{6x}{x^2+16}\,dx + \int \frac{4}{x^2+16}\,dx$

$= 3\int \frac{2x}{x^2+16}\,dx + \int \frac{4}{x^2+16}\,dx$ ✓

$= 3\ln(x^2+16) + 4 \cdot \frac{1}{4} \tan^{-1}\frac{x}{4} + C$

$= 3\ln(x^2+16) + \tan^{-1}\frac{x}{4} + c$ ✓

d) i) Let $\lambda = x + iy$

Let $(x+iy)^2 = -21-20i$

$$\left.\begin{aligned} x^2 - y^2 &= -21 \\ 2xy &= -20 \\ xy &= -10 \end{aligned}\right\}$$ ✓

Different signs

$x = 2 \quad y = -5 \quad$ or $\quad x = -2 \quad y = 5$

$\lambda = 2-5i \quad$ or $\quad \lambda = -(2-5i)$ ✓

ii) $z^2 - (4+i)z + (9+7i) = 0$

$a = 1$

$b = -(4+i)$

$c = 9+7i$

$\Delta = (4+i)^2 - 4 \times 1 \times (9+7i)$

$= 16 + 8i + i^2 - 36 - 28i$

$= 16 - 1 - 36 - 20i$

$= -21 - 20i$ ✓

$z = \frac{4+i+2-5i}{2}$ or $z = \frac{4+i-(2-5i)}{2}$

$= \frac{6-4i}{2}$

$z = 3-2i$

$= \frac{2+6i}{2}$

$z = 1+3i$ ✓

Q9 a) $\int \sin^4 x \cos^3 x \, dx$

$= \int \sin^4 x (1 - \sin^2 x) \cos x \, dx$

$= \int (\sin^4 x - \sin^6 x) \cos x \, dx$ ✓

let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$= \int u^4 - u^6 \, du$

$= \frac{u^5}{5} - \frac{u^7}{7} + c$

$= \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + c$ ✓

b) Prove by MI $n! \geq 2^{n-1}$ for positive integer n

Part A When $n = 1$ LHS $= 1!$ $= 1$ RHS $= 2^{1-1}$ $= 2^0$ $= 1$

LHS $\geq$ RHS

Statement is true for $n = 1$ ✓

Part B Assume statement is true for $n = k$ where k is a positive integer

ie. $k! \geq 2^{k-1}$ *

We must prove statement true for $n = k+1$

ie $(k+1)! \geq 2^k$ ✓

Beginning with the induction hypothesis *

$k! \geq 2^{k-1}$

$(k+1) \times k! \geq (k+1) 2^{k-1}$ ✓

$\geq 2 \times 2^{k-1}$ since $k \geq 1$

$= 2^k$

Part C It follows from Parts A & B by MI the statement holds true for all positive integers n.

Q9 c) $\int \frac{9x^2}{(x-1)^2(x+2)} dx \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$

$9x^2 = A(x-1)(x+2) + B(x+2) + C(x-1)^2$

if $x = 1$ $\quad 9 = 3B$

$\underline{B = 3}$ ✓

if $x = -2$ $\quad 36 = (-3)^2 C$

$\underline{C = 4}$

coefficient of x^2 $\quad A + C = 9$

$A + 4 = 9$

$\underline{A = 5}$ ✓

$\int \frac{9x^2}{(x-1)^2(x+2)} dx = \int \frac{5}{x-1} + \frac{3}{(x-1)^2} + \frac{4}{x+2} \, dx$ ✓

$= 5\ln(x-1) - \frac{3}{x-1} + 4\ln(x+2) + c$

$= \ln(x-1)^5 + \ln(x+2)^4 - \frac{3}{x-1} + c$

$= \ln\left[(x-1)^5(x+2)^4\right] - \frac{3}{x-1} + c.$

Q9 d) $I = \int x e^{2x}\,dx$

let $u = x$ $\qquad v' = e^{2x}$

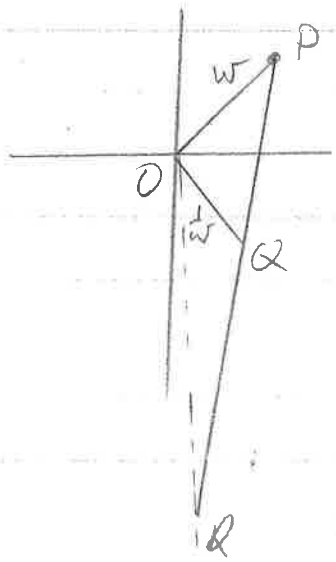
$u' = 1$ $\qquad v = \frac{e^{2x}}{2}$

$$I = \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}\,dx \quad \checkmark$$

