Student Name: ____________________

Teacher: ________________________

Sydney Technical High School

2023 | HIGHER SCHOOL CERTIFICATE ASSESSMENT 3

Mathematics Extension 2

General Instructions

- Working Time – 1 hour and 30 minutes
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless work or illegible writing.
- **Begin each question on a new page.**

Total marks: 58

Section I — 6 marks

- Attempt Questions 1 - 6
- Allow about 10 minutes for this part.

Section II — 52 marks

- Attempt Questions 7 - 10
- Allow about 75 minutes for this section.

SECTION I

Attempt Questions 1-6.
Answer the questions on the multiple-choice answer sheet located at the start of your answer booklet.
Allow approximately 15 minutes for this section.

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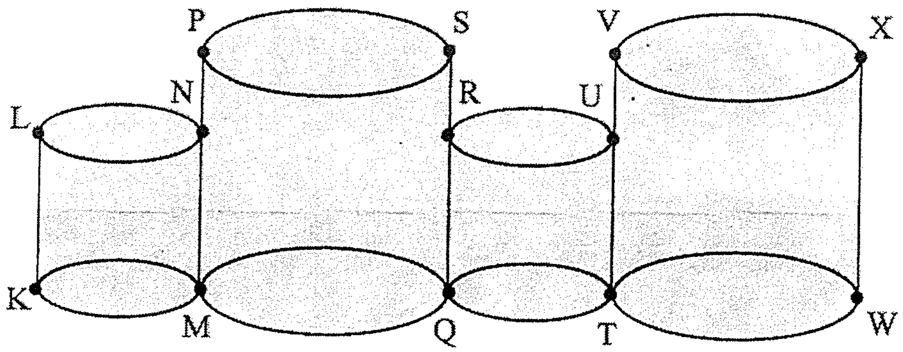
1. Two identical small cylinders and two identical large cylinders are placed next to each other as shown below.



Which of the following is equal to $\overrightarrow{KS}$?

A. $\overrightarrow{KN} + \overrightarrow{MS}$

B. $\overrightarrow{KM} + \overrightarrow{SM}$

C. $\overrightarrow{LN} + \overrightarrow{TX}$

D. $\overrightarrow{SM} + \overrightarrow{MK}$

2. The vector equation of a line joining the points $P(1,5,3)$ and $Q(a,b,c)$ is:

$$\underset{\sim}{r} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 7 \\ 5 \end{pmatrix}$$

Which of the following could be the coordinates of Q?

A. $(2,2,1)$

B. $(4,12,8)$

C. $(4,10,8)$

D. $(4,12,6)$

3. $OABC$ is a rectangle with $\overrightarrow{OA} = 3\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\overrightarrow{OC} = 6\underset{\sim}{i} + 4\underset{\sim}{j} + a\underset{\sim}{k}$ for some constant a.

What is the value of a?

A. -2

B. -3

C. -5

D. -6

4. $\int \frac{1}{\sqrt{3-2x-x^2}}dx$ equals:

A. $\sin^{-1}\frac{1}{2}(x-1)+C$

B. $\sin^{-1}\frac{1}{2}(x+1)+C$

C. $\sin^{-1}\frac{1}{\sqrt{2}}(x+1)+C$

D. $\sin^{-1}\frac{1}{\sqrt{2}}(x-1)+C$

5. Which of the following is the vector equation of a line that passes through the point $(1,3,-2)$ and is perpendicular to the line:

$$\underset{\sim}{r} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

A. $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$

B. $\underset{\sim}{r} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$

C. $\underset{\sim}{r} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$

D. $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix}$

6. Which expression is equal to $\int \frac{2x+4}{x^2+16}dx$?

A. $2\ln|x^2+16| + 4\tan^{-1}\frac{x}{4} + C$

B. $\ln|x^2+16| + \tan^{-1}\frac{x}{4} + C$

C. $\ln|x^2+16| + 4\tan^{-1}\frac{x}{4} + C$

D. $2\ln|x^2+16| + \tan^{-1}\frac{x}{4} + C$

END OF SECTION 1

SECTION II

Answer each question showing all necessary working and calculations in the answer booklet.
Start each question on the TOP of a NEW page.
Allow approximately 1 hour and 15 minutes for this section.

