Sydney Technical High School

Name: ....

Maths Class: ....

Year 12

Mathematics Extension 2

HSC Assessment 2

June 2020

Time allowed: 120 minutes

General Instructions:

1. Marks for each question are indicated on the question.
2. Approved calculators may be used
3. All necessary working should be shown
4. Full marks may not be awarded for careless work or illegible writing
5. ***Begin each question on a new page***
6. Write using black or blue pen
7. All answers are to be in the writing booklet provided
8. A reference sheet is provided at the rear of this Question Booklet, and may be removed at any time.

Section 1 Multiple Choice

Questions 1-8

8 Marks

Section II Questions 9-12

64 Marks

Total = 72 marks

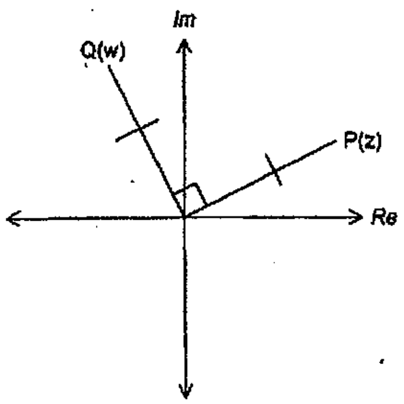
SECTION 1

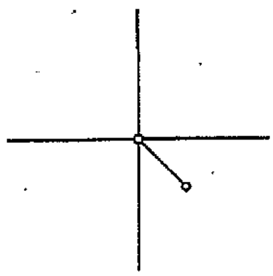
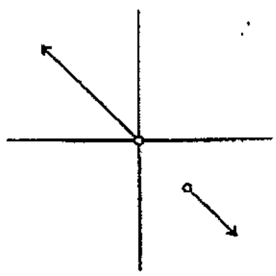
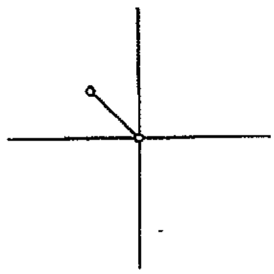
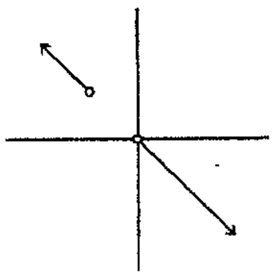
(Spend about 15 minutes completing this section)

MULTIPLE CHOICE (8 MARKS)

All questions are worth 1 mark each

Select one answer from among the 4 choices given

1	The exact value of $(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}})^{100}$ is A. 1 B. -1 C. $\frac{1}{2^{50}}$ D. $-\frac{1}{2^{50}}$
2	The centre and radius of the sphere $(x-1)^2 + (y+4)^2 + (z-3)^2 = 9$ are: A. (-1, 4, 3) and 9 units B. (-1,4,-3) and 3 units C. (1, -4, 3) and 9 units D. (1, -4, 3) and 3 units.
3	On the diagram P and Q represent the numbers z and w  Which of the following is not true? A. $\|z\|^2 + \|w\|^2 = \|z+w\|^2$ B. $z^2 - w^2 = 0$ C. $w = iz$ D. $z^2 + w^2 = 0$

4	If $\underline{u} = 3\underline{i} - 2\underline{j} + 4\underline{k}$ which of the following vectors is perpendicular to $\underline{u}$? A. $3\underline{i} + 2\underline{j} - 4\underline{k}$ B. $2\underline{i} + \underline{j} - \underline{k}$ C. $\frac{1}{3}\underline{i} - \frac{1}{2}\underline{j} + \frac{1}{4}\underline{k}$ D. $2\underline{i} - \underline{j} + \underline{k}$
5	If ω is a non-real complex cube root of unity, then $\frac{1}{1+\omega} + \frac{1}{1+\omega^2} =$ A. -1 B. 0 C. 1 D. 2
6	Which diagram best represents the solutions to the equation $arg\, z = arg(z + 1 - i)$ A.  B.  C.  D. 