$$= \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c \quad \checkmark$$

e) $\overrightarrow{PQ} = \frac{1}{w} - w$

Diagram is if P is in Quadrant 1



$|QR| = 2|PQ|$

$$\overrightarrow{PR} = 3\overrightarrow{PQ}$$

$$= 3\left(\frac{1}{w} - w\right)$$

$$= \frac{3}{w} - 3w \quad \checkmark$$

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$$

$$= w + \frac{3}{w} - 3w$$

$$= \frac{3}{w} - 2w. \quad \checkmark$$

Q10 a) $\int \frac{1}{x(\log_e x)^3}\,dx$

Let $u = \log_e x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{dx}{x}$

$= \int \frac{1}{x} \times (\log_e x)^{-3}\,dx$

$= \int u^{-3}\,du$

$= \frac{u^{-2}}{-2} + c$ ✓

$= -\frac{1}{2(\log_e x)^2} + c$.

b) i) $\frac{d}{d\theta} \sec\theta = \frac{d}{d\theta}\left(\frac{1}{\cos\theta}\right)$

$= \frac{d}{d\theta} (\cos\theta)^{-1}$ ✓

$= -(\cos\theta)^{-2} \times -\sin\theta$

$= \frac{\sin\theta}{\cos^2\theta}$

$= \frac{1}{\cos\theta} \times \frac{\sin\theta}{\cos\theta}$

$= \sec\theta \tan\theta$

ii) next page

Q10 b) ii) $x = \sec\theta - 1$

$$\frac{dx}{d\theta} = \sec\theta\tan\theta$$

$x = \sqrt{2}-1$
$\sqrt{2}-1 = \sec\theta - 1$
$\sqrt{2} = \sec\theta$
$\cos\theta = \frac{1}{\sqrt{2}}$
$\theta = \frac{\pi}{4}$

$x = 1$
$\sec\theta = 2$
$\cos\theta = \frac{1}{2}$
$\theta = \frac{\pi}{3}$

$$\int_{\sqrt{2}-1}^{1} \frac{2}{(x+1)\sqrt{x^2+2x}}\,dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\sec\theta\tan\theta\,d\theta}{(\sec\theta - 1 + 1)(\sqrt{(\sec\theta-1)^2 + 2(\sec\theta - 1)})}$$

award 1 mark if limits incorrect

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\cancel{\sec\theta}\tan\theta\,d\theta}{\cancel{\sec\theta}\sqrt{\sec^2\theta - \cancel{2\sec\theta} + 1 + \cancel{2\sec\theta} - 2}}$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\tan\theta\,d\theta}{\sqrt{\tan^2\theta}}$$

$1 + \tan^2\theta = \sec^2\theta$

must have correct limits.

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 2\,d\theta$$

$$= 2\left[\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= 2\left(\frac{4\pi - 3\pi}{12}\right)$$

$$= \frac{\pi}{6}$$

Q10 c) $I_n = \int_1^e \frac{(\ln x)^n}{x^2}\, dx \qquad n \geq 0$

By integration by parts

$u = (\ln x)^n \qquad v' = x^{-2}$

$u' = n(\ln x)^{n-1} \times \frac{1}{x} \qquad v = \frac{x^{-1}}{-1}$

$= \frac{n(\ln x)^{n-1}}{x} \qquad = -\frac{1}{x}$ ✓

$$I_n = \left[-\frac{1}{x} \times (\ln x)^n\right]_1^e - \int_1^e -\frac{n(\ln x)^{n-1}}{x^2}\, dx$$

$$= \left[-\frac{1}{e}(\ln e)^n - 1 \times (\ln 1)^n\right] + n\int_1^e \frac{(\ln x)^{n-1}}{x^2}\, dx$$ ✓

