Name: ....

Sydney Technical High School

Maths Class: ....

Year 12

Mathematics Extension 2

HSC Course

Assessment 3

June, 2022

Time allowed: 120 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A NESA reference sheet is provided.

Section 1 Multiple Choice

Questions 1-9

9 Marks

Allow approximately 15 minutes for this section

Section II Questions 10 – 13

60 Marks

Section 1

Attempt questions 1- 9

Answer the questions on the multiple-choice answer sheet located at the start of your answer booklet.

Allow approximately 15 minutes for this section.

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1. If $\boldsymbol{A}$ and $\boldsymbol{B}$ are points defined by the position vectors $\underset{\sim}{a} = 3\underset{\sim}{i} + 4\underset{\sim}{j} - \underset{\sim}{k}$ and

$\underset{\sim}{b} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$, then $\left|\overrightarrow{AB}\right|$ is equal to which of the following?

A $\sqrt{37}$

B $\sqrt{41}$

C $\sqrt{26}$

D 3

2. Let $\underset{\sim}{u} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\underset{\sim}{\boldsymbol{v}} = \underset{\sim}{\boldsymbol{i}} + 3\underset{\sim}{\boldsymbol{j}} - 4\underset{\sim}{\boldsymbol{k}}$. Which best describes $\underset{\sim}{u} \cdot \underset{\sim}{v}$?

A 0

B 1

C -1

D 2

3. Which expression is equal to $\int xsinx\, dx$?

A $xcosx - sinx + c$

B $-xcosx + sinx + c$

C $xsinx - sinx + c$

D $-\frac{1}{2}x^2cosx + c$

4. Which of the following represents the angle that the vector $\underset{\sim}{v} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$

makes with the $y - axis$?

A $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

B $\cos^{-1}\left(\frac{3}{\sqrt{29}}\right)$

C $\cos^{-1}\left(\frac{4}{\sqrt{29}}\right)$

D $\cos^{-1}\left(\frac{4}{29}\right)$

5. Which line does the point $A\ (-4, -1, 0\ )$ lie on?

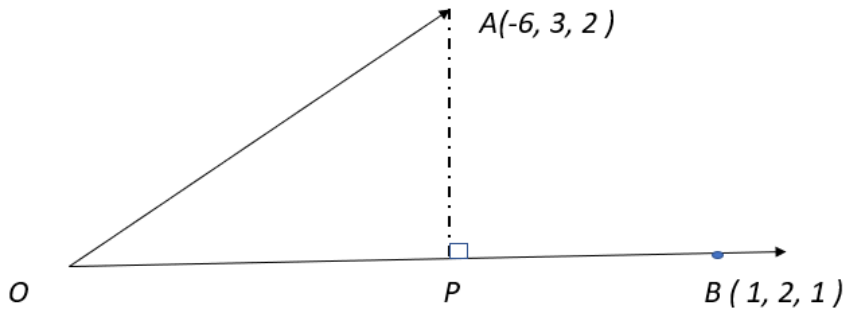
A $\underset{\sim}{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$

B $\underset{\sim}{r} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$

C $\underset{\sim}{r} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$

D $\underset{\sim}{r} = \begin{pmatrix} 4 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$

6. Consider the position vectors, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in the diagram below.



The distance *AP* is:

A $\sqrt{\frac{6}{9}}$

B 1

C $\sqrt{48\frac{1}{3}}$

D 7

7. The value of $\frac{d}{dx}\left[\int_x^{4x}\frac{dt}{1+t^2}\right]$ is which of the following?

A $\dfrac{1}{1+9x^2}$

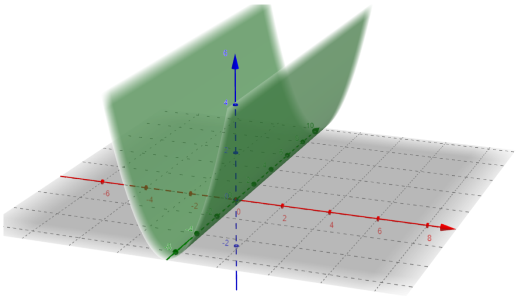
B $\dfrac{4}{1+16x^2}$

C $\dfrac{4}{1+16x^2}-\dfrac{1}{1+x^2}$

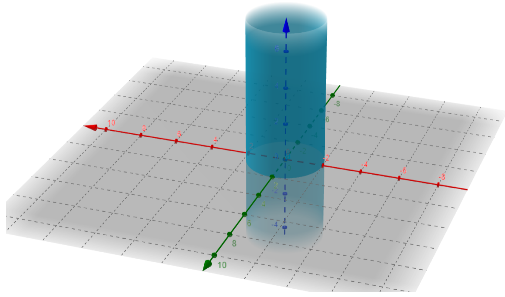
D $\dfrac{4}{1+x^2}-\dfrac{1}{1+x^2}$

8. Which of the following depicts the equation $z = 4x^2 + 4y^2$?

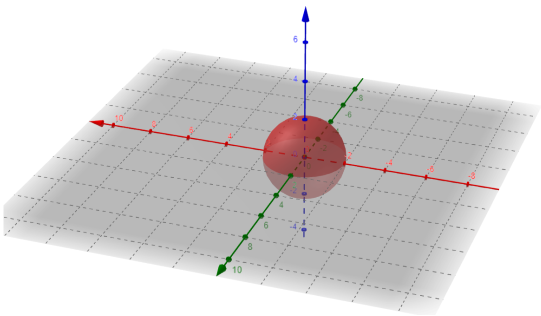
A



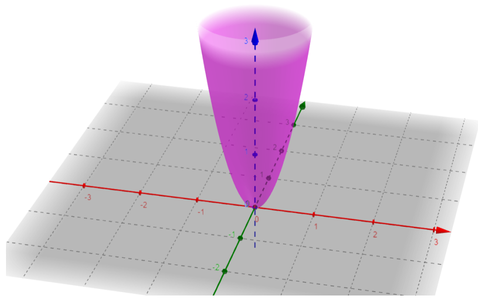
B



C



D



9. Which of the following integrals has the largest value?

A $\int_0^1 \cos^{-1} x \; dx$

B $\int_0^1 x\sqrt{1-x^2} \; dx$

C $\int_0^1 xe^{-x} \, dx$

D $\int_0^1 \frac{1}{x^2-2x+2} \; dx$

End of section 1

Section II 60 marks

Answer each question showing all necessary mathematical working and calculations in the answer booklet and starting each question on the TOP of a NEW page.

Allow about 1 hour and 45 minutes for this section

Start each question at the top of a new page in your answer booklet.

Question 10 15 marks

a. Point A has a position vector $\underset{\sim}{a} = -4\underset{\sim}{i} + 3\underset{\sim}{j} + 6\underset{\sim}{k}$ and point B has a position vector $\underset{\sim}{b} = 2\underset{\sim}{i} - 5\underset{\sim}{j} + 3\underset{\sim}{k}$

i. Find the vector $\overrightarrow{AB}$ **1**

ii. Find the unit vector $\hat{\underset{\sim}{b}}$ **1**

iii. Find $\left|proj_{\underset{\sim}{b}}\underset{\sim}{a}\right|$ **2**

iv. Point C lies on the vector $\overrightarrow{AB}$ and divides the interval AB in the ratio of 4 : 3. Find the position vector of C. **2**

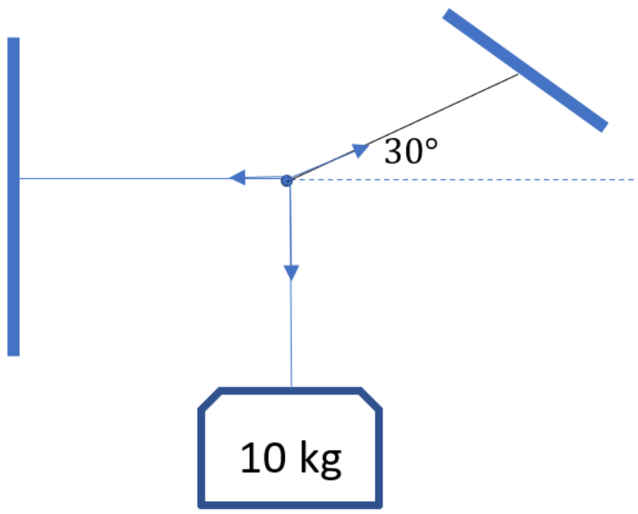
b. Evaluate $\int_0^2 x\sqrt{2-x}\ dx$ **3**

c. Use integration by parts to evaluate $\int_2^3 lnx\, dx$, expressing the solution as a single term. **3**

Question 10 continues on the next page .....

d. A 10 kg object is held in equilibrium by three ropes. One is horizontal and is attached to a vertical wall, the other is attached to a cross beam at an angle of 30^o, as shown in the diagram below. Taking gravity as 9.8 ms^{-2}, and using vectors, find the tension forces in the ropes.

3

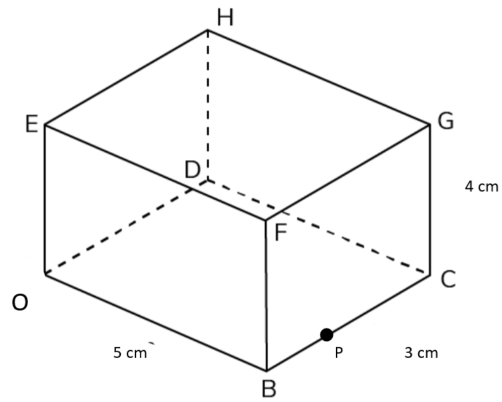


Start this question at the top of a new page in your answer booklet.

Question 11 **15 marks**

a. *OBCDEFGH* is a rectangular prism with dimensions $5\ cm\ \times 3\ cm \times 4\ cm$

Point *P,* lies on *BC*, such that, 2*BP* = *PC*



As O represents the origin, use vector methods to find angle *OGP*, correct to the nearest degree. **3**

b. Find,

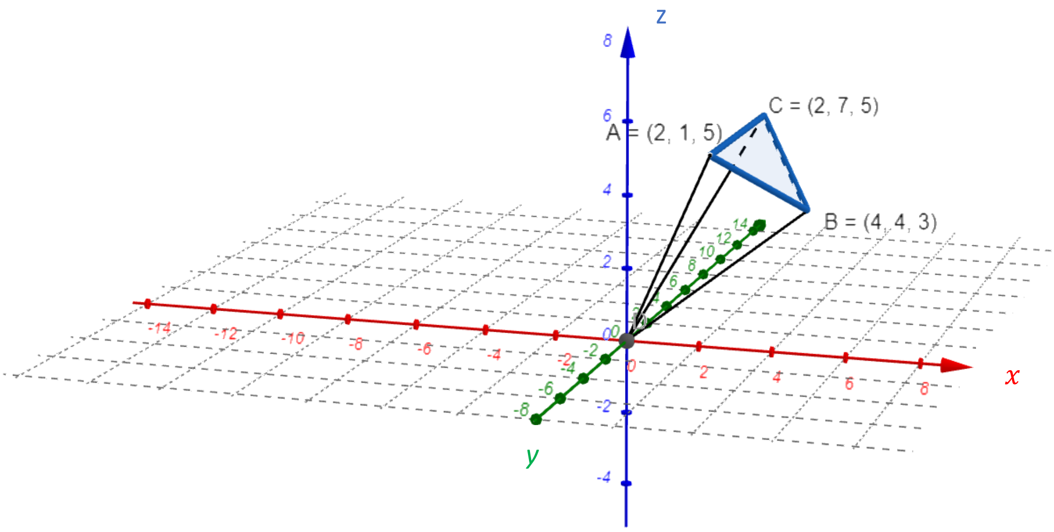
i. $\int \sin 3x \cos 4x \, dx$ **2**

ii. $\int \cos^3 x \, dx$ **2**

Question 11 continues on the next page……..