Question 7 **Marks**

a) Two vectors are such that $\underset{\sim}{u} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix}$. Evaluate $\left|\underset{\sim}{u}\right| \times \underset{\sim}{v} \cdot \underset{\sim}{v}$ **2**

b) Find the acute angle (to the nearest degree) between the vectors: **2**

$$\underset{\sim}{u} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \text{ and } \underset{\sim}{v} = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$$

c) Find $\int 2x\sqrt{x+2}\,dx$ **3**

d) Find: **3**

$$\int \frac{2x^2 - x + 1}{x - 2}\,dx$$

e) If $A = (1,1,1)$ and $B = (13,4,5)$, find the point P on $\overrightarrow{AB}$ such that $AP:PB = 3:1$ **3**

Question 8

a) A sphere has its centre at $(3,-3,4)$ and its radius is 6 units. A line has equation:

$$\underset{\sim}{r} = \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

i. Write down the vector equation of the sphere. **1**

ii. Determine whether the line is a tangent to the sphere, clearly justifying your conclusion. **3**

b) With respect to the origin O, the lines l_1 and l_2 are given by the equations: **4**

$l_1: \underset{\sim}{r} = \left(10\underset{\sim}{i} - 9\underset{\sim}{k}\right) + \lambda(-\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k})$

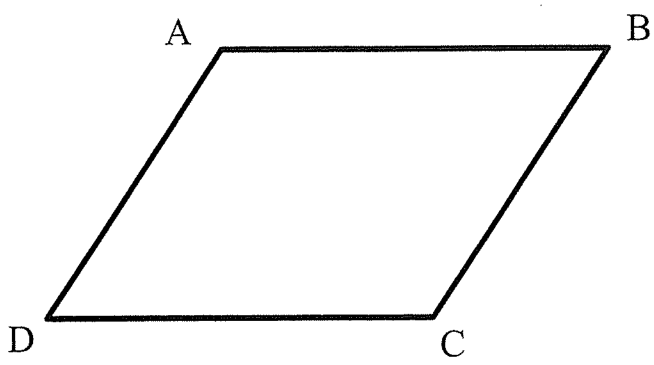
$l_2: \underset{\sim}{r} = \left(17\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}\right) + \mu(5\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k})$

where λ and μ are scalar parameters. Show that l_1 and l_2 meet and find the position vector of the point of intersection.

Marks

c) ABCD is a parallelogram. If $\angle CDB = \angle ADB$, use vector methods to prove that $AB = BC$ and hence that $ABCD$ is a rhombus. **3**

Let $\overrightarrow{DC} = \underset{\sim}{c}$, let $\overrightarrow{CB} = \underset{\sim}{b}$ and $\angle CDB = \angle ADB = \theta$.



d) Let $\underset{\sim}{a} = 14\underset{\sim}{i} + 4\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} - 2\underset{\sim}{j} + 2\underset{\sim}{k}$. **2**

Find the length of the projection of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$.

Question 9

a) Find the perpendicular distance from $P(2,1,1)$ to the line through $A(-1,0,2)$ and $B(1,1,4)$ **4**

b) Two lines which are not parallel to each other and do not intersect are called skew lines. **3**

If $\underset{\sim}{u} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 3 \\ 1 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$, show that $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are skew.

c)

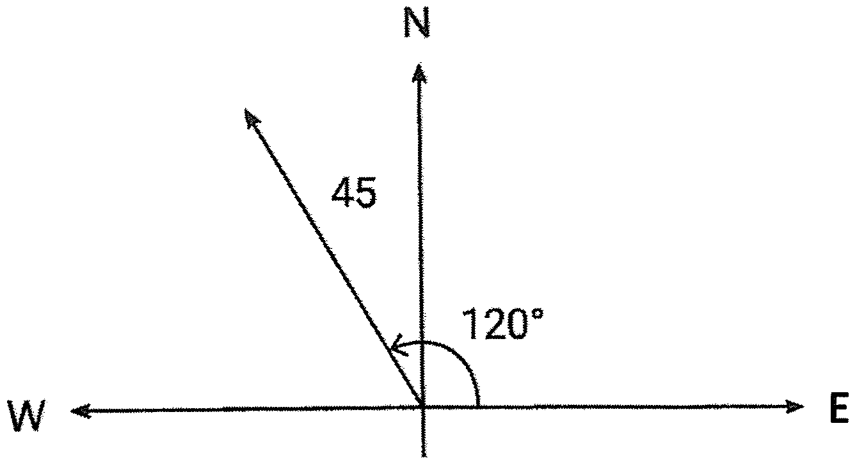
i. Express $\frac{5}{(x-1)(x^2+1)}$ as the sum of partial fractions over $\mathbb{R}$. **3**