$$= -\frac{1}{e} + 0 + n I_{n-1} \quad \text{for } n \geq 1$$

$I_2 = -\frac{1}{e} + 2 I_1$

$I_1 = -\frac{1}{e} + I_0$

$I_0 = \int_1^e \frac{1}{x^2}\, dx$

$= -\left[\frac{1}{x}\right]_1^e$

$= -\left(\frac{1}{e} - 1\right)$

$= 1 - \frac{1}{e}$ ✓

$I_1 = -\frac{1}{e} + \left(1 - \frac{1}{e}\right)$

$= 1 - \frac{2}{e}$

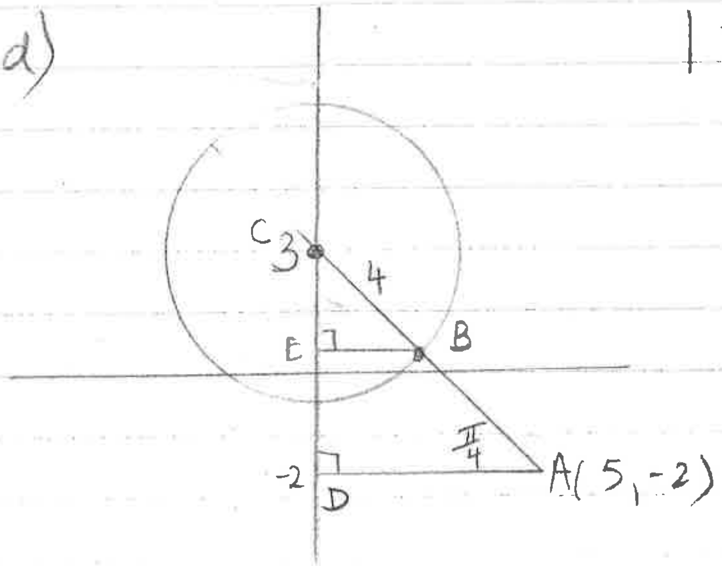
$I_2 = -\frac{1}{e} + 2\left(1 - \frac{2}{e}\right)$

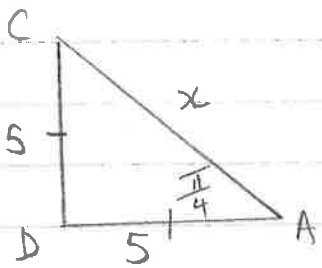
$= 2 - \frac{5}{e}$ ✓

Q10 d)

$|z - 3i| \leq 4$

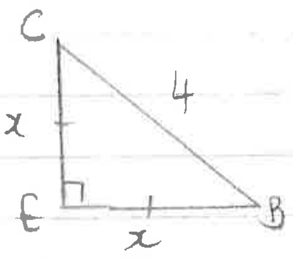
circle centre $3i$
radius 4





$x^2 = 5^2 + 5^2$
$= 50$
$\underline{x = 5\sqrt{2}}$ ✓

$AC = 5\sqrt{2}$
$BC = $ radius of circle
$= 4$
$AB = AC - BC$
$\underline{= 5\sqrt{2} - 4}$ ✓



Right angled isosceles
$x^2 + x^2 = 4^2$
$2x^2 = 16$
$x^2 = 8$
$x = 2\sqrt{2}$
$y = 3 - 2\sqrt{2}$ ✓

$B(2\sqrt{2}, 3 - 2\sqrt{2})$
The complex number z is $\underline{2\sqrt{2} + (3 - 2\sqrt{2})i}$

Q11(a)

$|z^2 - (\bar{z})^2| \leq 4$

Let $z = x + iy$

LHS $= |(x+iy)^2 - (x-iy)^2|$

$= |[x+iy-(x-iy)][x+iy+x-iy]|$

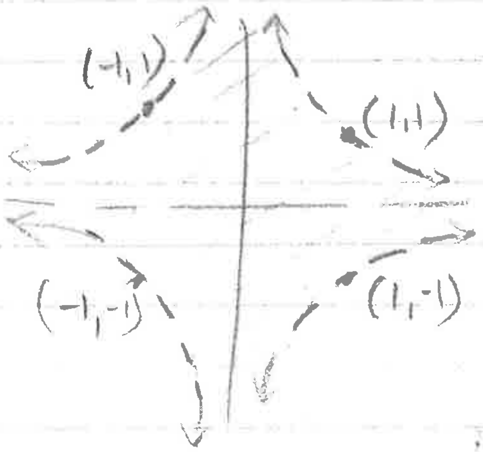
$= |(2iy)(2x)|$

$= |4ixy|$

$4|xy| < 4$

$|xy| < 1$ ✓

$-1 < xy < 1$



✓

Q 11

b) $\displaystyle\int \frac{1}{4+3\sin x}\,dx$

$t = \tan\frac{x}{2}$

$\frac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$

$= \frac{1}{2}\left(1+\tan^2\frac{x}{2}\right)$

$\frac{dx}{dt} = \frac{2}{1+t^2}$

$dx = \frac{2dt}{1+t^2}$

$$= \int \frac{\frac{2dt}{1+t^2}}{\frac{4(1+t^2)}{1+t^2} + \frac{3(2t)}{1+t^2}}$$ ✓