c. $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$ $\quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix}$ $\quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 7 \\ 5 \end{pmatrix}$ are shown on the diagram below.

O is the origin.



i. Show that ABC is an isosceles triangle. **3**

ii. Use a vector dot product to find angle CAB and hence, calculate the exact area of ΔABC. **3**

iii. Find the point D, such that, $ABCD$ is a parallelogram. **2**

Start this question at the top of a new page in your answer booklet.

Question 12 **15 marks**

a. Consider the position vectors $\underset{\sim}{a} = \underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = -\underset{\sim}{i} - 6\underset{\sim}{j}$ of the points A and B

i. Find the equation $\underset{\sim}{r}$ of the line through AB. **2**

ii. For what values of λ is the equation $\underset{\sim}{r}$ found in part i defined to be the line segment joining AB. **2**

iii. Find the Cartesian equation of the line in part i. **2**

b. Find the value of a which minimises the integral $\int_0^1 (x^2 - a)^2 dx$ **3**

c.

i. Find the values of A, B and C such that

$$\frac{4x+3}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$ **3**

ii. Hence, evaluate $\int_0^1 \frac{4x+3}{(x+2)(x^2+1)} dx$ **3**

Start this question at the top of a new page in your answer booklet.

Question 13 **15 marks**

a. The area under the curve $y = 4\sin^{-1} 2x$ to the $x - axis$ in the first quadrant is rotated about the $y - axis$.

Find the volume of the solid formed. **3**

b. Find $\int \frac{1}{2+cosx} dx$ using the substitution of $t = \tan\frac{x}{2}$ **3**

c. A line passes through the points $P(1, 3, -2)$ and $Q(2, -1, 5)$

i. Find the equation of the line PQ in the form

$\underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{b}$ $where\ \lambda \in R$ **1**

ii. Consider a line with parametric equations

$x = 1 - \mu, y = 2 + \mu,\ z = -1 + \mu\ \ where\ \mu \in R$

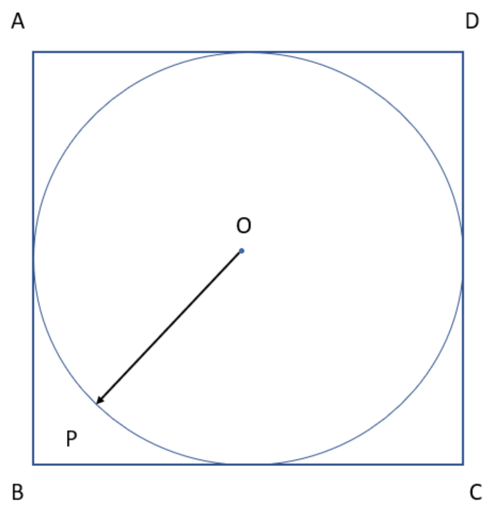
Determine, with working, whether this line intersects the line PQ from part i. **3**

Question 13 continues on the next page……………..

d. In the diagram below, O is the centre of the circle, radius r, inscribed in the square $ABCD$. Point P is any point on the circumference of the circle;

i. Show that $\overrightarrow{AP} \cdot \overrightarrow{AP} = 3r^2 - 2\overrightarrow{OP} \cdot \overrightarrow{OA}$ **3**

ii. Hence find $AP^2 + BP^2 + CP^2 + DP^2$ in terms of r **2**



End of Task

STHS

Extension 2 2022 Task 3

$\int$ and Vectors

Multiple Choice

1. B 2. C 3. B 4. B 5. B

6. C 7. C 8. D 9. A.

Working ~ section 1 ~ MC.

1. $\overrightarrow{AB} = \begin{pmatrix} -1 \\ -6 \\ 2 \end{pmatrix}$ dist $= \sqrt{(-1)^2+(-6)^2+2^2} = \sqrt{41}$ B

2. $\underset{\sim}{u} \cdot \underset{\sim}{v} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} = 0 + 3 + -4 = -1$ C

3. $\int x \sin x \, dx$ (PARTS)

$= x \cdot -\cos x - \int 1 \cdot -\cos x \, dx$

$= -x\cos x + \int \cos x \, dx$

$= -x\cos x + \sin x + C$ B

4. pt $(2, 3, -4)$ y value $= 3$ $\therefore \cos\theta = \frac{3}{\sqrt{29}}$ B.

dist $= \sqrt{4+9+16}$

5. A. $\begin{pmatrix} -4 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} \Rightarrow \begin{pmatrix} -5 \\ 2 \\ -2 \end{pmatrix} = \lambda \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ No λ exists

B. $\begin{pmatrix} -4 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} \Rightarrow \begin{pmatrix} -3 \\ -4 \\ 2 \end{pmatrix} = \lambda \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ $\therefore \lambda = -1$

$\therefore$ B.