7	If P is the point (1, -4, 2) then a unit vector for $\overrightarrow{OP}$ where O is the origin, is: A. $\underline{i} - 4\underline{j} + 2\underline{k}$ B. $\frac{1}{\sqrt{7}}(\underline{i} - 4\underline{j} + 2\underline{k})$ C. $\frac{1}{\sqrt{21}}(\underline{i} - 4\underline{j} + 2\underline{k})$ D. $\sqrt{21}(\underline{i} - 4\underline{j} + 2\underline{k})$
8	It is given that $z = 2 + i$ is a root of $z^3 + az^2 - 7z + 15 = 0$, where a is a real number. What is the value of a? A. -1 B. 1 C. 7 D -7

SECTION II

(64 Marks) (Start each question on a new page)

Spend about 1 hour 45 minutes on this section

QUESTION 9: (16 MARKS)

(a) Assuming that all variables are real, insert the correct symbol $\Rightarrow$ *or* $\Leftrightarrow$ in the statement below. **1**

$$a < b \ \square \ -b < -a$$

(b) If $z = 2 + 2\sqrt{3}i$, find

(i) $\frac{z}{1+i}$ in the form $a + ib$ **1**

(ii) $|z|$ **1**

(iii) $\arg z$ **1**

(iv) z in the form $re^{i\theta}$ **1**

(v) $\frac{1}{z^4}$, with positive argument **2**

(c) The position vectors of P, Q and R are **3**

$8\underset{\sim}{i} - 4\underset{\sim}{j} - 3\underset{\sim}{k}$, $6\underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k}$, and $7\underset{\sim}{i} + 5\underset{\sim}{j} - 5\underset{\sim}{k}$ respectively.

Find the angle between $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ to the nearest degree.

QUESTION 9 continues overleaf....)

QUESTION 9 continued....)

(d) z is a complex number where $|z| = 1$ and $arg(z) = A$. By using a suitable diagram, or otherwise, find, in terms of A, the value of $\arg(z + 1)$. **3**

Justify your answer.

(e) Prove, by the method of contradiction, that $log_2 5$ is irrational. **3**

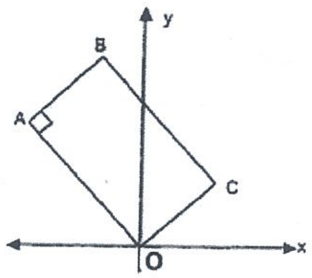
QUESTION 10: (16 MARKS) (Start a new page)

(a) Find the Cartesian equation for the vector $\underline{r}$ which has the parametric form **2**

$$\underline{r}(t) = 4t^2\underline{i} + 2t\underline{j}$$

and describe the function formed

(b) OABC is a rectangle with $OA = 3 \times OC$. O is the origin. **3**



If C represents the complex number z, find, in terms of z, the complex number represented by:

(i) the point A

(ii) the point B

(iii) the vector $\overrightarrow{AC}$

(c) (i) Find the vector equation of the line $\underline{r}$ through the point A(1, -2, 3) which is parallel to the vector **1**

$$\underline{u} = 2\underline{i} - 5\underline{j} + \underline{k}$$

(ii) Find the co-ordinates of the point C on $\underline{r}$ where the *x*-value of C is 2. **2**

QUESTION 10 continues overleaf....)

QUESTION 10 continued....)

(d) (i) Solve $z^5 = -1$ over the complex field, giving your answers in the form $re^{i\theta}$ **2**

(ii) If w is the complex root of $z^5 = -1$ with the smallest positive argument, show that the other complex roots are $-w^2, w^3$ and $-w^4$. **3**

(iii) Using w as in part (ii), simplify $(1 - w + w^2 - w^3)^8$, giving your answer in the form $re^{i\theta}$ **3**

QUESTION 11: (16 MARKS) (Start a new page)

(a) State whether the following statement is true ∀ a, b ∈R. **1**

"If $a < b$, then $a^2 < b^2$"

If you think it is false, provide a counter example.

(b) Write down the converse of this statement and state whether the converse is true or false: **1**

"If a triangle has 3 equal sides, then it has 3 equal angles".

(c) You are given the points P, Q and R whose position vectors are

$4\underset{\sim}{i} + \underset{\sim}{j}$, $\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$, and $3\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$ respectively.

(i) Show that ΔPQR is right-angled. **1**

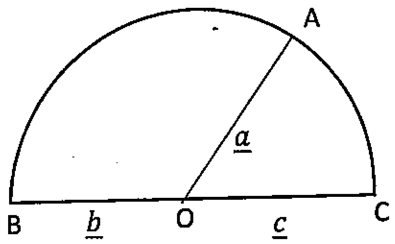
(ii) Find the area of ΔPQR in exact form **2**

(d) Prove that $a^3 + a$ is always even, $a \in Z$ (do not use induction). **3**

QUESTION 11 continues overleaf....)

