Hills Grammar

2025 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- Reading time - 10 minutes
- Working time - 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper.
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page
- Write your Centre Number and Student Number on the Question 13 Writing Booklet attached

Total marks: 100

Section I - 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II - 90 marks

- Attempt Questions 11-16
- Allow about 2 hours 45 minutes for this section

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Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 Consider the statement:

'If an animal can fly then it is a bird.'

Which of the following is the contrapositive of this statement?

A. If an animal is not a bird then it cannot fly.
B. If an animal is a bird then it can fly.
C. If an animal can fly then it is not a bird.
D. If an animal cannot fly then it is not a bird.

2 It is given that $|z - 1 - i| = 1$.

What is the maximum value of Arg (z)?

A. $-\dfrac{\pi}{2}$
B. 0
C. $\dfrac{\pi}{2}$
D. π

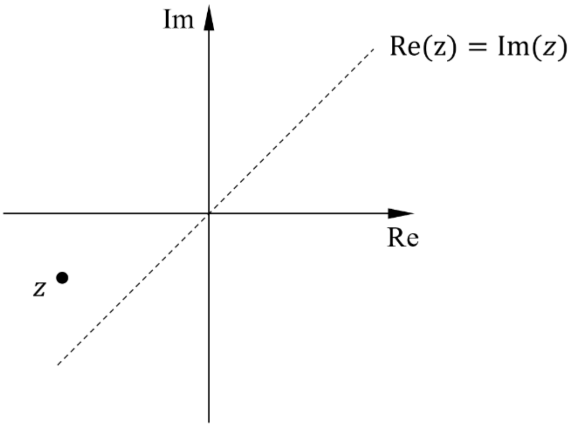
3 Consider the following proposition.

'If a positive integer is not a perfect square, then it is not prime.'

If a number was found which had the following properties, which would be a counterexample to the proposition?

A. A positive integer that is not a perfect square and is prime.
B. A positive integer that is not a perfect square and is not prime.
C. A positive integer that is a perfect square and is prime.
D. A positive integer that is a perfect square and is not prime.

4 The diagram shows the complex number $z = x + iy$ in the third quadrant, with $x < y$.



Given $z^2 = a + ib$, which of the following statements is true?

A. $a < 0,\ b < 0$
B. $a < 0,\ b > 0$
C. $a > 0,\ b < 0$
D. $a > 0,\ b > 0$

5 Which of these algebraic fractions cannot be written as partial fractions in the form

$$\frac{a}{x+b} + \frac{c}{x+d},$$

where $a,\ b,\ c$ and d are real numbers?

A. $\dfrac{2x+6}{x^2+6x+5}$

B. $\dfrac{2x-8}{x^2-8x+16}$

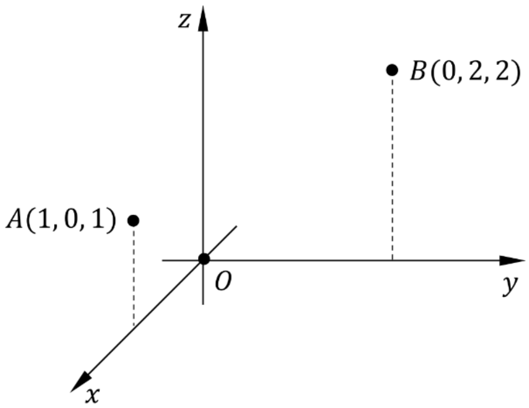
C. $\dfrac{2x+4}{x^2-4x+8}$

D. $\dfrac{2x+2}{x^2+2x-3}$

6 Which of the following is equivalent to $\left(\sqrt{2}e^{\frac{\pi}{4}i}\right)^3$?

A. $-2-2i$
B. $-2+2i$
C. $2-2i$
D. $2+2i$

7 Consider triangle AOB, where A is the point $(1,0,1)$ and B is the point $(0,2,2)$.



NOT TO SCALE

Which of the following is true?

A. Angle AOB is a right angle
B. Triangle AOB is isosceles
C. Angle OAB is twice the size of angle OBA
D. Angle AOB is twice the size of angle OBA

8 The spheres represented by $\left|\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right| = r_1$ and $\left|\begin{pmatrix} x-a \\ y-b \\ z-c \end{pmatrix}\right| = r_2$, where r_1, r_2 are prime numbers, intersect at one point only.

Let $d = \sqrt{a^2 + b^2 + c^2}$.

Which statement could be true?

A. $r_1 + r_2 + d = 0$
B. $r_1 - r_2 = d$
C. $(r_1)^2 + (r_2)^2 = d^2$
D. $r_1 \times r_2 = d$

9 Evaluate: . $\displaystyle\int_0^3 \frac{1+x}{9+x^2}\,dx$

A. $\frac{\pi}{4} + \ln 2\sqrt{3}$

B. $\frac{\pi}{4} + \ln \sqrt{2}$

C. $\frac{\pi}{12} + \ln 2\sqrt{3}$

D. $\frac{\pi}{12} + \ln \sqrt{2}$

10

The equation of the sphere with centre $(4, b, -1)$ is $x^2 - 8x + y^2 + 4y + z^2 + 2z = 4$.

What is the value of b?

A. -8

B. -4

C. -2

D. 2

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

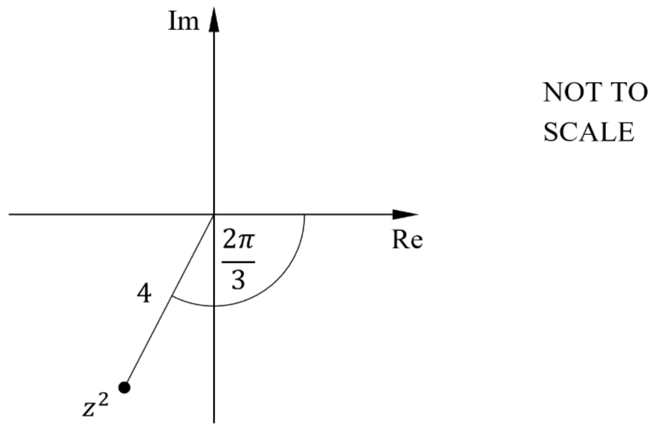
Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet

(a) Given $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = -5\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$, find $2\underset{\sim}{a} - \underset{\sim}{b}$ in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$. **2**

(b) The complex number z^2 is shown on the diagram below. **2**



Given that z lies in the fourth quadrant only, find z in mod-arg form.

Question 11 continues on page 8

Question 11 (continued)

(c) Consider the line $\underset{\sim}{r} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 3 \end{pmatrix}$ where $\lambda \in R$.

(i) Show that the point $(0, 6)$ lies on $\underset{\sim}{r}$. **1**

(ii) Show that the line through the points $A(2, 3)$ and $B(-1, -2)$ is perpendicular to $\underset{\sim}{r}$. **2**

(d) It is known that $1 + 2i$ is one of the square roots of $4i - 3$. **2**

Using this fact, or otherwise, solve $z^2 - 5z + 7 - i = 0$.

(e) A particle is initially at the origin with velocity 2 metres per second. **2**

Its acceleration given by

$$v\frac{dv}{dx} = v^2 - v,$$

where v is the velocity of the particle in metres per second and x is its displacement in metres.

Find an expression for x in terms of v.

(f) (i) Find the values of a and b such that **2**

$$\frac{x + 7}{(x - 2)(x + 1)} = \frac{a}{x - 2} + \frac{b}{x + 1}.$$

(ii) Show that **2**

$$\int_0^1 \frac{x + 7}{(x - 2)(x + 1)}\,dx = -5\ln 2\,.$$

End of Question 11

Question 12 (14 marks) Use the Question 12 Writing Booklet

(a) Consider the complex number $z = a + ib$. **2**

- The product of z and $\bar{z}$ is 25.
- The sum of z and $\bar{z}$ is 8.

Find z.

(b) Find **2**

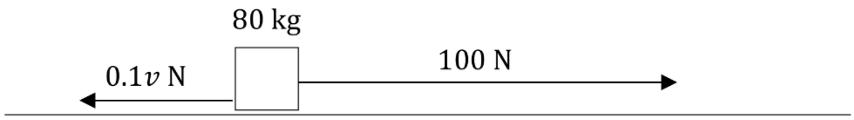
$$\int x \ln x \, dx.$$

(c) Prove that $(n+1)^n - n^{n+1}$ is odd for all positive integers n. **3**

(d) For the complex numbers z and w, it is known that $\frac{z}{w}$ is purely real with a modulus of 2. **2**

If $z = 4e^{\frac{\pi}{3}i}$, find w in exponential form.

(e) An 80-kilogram mass is being dragged along a horizontal surface by a force of 100 newtons.



The mass is subject to friction of $0.1v$ newtons, where v is the speed of the mass in metres per second. The mass is initially at rest.

The equation of motion is

$$\ddot{x} = \frac{1000 - v}{800}. \qquad \text{(Do NOT prove this.)}$$

(i) Show that the equation for time as a function of velocity is **3**

$$t = 800 \ln\left(\frac{1000}{1000 - v}\right).$$

(ii) Find the velocity of the mass after 10 seconds, correct to 1 decimal place. **2**

Question 13 (16 marks) Use the Question 13 Writing Booklet

(a) Find **3**

$$\int \frac{4x-3}{4x^2-4x+5}\,dx.$$

(b) It is known that for all positive real numbers $x,\ y$ **2**

$$x+y \geq 2\sqrt{xy}. \qquad \text{(Do NOT prove this.)}$$

Using this result, or otherwise, show that for all real a

$$|a| \leq \frac{a^2+1}{2}.$$

(c) Given $a_{n+1} = \sqrt{2a_n}$ and $a_1 = 1$, prove by mathematical induction that for all integers $n \geq 1$ **3**

$$a_n = 2^{\left(1-2^{(1-n)}\right)}.$$

(d) The complex number $1+i$ is a zero of the polynomial **3**

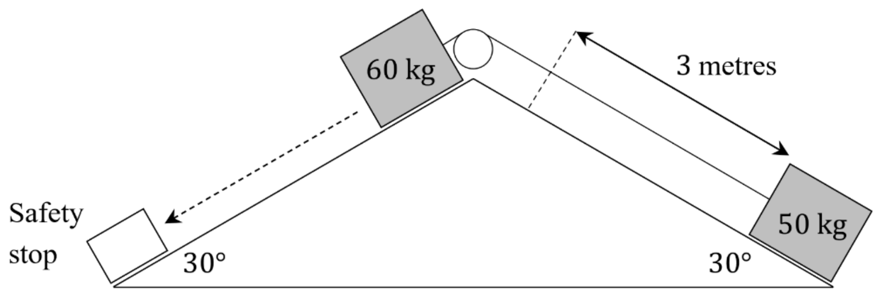
$$P(z) = z^4 - 2z^3 + z^2 + 2z - 2.$$

Find the remaining zeros of the polynomial.