2. $\overrightarrow{OP} = \text{proj}_{\overrightarrow{OB}} \overrightarrow{OA} = \frac{1}{3}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ $\therefore |\overrightarrow{OP}| = \sqrt{1/9 + 4/9 + 1/9} = \sqrt{6}/3$

$|\overrightarrow{OA}| = 7$ Pyth $\therefore |\overrightarrow{AP}| = \sqrt{|\overrightarrow{OA}|^2 - |\overrightarrow{OP}|^2} = \sqrt{48\frac{1}{3}}$ C

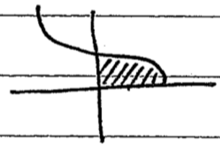
7. $\frac{d}{dx}\left[\int_x^{4x} \frac{dt}{1+t^2}\,dt\right] = \frac{d}{dx}\left[\tan^{-1}t\right]_x^{4x}$

$= \frac{d}{dx}\left(\tan^{-1}(4x) - \tan^{-1}(x)\right)$

$= \frac{4}{1+16x^2} - \frac{1}{1+x^2}$ $\therefore$ C

8. x^2+y^2 = circle But different z values as x & y change $\therefore$ D.

9.

A. $\int_0^1 \cos^{-1}x\,dx$  $= \int_0^{\pi/2} \cos y\,dy$

$= \sin y\Big]_0^{\pi/2} = 1$

B. $-\frac{1}{2}\int_0^1 -2x\sqrt{1-x^2}\,dx = -\frac{1}{\cancel{2}}(1-x^2)^{3/2}\cdot\frac{\cancel{2}}{3}\Big]_0^1$

$= -\frac{1}{3}\left[0-1\right] = 1/3$

C. $\int_0^1 x e^{-x}\,dx = x.\frac{e^{-x}}{-1} - \int 1.\frac{e^{-x}}{-1}\,dx$

$= -xe^{-x} + \int e^{-x}\,dx$

$= -xe^{-x} - e^{-x}\Big]_0^1$

$= -1.e^{-1} - e^{-1} - (0 - e^0)$

$\doteq 0.264...$

D. $\int_0^1 \frac{1}{(x-1)^2+1}\,dx = \tan^{-1}(x-1)\Big]_0^1$

$= \tan^{-1}0 - \tan^{-1}(-1)$ $\therefore$ A

$= 0.7853....$

Question 10

a. i. $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix} - \begin{pmatrix} -4 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \\ -3 \end{pmatrix}$ or $6\underset{\sim}{i} - 8\underset{\sim}{j} - 3\underset{\sim}{k}$

ii. $\hat{\underset{\sim}{b}} = \dfrac{\underset{\sim}{b}}{|\underset{\sim}{b}|} = \dfrac{1}{\sqrt{4+25+9}} \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$

$= \dfrac{1}{\sqrt{38}} \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$

iii. $\text{proj}_{\underset{\sim}{b}} \underset{\sim}{a} = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \left(\underset{\sim}{b}\right)$

$= \dfrac{\begin{pmatrix} -4 \\ 3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}}{\begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}} \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$

$= \dfrac{-8 - 15 + 18}{4 + 25 + 9} \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$

$= \dfrac{-5}{38} \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$ magnitude = $\sqrt{}$

$= -10/38\, \underset{\sim}{i} + \dfrac{25}{38} \underset{\sim}{j} - \dfrac{15}{38} \underset{\sim}{k}$

$=$

$\therefore$ length =

iv. $A \begin{pmatrix} -4 \\ 3 \\ 6 \end{pmatrix}$ $B \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$ $C = \left[\dfrac{-12+8}{7}, \dfrac{9-20}{7}, \dfrac{18+12}{7} \right]$

4 : 3

$C = \left[\dfrac{-4}{7}, -\dfrac{11}{7}, \dfrac{30}{7} \right]$

$\overrightarrow{OC} = \dfrac{1}{7} \left(-4\underset{\sim}{i} - 11\underset{\sim}{j} + 30\underset{\sim}{k} \right)$