ii. Find: **3**

$$\int \frac{5}{(x-1)(x^2+1)}\,dx$$

Question 10 **Marks**

a) A drone is set to fly west at 40km/h. A cross wind diverts its path so that it travels with a speed of 45km/h in the direction shown below.



(i) By taking the starting point to be the origin and the path above to be the resultant, use vector expressions to express the cross wind as a vector. **3**

(ii) Calculate the speed and bearing of the cross wind. **2**

b) Find: **4**

$$\int \frac{x^3}{2x+1}\,dx$$

c) Two submarines, A and B have position vectors $(-120,210,-265)$ and $(100,458,-151)$ respectively. Submarine A sets off at 12pm while submarine B sets off at 12:30pm. The velocity vector for submarine A is $\begin{pmatrix} 30 \\ 45 \\ -10 \end{pmatrix}$ km/h and the velocity vector for submarine B is $\begin{pmatrix} -15 \\ -\frac{9}{2} \\ 13 \end{pmatrix}$ km/h.

(i) Using parameter t, write vector equations for both lines. **1**

(ii) Find the time when one submarine is directly above the other. **3**

END OF EXAM

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2023 Ext. 2 Task 3 Solutions

1. $\overrightarrow{LN} = \overrightarrow{KM}$

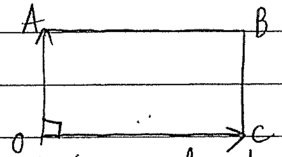
$\overrightarrow{TX} = \overrightarrow{MS}$

$\therefore \overrightarrow{KM} + \overrightarrow{MS}$

C

2. If Q lies on $\underset{\sim}{r}$, look for constant λ to satisfy this. When $\lambda = 1$, you have $(4, 12, 8)$

$\therefore$ B

3. 

Dot product $= 0$

$\therefore 3 \times 6 + -2 \times 4 + 2a = 0$

$18 - 8 + 2a = 0$

$a = -5$

C

4. $\int \frac{1}{\sqrt{3 - 2x - x^2}}\, dx$

$\int \frac{1}{\sqrt{3 - (x^2 + 2x + 1) + 1}}\, dx$

$\int \frac{1}{\sqrt{4 - (x+1)^2}}\, dx$

$= \sin^{-1}\left(\frac{x+1}{2}\right) + c$

B

5. ~~Look for $\perp$ direction vector where dot product $= 0$ Only in C or D But passes through $(1, 3, -2)$ $\therefore$~~

Accept C or D

6. $\int \frac{2x + 4}{x^2 + 16}\, dx$

$\int \frac{2x}{x^2 + 16} + \frac{4}{x^2 + 16}\, dx$

$= \log_e |x^2 + 16| + \tan^{-1}\frac{x}{4}$

B

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7a. $|\underset{\sim}{v}| = \sqrt{2^2 + 1^2 + 3^2}$

$= \sqrt{14}$

$\underset{\sim}{v} \cdot \underset{\sim}{v} = |\underset{\sim}{v}|^2$

$= \left(\sqrt{4^2 + 2^2 + 4^2}\right)^2$

$= 36$

$\therefore 36\sqrt{14}$

b. $\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}| \times |\underset{\sim}{v}| \times \cos\theta$