QUESTION 11 continued....)

(e) The following diagram shows a semi-circle of diameter BC, centre O and a point on the circumference, A. The line OA has vector $\underset{\sim}{a}$, OB has vector $\underset{\sim}{b}$, and OC has vector $\underset{\sim}{c}$



(i) Explain why $\underset{\sim}{b} = -\underset{\sim}{c}$ **1**

(ii) Why is $|\underset{\sim}{a}| = |\underset{\sim}{b}|$? **1**

(iii) Prove that

"The angle subtended on the circumference by a diameter is 90^o " **2**
(ie., Prove that $\angle BAC$ is a right angle)

(f) Prove by mathematical induction that for all positive integers $n > 2$, **4**

$$(1+x)^n > 1 + nx + nx^2, \quad \text{provided that } x > 0$$

QUESTION 12: (16 MARKS) (Start a new page)

(a) Consider the statement: "If a number ends in the digit 2, then it is even" **1**

Write the contrapositive statement which needs to be proven to make the above statement true.

(b) (i) If A is the point (3, 2, 1) P the point (-4, 3, -1) and B the point (0, -4, 1), prove that the projection of the line $\overrightarrow{AP}$ onto the line $\overrightarrow{AB}$ is given by $-\underset{\sim}{i} - 2\underset{\sim}{j}$ **2**

(ii) Hence find the perpendicular distance of the point P from the line AB. **2**

(c) On an Argand diagram sketch the locus specified by **2**

$$|z - 1 - i| \geq |z|$$

(d) If $1 < x \leq y$ show that

(i) Show that $x - 1$ is positive. **1**

(ii) Prove that $y(x-1) \geq x(x-1)$ **2**

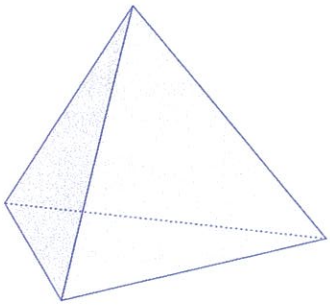
(iii) Hence deduce that $x(y - x + 1) \geq y$ **1**

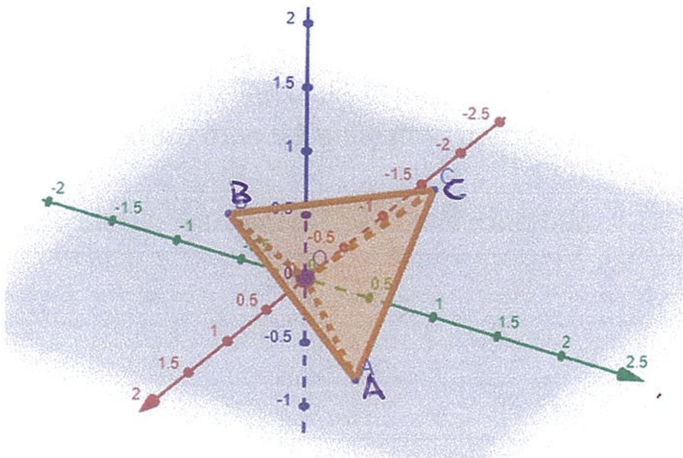
QUESTION 12 continues overleaf....)

QUESTION 12 continued....)

(e) The points O(0, 0, 0), A (1, 1, 0), B (1, 0, 1) and C(0, 1, 1) form a regular tetrahedron.

(A regular tetrahedron is a triangular pyramid with 4 congruent faces)





(i) Find the length of the edges of the tetrahedron **1**

(ii) Find the angle between any two edges of the figure **2**

(iii) The point M, is the midpoint of the edge BC. Prove that AM is perpendicular to MB. **2**

STHS - EXTENSION 2 - 2020

ASSESSMENT 2 SOLUTIONS

SECTION I - M/C.