Question 10

b) $\int_0^2 x\sqrt{2-x}\,dx$ let $u = 2-x$

$x = 2 \quad u = 0$

$x = 0 \quad u = 2$

$= -\int_2^0 (2-u)\sqrt{u}\,du \qquad -du = dx$

$= +\int_0^2 2u^{1/2} - u^{3/2}\,du$

$= \left. 2u^{3/2} \cdot \frac{2}{3} - u^{5/2} \cdot \frac{2}{5} \right]_0^2$

$= \frac{4}{3} \times 2^{3/2} - \frac{2}{5} \cdot 2^{5/2} - (0-0)$

$= \frac{4}{3} \times 2\sqrt{2} - \frac{2}{5}\, 4\sqrt{2}$

$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$

$= \frac{16\sqrt{2}}{15}.$

c) $\int_2^3 \ln x\,dx = \int_2^3 1 \times \ln x\,dx$

$= (\ln x) \cdot x - \int \frac{1}{x} \cdot x\,dx$

$= \Big[x\ln x - x \Big]_2^3$

$= 3\ln 3 - 3 - 2\ln 2 + 2$

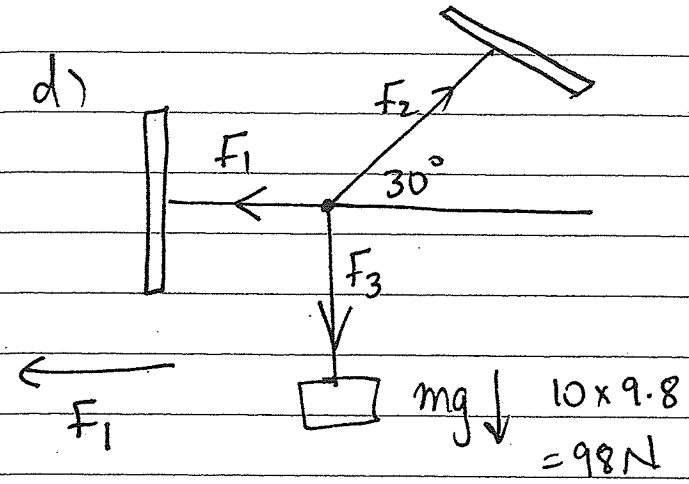
$= \ln 27 - \ln 4 - 1$

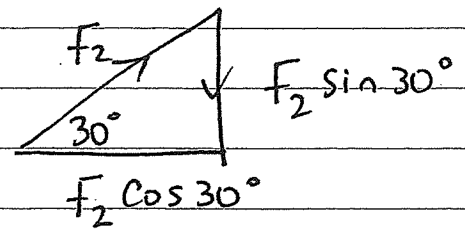
$= \ln 27 - \ln 4 - \ln e$

$= \ln\left(\frac{27}{4e}\right).$

Question 10

d)





Across

$F_1 = F_2 \cos 30°$

Down

$F_2 \sin 30° = 98\ N$

$F_2 = \frac{98}{\sin 30°}$

$= 196\ N$

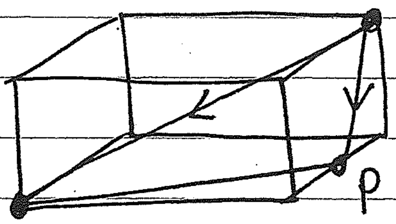
$\therefore F_1 = 196 \times \cos 30°$

$= 196 \times \frac{\sqrt{3}}{2}$

$= 98\sqrt{3}\ N$

$\therefore F_1 = 98\sqrt{3}\ N$, $F_2 = 196N$ and $F_3 = 98N$.

Question 11



$$\vec{OG} = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix}$$

$$\vec{OP} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}$$

$$\therefore \vec{GO} = \begin{pmatrix} -5 \\ -3 \\ -4 \end{pmatrix}$$

$$\vec{GP} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ -4 \end{pmatrix}$$

NOW $\left|\vec{GO}\right| = \sqrt{25+9+16}$ $= \sqrt{50}$ $\qquad \left|\vec{GP}\right| = \sqrt{0+4+16}$ $= \sqrt{20}$

NOW $\vec{GO} \cdot \vec{GP} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\theta$

$$\begin{pmatrix} -5 \\ -3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -2 \\ -4 \end{pmatrix} = \sqrt{50} \times \sqrt{20} \cos\theta$$

$$0 + 6 + 16 = 10\sqrt{10} \cos\theta$$

Alternative is Cosine rule with vectors in ΔOGP

$$\theta = \cos^{-1}\left(\frac{22}{10\sqrt{10}}\right)$$

$$\theta \doteq 45.9°.$$

b) $\int \sin 3x \cos 4x \, dx \implies$ use Reference Sheet.

$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$= \int \cos 4x \sin 3x \, dx \qquad A = 4x$, $B = 3x$

$= \frac{1}{2}\int \sin(4x+3x) - \sin(4x-3x) \, dx$

$= \frac{1}{2}\int \sin 7x - \sin x \, dx$

$= \frac{1}{2}\left[-\frac{1}{7}\cos 7x + \cos x\right] + C.$

b)

11 $\int \cos^3 x \, dx = \int \cos x \cos^2 x \, dx$

$= \int \cos x \left(1 - \sin^2 x\right) dx$

$= \int \cos x - \cos x \left(\sin x\right)^2 dx$

$= \sin x - \frac{1}{3} \sin^3 x + C$

Question 11

c) $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$ $\quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix}$ $\quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 7 \\ 5 \end{pmatrix}$

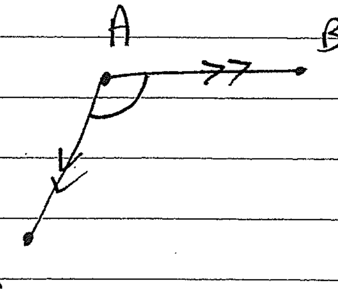
Find $|\overrightarrow{AB}|$ $\quad |\overrightarrow{AC}|$ and $|\overrightarrow{BC}|$

$$|\overrightarrow{AB}| = \left|\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}\right| = \sqrt{4+9+4} = \sqrt{17}$$

$$|\overrightarrow{AC}| = \left|\begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}\right| = \sqrt{36} = 6$$

$$|\overrightarrow{BC}| = \left|\begin{pmatrix} 2-4 \\ 7-4 \\ 5-3 \end{pmatrix}\right| = \left|\begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}\right| = \sqrt{4+9+4} = \sqrt{17}$$

$\therefore$ AB = BC and ΔABC is isosceles.

11. 