Question 13 continues on page 11

Question 13 (continued)

(e) A tiler is working on the roof of a house which has an angle to the horizontal of 30° on both sides as shown.



The tiler uses a rope and pulley to move a 50-kilogram box 3 metres up the roof, with a 60-kilogram counterweight attached to the other end. Both masses are initially at rest.

When the masses are released, the 60-kilogram mass slides down the roof until it hits the safety stop, pulling the 50-kilogram up the roof exactly 3 metres.

Assume that there is no friction and no air resistance. The acceleration due to gravity is 10 m s^{-2}.

(i) Prove that **3**

$$v = \frac{5t}{11}.$$

(ii) Hence, or otherwise, find the time it takes for the 50-kilogram mass to move 3 metres up the roof, correct to 1 decimal place. **2**

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet

(a) Find **2**

$$\int \frac{e^{2x} - e^{x}}{e^{x} + 1}\, dx\,.$$

(b) A 2-kilogram mass is moving in simple harmonic motion along the x-axis about the origin, with a period of π seconds.

The greatest magnitude of the force acting on the mass during its motion is 24 newtons.

(i) Show that $\ddot{x} = -4x$ and the extremities of the mass' motion are 3 metres from the centre of its motion. **2**

(ii) By integrating $\ddot{x} = -4x$, prove that $v^2 = -4(x^2 - 9)$. **2**

(c) (i) Let $z = \cos\theta + i\sin\theta$. **2**

Show that $z^n + z^{-n} = 2\cos n\theta$.

(ii) Hence prove that **3**

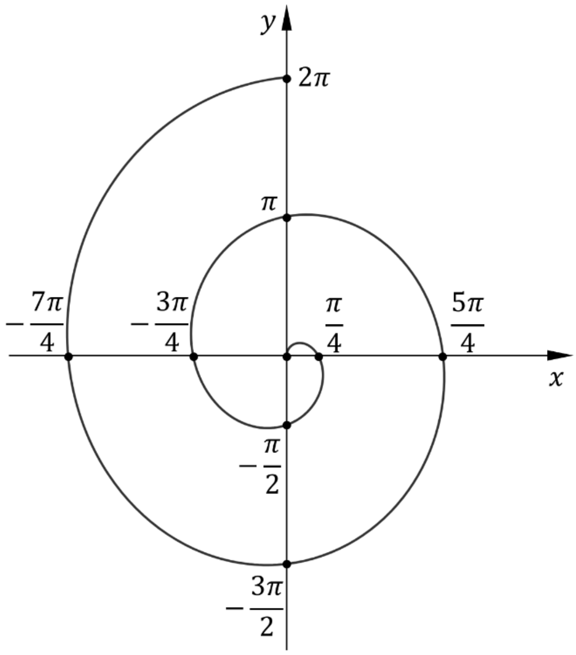
$$8\cos^4\theta - 10\cos^2\theta + 2 = \cos 4\theta - \cos 2\theta\,.$$

Question 14 continues on page 13

Question 14 (continued)

(d) A spiral starts at the origin and completes two clockwise spirals to end at $(0, 2\pi)$.

Its position from $t = 0$ to $t = T$ is shown in the diagram.



The parametric equations of the spiral are

$$\begin{cases} x = f(t) \\ y = g(t) \end{cases} \quad \text{for} \quad \left[0, T\right].$$

(i) Find $f(t)$, $g(t)$ and T. **3**

(ii) The parametric equations of a second spiral are **1**

$$\begin{cases} x = g(t) \\ y = f(t) \end{cases} \quad \text{for} \quad \left[0, \frac{T}{2}\right].$$

If the second spiral was drawn on the axes above, how many points of intersection would it have with the first spiral?

End of Question 14

Question 15 (14 marks) Use the Question 15 Writing Booklet

(a) For the positive real numbers a and b, $a \neq b$, it is given that

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ and $(a - b)^2 > 0$.

(i) Using these facts, or otherwise, prove **2**

$$a^3 + b^3 > ab(a + b).$$

(ii) Hence, or otherwise, show that if c is also a positive real number that is not equal to a or b, that **2**

$$2(a^3 + b^3 + c^3) > a^2(b + c) + b^2(a + c) + c^2(a + b).$$

(b) For the positive real numbers a and b, with $a \neq 2b$, prove that **3**

$$\frac{1}{a - 2b} \neq \frac{1}{a} - \frac{1}{2b}.$$

(c) The vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are defined as $\underset{\sim}{u} = \underset{\sim}{a} + m\underset{\sim}{b}$ and $\underset{\sim}{v} = m\underset{\sim}{a} - \underset{\sim}{b}$, where $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are unit vectors and m is a real constant. **3**

Given that the dot product of $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is 1, and that the acute angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is $\frac{\pi}{3}$, find the value of m.

(d) A unit mass is projected vertically upward under gravity with a speed S metres per second in a medium that exerts a resistance to motion equal to kv newtons, where k is a positive constant and v is the upward velocity of the mass in metres per second. **4**

The acceleration due to gravity is g m s^{-2}, where $g > 0$.

If the particle reaches its greatest height in time T seconds, show for $0 \leq t \leq T$ that

$$v = \frac{g\left(e^{k(T-t)} - 1\right)}{k}.$$

Question 16 (16 marks) Use the Question 16 Writing Booklet

(a) Consider the integral

$$I_n = \int_0^1 \sqrt{x}(1-x)^n \, dx, \qquad n = 0, 1, 2, 3, \dots$$

(i) Show that **3**

$$I_n = \frac{2n}{2n+3} I_{n-1}, \qquad n = 1, 2, 3, \dots$$

(ii) Evaluate I_1. **1**

(b) The complex number w lies on the unit circle with $\operatorname{Arg}(w) = \theta$.

(i) Show that **3**

$$\frac{1+w}{1-w} = i \cot \frac{\theta}{2}.$$

(ii) Hence solve **3**

$$\left(\frac{z-1}{z+1}\right)^8 = -1.$$

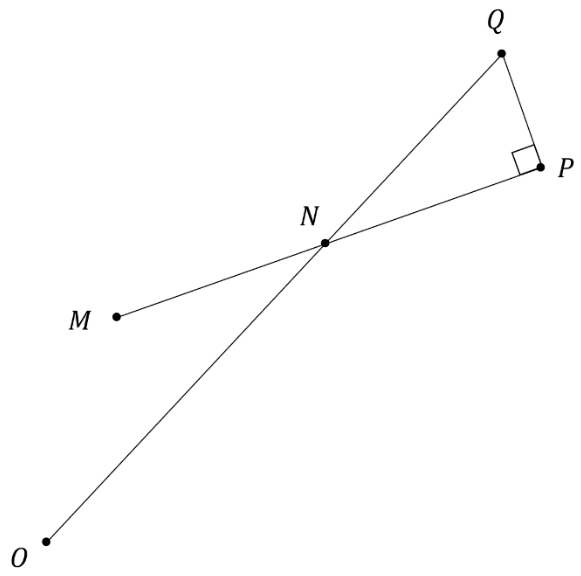
Question 16 continues on page 16

Question 16 (continued)

(c) M and P are two points with position vectors $\overrightarrow{OM} = \underset{\sim}{m}$ and $\overrightarrow{OP} = \underset{\sim}{p}$ respectively. **3**

N is the midpoint of MP and has position vector $\overrightarrow{ON} = \underset{\sim}{n}$.

PQ is perpendicular to MN and $\overrightarrow{NQ} = \lambda\overrightarrow{ON}$, where $\lambda \in R$.



NOT TO SCALE

Show that

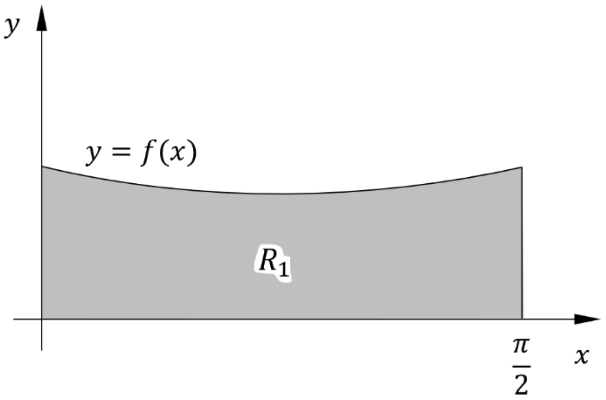
$$\lambda = \frac{\left|\underset{\sim}{m}\right|^2 + \left|\underset{\sim}{n}\right|^2 - 2\underset{\sim}{m} \cdot \underset{\sim}{n}}{\left|\underset{\sim}{n}\right|^2 - \underset{\sim}{m} \cdot \underset{\sim}{n}}.$$

Question 16 continues on page 17

Question 16 (continued)

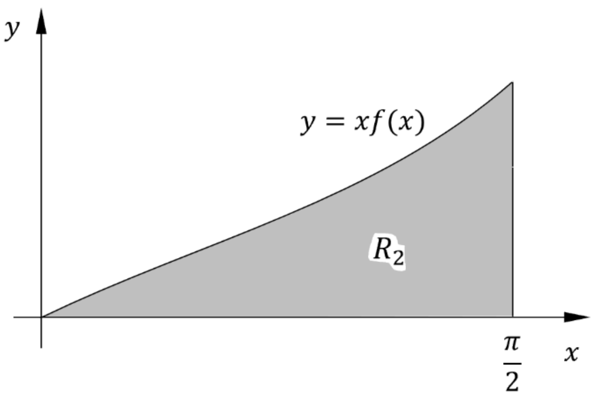
(d) Consider the curve $y = f(x)$, where $f(x) = \frac{1}{1+\cos x+\sin x}$. **3**

The area of the region R_1, bounded by the x-axis, the y-axis, the line $x = \frac{\pi}{2}$ and the curve $y = f(x)$, is $\ln 2$ units^2. (Do NOT prove this.)



NOT TO SCALE

The region R_2 is bounded by the x-axis, the line $x = \frac{\pi}{2}$ and the curve $y = x\,f(x)$.



NOT TO SCALE

Prove that the ratio of the area of R_1 to the area of R_2 is $4{:}\,\pi$.

End of paper

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MATHEMATICS: MULTIPLE CHOICE ANSWER SHEET

Student Number:____________________

Teacher:_____________________

Select the alternative A, B, C or D that best answers the question. Fill in the response circle completely.

Sample: $2 + 4 =$ A. 2 B. 6 C. 8 D. 9

A	●	C	D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A	●	C	~~●~~

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow towards the correct answer.