Q1 $\left[\operatorname{cis}\left(-\frac{\pi}{4}\right)\right]^{100} = \operatorname{cis}(-25\pi)$

$= \operatorname{cis}(-\pi)$ $\therefore$ B

$= -1$

Q2 $C: (1, -4, 3)$ $r = 3$ $\therefore$ D

Q3 POINT Q: $w = iz \to$ "C"

$|w| = |z|$ and diagonal is $|w+z|$

By Pythagoras $|w|^2 + |z|^2 = |w+z|^2 \to$ "A"

$w^2 = -z^2$ from C, $\to$ "D" $\therefore$ B

Q4. Check on dot products gives B

Q5 $\dfrac{1+w^2+1+w}{(1+w)(1+w^2)} = \dfrac{1+(1+w+w^2)}{1+w+w^2+w^3}$

$= \dfrac{1+0}{0+w^3}$ (since if w is non-real $1+w+w^2=0$)

$= \frac{1}{1}$ (since $w^3 = 1$)

$\therefore$ C

Q6 D

Q7 $\overrightarrow{OP} = \underset{\sim}{i} - 4\underset{\sim}{j} + 2\underset{\sim}{k}$ and $|\overrightarrow{OP}| = \sqrt{21}$

$\therefore \hat{\underset{\sim}{OP}} = \frac{1}{\sqrt{21}}(\underset{\sim}{i} - 4\underset{\sim}{j} + 2\underset{\sim}{k})$ $\therefore$ C

Q8 Roots are $2+i$, $2-i$, α (say)

Sum of roots $= 4 + \alpha = -a$

Product of roots $= 5\alpha = -15$

$\therefore \alpha = -3$

$\therefore 4 + (-3) = -a$

$\therefore a = -1$. $\therefore$ A

SECTION 2

QUESTION 9:

(a) $\Leftrightarrow$

(b) (i) $\dfrac{2+2\sqrt{3}i}{1+i} = \dfrac{2 - 2i + 2\sqrt{3}i + 2\sqrt{3}}{2}$

$= (1+\sqrt{3}) + i\,(\sqrt{3}-1)$.

(ii) $|z| = \sqrt{4+12}$

$= 4$

(iii) Let $\theta = \arg z$

$\tan\theta = \sqrt{3}$

$\therefore \theta = \pi/3$

$\therefore \arg z = \pi/3$

(iv) $z = 4e^{\frac{\pi}{3}i}$

(v) $z^{-4} = \dfrac{1}{256} e^{-\frac{4\pi}{3} i}$

$= 1/256\, e^{\frac{2\pi}{3} i}$

OR, $z^{-4} = \dfrac{1}{256} \operatorname{cis} \dfrac{2\pi}{3}$

(c) $\overrightarrow{PQ} = -2\underset{\sim}{i} + 7\underset{\sim}{j} - \underset{\sim}{k}$ $\qquad \overrightarrow{OR} = \underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$

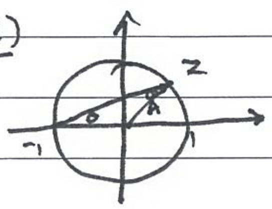
$\cos\theta = \dfrac{\overrightarrow{PQ}\cdot\overrightarrow{OR}}{|PQ||OR|}$

$= \dfrac{-2+14+1}{\sqrt{54}\sqrt{6}}$

$= 13/18$

$\therefore \theta \doteqdot 44^\circ$

(d)



since the triangle is isosceles

$\theta = A/2$ (external angle is sum of 2 internal angles)

$\therefore \arg(z+1) = A/2$

(e) Assume that $\log_2 5$ is rational

$\therefore \log_2 5 = p/q$

$\therefore 2^{p/q} = 5$

$\therefore 2^p = 5^q$

BUT 2^p is even and 5^q is odd

$\therefore$ CONTRADICTION $\Rightarrow \log_2 5$ is irrational

QUESTION 10:

(a) $x = 4t^2$ and $y = 2t$

$\therefore x = y^2$ which is a sideways parabola.

(b) (i) $A = 3iz$

(ii) $B = z + 3iz$

(iii) $\vec{AC} = z - 3iz$.