$\overrightarrow{AC} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$ $\quad |\overrightarrow{AC}| = 6$

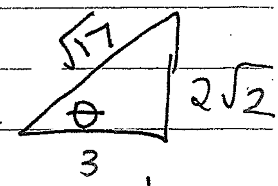
$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$ $\quad |\overrightarrow{AB}| = \sqrt{17}$

$$\therefore \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = |a||b| \cos\theta$$

$$0 + 18 - 0 = 6\sqrt{17}\cos\theta$$

$$\cos\theta = \frac{18}{6\sqrt{17}}$$

$$\cos\theta = \frac{3}{\sqrt{17}}$$



$$\theta = 43.31^\circ$$

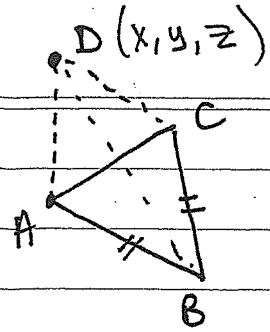
exact area

$\therefore$ Area $= \frac{1}{2} \times 6 \times \sqrt{17} \sin 43.31^\circ$
$= 8.485\ u^2$ accept

But exact $\Longrightarrow$ $\frac{1}{2} \times 6 \times \sqrt{17} \times \frac{2\sqrt{2}}{\sqrt{17}}$

$$A = \sqrt{72}\ u^2$$

$6\sqrt{2}\ u^2$



$A(2,1,5)$ $\quad B(4,4,3)$ $\quad C(2,7,5)$

Midpt AC $\quad (2, 4, 5)$

Midpt method:

$$\therefore \quad \frac{x+4}{2} = 2 \qquad \frac{y+4}{2} = 4 \qquad \frac{z+3}{2} = 5$$

$$D(0,4,7)$$

or vector addition

$$\vec{OD} = \vec{OA} + \vec{AD} \qquad \text{But } \vec{AD} = \vec{BC}$$

$$= \vec{OA} + \vec{BC}$$

$$= \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 4 \\ 7 \end{pmatrix} \qquad \therefore \quad D = (0,4,7)$$

Question 12

a) I. $\overrightarrow{AB} = \begin{pmatrix}-1\\-6\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}-2\\-8\end{pmatrix}$

$\therefore$ $\underset{\sim}{r} = \begin{pmatrix}1\\2\end{pmatrix} + \lambda\begin{pmatrix}-2\\-8\end{pmatrix}$.

II. $A(1,2)$ $\quad B(-1,-6)$

$\lambda = 0$

$\begin{pmatrix}-1\\-6\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix} + \lambda\begin{pmatrix}-2\\-8\end{pmatrix}$

$\lambda = 1$ $\quad \therefore \; 0 \le \lambda \le 1$

III. $x = 1 + -2\lambda \longrightarrow 2\lambda = 1 - x$

$y = 2 + -8\lambda$

$\lambda = \frac{1-x}{2}$

$\therefore \; y = 2 - 8\left(\frac{1-x}{2}\right)$

$y = 2 - 4(1-x)$

$y = 2 - 4 + 4x$

$y = 4x - 2$.

b) $\int_0^1 (x^2 - a)^2 \, dx$

$\int_0^1 x^4 - 2ax^2 + a^2 \, dx$

$= \left.\frac{x^5}{5} - \frac{2ax^3}{3} + a^2 x \right]_0^1$

$= 1/5 - \frac{2a}{3} + a^2 - (0)$

$A = a^2 - \frac{2a}{3} + \frac{1}{5}$

$\therefore$ min area $\Rightarrow 2a - 2/3 = 0$ $\quad$ ie $dA/da = 0$

$2a = 2/3$

$a = 1/3$ $\quad$ Concave up

$\therefore$ minimum.

Question 12

c) (i) $4x+3 = A(x^2+1) + (Bx+c)(x+2)$

let $x=-2$

$-8+3 = A(4+1) + 0$

$-5 = 5A$

$\underline{A = -1}$

let $x = 0$

$3 = A(1) + (0+c)(2)$

$3 = -1\times 1 + 2c$

$4 = 2c$

$\underline{c = 2}$

or equivalent method such as equating!

let $x = 1$

$7 = (-1)(2) + (B+2)(3)$

$9 = 3(B+2)$

$3 = B+2$

$\underline{B = 1}$

$\therefore$ (ii) $\int_0^1 f(x)\,dx = \int \frac{-1}{x+2} + \frac{x+2}{x^2+1}\,dx$

$= \int \frac{-1}{x+2} + \frac{x}{x^2+1} + \frac{2}{x^2+1}\,dx$

$= \left[-\ln|x+2| + \frac{1}{2}\ln|x^2+1| + 2\tan^{-1}x\right]_0^1$

$= -\ln 3 + \frac{1}{2}\ln 2 + 2\tan^{-1}(1) - (-\ln 2 + 0 + 0)$

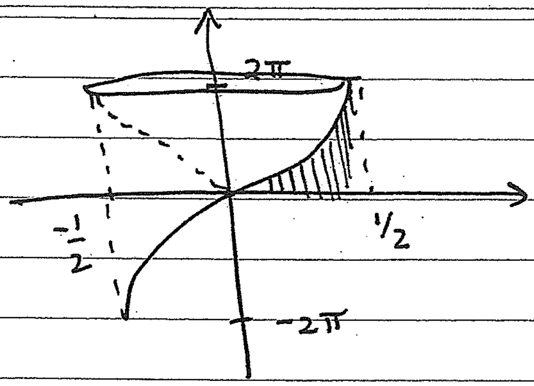
$= -\ln 3 + \ln\sqrt{2} + 2(\pi/4) + \ln 2$

$= \ln\left(\frac{2\sqrt{2}}{3}\right) + \frac{\pi}{2}$

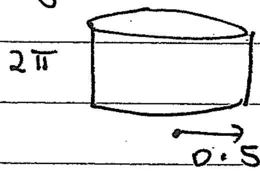
Question 13

a)