1	A B C D
2	A B C D
3	A B C D
4	A B C D
5	A B C D
6	A B C D
7	A B C D
8	A B C D
9	A B C D
10	A B C D

2025 Year 12 Trial HSC Mathematics Extension 2 Marking Guidelines

THGS

Section I

Multiple Choice Answer Key

Question	Answer
1	A
2	C
3	A
4	D
5	C
6	B
7	D
8	B
9	~~C~~ D
10	C

Worked Solutions

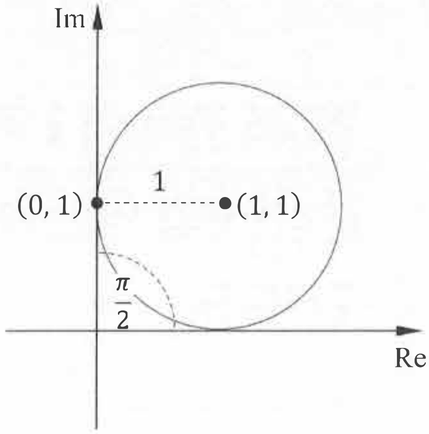
1 A

Reverse the two parts of the statement and negate each part, so A is correct.
B is the converse, C is a negation of the conclusion and D is the inverse.

2 C

$z - 1 - i = 1$ is a circle of radius one unit centred at $(1,1)$.

The maximum value of $\text{Arg}(z)$ occurs when z is directly above zero at $(0,1)$, where it takes a value of $\frac{\pi}{2}$ as shown.



3 A

The negation of $P \Rightarrow Q$ is P and $\neg Q$, so in this case 'not a perfect square and is prime'.

4 D

The principal argument of z is between $-\frac{3\pi}{4}$ and $-\pi$, so when it is squared we get an argument between $-\frac{3\pi}{2}$ and -2π, which places it in the first quadrant, where a and b are both positive.

Alternative solution I:

$$z^2 = (x + iy)^2$$
$$= x^2 - y^2 + 2xyi$$
$$\therefore \quad a = x^2 - y^2 \quad \text{and} \quad b = 2xy$$

Now $x^2 > y^2$ since $x < y$ and they are both negative, so $a > 0$.
$2xy > 0$ since x and y are both negative, so $b > 0$.

5 C

A. $\dfrac{2x+6}{x^2+6x+5} = \dfrac{1}{x+5} + \dfrac{1}{x+1}$

B. $\dfrac{2x-8}{x^2-8x+16} = \dfrac{1}{x-4} + \dfrac{1}{x-4}$

C. $\dfrac{2x+4}{x^2-4x+8} = \dfrac{2x+4}{(x-2)^2+2^2}$

The denominator cannot be written as the product of two real linear factors as it is not the difference of two squares.

D. $\dfrac{2x+2}{x^2+2x-3} = \dfrac{1}{x-1} + \dfrac{1}{x+3}$

Alternative solution I:

If the denominator is the product of two real linear factors, then it can be written in this form. We can do this if the discriminant is non-negative.

A. $\Delta = 6^2 - 4(1)(5) = 16 \geq 0$ ✓

B. $\Delta = (-8)^2 - 4(1)(16) = 0 \geq 0$ ✓

C. $\Delta = (-4)^2 - 4(1)(8) = -16 < 0$ ✗

D. $\Delta = 2^2 - 4(1)(-3) = 16 \geq 0$ ✓

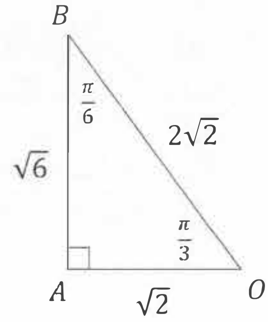
6 B

$$\begin{aligned}\left(\sqrt{2}e^{\frac{\pi}{4}i}\right)^3 &= 2\sqrt{2}e^{\frac{3\pi}{4}i}\\ &= 2\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\\ &= 2\sqrt{2}\left(-\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\\ &= 2\sqrt{2}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)\\ &= -2+2i\end{aligned}$$

Alternative solution I:

$$\begin{aligned}\left(\sqrt{2}e^{\frac{\pi}{4}i}\right)^3 &= \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^3\\ &= \left[\sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)\right]^3\\ &= (1+i)^3\\ &= 1+3i+3i^2+i^3\\ &= 1+3i-3-i\\ &= -2+2i\end{aligned}$$

7 D



$|\overrightarrow{OA}| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$
$|\overrightarrow{OB}| = \sqrt{0^2 + 2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$
$|\overrightarrow{AB}| = |(-1, 2, 1)| = \sqrt{(-1)^2 + 2^2 + 1^2} = \sqrt{6}$
$|\overrightarrow{OA}|:|\overrightarrow{AB}|:|\overrightarrow{OB}| = \sqrt{2}:\sqrt{6}:2\sqrt{2} = 1:\sqrt{3}:2$
This is the $\frac{\pi}{6}, \frac{\pi}{3}$ exact triangle (scalene) with $\angle BAO = \frac{\pi}{2}, \angle AOB = \frac{\pi}{3}$ and $\angle OBA = \frac{\pi}{6}$, so angle AOB is twice the size of angle OBA.

Alternative solution I:
$|\overrightarrow{OA}| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$
$|\overrightarrow{OB}| = \sqrt{0^2 + 2^2 + 2^2} = \sqrt{8}$
$|\overrightarrow{AB}| = |(-1, 2, 1)| = \sqrt{(-1)^2 + 2^2 + 1^2} = \sqrt{6}$
$\therefore$ ΔOAB is not isosceles, so eliminate B.
$|\overrightarrow{OB}|^2 = |\overrightarrow{OA}|^2 + |\overrightarrow{AB}|^2$ so $\angle\ OAB = 90°$, so $\angle AOB \neq 90°$, so eliminate A.
Now since ΔOAB is not isosceles, $\angle OBA \neq 45°$, so eliminate C.
By elimination, $\angle AOB = 2\angle OBA$

8 B

Case 1: If the two spheres touch externally the distance between the centres equals the sum of the radii, so $d = r_1 + r_2$.
Case 2: If the two spheres touch internally the distance between the centres equals the difference of the radii, so $d = r_1 - r_2$ or $d = r_2 - r_1$. Since the second one matches B, this is the answer.

Explanation of why A, C and D are not possible:

- For A, it could not be true as at least one of the three would have to be negative and they are all positive.
- For C, we would have:
  Case 1: $d^2 = (r_1 + r_2)^2 = (r_1)^2 + 2r_1r_2 + (r_2)^2 > (r_1)^2 + (r_2)^2$ since $r_1r_2 > 0$, so false.
  Case 2a: $d^2 = (r_1 - r_2)^2 = (r_1)^2 - 2r_1r_2 + (r_2)^2 < (r_1)^2 + (r_2)^2$ since $r_1r_2 > 0$, so false.
  Case 2b: Similarly if $d = r_2 - r_1$ this is false.
- For D, we would have
  Case 1: $r_1 \times r_2 = r_1 + r_2 \quad \rightarrow \quad r_1 = \frac{r_1}{r_2} + 1$
  Now this is a contradiction, since if r_1 and r_2 are prime numbers with $r_1 \neq r_2$ then $\frac{r_1}{r_2}$ is not an integer so r_1 is not an integer.
  Cases 2a and 2b: $r_1 \times r_2 = r_1 - r_2 \quad \rightarrow \quad r_1 = \frac{r_1}{r_2} - 1, r_1 \times r_2 = r_2 - r_1 \quad \rightarrow \quad r_2 = \frac{r_2}{r_1} - 1$ are similar.

9 D

$$\int_0^3 \frac{1 + x}{9 + x^2}\, dx = \int_0^3 \left(\frac{1}{9 + x^2} + \frac{x}{9 + x^2}\right) dx$$

$$= \left[\frac{1}{3} tan^{-1} \frac{x}{3} + \frac{1}{2} ln(9 + x^2)\right]_0^3$$

$$= \frac{1}{3}[tan^{-1} 1 - tan^{-1} 0] + \frac{1}{2}[ln\, 18 - ln\, 9]$$

$$= \frac{1}{3}\left(\frac{\pi}{4}\right) + \frac{1}{2} \ln 2$$

$$= \frac{\pi}{12} + ln\sqrt{2}$$

10. C

$$x^2 - 8x + y^2 + 4y + z^2 + 2z = 4$$

$$x^2 - 8x + 16 + y^2 + 4y + 4 + z^2 + 2z + 1 = 4 + 16 + 4 + 1$$

$$(x - 4)^2 + (y + 2)^2 + (z + 1)^2 = 25$$

$\therefore b = -2$

section II

$\boxed{A}$ $\boxed{B}$ etc in the sample answers indicate the point where each mark is awarded.

Question 11 (a)

Criteria	Marks
• Provides correct solution	2
• Attempts to find $2\underset{\sim}{a} - \underset{\sim}{b}$, or equivalent merit	1

Suggested solution(s)	comments
$2\underset{\sim}{a} - \underset{\sim}{b} = 2(2\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}) - (-5\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k})$ $\boxed{A}$ $= 4\underset{\sim}{i} + 6\underset{\sim}{j} + 2\underset{\sim}{k} + 5\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ $= 2\begin{pmatrix}2\\3\\1\end{pmatrix} - \begin{pmatrix}-5\\1\\-2\end{pmatrix}$ $= 9\underset{\sim}{i} + 5\underset{\sim}{j} + 4\underset{\sim}{k}$ $=$ $= \begin{pmatrix}9\\5\\4\end{pmatrix}$ $\boxed{B}$	· Very well attempted

Question 11 (b)

Criteria	**Marks**
• Provides correct solution	2
• Writes z^2 in mod-arg form, or attempts to take the square root of the modulus or halve the argument, or finds the correct modulus or the correct argument, or equivalent merit	1

Suggested solution(s)	comments
$z^2 = 4\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$ $\boxed{A}$ $z = \sqrt{4}\left(\cos\left(-\frac{2\pi}{3} \div 2\right) + i\sin\left(-\frac{2\pi}{3} \div 2\right)\right)$ $= 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$ $\boxed{B}$ Accept $\frac{5\pi}{3}$ for arg Accept $z = 2e^{-\frac{\pi}{3}i}$.	· Reasonably well attempted · 'cis' is Not Mod-Arg form [Polar]

Question 11 (c) (i)

Criteria	Marks
• Provides correct solution	1

Suggested solution(s)	comments
By inspection let $\lambda = 1$? * show that ∃ a λ-value i.e λ will be 1 $\underset{\sim}{r} = \binom{5}{3} + \binom{-5}{3} = \binom{0}{6}$ so $(0, 6)$ is on $\underset{\sim}{r}$ $\boxed{A}$	. well attempted . Solution is not the best

Question 11 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds $\overrightarrow{AB}$ or $\overrightarrow{BA}$, or equivalent merit	1

Suggested solution(s)	comments
$\overrightarrow{AB} = \binom{-1-2}{-2-3} = \binom{-3}{-5}$ brief workings $\boxed{A}$ $\binom{-3}{-5} \cdot \binom{-5}{3} = 15 - 15 = 0$, so the lines are perpendicular. $\boxed{B}$ as $\cos 90° = 0$ Alternative solution I: $\overrightarrow{BA} = \binom{2+1}{3+2} = \binom{3}{5}$ same $\boxed{A}$ $\binom{3}{5} \cdot \binom{-5}{3} = -15 + 15 = 0$, so the lines are perpendicular. $\boxed{B}$	. Very well attempted