$6 - 2 + 15 = \sqrt{14} \times \sqrt{38} \cos\theta$

$\therefore \cos\theta = \frac{19}{\sqrt{14} \times \sqrt{35}}$

$\theta = 35^\circ$

c) $\int 2x\sqrt{x+2}\, dx$

Let $u = x + 2$

$\therefore x = u - 2$

$2x = 2u - 4$

and $du = dx$

change limits

$\int (2u - 4)\sqrt{u}\, du$

$\int 2u^{\frac{3}{2}} - 4u^{\frac{1}{2}}\, du$

$= \left[\frac{4}{5}u^{\frac{5}{2}} - \frac{8}{3}u^{\frac{3}{2}}\right]$

$= \frac{4}{5}(x+2)^{\frac{5}{2}} - \frac{8}{3}(x+2)^{\frac{3}{2}} + c$

d) $\int \frac{2x^2 - x + 1}{x - 2}\, dx$

$$\begin{array}{r} 2x + 3 \\ x - 2 \enclose{longdiv}{2x^2 - x + 1} \\ 2x^2 - 4x \\ \hline 3x + 1 \\ 3x - 6 \\ \hline 7 \end{array}$$

$\therefore \int 2x + 3 + \frac{7}{x - 2}\, dx$

$= x^2 + 3x + 7\log_e|x - 2| +$

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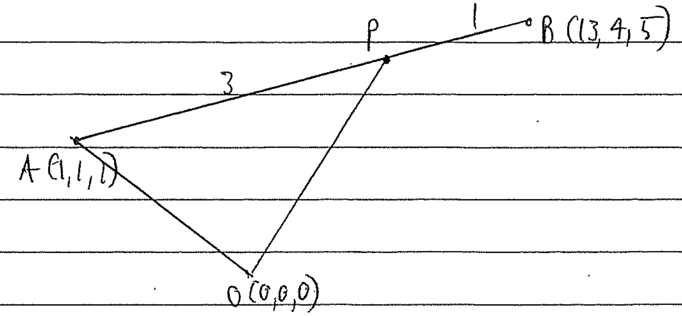
e)



P is $\frac{3}{4}$ of the way from A to B

$$\vec{OP} = \vec{OA} + \vec{AP}$$
$$= (1,1,1) + \tfrac{3}{4}\vec{AB}$$
$$= (1,1,1) + \tfrac{3}{4}(13-1, 4-1, 5-1)$$
$$= (1,1,1) + \tfrac{3}{4}(12, 3, 4)$$
$$= (1,1,1) + (9, \tfrac{9}{4}, 3)$$
$$= (10, \tfrac{13}{4}, 4)$$

8 a)(i) $\left| \underset{\sim}{v} - \begin{pmatrix} 3 \\ -3 \\ 4 \end{pmatrix} \right| = 6$ (ii) Line is $\begin{pmatrix} 1+2\lambda \\ 5-\lambda \\ 4-\lambda \end{pmatrix}$

Sub this into sphere:

$$\left| \begin{matrix} 1+2\lambda-3 \\ 5-\lambda+3 \\ 4-\lambda-4 \end{matrix} \right| = 6 \quad \text{is} \quad \left| \begin{matrix} 2\lambda-2 \\ 8-\lambda \\ -\lambda \end{matrix} \right| = 6$$

$$\Rightarrow (2\lambda-2)^2 + (8-\lambda)^2 + (-\lambda)^2 = 36$$
$$\cancel{4}\lambda^2 - \cancel{8}\lambda + 4 + 64 - \cancel{16}\lambda + \cancel{\lambda^2} + \cancel{\lambda^2} = 36$$
$$6\lambda^2 - 24\lambda + 32 = 0$$
$$3\lambda^2 - 12\lambda + 16 = 0$$
$$\Delta = 144 - 4\times3\times16 < 0 \therefore \text{no solution}$$

so line does not cross sphere.

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b) Lines meet if components are equal:

$$l_1: \quad r = (10-\lambda)\underset{\sim}{i} + \lambda\underset{\sim}{j} + (2\lambda-9)\underset{\sim}{k}$$
$$l_2: \quad r = (17+5\mu)\underset{\sim}{i} + (1-\mu)\underset{\sim}{j} + (3+3\mu)\underset{\sim}{k}$$

Equating gives

$$10-\lambda = 17+5\mu \quad ①$$
$$\lambda = 1-\mu \quad ②$$
$$2\lambda-9 = 3+3\mu \quad ③$$

Solve ① and ② simultaneously:

$$10-(1-\mu) = 17+5\mu$$
$$10-1+\mu = 17+5\mu$$
$$-8 = 4\mu$$
$$\mu = -2 \quad \therefore \lambda = 3$$