(c)(i) $\underset{\sim}{r} = \begin{pmatrix}1\\-2\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\-5\\1\end{pmatrix}$ or $\underset{\sim}{r} = (\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}) + \lambda(2\underset{\sim}{i} - 5\underset{\sim}{j} + \underset{\sim}{k})$

(ii) $x = 2 \Rightarrow 2\lambda + 1 = 2$

$\lambda = \frac{1}{2}$

$y = -2 - 5\lambda$ and $z = 3 + \lambda$

$\therefore y = -\frac{9}{2}$ $\quad z = \frac{7}{2}$

$\therefore C$ is $(2, -\frac{9}{2}, \frac{7}{2})$

(d) (i) $z = -1, \text{cis}(\pi + \frac{2\pi}{5}), \text{cis}(\pi + \frac{4\pi}{5})$

$\text{cis}(\pi + \frac{6\pi}{5}), \text{cis}(\pi + \frac{8\pi}{5})$

$= -1, \text{cis}\frac{7\pi}{5}, \text{cis}\frac{9\pi}{5}, \text{cis}\frac{11\pi}{5}, \text{cis}\frac{13\pi}{5}$

$= -1, \text{cis}\frac{7\pi}{5}, \text{cis}\frac{9\pi}{5}, \text{cis}\frac{\pi}{5}, \text{cis}\frac{3\pi}{5}$

$= e^{i\pi/5}, e^{i3\pi/5}, e^{i\pi}, e^{i7\pi/5}, e^{i9\pi/5}$

(ii) If $\omega = e^{i\pi/5}$ then

$\omega^2 = e^{\frac{2\pi}{5}i}$ $\quad \omega^3 = e^{\frac{3\pi}{5}i}, \quad \omega^4 = e^{\frac{4\pi}{5}i}$ $\quad e^{i\pi} = -1$

$-\omega^2 = (e^{\frac{2\pi}{5}i})(e^{i\pi})$ $\quad -\omega^4 = (e^{\frac{4\pi}{5}i})(e^{i\pi})$

$= e^{i7\pi/5}$ $\quad \omega^3 = e^{\frac{3\pi}{5}i}$ $\quad = e^{\frac{9\pi}{5}i}$ $\quad e^{i\pi} = -1$

$\therefore$ which are the roots from part (i)

(iii) Sum of the roots is 0

$\therefore -1 + \omega - \omega^2 + \omega^3 - \omega^4 = 0$

$\therefore 1 - \omega + \omega^2 - \omega^3 = -\omega^4$

$\therefore (1 - \omega + \omega^2 - \omega^3)^8 = (-\omega^4)^8$

$= \omega^{32}$

$= \omega^2$

$= e^{\frac{2i\pi}{5}}$.

QUESTION 11:

(a). FALSE

$-3 < -2$ but $(-3)^2 \nless (-2)^2$

(b) If a triangle has 3 equal angles,
then it has 3 equal sides
$\therefore$ TRUE

(c) $\vec{OP} = 4\underset{\sim}{i} + \underset{\sim}{j}$ $\quad \vec{OQ} = \underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$

$\vec{OR} = 3\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$

(i) $\vec{PQ} = -3\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$

$\vec{RQ} = -2\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k}$

$\vec{PQ} \cdot \vec{RQ} = 6 - 3 - 3$
$= 0$

$\therefore PQ \perp RQ$ $\therefore \triangle PQR$ is a right-angled

(ii) $|PQ| = \sqrt{9+9+1}$
$= \sqrt{19}$

$|RQ| = \sqrt{4+1+9}$
$= \sqrt{14}$

$\therefore$ Area $= \frac{1}{2}\sqrt{266}$ u^2

(d) $a^3 + a = a(a^2+1)$
If a is even, $a(a^2+1)$ is even.
If a is odd, a^2+1 is even $\Rightarrow a(a^2+1)$ is even
$\therefore a^3 + a$ is always even

(e)(i) They are both radii, but OB is in the opposite direction to OC

(ii) $|OB|$ and $|OA|$ have same length because they are both radii

(iii)

Q 11 (e) (ii)

$\overrightarrow{BA} = \underset{\sim}{a} - \underset{\sim}{b}$ and $\overrightarrow{CA} = \underset{\sim}{c} - \underset{\sim}{a}$

$\therefore \overrightarrow{BA} \cdot \overrightarrow{CA} = (\underset{\sim}{a} - \underset{\sim}{b})(\underset{\sim}{c} - \underset{\sim}{a})$

$= (\underset{\sim}{a} - \underset{\sim}{b})(-\underset{\sim}{b} - \underset{\sim}{a})$ since $\underset{\sim}{c} = -\underset{\sim}{b}$

$= -\underset{\sim}{a}\underset{\sim}{b} - \underset{\sim}{a}\underset{\sim}{a} + \underset{\sim}{b}\underset{\sim}{b} + \underset{\sim}{a}\underset{\sim}{b}$

$= |b|^2 - |a|^2$

$= 0$ since, from part (ii) $|a| = |b|$

$\therefore \angle BAC = 90°$

[THIS CAN ALSO BE PROVEN by geometric methods]

(f) For $n=3$, $(1+x)^3 = 1 + 3x + 3x^2 + x^3$

$> 1 + 3x + 3x^2$ since $x^3 > 0$

$\therefore$ true for $n=3$.