Question 11 (d)

Criteria	Marks
• Provides correct solution	2
• Attempts to use the quadratic formula or to complete the square or to factorise the quadratic, or equivalent merit	1

Suggested solution(s)	comments

$$z^2 - 5z + 7 - i = 0$$

$$\begin{aligned} z &= \frac{5 \pm \sqrt{(-5)^2 - 4(1)(7-i)}}{2} \qquad \boxed{A} \\ &= \frac{5 \pm \sqrt{25 - 28 + 4i}}{2} \\ &= \frac{5 \pm \sqrt{4i - 3}}{2} \\ &= \frac{5 \pm (1+2i)}{2} \\ &= \frac{4-2i}{2}, \frac{6+2i}{2} \\ &= 2 - i \text{ or } 3 + i \qquad \boxed{B} \end{aligned}$$

Alternative solution I:

$$z^2 - 5z + 7 - i = 0$$

$$\begin{aligned} z^2 - 5z + \frac{25}{4} &= i - 7 + \frac{25}{4} \qquad \boxed{A} \\ \left(z - \frac{5}{2}\right)^2 &= \frac{4i - 3}{4} \\ &= \frac{(1+2i)^2}{4} \\ z - \frac{5}{2} &= \pm\frac{1+2i}{2} \\ z &= \frac{5}{2} \pm \frac{1+2i}{2} \\ &= \frac{4-2i}{2}, \frac{6+2i}{2} \\ z &= 2 - i \text{ or } 3 + i \qquad \boxed{B} \end{aligned}$$

Alternative solution II:

$$z^2 - 5z + 7 - i = 0$$

$$(z - 2 + i)(z - 3 - i) = 0 \qquad \boxed{A}$$

$$z = 2 - i \text{ or } 3 + i \qquad \boxed{B}$$

Comments:

• Very well attempted

• Note

$\pm\sqrt{4i-3} \equiv \pm(1+2i)$

$\sqrt{4i-3} \neq 1+2i$

Question 11 (e)

Criteria	Marks
• Provides correct solution	2
• Separates the variables, or equivalent merit	1

Suggested solution(s)	comments
$v\dfrac{dv}{dx} = v^2 - v$ $= v(v-1)$ $\displaystyle\int_2^v \frac{dv}{v-1} = \int_0^x dx$ $\boxed{A}$ $\left[\ln\lvert v-1\rvert\right]_2^v = \left[x\right]_0^x$ or $\ln\lvert v-1\rvert = x + c$ $\ln\lvert v-1\rvert - \ln 1 = x - 0$ $x = \ln\lvert v-1\rvert$ $\boxed{B}$ Accept $x = \ln(v-1)$ since $v \geq 2$ given initial conditions. Alternative solution I: $v\dfrac{dv}{dx} = v^2 - v$ $\displaystyle\int \frac{dv}{v-1} = \int dx$ $\boxed{A}$ $\ln\lvert v-1\rvert = x + c$ Let $x = 0, v = 2$ $\ln\lvert 2-1\rvert = 0 + c$ $c = 0$ $\therefore\ x = \ln\lvert v-1\rvert$ $\boxed{B}$ Accept $x = \ln(v-1)$ since $v \geq 2$ given initial conditions.	• Note $v \neq 0$ $\therefore \dfrac{dv}{dx} = v - 1$ • Need to use data (I.V. Property) $v \geq 2$ $t = 0$ $v = 2$ to get to $\ln(v-1)$

Question 11 (f) (i)

Criteria	Marks
• Provides correct solution	2
• Finds a or b, or equivalent merit	1

Suggested solution(s)	comments
Let $\dfrac{x+7}{(x-2)(x+1)} = \dfrac{a}{x-2} + \dfrac{b}{x+1}$ $a = \dfrac{2+7}{2+1} = 3$ $\boxed{A}$ $b = \dfrac{-1+7}{-1-2} = -2$ $\boxed{B}$ Alternative solution I: $\dfrac{x+7}{(x-2)(x+1)} = \dfrac{a(x+1)+b(x-2)}{(x-2)(x+1)}$ $= \dfrac{(a+b)x + a - 2b}{(x-2)(x+1)}$ $a + b = 1 \quad (1)$ $a - 2b = 7 \quad (2)$ (1) − (2): $3b = -6 \to b = -2$ $\boxed{A}$ sub in (1): $a - 2 = 1 \to a = 3$ $\boxed{B}$ Alternative solution II: ✓ $\dfrac{x+7}{(x-2)(x+1)} = \dfrac{a(x+1)+b(x-2)}{(x-2)(x+1)}$ $x + 7 = a(x+1) + b(x-2)$ Let $x = -1$: $-1 + 7 = b(-1-2) \to 6 = -3b \to b = -2$ $\boxed{A}$ Let $x = 2$: $2 + 7 = a(2+1) \to 9 = 3a \to a = 3$ $\boxed{B}$	• Very well attempted • Solution II the 'best' method

Question 11 (f) (ii)

Criteria	Marks
• Provides correct solution	2
• Obtains the correct antiderivative, or equivalent merit	1

Suggested solution(s)	comments
$\int_0^1 \frac{x+7}{(x-2)(x+1)}\,dx = \int_0^1 \left(\frac{3}{x-2} - \frac{2}{x+1}\right) dx$ $= \Big[3\ln\lvert x-2\rvert - 2\ln\lvert x+1\rvert\Big]_0^1$ $\boxed{A}$ $= (3\ln 1 - 2\ln 2) - (3\ln 2 - 2\ln 1)$ $= 0 - 2\ln 2 - 3\ln 2 + 0$ $= -5\ln 2$ $\boxed{B}$	• Very well attempted

Question 12 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the value of a, or creates a quadratic which will have z and $\bar{z}$ as solutions, or equivalent merit	1

Suggested solution(s)	comments
$z\bar{z} = \lvert z\rvert^2$ $\therefore 25 = a^2 + b^2 \quad (1)$ $z + \bar{z} = 2\mathrm{Re}(z)$ $8 = 2a$ $a = 4 \quad \boxed{A}$ sub in (1): $25 = 4^2 + b^2$ $b^2 = 9b$ $= \pm 3 \quad \boxed{B}$ $z = 4 \pm 3i$ Alternative solution I: Not common to this cohort Let z and $\bar{z}$ be the solutions of the quadratic $P(\omega) = 0$. $\therefore (\omega - z)(\omega - \bar{z}) = 0$ $\omega^2 - (z + \bar{z})\omega + z\bar{z} = 0$ $\omega^2 - 8\omega + 25 = 0 \quad \boxed{A}$ $\omega^2 - 8\omega + 16 = 16 - 25$ $(\omega - 4)^2 = -9$ $\omega - 4 = \pm 3i$ $\omega = 4 \pm 3i$ $\therefore z = 4 \pm 3i \quad \boxed{B}$	• Very well attempted •

Question 12 (b)

Criteria	Marks
• Provides correct solution	2
• Attempts integration by parts, or equivalent merit	1

Suggested solution(s)	comments
$\int x \ln x \, dx$ $u = \ln x$, $\frac{dv}{dx} = x$, $\frac{du}{dx} = \frac{1}{x}$, $v = \frac{x^2}{2}$ $\boxed{A}$ $= \frac{x^2 \ln x}{2} - \frac{1}{2}\int x \, dx$ $= \frac{x^2 \ln x}{2} - \frac{1}{2}\left(\frac{x^2}{2}\right) + c$ $= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + c$ $\boxed{B}$ $u = \ln x \quad dv = x\,dx$ $du = \frac{dx}{x} \quad v = \frac{1}{2}x^2$ $\therefore \quad I = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int \cancel{x}^2 \cdot \frac{1}{x} dx$	• Very well attempted • But some setting out was strange!

Question 12 (c)

Criteria	Marks
• Provides correct solution	3
• Proves that the result is true for all odd numbers or all even numbers, or equivalent merit	2
• Attempts to prove the result for an odd or even number, or proves the result is true for $n = 1$, or equivalent merit	1

Suggested solution(s)	comments
Case 1 – if n is odd Let $n = 2k +$ 1 $\boxed{A}$ $(n+1)^n - n^{n+1} = (2k+1+1)^{2k+1} - (2k+1)^{2k+1+1}$ $= (2k+2)^{2k+1} - (2k+1)^{2k+2}$ • $2k + 2 = 2(k+1)$ is even, and an even number to any integer power is even. • $2k + 1$ is odd, and an odd number to any integer power is odd. The difference between an even number and an odd number is odd. $\therefore (n+1)^n - n^{n+1}$ is odd if n is odd. $\boxed{B}$ Case 2 – if n is even Let $n = 2k$ $(n+1)^n - n^{n+1} = (2k+1)^{2k} - (2k)^{2k+1}$ • $2k + 1$ is odd, and an odd number to any integer power is odd. • $2k$ is even, and an even number to any integer power is even. The difference between an odd number and an even number is odd. $\therefore (n+1)^n - n^{n+1}$ is odd if n is even. $\boxed{C}$ $\therefore (n+1)^n - n^{n+1}$ is odd for all positive integers n. Alternative solution I: If n is even then $n + 1$ is odd $\boxed{A}$ $\Rightarrow (n+1)^n$ is odd and n^{n+1} is even as an odd power to an integer power is odd and an even number to an integer power is even However an odd number minus an even number is odd $\therefore (n+1)^n - n^{n+1}$ is odd if n is even $\boxed{B}$ If n is odd then $n + 1$ is even $\Rightarrow (n+1)^n$ is even and n^{n+1} is odd as above	• Fairly well attempted • Most consider two cases • But more practice on Proofs Qs will be useful

PTO P15 too

However an even number minus an odd number is odd $\therefore\ (n+1)^n - n^{n+1}$ is odd if n is odd $\boxed{C}$ $\therefore\ (n+1)^n - n^{n+1}$ is odd for $\forall n \in \mathbb{Z}^+$.	

Question 12 (d)

Criteria	Marks
• Provides correct solution	2
• Finds that $w = \pm\frac{1}{2}z$, or equivalent merit	1

Suggested solution(s)	comments
$\Rightarrow \left\lvert \frac{z}{w} \right\rvert = 2$ $\therefore \frac{z}{w} = \pm 2$ $z = \pm 2w$ $w = \pm\frac{1}{2}z$ $\boxed{A}$ $= \pm\frac{1}{2}\left(4e^{\frac{\pi}{3}i}\right)$ $= \pm 2e^{\frac{\pi}{3}i}$ $= 2e^{\frac{\pi}{3}i}, 2e^{\left(\frac{\pi}{3}-\pi\right)i}$ $w = 2e^{\frac{\pi}{3}i}$, or $2e^{-\frac{2\pi}{3}i}$ $\boxed{B}$	• Fairly well attempted • Did not realise it was/is ± 2 R.T.Q carefully

Question 12 (e) (i)

Criteria	Marks
• Provides correct solution	2
• Separates the variables, or equivalent merit	1

Q11

Suggested solution(s)

Force Diagram ?