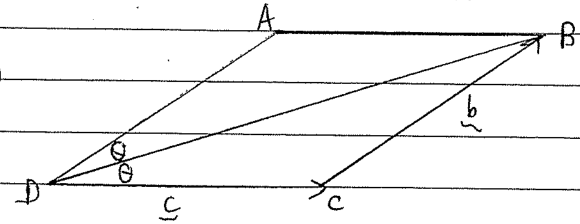
Check ③ is satisfied $2\times3-9 = 3+3\times-2$

$$-3 = -3 \checkmark$$

$\therefore$ They meet. Sub $\lambda = 3$ into l_1

$$\therefore r = 7\underset{\sim}{i} + 3\underset{\sim}{j} - 3\underset{\sim}{k}$$

c)



Let $\angle BDC = \angle DAB = \theta$ given

In $\triangle DCB$

$$\cos\theta = \frac{(\underset{\sim}{c}+\underset{\sim}{b}).\underset{\sim}{c}}{|\underset{\sim}{c}+\underset{\sim}{b}||\underset{\sim}{c}|}$$

In $\triangle ADB$

$$\cos\theta = \frac{(\underset{\sim}{c}+\underset{\sim}{b}).\underset{\sim}{b}}{|\underset{\sim}{c}+\underset{\sim}{b}||\underset{\sim}{b}|}$$ since $\vec{AD} = \underset{\sim}{b}$

$$\therefore \frac{\underset{\sim}{c}.\underset{\sim}{c} + \underset{\sim}{b}.\underset{\sim}{c}}{|\underset{\sim}{c}+\underset{\sim}{b}||\underset{\sim}{c}|} = \frac{\underset{\sim}{c}.\underset{\sim}{b} + \underset{\sim}{b}.\underset{\sim}{b}}{|\underset{\sim}{c}+\underset{\sim}{b}||\underset{\sim}{b}|}$$

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$$\frac{|\underset{\sim}{c}|^2 + \underset{\sim}{b}\cdot\underset{\sim}{c}}{|\underset{\sim}{c}|} = \frac{\underset{\sim}{b}\cdot\underset{\sim}{c} + |\underset{\sim}{b}|^2}{|\underset{\sim}{b}|}$$

is only true if $|\underset{\sim}{c}| = |\underset{\sim}{b}|$ ∴ since adjacent sides in a parallelogram are equal, it must be a rhombus.

d) $$\text{proj}_{\underset{\sim}{b}}\, \underset{\sim}{a} = \frac{\underset{\sim}{b}\cdot\underset{\sim}{a}}{\underset{\sim}{b}\cdot\underset{\sim}{b}}\, \underset{\sim}{b}$$

$$= \left|\frac{14\times-2+4\times-2+4\times2}{2^2+2^2+2^2}\times\begin{pmatrix}-2\\-2\\2\end{pmatrix}\right|$$

$$= \left|\frac{8}{12}\begin{pmatrix}-2\\-2\\2\end{pmatrix}\right|$$

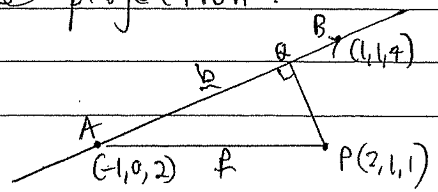
$$= \left|\begin{matrix}\frac{14}{3}\\ \frac{14}{3}\\ -\frac{14}{3}\end{matrix}\right|$$

$$= \sqrt{3\times\frac{196}{9}}$$

$$=$$

$$= \frac{14\sqrt{3}}{3}$$

9.a. Use projection:



Let $\underset{\sim}{p} = \overrightarrow{AP}$

$\underset{\sim}{p} = 3\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$

and $\underset{\sim}{b} = \overrightarrow{AB}$

$= 2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$

then $|\overrightarrow{AB}|^2 = 9$ $\quad \underset{\sim}{b}\cdot\underset{\sim}{p} = 5$