Assume it is true for $n=k$

i.e. $(1+x)^k > 1 + kx + kx^2$

For $n=k+1$

$(1+x)^{k+1} = (1+x)^k(1+x)$

$> (1+kx+kx^2)(1+x)$ from assumption

$= 1 + kx + kx^2 + x + kx^2 + kx^3$

$> 1 + (k+1)x + 2kx^2$ since $kx^3 > 0$

$> 1 + (k+1)x + (k+1)x^2$ since $2k > k+1$ $\forall k > 1$

which is the required form

$\therefore$ If it is true for $n=k$, it is true for $n=k+1$

But it is true for $n=3$,

$\therefore$ " " " " $n=4$ and so on..

ie True $\forall n \geq 3$.

QUESTION 12:

(a) "If a number is odd (not even) then it does not end in 2"

(b) (i) $A(3,2,1)$ $P(-4,3,-1)$ $B(0,-4,1)$

Let $\underset{\sim}{a} = \overrightarrow{AP}$ and $\underset{\sim}{b} = \overrightarrow{AB}$

$= -7\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ $\quad = -3\underset{\sim}{i} - 6\underset{\sim}{j}$

$$\text{Proj}_{\underset{\sim}{b}}\,\underset{\sim}{a} = \left(\frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|b|^2}\right)\underset{\sim}{b}$$

$$= \left(\frac{21-6}{45}\right)(-3\underset{\sim}{i} - 6\underset{\sim}{j})$$

$$= -\underset{\sim}{i} - 2\underset{\sim}{j}$$

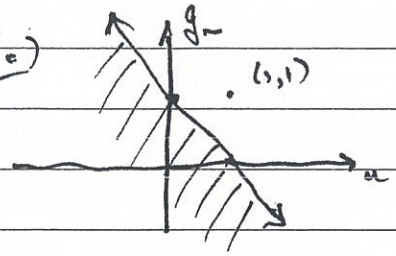
(ii) $\left|\text{proj}_{\underset{\sim}{b}}\,\underset{\sim}{a} - \underset{\sim}{a}\right| = \left|-\underset{\sim}{i} - 2\underset{\sim}{j} - (-7\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k})\right|$

$= \left|6\underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}\right|$

$= \sqrt{36+9+4}$

$= 7$ units.

(c)



The perpendicular bisector $x + y = 1$ is included.

(d) (i) $1 < x \Rightarrow x - 1 > 0$

(ii) $y \geq x$

$\therefore$ $y(x-1) \leq x(x-1)$ since $x - 1 > 0$

$\therefore$

(iii) $yx - y \leq x^2 - x$

$yx - x^2 + x \leq y$.

$\therefore$ $x(y - x + 1) \leq y$.

Q 12 (e) (i) $|AB| = \sqrt{0^2+1^2+1^2}$
$= \sqrt{2}$

(ii) $\overrightarrow{AB} = -\underset{\sim}{j} + \underset{\sim}{k}$ $\quad \overrightarrow{CB} = \underset{\sim}{i} - \underset{\sim}{j}$

$$\cos\theta = \frac{\overrightarrow{AB}\cdot\overrightarrow{CB}}{|AB||CB|}$$
$$= \frac{0+1+0}{\sqrt{2}\sqrt{2}}$$
$$= \tfrac{1}{2}$$
$\therefore \theta = \pi/3$

(iii) 2 POSSIBLE SOLUTIONS

(I) M is $(\tfrac{1}{2}, \tfrac{1}{2}, 1)$ $\quad \overrightarrow{AM} = -\tfrac{1}{2}\underset{\sim}{i} - \tfrac{1}{2}\underset{\sim}{j} + \underset{\sim}{k}$

$\overrightarrow{BM} = -\tfrac{1}{2}\underset{\sim}{i} + \tfrac{1}{2}\underset{\sim}{j}$