Data: $t=0$ $x=0$ $v=0$ $\ddot{x} = \quad > 0$

$\therefore 1000 - v > 0$

$v < 1000$

$$\ddot{x} = \frac{1000 - v}{800}$$ given

$$\frac{dv}{dt} = \frac{1000 - v}{800}$$

$$\frac{dt}{dv} = \frac{800}{1000 - v}$$ No Need

$$\int_0^t dt = \int_0^v \frac{800}{1000 - v}\,dv$$ sep. Var $\boxed{A}$

$$\Big[t\Big]_0^t = -800\Big[\ln|1000 - v|\Big]_0^v$$

$$t - 0 = -800\left(\ln\left|1000 - v\right| - \ln 1000\right)$$ $\boxed{B}$

$$t = 800\ln\left(\frac{1000}{1000 - v}\right) \qquad \text{since } 1000 - v > 0$$

Alternative solution I:

$$\ddot{x} = \frac{1000 - v}{800}$$

$$\frac{dv}{dt} = \frac{1000 - v}{800}$$ $\boxed{A}$

$$\frac{dt}{dv} = \frac{800}{1000 - v}$$??

$$\int dt = \int \frac{800}{1000 - v}\,dv$$

$$t = -800\ln\left|1000 - v\right| + c$$

Let $t = 0, v = 0$

$0 = -800\ln 1000 + c$

$c = 800\ln 1000$

$$t = -800\ln\left|1000 - v\right| + 800\ln 1000$$

$$= 800\ln\left(\frac{1000}{1000 - v}\right) \qquad \text{since } 1000 - v > 0$$

comments

- Reasonably well attempted
- Need to justify why $|\ldots|$ becomes $(\ldots)$

  $0 \le v < 1000$
- Use separation of variables

Question 12 (e) (ii)

Criteria	Marks
• Provides correct solution	3
• Correctly removes the logarithm from the equation, or equivalent merit	2
• Substitutes $t = 10$ into the result from (i), or equivalent merit	1

<table>
<tr><th>Suggested solution(s)</th><th>comments</th></tr>
<tr><td>

$$10 = 800\ln\left(\frac{1000}{1000-v}\right)$$
$$0.0125 = \ln\left(\frac{1000}{1000-v}\right)$$
$$e^{0.0125} = \frac{1000}{1000-v} \qquad \boxed{A}$$
$$\frac{1000-v}{1000} = e^{-0.0125}$$
$$1000 - v = 1000e^{-0.0125}$$
$$v = 1000(1 - e^{-0.0125}) \qquad \boxed{B}$$
$$\checkmark \quad = 12.422199\ldots$$
$$\text{velocity} = 12.4\ \text{m s}^{-1}\ (1\text{ dp})$$

</td><td>

• Very well attempted

• But Note $v = 12.4\ldots\ldots$ so velocity is ≈ 12.4 m/s

</td></tr>
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Question 13 (a)

Criteria	Marks
• Provides correct solution	3
• Completes the square on the denominator and rearranges the numerator in terms of $8x-4$, or equivalent merit	2
• Completes the square on the denominator, or rearranges the numerator in terms of $8x-4$, or equivalent merit	1

ln tan^{-1}

Suggested solution(s)	comments
$\int \frac{4x-3}{4x^2-4x+5}dx$ $=\int \frac{4x-3}{(2x-1)^2+2^2}dx$ $\boxed{A}$ $=\int \frac{\frac{1}{2}(8x-4)-1}{(2x-1)^2+2^2}dx$ $\boxed{B}$ $=\frac{1}{2}\int \frac{8x-4}{4x^2-4x+5}dx-\frac{1}{2}\int \frac{2}{(2x-1)^2+2^2}dx$ $=\frac{1}{2}\ln\left\|4x^2-4x+5\right\|-\frac{1}{2}\times\frac{1}{2}\tan^{-1}\left(\frac{2x-1}{2}\right)+c$ $\boxed{C}$ $=\frac{1}{2}\ln\left\|4x^2-4x+5\right\|-\frac{1}{4}\tan^{-1}\left(\frac{2x-1}{2}\right)+c$ Since $4x^2-4x+5>0$ for all x, the absolute value signs are not required for full marks.	• Very well attempted

Question 13 (b)

Criteria	Marks
• Provides correct solution	2
• Attempts to use a or a^2 in the AM-GM inequality, or finds LHS minus RHS, or equivalent merit	1

Suggested solution(s)	**comments**
Let $x = a^2,\ y = 1$ in the given result $a^2 + 1 \geq 2\sqrt{a^2 \times 1}$ $\boxed{A}$ $a^2 + 1 \geq 2\lvert a\rvert$ since $\sqrt{a^2} = \lvert a\rvert$ ✓ $\therefore \lvert a\rvert \leq \dfrac{a^2+1}{2}$ $\boxed{B}$ Alternative solution I: $\text{LHS} - \text{RHS} = \lvert a\rvert - \dfrac{a^2+1}{2}$ $\boxed{A}$ $= -\dfrac{a^2 - 2\lvert a\rvert + 1}{2}$ $= -\dfrac{(\lvert a\rvert - 1)^2}{2}$ ≤ 0 since $\mathbb{R}^2 \geq 0$ $\therefore \text{LHS} \leq \text{RHS}$ $\therefore \lvert a\rvert \leq \dfrac{a^2+1}{2}$ $\boxed{B}$	• Very well attempted.

Question 13 (c)

Criteria	Marks
• Provides correct solution	3
• Proves the base case and uses $P(k)$ correctly in the body of the proof, or equivalent merit	2
• Proves the base case, or uses $P(k)$ correctly in the body of the proof without proving the base case, or equivalent merit	1

Suggested solution(s)	comments
Let $P(n)$ represent the proposition. $P(1)$ is true since $a_1 = 2^{1-2^{1-1}} = 2^{1-1} = 1$ and a_1 is 1 $\boxed{A}$ ✓ If $P(k)$ is true for some arbitrary $k \geq 1$ then $a_k = 2^{(1-2^{(1-k)})}$ ... (*) RTP $P(k+1)$ $\quad a_{k+1} = 2^{(1-2^{-k})}$ $\text{LHS} = a_{k+1}$ $= \sqrt{2a_k}$ $= \sqrt{2 \times 2^{(1-2^{(1-k)})}}$ from $P(k)$ using inductive hypothesis (*) $\boxed{B}$ ✓ $= \sqrt{2^{1+1-2^{1-k}}}$ $= \sqrt{2^{2-2^{1-k}}}$ $= \sqrt{2^{2(1-2^{-k})}}$ $= 2^{1-2^{-k}}$ ✓ $\boxed{C}$ $= \text{RHS}$ $\therefore P(k) \Rightarrow P(k+1)$ Hence $P(n)$ is true for $n \geq 1$ by induction 3	• well attempted • Q had its flaws !!! • should be verifying P(2)

* should be:

$a_1 = 1$ and $a_n = \sqrt{2a_{n-1}}$ for $n = 2, 3, 4, \ldots$

Test/Verify P(1)
P(2) for the link
then P(k) can be accepted

$0 = \frac{5}{11}(0) + c$ $c = 0$ v $= \frac{5}{11}t$ $\boxed{C}$ Alternative solution II: $\therefore v = \frac{5}{11}t$ since the gradient of $\frac{dv}{dt}$ is constant and when $t = 0, v = 0$ $\boxed{C}$ Note: students who do not use calculus need to refer to both the gradient and v-intercept to gain the third mark.	ok

Question 13 (e) (ii)

Criteria	Marks
• Provides correct solution	2
• Finds the primitive correctly	1

Suggested solution(s)	comments
$\frac{dx}{dt} = \frac{5}{11}t$ $\int_0^3 dx = \frac{5}{11}\int_0^t t\,dt$ $\left[x\right]_0^3 = \frac{5}{11}\left[\frac{t^2}{2}\right]_0^t$ $\boxed{A}$ $3 - 0 = \frac{5}{22}(t^2 - 0)$ $3 = \frac{5}{22}t^2$ $t^2 = \frac{66}{5}$ $t = 3.63318 \quad (t > 0)$ time $= 3.6$ seconds (1 dp) $\boxed{B}$ Alternative solution I: $\frac{dx}{dt} = \frac{5}{11}\mathrm{t}$ $\int dx = \frac{5}{11}\int t\,dt$ $x = \frac{5}{22}t^2 + c$ $\boxed{A}$ Let $x = 0, t = 0$ $0 = 0 + c$ $c = 0$ $x = \frac{5t^2}{22}$ Let $x = 3$ $3 = \frac{5}{22}t^2$ $t^2 = \frac{66}{5}$ $t = 3.633180 \quad (t > 0)$ ATQ time $= 3.6$ seconds (1 dp) $\boxed{B}$	• well attempted • Note: $t = 3.62...$ time taken is 3.6 second

Question 14 (a)

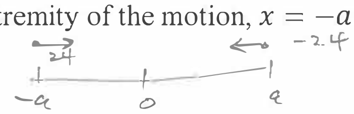
Criteria	Marks
• Provides correct solution	2
• Rearranges the numerator in terms of $e^x + 1$, or correctly rewrites the integral in terms of their u, or equivalent merit	1

Suggested solution(s)	comments
$\int \frac{e^{2x} - e^x}{e^x + 1} dx$ $= \int \frac{(e^x(e^x + 1) - 2e^x)}{e^x + 1} dx$ $\boxed{A}$ $= \int \left(e^x - \frac{2e^x}{e^x + 1}\right) dx$ $= e^x - 2\ln\left\|e^x + 1\right\| + c$ $\boxed{B}$ The absolute value signs are not needed as $e^x + 1 > 0$ for all x, so $e^x - 2\ln(e^x + 1) + c$ is acceptable for full marks. Alternative solution I: $\int \frac{e^{2x} - e^x}{e^x + 1} dx$ $\boxed{u = e^x,\ \frac{du}{dx} = e^x,\ dx = \frac{du}{u}}$ $= \int \frac{u^2 - u}{u + 1} \times \frac{du}{u}$ $\boxed{A}$ $= \int \frac{u - 1}{u + 1} du$ $= \int \frac{u + 1 - 2}{u + 1} du$ $= \int \left(1 - \frac{2}{1 + u}\right) du$ $= u - 2\ln\left\|1 + u\right\| + c$ $= e^x - 2\ln\left\|1 + e^x\right\| + c$ $\boxed{B}$	• Very well attempted

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Shows that $\ddot{x} = -4x$, or equivalent merit	1

Suggested solution(s)	comments
$T = \frac{2\pi}{n} \quad \rightarrow \quad \pi = \frac{2\pi}{n} \quad \rightarrow \quad n = 2$ ✓ mag. of greatest force stem of Q correct now $c = 0$ $\ddot{x} = -n^2(x - c)$ $\ddot{x}$ $= -4x$ what?? $\boxed{A}$ The maximum force is at the left-hand extremity of the motion, $x = -a$. $\ddot{x}$ $F_{max} = m\ddot{x}_{max}$ $24 = 2 \times (-4(-a))$ $24 = 8a$ $a = 3$ $\boxed{B}$ The extremities of the mass' motion are 3 metres from the centre of its motion.	• Reasonably well attempted. • Max. magnitude of force at end points so $-4a = -24$ $x = a$ $4a = 24$ $x = -a$



Question 14 (b) (ii)

Criteria	Marks
• Provides correct solution	2
• Separates the variables, or equivalent merit	1

Suggested solution(s)	comments
$\ddot{x} = -4x$ $v\frac{dv}{dx} = -4x$ $\int_0^v v\,dv = -4\int_{-3}^x x\,dx$ $\boxed{A}$ $\left[\frac{v^2}{2}\right]_0^v = -4\left[\frac{x^2}{2}\right]_{-3}^x$ $\frac{v^2}{2} - 0 = -2(x^2 - 9)$ $v^2 = -4(x^2 - 9)$ $\boxed{B}$ Alternative solution I:	• Very well attempted.