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then $$\text{proj}_{\underset{\sim}{b}}\, \underset{\sim}{p} = \frac{\underset{\sim}{b}\cdot\underset{\sim}{p}}{|\underset{\sim}{b}|^2}\,\underset{\sim}{b} = \frac{5}{9}\begin{pmatrix}2\\1\\2\end{pmatrix}$$

Thus if Q is the point on AB closest to P then

$$\overrightarrow{PQ} = \text{proj}_{\underset{\sim}{b}}\, \underset{\sim}{p} - \underset{\sim}{p}$$

$$= \frac{5}{9}\begin{pmatrix}2\\1\\2\end{pmatrix} - \begin{pmatrix}3\\1\\-1\end{pmatrix}$$

$$= \begin{pmatrix}10/9 - 3\\ 5/9 - 1\\ \frac{10}{9} + 1\end{pmatrix}$$

$$= \begin{pmatrix}-\frac{17}{9}\\ -\frac{4}{9}\\ 19/9\end{pmatrix} = \frac{1}{9}\begin{pmatrix}-17\\-4\\19\end{pmatrix}$$

∴ Perp. distance

$= |\overrightarrow{PQ}|$

$= \sqrt{\left(\frac{1}{9}\right)^2\left[17^2+4^2+19^2\right]}$

$= \frac{\sqrt{666}}{9}$

b) $\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\begin{pmatrix}2\\3\\-1\end{pmatrix}$ are not scalar multiples ∴ are not parallel.

Now try solving simultaneously for a point of intersection.

$2+\lambda = 3+2\mu$ ①

$-1+2\lambda = 1+3\mu$ ②

$4+3\lambda = 6-\mu$ ③

①×2 − ② ⇒

$5 = 5+\mu$ ∴ $\mu = 0$

if $\mu = 0$, $\lambda = 1$

do not satisfy ③

∴ No point of intersection ∴ lines must be skew.

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c (i) $\frac{5}{(x-1)(x^2+1)} = \frac{a}{x-1} + \frac{bx+c}{x^2+1}$

$5 = a(x^2+1) + (bx+c)(x-1)$

$= ax^2 + a + bx^2 - bx + cx - c$

$= (a+b)x^2 + (c-b)x + (a-c)$

$\therefore a+b=0$ ①   ① + ② $\Rightarrow a+c=0$

$c-b=0$ ②   but $a-c=5$ ③

$a-c=5$ ③ $\therefore 2a = 5$

$a = 2\frac{1}{2}$

$\therefore c = -2\frac{1}{2}$

$\therefore b = -2\frac{1}{2}$

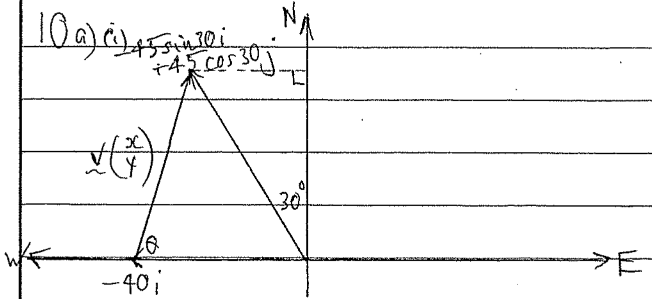
$\therefore \frac{\frac{5}{2}}{x-1} + \frac{-\frac{5}{2}x - \frac{5}{2}}{x^2+1}$

(ii) $\int \frac{\frac{5}{2}}{x-1}\,dx - \frac{5}{2}\int \frac{x+1}{x^2+1}\,dx$

$= \frac{5}{2}\log_e|x-1| - \frac{5}{2}\times\frac{1}{2}\int\frac{2x}{x^2+1}\,dx - \frac{5}{2}\int\frac{1}{x^2+1}\,dx$

$= \frac{5}{2}\log_e|x-1| - \frac{5}{4}\log_e|x^2+1| - \frac{5}{2}\tan^{-1}x + c$

10 a) (i)



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$\begin{pmatrix}-40\\0\end{pmatrix} + \begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-45\sin 30\\45\cos 30\end{pmatrix}$

$\therefore$ Cross wind $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-\frac{45}{2}+40\\45\times\frac{\sqrt{3}}{2}\end{pmatrix}$

$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\frac{35}{2}\\\frac{45\sqrt{3}}{2}\end{pmatrix}$