$\overrightarrow{AM}\cdot\overrightarrow{BM} = \tfrac{1}{4} - \tfrac{1}{4}$
$= 0$

$\therefore AM \perp BM$

$\boxed{\text{OR}}$ (II) M is $(\tfrac{1}{2}, \tfrac{1}{2}, 1)$

$|MB|^2 = \tfrac{1}{4} + \tfrac{1}{4}$ $\quad |AM|^2 = \tfrac{1}{4} + \tfrac{1}{4} + 1$
$= \tfrac{1}{2}$ $\quad = \tfrac{3}{2}$

and $|AB|^2 = (\sqrt{2})^2$
$= 2.$

$\therefore |AB|^2 = 2 = |MB|^2 + |AM|^2$

$\therefore$ by Pythagoras, $\triangle AMB$ is right angled
with $\underline{\angle BMA} = 90^\circ$

$\therefore AM \perp MB$

Year 12 Extension 2 Mathematics Task 2 Marker's Comments

Question 9

b) i) PLEASE check your expansion, there were many strange negatives in places where they did not belong.

v) Students struggled with the negative indices and applying De Moivre's Theorem

c. Some people found the obtuse angle THAT DID NOT LIE between PQ and QR by using the wrong vector (RQ instead of QR).

d. A diagram and some simple plane geometry would have made the question easier rather than trying to do it using algebra

e. Getting to $2^a = 5^b$ was not enough to determine a contradiction, you have to EXPLAIN why it's a false statement – e.g. odd and even, no common factors etc etc

Question 10

a) A number of you didn't take the +/- case when you square rooted so it was not just the top half or "root" curve, it was a whole sideways parabola.

c)A small number used the point A twice in the vector equation formula ☹. Some sloppiness in evaluating correct numerical values of the point.

d)(ii) Working, explanation or diagram needed to be shown, not just $cis(\frac{4\pi}{5}) = -cis(\frac{\pi}{5})$

(iii) This is a sum of roots question. You need to recognise this type in a question.

Question 11

(a) and (b) Please note the difference between contrapositive and converse. You can find a Youtube presentation on contraposition on https://www.youtube.com/watch?v=0YqZIHFmVzg and one on converse on https://www.youtube.com/watch?v=TCBu8PD4LIs

(c) (ii) Oh dear! At least 7 people think that $\sqrt{14} \times \sqrt{19} = \sqrt{33}$

(e) Just because $|\underset{\sim}{c}| = |\underset{\sim}{a}|$ does NOT mean $\underset{\sim}{c}.\underset{\sim}{a} = \underset{\sim}{a}^2$ They are not the same vector, they just have the same length. (as an example try $\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$ and $3\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$)

(f) In Induction proofs, it is <u>not</u> okay to deal with <u>both</u> sides of an equation/inequality.

(ie, in this case, you cannot assume the statement for both $n = k$ and $n = k+1$ and then work towards a conclusion that says, " $kx^3 + kx^2 - x^2 > 0$, which is true")

ALSO, You need to justify lines that involve some thought process:

e.g.,

$1 + x + kx + 2kx^2 + kx^3 > 1 + (k+1)x + 2kx^2$ **<u>since</u> $kx^3 > 0$ *for all, k, x >0***

AND It is far too easy to just say LHS > RHS in Step 1.

Again you have to justify it. (ie., $1 + 3x + 3x^2 + x^3 > 1 + 3x + 3x^2$ <u>***since***</u> $\boldsymbol{x^3 > 0}$

Question 12

a. READ THE QUESTION – many students went further than was necessary also becare with language the contra positive of A→B
Is not B→ not A

b. Most found the correct vector of AB but then did not project it correctly onto it – a diagram will help
ii. the perpendicular distance can be found using a diagram rather than a formula, as projecting AP onto AB creates an angle AMP – many who did a formula had the vectors in the incorrect order.

c. When in doubt sub in z = x + iy and do the algebra and check which side of the line to shade. ALSO please label points

d. The major issue here in ALL of part D is the setting out !!!!! – try using some linkage words so that what u write makes sense – do not assume what you are asked to find that should be the conclusion

e. Ii. Students could have just done an equilateral triangle here to get the pi/3 angle

iii. CARE and remembering to show the correct vectors and the actual calculation of the DOT product would be good.