$$= \int \frac{2}{\cancel{1+t^2}} \times \frac{\cancel{1+t^2}}{4+4t^2+6t}\,dt$$

$$= \int \frac{2}{4t^2+6t+4}\,dt$$

$$= \int \frac{\cancel{2}}{_2\cancel{4}\left(t^2+\frac{3}{2}t+1\right)}\,dt$$

$$= \frac{1}{2}\int \frac{dt}{\left(t^2+\frac{3}{2}t+\left(\frac{3}{4}\right)^2\right)-\frac{9}{16}+1}$$

$$= \frac{1}{2}\int \frac{dt}{\left(t+\frac{3}{4}\right)^2+\frac{7}{16}}$$ ✓

$a^2 = \frac{7}{16}$

$a = \frac{\sqrt{7}}{4}$

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{4}^{\,2}}{\sqrt{7}}\tan^{-1}\left(\frac{t+\frac{3}{4}}{\frac{\sqrt{7}}{4}}\right)+c$$

$$= \frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{4t+3}{\sqrt{7}}\right)+c$$

$$= \frac{2}{\sqrt{7}}\tan^{-1}\left(\frac{4\tan\frac{x}{2}+3}{\sqrt{7}}\right)+c.$$ ✓

must back substitute

Q11 c

i) $\overrightarrow{OD}$ is $\overrightarrow{OC}$ rotated anticlockwise about the origin by θ

$$d = c \times cis\theta$$
$$= c \times w$$
$$= wc$$

✓

ii) $\overrightarrow{OA}$ is $\overrightarrow{OB}$ rotated clockwise about the origin by θ $(= -\theta)$

$$a = [\cos(-\theta) + i\sin(-\theta)] \times b$$
$$= (\cos\theta - i\sin\theta)\, b$$
$$= \overline{w}\, b$$ ✓

Also $b = \dfrac{a}{\overline{w}}$
$= wa$

iii) $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$
$= -a + c \quad = c - a$

$w\overrightarrow{AC} = wc - wa$
$= d - b$ ✓
$= \overrightarrow{BD}$

$|\overrightarrow{BD}| = |w\overrightarrow{AC}|$ ✓
$= |w||\overrightarrow{AC}|$
$= 1 \times |\overrightarrow{AC}|$ since $|w| = 1$

$\therefore |\overrightarrow{BD}| = |\overrightarrow{AC}|$ as required.

$$I = \int x e^{x} \sin x \, dx$$

$$= \int x \sin x \, e^{x} \, dx$$

$u = x \sin x \qquad v' = e^{x}$

$u' = \sin x + x \cos x \qquad v = e^{x}$

$$I = x e^{x} \sin x - \int e^{x} \sin x + x e^{x} \cos x \, dx$$

$$= x e^{x} \sin x - \int e^{x} \sin x \, dx - \int x e^{x} \cos x \, dx$$

Let $I_1 = \int e^{x} \sin x \, dx$

$u = \sin x \qquad v' = e^{x}$

$u' = \cos x \qquad v = e^{x}$

$$I_1 = e^{x} \sin x - \int e^{x} \cos x \, dx$$

Let $I_2 = \int e^{x} \cos x \, dx$

$u = \cos x \qquad v' = e^{x}$

$u' = -\sin x \qquad v = e^{x}$

$$I_2 = e^{x} \cos x - \int -e^{x} \sin x \, dx$$

$$= e^{x} \cos x + I_1$$

$$I_1 = e^{x} \sin x - (e^{x} \cos x + I_1)$$

$$= e^{x} \sin x - e^{x} \cos x - I_1$$

$$2 I_1 = e^{x} (\sin x - \cos x)$$

$$I_1 = \frac{e^{x}}{2} (\sin x - \cos x) \quad \checkmark$$

$$I_3 = \int x e^x \cos x \, dx$$
$$= \int x \cos x \, e^x \, dx$$

$u = x \cos x \qquad v' = e^x$

$u' = \cos x + x(-\sin x) \qquad v = e^x$

$$I_3 = x e^x \cos x - \int e^x \cos x - x e^x \sin x \, dx$$
$$= x e^x \cos x - I_2 + \int x e^x \sin x \, dx$$
$$= x e^x \cos x - e^x \cos x - I_1 + I$$

$$I = x e^x \sin x - I_1 - I_3$$
$$= x e^x \sin x - \cancel{I_1} - (x e^x \cos x - e^x \cos x - \cancel{I_1} + I)$$

$$I = x e^x \sin x - x e^x \cos x + e^x \cos x - I$$

$$2I = e^x (x \sin x - x \cos x + \cos x)$$

$$I = \frac{e^x}{2}(x \sin x - x \cos x + \cos x) + C$$