(ii) Magnitude $= \left|\begin{matrix}x\\y\end{matrix}\right| = \sqrt{\left(\frac{35}{2}\right)^2 + \left(\frac{45\sqrt{3}}{2}\right)^2}$

$= 42.72$ km/h

$\tan\theta = \frac{\frac{45\sqrt{3}}{2}}{17.5}$

$\theta = 65°49'$

$\therefore$ Bearing is 024°T to nearest degree

b) $\int \frac{x^3}{2x+1}\,dx$

$\int \frac{x^2 \cdot x}{2x+1}\,dx$

Let $u = 2x+1$

$du = 2\,dx$

$\therefore dx = \frac{1}{2}du$

$\frac{1}{2}\int \frac{\left(\frac{u-1}{2}\right)^2\left(\frac{u-1}{2}\right)}{u}\,du$

$\frac{1}{8}\times\frac{1}{2}\int\frac{(u^2-2u+1)(u-1)}{u}\,du$

$\frac{1}{16}\int\frac{u^3-u^2-2u^2+2u+u-1}{u}\,du$

$\frac{1}{16}\int u^2 - 3u + 3 - \frac{1}{u}\,du$

$\frac{1}{16}\left[\frac{u^3}{3} - \frac{3u^2}{2} + 3u - \log_e u\right]$

$\frac{1}{16}\left[\frac{(2x+1)^3}{3} - \frac{3(2x+1)^2}{2} + 3(2x+1) - \log_e|2x+1|\right] + c$

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c) Vector equations for both paths where the direction vector is the velocity:

$$A = \begin{pmatrix} -120 \\ 210 \\ -265 \end{pmatrix} + t\begin{pmatrix} 30 \\ 45 \\ -10 \end{pmatrix} \qquad B = \begin{pmatrix} 100 \\ 458 \\ -151 \end{pmatrix} + (t-0.5)\begin{pmatrix} -15 \\ -\frac{9}{2} \\ 13 \end{pmatrix}$$

where parameter t is time in hours.
One sub is above the other when i and j components are equal.

$\therefore -120 + 30t = 100 + (t-0.5)(-15)$ and

$210 + 45t = 458 + (t-0.5)\left(\frac{-9}{2}\right)$

Solve ~~i~~ component.

$-120 + 30t = 100 - 15t + 7.5$

$45t = 227.5$

$t = 5.05$ hours.

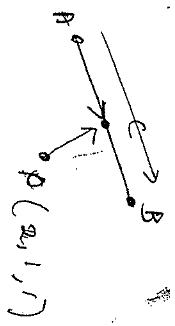
Checking with y

$210 + 45 \times 5.05 = 458 + (5.05-0.5)\left(\frac{-9}{2}\right)$ ✓✓

$\therefore$ 12pm + 5.05 = 5:03 pm.

let C lie on AB such that

$AB \perp PC$



$\therefore \vec{AB} \cdot \vec{PC} = 0$ ✓

$$\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2\lambda-3 \\ \lambda-1 \\ 2\lambda+1 \end{pmatrix} = 0$$

Line $\vec{AB}$

$$\begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$$\vec{PC} = \begin{bmatrix} -1+2\lambda-2 \\ 0+\lambda-1 \\ 2+2\lambda-1 \end{bmatrix} = \begin{bmatrix} 2\lambda-3 \\ \lambda-1 \\ 2\lambda+1 \end{bmatrix}$$

$(4\lambda-6) + (\lambda-1) + (4\lambda+2) = 0$

$9\lambda = 5$

$\lambda = \frac{5}{9}$ ✓

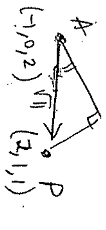
$$\therefore \vec{PC} = \begin{bmatrix} 2(5/9)-3 \\ 5/9-1 \\ 2(\frac{5}{9})+1 \end{bmatrix} = \begin{bmatrix} -\frac{17}{9} \\ -\frac{4}{9} \\ \frac{19}{9} \end{bmatrix}$$ ✓

$\therefore |\vec{PC}| = \sqrt{\frac{74}{9}}$

$= \frac{\sqrt{74}}{3}$

$= 2.867$ ✓

(4)