$v\frac{dv}{dx} = -4x$ $\int v\,dv = -4\int x\,dx$ $\boxed{A}$ $\frac{v^2}{2} = -2x^2 + c$ Let $x = -3, v = 0$ $0 = -2(-3)^2 + c$ $c = 18$ $\frac{v^2}{2} = -2x^2 + 18$ $v^2 = -4x^2 + 36$ $= -4(x^2 - 9)$ $\boxed{B}$ Alternative solution II: $\ddot{x} = -4x$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4x$ $\frac{v^2}{2} = -4\int x\,dx$ $\boxed{A}$ $= -2x^2 + c$ Let $x = -3, v = 0$ $0 = -2(-3)^2 + c$ $c = 18$ $\frac{v^2}{2} = -2x^2 + 18$ $v^2 = -4x^2 + 36$ $= -4(x^2 - 9)$ $\boxed{B}$	

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Uses De Moivre's theorem correctly, or equivalent merit	1

Suggested solution(s)	**comments**
$z^n + z^{-n} = (\cos\theta + i\sin\theta)^n + (\cos\theta + i\sin\theta)^{-n}$ DMT $= \cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta)$ $\boxed{A}$ $= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$ since cosine is even and sine is odd $= 2\cos n\theta$ $\boxed{B}$	Need
Alternative solution I: $z^n + z^{-n} = (\cos\theta + i\sin\theta)^n + (\cos\theta + i\sin\theta)^{-n}$!! $= (\cos\theta + i\sin\theta)^n + \overline{(\cos\theta + i\sin\theta)^n}$ $= \cos n\theta + i\sin n\theta + \overline{\cos n\theta + i\sin n\theta}$ $z + \bar{z} = 2\mathrm{Re}(z)$ $\boxed{A}$ $= 2\mathrm{Re}(\cos n\theta + i\sin n\theta)$ Property?! $= 2\cos n\theta$ $\boxed{B}$	• Quite well attempted. • But must justify why z^{-n} becomes $+\cos n\theta - i\sin n\theta$
Alternative solution II: -- Not use $z^n + z^{-n} = (e^{i\theta})^n + (e^{i\theta})^{-n}$ $= e^{in\theta} + e^{-in\theta}$ $\boxed{A}$ $= e^{in\theta} + \overline{e^{in\theta}}$ property !? $= 2\mathrm{Re}(e^{in\theta})$ $= 2\cos n\theta$ $\boxed{B}$	

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	3
• Rearranges the expression in preparation to use the result from (i) with $n = 2$ and $n = 4$ or $n = 1$ respectively, or proves the result using the double angle result without using the result from (i), or equivalent merit	2
• Attempts to use the result from (i) with $n = 1$, or attempts to use the result from (i) with $n = 2$ and $n = 4$, or equivalent merit	1

$\cos n\theta = \frac{z^n + z^{-n}}{2}$, $\cos\theta = \frac{z+z^{-1}}{2}$

Suggested solution(s)	comments
LHS $8\cos^4\theta - 10\cos^2\theta + 2$ $= 8\left[\frac{1}{2}(z+z^{-1})\right]^4 - 10\left[\frac{1}{2}(z+z^{-1})\right]^2 + 2$ $\boxed{A}$ missing $\frac{8}{16}[z^4+4z^2+16+4z^{-2}+z^{-4}] - \frac{10}{4}[z^2+2+z^{-2}]+2$ $= \frac{1}{2}[(z^4+z^{-4}) + 4(z^2+z^{-2}) + 6] - \frac{5}{2}[(z^2+z^{-2}) + 2] + 2$ $= \frac{1}{2}(z^4+z^{-4}) - \frac{1}{2}(z^2+z^{-2})$ $\boxed{B}$ $= \frac{1}{2}(2\cos 4\theta) - \frac{1}{2}(2\cos 2\theta)$ $= \cos 4\theta - \cos 2\theta$ RHS $\boxed{C}$ Alternative solution I: RHS: $\cos 4\theta - \cos 2\theta$ $= \frac{1}{2}(z^4+z^{-4}) - \frac{1}{2}(z^2+z^{-2})$ $\boxed{A}$ $= \frac{1}{2}z^4 + \frac{1}{2}z^{-4} - \frac{1}{2}z^2 - \frac{1}{2}z^{-2}$ 'Rearrangement' $= \left[\frac{1}{2}z^4 + 2z^2 + 3 + 2z^{-2} + \frac{1}{2}z^{-4}\right] - \left[\frac{5}{2}z^2 + 5 + \frac{5}{2}z^{-2}\right] + 2$ tricky method. $= \frac{1}{2}(z+z^{-1})^4 - \frac{5}{2}(z+z^{-1})^2 + 2$ $\boxed{B}$ $= \frac{1}{2}(2\cos\theta)^4 - \frac{5}{2}(2\cos\theta)^2 + 2$ $= 8\cos^4\theta - 10\cos^2\theta + 2$ LHS $\boxed{C}$ $\boxed{B}$	• Fairly well attempted. • Some became confused: in setting out • LHS $\rightarrow$ RHS • RHS $\rightarrow$ LHS

Question 14 (d) (i)

Criteria	Marks
• Provides correct answers	3
• Finds the parametric equations without finding T, or finds T and creates a pair of parametric equations with an increasing radius or a clockwise spiral, or equivalent merit	2
• Attempts to find the parametric equations of a spiral, or attempts to find the value of T, or equivalent merit	1

Suggested solution(s)	comments
$\begin{cases} x = \frac{t}{2}\sin t \\ y = \frac{t}{2}\cos t \end{cases}$ for $[0,4\pi]$ $\boxed{A}$ $\boxed{B}$ $\boxed{C}$ *Three bald answers is worth three marks.* Alternative solution I: Every rotation by $\frac{\pi}{2}$ increases the distance from the origin by $\frac{\pi}{4}$, so $f(t) = \frac{t}{2} \times p(t)$ and $g(t) = \frac{t}{2} \times q(t)$. $\boxed{A}$ Clockwise rotation is the opposite to the normal rotation of the unit circle, so instead of $p(t) = \cos t$ and $q(t) = \sin t$ we swap them around to get $p(t) = \sin t$ and $q(t) = \cos t$. Combining the above, we have $\begin{cases} x = \frac{t}{2}\sin t \\ y = \frac{t}{2}\cos t \end{cases}$ for $[0, T]$ $\boxed{B}$ Checking a few points: When $t = \pi$, $x = \frac{\pi}{2}\sin\pi = 0$, $y = \frac{\pi}{2}\cos\pi = -\frac{\pi}{2}$ ✓ When $t = \frac{3\pi}{2}$, $x = \frac{3\pi}{4}\sin\frac{3\pi}{2} = -\frac{3\pi}{4}$, $y = \frac{3\pi}{4}\cos\frac{3\pi}{2} = 0$ ✓ The distance from the origin is always half the value of t. At $t = T$, the distance from the origin is 2π, so $T = 4\pi$. $\boxed{C}$ better method Alternative solution II: * An anticlockwise spiral is typically in the form $\begin{cases} x = at\cos t \\ y = at\sin t \end{cases}$ $\boxed{A}$ * As this is in a clockwise direction we need as $t=0$ $\begin{cases} x=0 \\ y=0 \end{cases}$ $\begin{cases} x = at\sin t \\ y = at\cos t \end{cases}$ The spiral completes two revolutions	• Not well attempted Poorly attempted • Confused in setting out

$\therefore 0 \le t \le 4\pi \to T = 4\pi$ $\boxed{B}$ When $t = 4\pi, y = 2\pi$ $\therefore\ 2\pi = a(4\pi)\cos 4\pi$ $2\pi = a \times 4\pi \times 1$ $a = \dfrac{1}{2}$ $\therefore \begin{cases} x = \dfrac{t}{2}\sin t \\ y = \dfrac{t}{2}\cos t \end{cases}$ for $\left[0,4\pi\right]$ $\boxed{C}$	

Question 14 (d) (ii)

Criteria	Marks
• Provides correct solution	1

Suggested solution(s) | **comments**

There are 3 points of intersection.

$\boxed{A}$

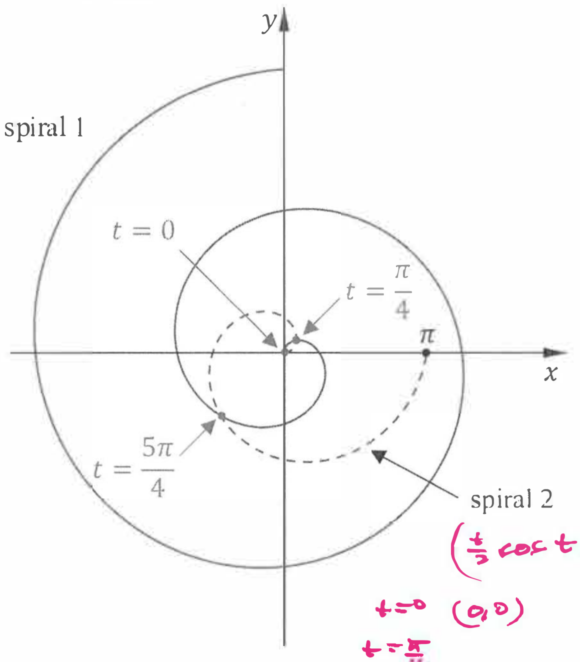
A bald answer is acceptable for this question.

Explanation – not needed for the mark.
The domain of the second spiral is $[0,\ 2\pi]$, so it is a single anticlockwise spiral ending at $(\pi,\ 0)$.
The graphs intersect when $\frac{t}{2}\sin t = \frac{t}{2}\cos t$ for $[0, 2\pi]$, which is when $t = 0, \frac{\pi}{4}$ and $\frac{5\pi}{4}$.
The points of intersection occur when $t = 0$ (they both start at the origin) and when $\sin t = \cos t \rightarrow \tan t = \frac{\pi}{4}$.