$y = 4\sin^{-1} 2x$



Cylinder volume $= \pi \times \left(\frac{1}{2}\right)^2 \times 2\pi$



$= \frac{\pi^2}{2}$

$\frac{y}{4} = \sin^{-1} 2x$

$2x = \sin\left(\frac{y}{4}\right)$

$x = \frac{1}{2}\sin\frac{y}{4}$

$x^2 = \frac{1}{4}\sin^2 y/4$

$V_y = \pi \int x^2 \, dy$

$= \pi \int_0^{2\pi} \frac{1}{4}\sin^2\frac{y}{4} \, dy$

$= \frac{\pi}{4}\int_0^{2\pi} \frac{1}{2}\left(1 - \cos 2\left(\frac{y}{4}\right)\right) dy$

$= \frac{\pi}{8}\int_0^{2\pi} 1 - \cos\frac{y}{2} \, dy$

$= \frac{\pi}{8}\left[y - \frac{\sin\frac{y}{2}}{1/2}\right]_0^{2\pi}$

$= \frac{\pi}{8}\left[2\pi - 2\sin\pi - (0 - 0)\right]$

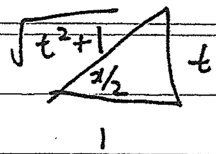
$= \frac{\pi}{8} \times 2\pi$

$= \frac{\pi^2}{4}$

$\therefore$ Required volume $= \frac{\pi^2}{2} - \frac{\pi^2}{4} = \frac{\pi^2}{4} \, u^3.$

b) $\displaystyle\int \frac{1}{2+\cos x}\,dx$

$t = \tan\frac{x}{2}$



$\cos\frac{x}{2} = \frac{1}{\sqrt{t^2+1}}$

$\cos x = \frac{1-t^2}{1+t^2}$

$\dfrac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$

$dt = \dfrac{1}{2\cos^2\frac{x}{2}}\,dx$

$dt = \dfrac{1}{\frac{2}{t^2+1}}\,dx$

$dx = \dfrac{2}{t^2+1}\,dt$

$$\int \frac{1}{2+\frac{1-t^2}{1+t^2}}\cdot\frac{2\,dt}{t^2+1}$$

$$= \int \frac{2\,dt}{2(t^2+1)+1-t^2}$$

$$= \int \frac{2\,dt}{2t^2+2+1-t^2}$$

$$2\int \frac{1}{t^2+3}\,dt$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\frac{t}{\sqrt{3}} + c$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{3}}\right) + c.$$

c) $\underset{\sim}{r} = \begin{pmatrix}1\\3\\-2\end{pmatrix} + \lambda\begin{pmatrix}2-1\\-1-3\\5--2\end{pmatrix}$

PQ: $\underset{\sim}{r} = \begin{pmatrix}1\\3\\-2\end{pmatrix} + \lambda\begin{pmatrix}1\\-4\\7\end{pmatrix}$

PQ in Parametric form $\Rightarrow$ $x = 1 + \lambda$ $\quad y = 3 - 4\lambda$ $\quad z = -2 + 7\lambda$

given $x = 1 - \mu$ $\quad y = 2 + \mu$ $\quad z = -1 + \mu$

Solving for μ and λ and see if they work for z.

$$1 - \mu = 1 + \lambda \qquad\qquad 2 + \mu = 3 - 4\lambda$$
$$\mu = -\lambda \qquad\qquad 2 - \lambda = 3 - 4\lambda$$
$$3\lambda = 1$$
$$\lambda = \frac{1}{3}$$
$$\therefore \mu = \frac{-1}{3}$$

NOW $z = -2 + 7\lambda$ for PQ
$$= -2 + 7(1/3)$$
$$= \frac{1}{3}$$

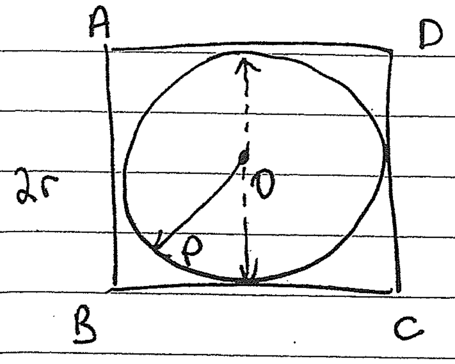
$z = -1 + \mu$ other line.
$$= -1 + {}^{-1}/3$$
$$= -1\tfrac{1}{3}$$

$\therefore$ the values of μ & λ don't work for x, y & z

$\therefore$ Lines don't intersect

Note as direction vectors are not $=$ they are also not $\parallel$ $\therefore$ they are Skew Lines.

Question 13 d)



$\therefore \; |\overrightarrow{AB}| = 2r$

$$|\overrightarrow{AC}| = \sqrt{(2r)^2 + (2r)^2}$$
$$= \sqrt{4r^2 + 4r^2}$$
$$= \sqrt{8r^2}$$
$$|\overrightarrow{AC}| = 2\sqrt{2}\,r \qquad \therefore \; |\overrightarrow{OA}| = \sqrt{2}\,r$$

$$\overrightarrow{AP} \cdot \overrightarrow{AP} = |\overrightarrow{AP}|^2$$
$$= |\overrightarrow{AO} + \overrightarrow{OP}|^2$$
$$= |\overrightarrow{AO}|^2 + 2|\overrightarrow{AO}||\overrightarrow{OP}| + |\overrightarrow{OP}|^2$$
$$= 2r^2 - 2\,\overrightarrow{OA} \cdot \overrightarrow{OP} + r^2$$
$$= 3r^2 - 2\,\overrightarrow{OA} \cdot \overrightarrow{OP} \qquad \text{QED.}$$