$B(1,1,4)$

$$\vec{AB} = \begin{pmatrix} 1--1 \\ 1-0 \\ 4-2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$$\vec{AP} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = \sqrt{11}$$

$6 + 1 + -2 = 3 \times \sqrt{11}\cos\theta$

$\cos\theta = \frac{5}{3\sqrt{11}}$

$\theta = 59.833°$

$|\vec{BP}| = \sqrt{11}\sin 59.833°$

$= 2.867$

a = (4)

Extension 2 Markers Comments Task 3 2023

Question 7

a) If you couldn't do these ....????????????
b) If you couldn't do these ....????????????
c) Remember to express your final answer back in terms of x, don't leave it in u.
d) Whenever degree of numerator is larger than denominator, you must divide by long division or reduce by inspection.
e) Many couldn't do this one, check solution carefully. A diagram is very helpful with these types of questions.

Question 8

a) i) A smattering of students left their answers in Cartesian Form
ii) There were two methods for those. Students who substituted the vector into the sphere needed to **show** that the quadratic had no solutions. We don't simply trust that $\Delta < 0$. Guesses without justification were not awarded marks.

b) The most common error was misreading the vector $\begin{bmatrix} 10 \\ -9 \\ 0 \end{bmatrix}$ as $\begin{bmatrix} 10 \\ 0 \\ -9 \end{bmatrix}$. The next was forgetting to **check your values of λ and μ in all coordinates**. This allows you to verify that the lines actually intersect in 3D. Finally, some students did all of the work and then didn't find the final vector.

c) This question was a real challenge for a large portion of the cohort. Students could either work entirely with vector methods, or borrow some strategies from Year 9 Geometry. Both strategies had students making the outstanding claim that "if it is a rhombus then it is a rhombus". The worst case of this was assuming that diagonals are perpendicular to prove that sides are equal: both are rhombus properties that shouldn't be assumed!

Those who used geometry had an easier pathway, but many left very little justification in their answer, making it impossible to follow while awarding full marks. Students who want to use these methods may have forgotten about alternate angles, congruent triangles, and isosceles triangles – the diagram is not enough to say that.

Starting correctly vectors involved equating two forms of $\cos\theta$, found with dot products. Many students who did this either incorrectly "cancelled" dot products or assumed that the dot products were equal (which is dependent on lengths being equal, which you're trying to prove!). If you found that $\underset{\sim}{b} = \underset{\sim}{c}$, that's a really good hint that you're on the wrong track: they are very clearly not parallel.

d) Mostly done well, some students forgot to find the length after finding the projection.

Question 9

a. Students knew how to attack the question – or they did not – it was hard to get more than 2/4 without a concrete method (note there is 3 different methods so chat to your teacher about it if you need) - suggest students who did not obtain full marks revise this skill.
b. Should note that the lines were not parallel then show that they did not intersect by finding $\mu \; and \; \lambda$ for a pair of the equations then show that they do not work for the third – a conclusion stating that they are then skew is required. Mostly well done – no mark was deducted this time for not stating NOT parallel.
c. i. resolve into partial fractions – if the denominator is quadratic then the numerator needs to be linear – often students forgot this. - care with + and – sign as usual a concern

   Ii. Unfortunately, if the Bx+C was not used in part I students made the question easier and were not then eligible for 3/3 in part ii. Again, general care with sign and fractions is Required.

Question 10

Overall, students struggled with this question – probably because timing was an issue.

a)
   i. Many made the assumption that the crosswind vector was North. Successful students wrote down the resultant vector (broken into components) as the sum of the original path (broken into components) and crosswind (broken into components) before manipulating.
   ii. Draw a picture to find the bearing.

b) Careful with algebraic manipulation. Negatives and positive signs kept swapping at random.

c)
   i. If it tells you to use the parameter t, don't make up your own parameters. If you defined your new parameters with respect to t, then you still got the mark. The velocity vectors were in km/h, hence your 30 mins had to be converted into hours first. You could have used t as time after 12pm, or t as time after 12.30pm
   ii. Directly on top of each other does not mean point of intersection. That means the two submarines have crashed. Only the x and y components are equal. Some students got confused with what their t meant and added time onto 12pm/12:30pm incorrectly.