$$AP^2 = 3r^2 - 2\overrightarrow{OA} \cdot \overrightarrow{OP}$$

$$BP^2 = 3r^2 - 2\overrightarrow{OB} \cdot \overrightarrow{OP}$$

$$CP^2 = 3r^2 - 2\overrightarrow{OC} \cdot \overrightarrow{OP}$$

$$DP^2 = 3r^2 - 2\overrightarrow{OD} \cdot \overrightarrow{OP}$$

$+$

$$AP^2 + BP^2 + CP^2 + DP^2 = 12r^2 - 2\overrightarrow{OP}\left[\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}\right]$$

$$= 12r^2 - 2r\left[\overrightarrow{OA} - \overrightarrow{OA} + \overrightarrow{OC} - \overrightarrow{OC}\right]$$

$$= 12r^2 - 2r(0)$$

$$= 12r^2$$

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Question 10:

a)
Students generally knew their fundamental vector formulas. Several students did not realise that iii asked for the **length** of the projection, it had to be both positive and a scalar. Many strange errors occurred in iv, the most common mistake was finding AC, rather than the position vector OC.

b) Many methods worked for this question, but a substitution of $u = 2 - x$ was most effective. Bar silly negative errors those who used this technique were successful. Some students got it by parts, or by substituting $u = \sqrt{(2-x)}$, but this led to more mess and therefore a higher chance of errors.

c) Most students know how to integrate by parts. Simplifying it as one expression caused greater difficulty, as it required seeing that $1 = \ln e$. Many students also claimed that $3\ln 3 = \ln 9$.

d) It was clear which students understood the notion of a Force as a vector in this question. Use of $F = ma$ and carefully drawing diagrams led to a simple exercise in Trigonometry. Aside from non-attempts, the most common error was incorrectly applying vector addition.

Question 11:

a) This question was deceptively tricky. To find the $\angle OGP$ the tails of the vectors must be at G so the relevant vectors are $\vec{GO}$ and $\vec{GP}$. Most students used $\vec{OG}$ as most questions tend to be from the Origin so it is easy to develop the habit to always start from O. The wrong direction will result in an obtuse angle which some students left as their answer. Students need to always make a common-sense check of their answer. Students who did realise his and found the supplementary angle for the correct angle scored full marks.

b) i) One of the skill required for Extension 2 integration is the ability and knowledge to apply the most efficient technique for a selected integral. $\int \sin 3x \cos 4x dx$ is an obvious application of products to product to sum formulae (on the NESA reference sheet) which turn the question into basic 2 line Advanced trig integral. Many students tried to use integration by parts, only one of whom managed to correctly complete the ¾ page integration for full marks. (The others wasted effort and paper for no reward).

Students also need to look at the location, and context of equation as well as number of allocated marks to realise that it was probably not a killer integration by parts intergral.

c) i) students need to know their basic year 8 properties of quadrilaterals and triangles. A triangle is isosceles if and only if it has 2 equal sides. This was possibly the easiest and shortest 3 marks in the paper which several students over-complicated. All that was required was to find 2 equal sides. If you use the diagram or get lucky and find the 2 equal sides first you are done and do not need to show that the third side is not equal since an equilateral triangle is also an isosceles as the definitions are inclusive. Some nervous souls then went on to waste more effort finding the angles which was completely unnecessary but in this case at least helpful for the next part of the question.

ii) You do not need to find the base for the area of a triangle if you have 2 sides and an included angle (Year 10 concept , also on formula sheet)

d) If a question does not supply a diagram, DRAW YOUR OWN and label, that way if you misinterpret you will be allocated full marks as long as you apply the concept being tested correctly and have not made the question easier. ADDITIONALLY, diagrams will frequently help you see the answer with geometry questions. Strictly speaking the question was after the parallelogram where $AD \,||\, BC$ as shapes are labelled in cyclic order so parallelogram $ABCD$ has a line going from A to B to C and then back to A. No point order was penalised as long as it was correct.

Question 12:

(a) There were various answers to this depending upon the vector AB being taken as $\overrightarrow{AB}$ or $\overleftarrow{AB}$. All were considered.

(ii) For whatever form you had in part (i), there are values of λ which will give you point A and point B. All the points in between A and B give you the interval AB <u>and</u> these entail the use of values of λ between the initial values, whatever they were. The most simple answers were $0 \leq \lambda \leq 1$ or possibly $-1 \leq \lambda \leq 0$. Whichever you do, if you find values of λ outside this range, then the point found lies outside the interval AB.

(b) All students found this hard. It was actually quite easy. Just integrate w.r.t. *x* and then you have a quadratic in a. Differentiate this and proceed as usual.

Question 13:

a) It was surprising that more students did not start with a diagram here, as most students did not find the correct volume. It is a question adapted from a tech extension 1 question from years ago just involving inverse sine. Students had incorrect limits as they did not successfully change the domain and range of the curve. Errors also made when changing inverse back to sine it should be as follows…

$$y = 4\sin^{-1} 2x$$

$$\frac{y}{4} = \sin^{-1} 2x$$

$$2x = \sin\frac{y}{4}$$

$$x = \frac{1}{2}\sin\frac{y}{4}$$

Please also look at the reference sheet it has many rules on it

b) A standard no trick integration using $t - results$ - errors here included not subbing in for the dx and not knowing the $t - results$ subs (CHECK the reference sheet) it ended up as inverse tan and was easy if you used the reference sheet!!

c) i. learn the rule simple way is a vector line through PQ is r = P + k(PQ)

ii. mostly ok in method but some careless errors occurring, maybe due to time – however the time came down to the fact that students did not use sensible methods for integrals that were on the reference sheet.

d) the last question is meant to be different and challenging and most found it so – a diagram and finding the lengths of AB, AC and hence OA was the best place to start – then look at rewriting AP as two vectors

ii. explain the steps for those making it here is what is recommended.