$t(\cos t - \sin t) = 0$
$t(\tan t - 1) = 0$

Not required as part of the solution, but here we can see the two curves together, with points of intersection when $t = 0, \frac{\pi}{4}, \frac{5\pi}{4}$.

$t = 0 \quad \tan t = 1$
$t = \frac{\pi}{4}$ or $\frac{5\pi}{4}$



$\left(\frac{t}{2}\cos t,\ \frac{t}{2}\sin t\right)$
$t = 0 \quad (0,0)$
$t = \frac{\pi}{4}$
$t = \frac{5\pi}{4}$

· Poorly answered.

Question 15 (a) (i)

PTO

Criteria	Marks
• Provides correct solution	2

• Finds $a^2 - ab + b^2 > ab$, or shows $a^2 - ab + b^2 = a^2 - 2ab + b^2 + ab$, or factorises LHS – RHS, or equivalent merit	1

Suggested solution(s)	**comments**
$(a-b)^2 > 0$ since $a \neq b$ $a^2 - 2ab + b^2 > 0$ $a^2 - ab + b^2 > ab$ (1) $\boxed{A}$ $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ given $\therefore a^3 + b^3 > (a+b)ab$ using (1) $\boxed{B}$ as $a+b>0$ $\therefore$ when multiply stays ">" Alternative solution I: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $= (a+b)(a^2 - 2ab + b^2 + ab)$ $\boxed{A}$ $= (a+b)[(a-b)^2 + ab]$ $> (a+b)[0 + ab]$ since $\mathbb{R}^2 \geq 0, a \neq b$ $= ab(a+b)$ as $a+b>0$ $\boxed{B}$ Alternative solution II: LHS − RHS $= a^3 + b^3 - a^2b - ab^2$ $= a(a^2 - b^2) - b(a^2 - b^2)$ $= (a-b)(a^2 - b^2)$ $\boxed{A}$ $= (a-b)(a+b)(a-b)$ $= (a-b)^2(a+b)$ > 0 since $\mathbb{R}^2 \geq 0$ and $a, b > 0$ ie $a+b>0$ $\therefore$ LHS > RHS $\therefore a^3 + b^3 > ab(a+b)$ $\boxed{B}$ Alternative solution III: LHS − RHS $= a^3 + b^3 - ab(a+b)$ $= (a+b)(a^2 - ab + b^2) - ab(a+b)$ $= (a+b)(a^2 - ab + b^2 - ab)$ $\boxed{A}$ $= (a+b)(a^2 - 2ab + b^2)$ $= (a+b)(a-b)^2$ > 0 since $\mathbb{R}^2 \geq 0$ and $a, b > 0$ ie as $a+b>0$ $\therefore$ LHS > RHS $\therefore a^3 + b^3 > ab(a+b)$ $\boxed{B}$	• Quite well attempted • must justify if/when multiplying by $a+b$ method

Question 15 (a) (ii)

Criteria	Marks
• Provides correct solution	2
• Use the result from (i), or attempting to use the AM-GM for two or three terms, or equivalent merit	1

Suggested solution(s)	**comments**
Similarly from (i) $a^3 + c^3 > ac(a + c)$ $\boxed{A}$ $b^3 + c^2 > bc(b + c)$ Adding the inequalities $2(a^3 + b^3 + c^3) > ab(a + b) + ac(a + c) + bc(b + c)$ $2(a^3 + b^3 + c^3) > a^2b + ab^2 + a^2c + ac^2 + b^2c + bc^2$ $2(a^3 + b^3 + c^3) > a^2(b + c) + b^2(a + c) + c^2(a + b)$ $\boxed{B}$	• Very well attempted

Question 15 (b)

Criteria	Marks
• Identifies the contradiction, or provides correct solution	3
• Rearranges the equation to eliminate all fractions, or equivalent merit	2
• Attempts a proof by contradiction, or equivalent merit	1

Suggested solution(s) | **comments**

Suppose by contradiction that the negation is true $\boxed{A}$

$$\frac{1}{a-2b}=\frac{1}{a}-\frac{1}{2b}$$
$$\frac{1}{a-2b}=\frac{2b-a}{2ab}$$
$$\frac{1}{a-2b}=-\frac{a-2b}{2ab}$$
$$-2ab=(a-2b)^2 \qquad \boxed{B}$$

This is a contradiction as the LHS is negative as $a, b > 0$ and the RHS is positive as $\mathbb{R}^2 \geq 0$ $\boxed{C}$

$$\therefore \frac{1}{a-2b} \neq \frac{1}{a}-\frac{1}{2b} \text{ for } a,\ b > 0 \text{ and } a \neq 2b$$

• Very well attempted.

Alternative solution I:

Suppose by contradiction that the negation is true $\boxed{A}$

$$\frac{1}{a-2b}=\frac{1}{a}-\frac{1}{2b}$$
$$\frac{1}{a-2b}=\frac{2b-a}{2ab}$$
$$\frac{1}{a-2b}=-\frac{a-2b}{2ab}$$
$$-2ab=(a-2b)^2 \qquad \boxed{B}$$
$$-2ab=a^2-4ab+4b^2$$
$$\therefore\ a^2-2ab+4b^2=0$$
$$a^2-2ab+b^2+3b^2=0$$
$$(a-b)^2+3b^2=0$$

Now $(a-b)^2 \geq 0$ since $\mathbb{R}^2 \geq 0$, and $3b^2 > 0$ since $b > 0$, so the LHS > 0.

This is a contradiction as the RHS $= 0$. $\boxed{C}$

$$\therefore \frac{1}{a-2b} \neq \frac{1}{a}-\frac{1}{2b} \text{ for } a,\ b > 0 \text{ and } a \neq 2b$$

2 cases

* $(a-b)^2+3b^2=0$
* $(a-2b)^2+2ab$

to ~~[illegible]~~ conclusions

Alternative solution II:

Suppose by contradiction that the negation is true $\boxed{A}$

II

$\frac{1}{a-2b} = \frac{2b-a}{2ab}$ $-2ab = (a-2b)^2$ $-2ab = a^2 - 4ab + 4b^2$ $\therefore\ a^2 - 2ab + 4b^2 = 0$ $\boxed{B}$ Using the quadratic formula as a shortcut for completing the square - taking a as the variable and b as a constant $\Delta = (-2b)^2 - 4(1)(4b^2)$ $= -12b^2$ ≤ 0 since $b^2 \geq 0$ $\therefore$ no solutions, which is a contradiction, so the negation is false $\boxed{C}$ $\therefore \frac{1}{a-2b} \neq \frac{1}{a} - \frac{1}{2b}$ for $a,\ b > 0$ and $a \neq 2b$	

Question 15 (c)

Criteria	Marks
• Provides correct solution	3
• Finds $\underset{\sim}{a} \cdot \underset{\sim}{b} = \frac{1}{m^2-1}$, or equivalent merit	2
• Finds an expression for the dot product of $\underset{\sim}{u}$ and $\underset{\sim}{v}$ in terms of m, $\underset{\sim}{a}$ and $\underset{\sim}{b}$, or equivalent merit	1

Suggested solution(s)	comments

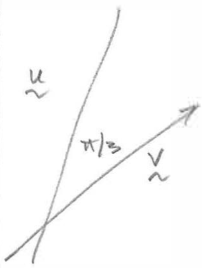
$$\left(\underset{\sim}{a} + m\underset{\sim}{b}\right) \cdot \left(m\underset{\sim}{a} - \underset{\sim}{b}\right) = 1 \qquad \boxed{A}$$

$$m\,\underset{\sim}{a} \cdot \underset{\sim}{a} - \underset{\sim}{a} \cdot \underset{\sim}{b} + m^2\,\underset{\sim}{a} \cdot \underset{\sim}{b} - m\,\underset{\sim}{b} \cdot \underset{\sim}{b} = 1$$

$$m\left|\underset{\sim}{a}\right|^2 + (m^2 - 1)\,\underset{\sim}{a} \cdot \underset{\sim}{b} - m\left|\underset{\sim}{b}\right|^2 = 1$$

$$m + (m^2 - 1)\,\underset{\sim}{a} \cdot \underset{\sim}{b} - m = 1$$

$$(m^2 - 1)\underset{\sim}{a} \cdot \underset{\sim}{b} = 1$$



$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \frac{1}{m^2 - 1} \qquad \boxed{B}$$

$$\cos\frac{\pi}{3} = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|}$$

$$\cos\frac{\pi}{3} = \frac{\frac{1}{m^2 - 1}}{1 \times 1}$$

$$\frac{1}{2} = \frac{1}{m^2 - 1}$$

$$m^2 - 1 = 2$$

$$m^2 = 3$$

$$m = \pm\sqrt{3} \qquad \boxed{C}$$

comments:

• A basic diagram would be helpful !!

• Fairly well attempted.

Question 15 (d)

Criteria	Marks
• Provides correct solution	4
• Finds a correct expression for v in terms of t or S in terms of T, or finds an equation linking g, T, k and S, or equivalent merit	3
• Finds the correct primitive for T or t, or equivalent merit	2
• Attempts to find a force equation, or equivalent merit	1

<table>
<tr><th>Suggested solution(s)   unit mass</th><th>?comments</th></tr>
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Force Diagram

Data:

$t=0 \quad x= \quad v=S \quad \ddot{x}=$

$t=T \quad x=H \quad v=0 \quad \ddot{x}=$

$t=t \quad x=x \quad v=V \quad \ddot{x}=$

$$\ddot{x} = -g - kv = -(g+kv) \qquad \boxed{A}$$

$$\frac{dv}{dt} = -(g+kv)$$

$$\int_0^t dt = -\int_S^v \frac{dv}{g+kv} \qquad \text{Sep. Var.}$$

$$\Big[t\Big]_0^t = \frac{1}{k}\Big[\ln(g+kv)\Big]_v^S \qquad \boxed{B}$$

$$t - 0 = \frac{1}{k}(\ln(g+kS) - \ln(g+kv))$$

$$kt = \ln\left(\frac{g+kS}{g+kv}\right)$$

$$e^{kt} = \frac{g+kS}{g+kv}$$

$$ge^{kt} + kve^{kt} = g + kS \qquad (1)$$

$$kve^{kt} = g(1-e^{kt}) + kS$$

$$v = \frac{g(e^{-kt}-1)}{k} + Se^{-kt} \qquad (2) \qquad \boxed{C}$$

From (1): $kS = g(e^{kt}-1) + kve^{kt}$

$$S = \frac{g(e^{kt}-1)}{k} + ve^{kt}$$

Let $t = T$, $v = 0$

$$S = \frac{g(e^{kT}-1)}{k} \qquad (3) \qquad \boxed{C}$$

Substitute (3) into (2):

$$v = \frac{g(e^{-kt}-1)}{k} + \frac{g(e^{kT}-1)}{k}e^{-kt}$$

$$= \frac{g(e^{-kt}-1)}{k} + \frac{g(e^{k(T-t)}-e^{-kt})}{k}$$

$$= \frac{g(e^{-kt}-1+e^{k(T-t)}-e^{-kt})}{k}$$

$$= \frac{g(e^{k(T-t)}-1)}{k} \qquad \boxed{D}$$

</td><td>

• Fairly well attempted

• Need to show
1) Force diagram
2) Force equation with Data
this helps out

</td></tr>
</table>

better option

Alternative solution I:

$$\ddot{x} = -g - kv \qquad \boxed{A}$$
$$= -(g + kv)$$
$$\frac{dv}{dt} = -(g + kv)$$
$$\int_0^t dt = -\int_S^v \frac{dv}{g + kv}$$
$$\Big[t\Big]_0^t = \frac{1}{k}\Big[\ln(g + kv)\Big]_v^S \qquad \boxed{B}$$
$$t - 0 = \frac{1}{k}(\ln(g + kS) - \ln(g + kv))$$
$$kt = \ln\left(\frac{g + kS}{g + kv}\right)$$
$$e^{kt} = \frac{g + kS}{g + kv}$$
$$ge^{kt} + kve^{kt} = g + kS \qquad (1)$$
$$g + kv = (g + kS)e^{-kt}$$
$$kv = (g + kS)e^{-kt} - g$$
$$v = \frac{(g + kS)e^{-kt} - g}{k} \qquad (2)$$

Sub $t = T, v = 0$ in (1)

$$\therefore \; ge^{kT} = g + kS \qquad (3) \qquad \boxed{C}$$

Sub (3) in (2): $v = \dfrac{ge^{kT} \times e^{-kt} - g}{k}$

$$= \frac{g\left(e^{k(T-t)} - 1\right)}{k} \qquad \boxed{D}$$

Question 16 (a) (i)

Criteria	Marks
• Provides correct solution	3
• Correctly splits the integrand, or equivalent merit	2
• Correctly applies integration by parts, or equivalent merit	1

Suggested solution(s)	comments
$I_n = \int_0^1 \sqrt{x}(1-x)^n\,dx$ $u = (1-x)^n$, $\frac{dv}{dx} = x^{\frac{1}{2}}$, $\frac{du}{dx} = -n(1-x)^{n-1}$, $v = \frac{2}{3}x^{\frac{3}{2}}$	• Reasonably well done.
$= \frac{2}{3}\left[x^{\frac{3}{2}}(1-x)^n\right]_0^1 + \frac{2n}{3}\int_0^1 x^{\frac{3}{2}}(1-x)^{n-1}\,dx$ $\boxed{A}$	• Some Not clearly converting: $x \equiv 1-(1-x)$ to progress
$= \frac{2}{3}(0-0) + \frac{2n}{3}\int_0^1 x\sqrt{x}(1-x)^{n-1}\,dx$ rearranging	
$= \frac{2n}{3}\int_0^1 \underline{(1-(1-x))}\sqrt{x}(1-x)^{n-1}\,dx$	
$= \frac{2n}{3}\int_0^1 \sqrt{x}(1-x)^{n-1}\,dx - \frac{2n}{3}\int_0^1 \sqrt{x}(1-x)^n\,dx$ $\boxed{B}$	
$\therefore I_n = \frac{2n}{3}I_{n-1} - \frac{2n}{3}I_n$	
$3I_n = 2nI_{n-1} - 2nI_n$	
$(2n+3)I_n = 2nI_{n-1}$	
$I_n = \frac{2n}{2n+3}I_{n-1}$ $\boxed{C}$	

Question 16 (a) (ii)

Criteria	Marks
• Provides correct solution	1

Suggested solution(s)	comments
$I_1 = \frac{2}{2+3}I_0$	• Very well done
$= \frac{2}{5}\int_0^1 \sqrt{x}\,dx$	
$= \frac{2}{5}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^1$	
$= \frac{4}{15}$ $\boxed{A}$	

Question 16 (b) (i)

Criteria	Marks
• Provides correct solution	3
• Simplifies the expression in terms of trig ratios of $\frac{\theta}{2}$ and i, or in terms of t and i, or shows $\operatorname{Arg}\left(\frac{1+w}{1-w}\right) = \pm\frac{\pi}{2}$, or shows $\left\|\frac{1+w}{1-w}\right\| = \left\|\cot\frac{\theta}{2}\right\|$, or equivalent merit	2
• Substitutes $w = e^{i\theta}$ or $w = \cos\theta + i\sin\theta$, or draws a diagram showing vectors $1-w$ and $1+w$ where w is on the unit circle, or equivalent merit	1

Suggested solution(s)	comments

$$\frac{1+w}{1-w} = \frac{1+\cos\theta + i\sin\theta}{1-\cos\theta - i\sin\theta} \qquad \boxed{A}$$

$$= \frac{1+\frac{1-t^2}{1+t^2} + i\frac{2t}{1+t^2}}{1-\frac{1-t^2}{1+t^2} - i\frac{2t}{1+t^2}} \quad ✓$$

$$= \frac{1+t^2+1-t^2+2ti}{1+t^2-1+t^2-2ti}$$

$$= \frac{2+2ti}{2t^2-2ti}$$

$$= \frac{1+ti}{t(t-i)} \qquad \boxed{B}$$

$$= \frac{1+ti}{t(t-i)} \times \frac{t+i}{t+i}$$

$$= \frac{t+i+t^2i-t}{t(t^2+1)}$$

$$= \frac{i(1+t^2)}{t(1+t^2)}$$

$$= \frac{i}{t}$$

$$= i\cot\frac{\theta}{2} \qquad \boxed{C}$$

using t-results/weisstrass

Half Angles easiest ✓

• Reasonably well attempted.

• Best was by using Half Angles

$\sin\theta = \frac{1}{2}\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$1+\cos\theta = 1+2\cos^2\frac{\theta}{2} - 1$

$1-\cos\theta = 1-(1-2\sin^2\frac{\theta}{2})$

Alternative solutions next two pages

II

$w\bar{w} = |w|^2 = 1$

$$\frac{1+w}{1-w} \times \frac{1-\bar{w}}{1-\bar{w}} = \frac{1-\bar{w}+w-w\bar{w}}{1-\bar{w}-w+w\bar{w}}$$

$$= \frac{w-\bar{w}}{2-(w+\bar{w})}$$

$$= \frac{2is}{2(1-c)} = \frac{is}{1-c} = \frac{2isc\,\theta/2}{1-(1-2s^2\theta/2)}$$

$$= i\cot\frac{\theta}{2}$$

Alternative solution I:

$$\frac{1+w}{1-w} = \frac{1+e^{i\theta}}{1-e^{i\theta}} \qquad \boxed{A}$$

$$= \frac{e^{-\frac{i\theta}{2}} + e^{\frac{i\theta}{2}}}{e^{-\frac{i\theta}{2}} - e^{\frac{i\theta}{2}}}$$

$$= \frac{2\mathrm{Re}\left(e^{-\frac{i\theta}{2}}\right)}{2i\,\mathrm{Im}\left(e^{-\frac{i\theta}{2}}\right)}$$

$$= \frac{\cos\left(-\frac{\theta}{2}\right)}{i\sin\left(-\frac{\theta}{2}\right)} \qquad \boxed{B}$$

$$= \frac{\cos\left(-\frac{\theta}{2}\right)}{i\sin\left(-\frac{\theta}{2}\right)} \times \frac{i}{i}$$

$$= \frac{i\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$ since cosine is even, sine is odd and $i^2 = -1$

$$= i\cot\frac{\theta}{2} \qquad \boxed{C}$$

Not common

GEOMETRY

Not Common

Alternative solution II:

$$\mathrm{Arg}(1-w) = \mathrm{Arg}(w+1) - \frac{\pi}{2} \qquad \boxed{A}$$

$$\mathrm{Arg}(w+1) - \mathrm{Arg}(1-w) = \frac{\pi}{2}$$

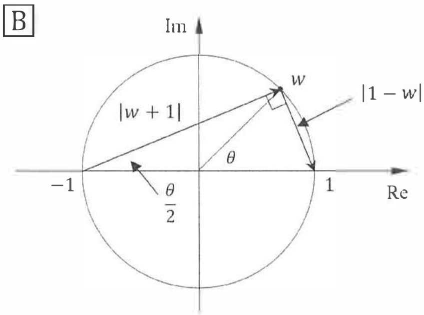
$$\mathrm{Arg}\left(\frac{w+1}{1-w}\right) = \frac{\pi}{2} \qquad \boxed{B}$$

$\therefore \dfrac{1+w}{1-w}$ is purely imaginary

Now $\left|\dfrac{1+w}{1-w}\right| = \dfrac{|1+w|}{|1-w|}$

$= \cot\dfrac{\theta}{2}$ from diagram

$$\therefore \frac{1+w}{1-w} = i\cot\frac{\theta}{2} \qquad \boxed{C}$$



- Angle in a semicircle is a right-angle
- Angle at the circumference is half the angle at the centre

Alternative solution III:

was the problem

$$\begin{aligned}
\operatorname{Arg}\left(\frac{1+w}{1-w}\right) &= \operatorname{Arg}(1+w) - \operatorname{Arg}(1+w) \\
&= \operatorname{Arg}(1+\cos\theta + i\sin\theta) - \operatorname{Arg}(1-\cos\theta - i\sin\theta) \qquad \boxed{A} \\
&= \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right) - \tan^{-1}\left(\frac{-\sin\theta}{1-\cos\theta}\right) \\
&= \tan^{-1}\left[\tan\left\{\tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right) - \tan^{-1}\left(\frac{-\sin\theta}{1-\cos\theta}\right)\right\}\right] \\
&= \tan^{-1}\left[\frac{\tan\left\{\tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)\right\} - \tan\left\{\tan^{-1}\left(\frac{-\sin\theta}{1-\cos\theta}\right)\right\}}{1+\tan\left\{\tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)\right\}\times\tan\left\{\tan^{-1}\left(\frac{-\sin\theta}{1-\cos\theta}\right)\right\}}\right] \\
&= \tan^{-1}\left[\frac{\frac{\sin\theta}{1+\cos\theta} - \frac{-\sin\theta}{1-\cos\theta}}{1+\frac{\sin\theta}{1+\cos\theta}\times\frac{-\sin\theta}{1-\cos\theta}}\right] \\
&= \tan^{-1}\left[\frac{\frac{\sin\theta - \sin\theta\cos\theta + \sin\theta + \sin\theta\cos\theta}{1-\cos^2\theta}}{1-\frac{\sin^2\theta}{1-\cos^2\theta}}\right] \\
&= \tan^{-1}\left[\frac{\frac{2\sin\theta}{1-\cos^2\theta}}{\frac{1-\cos^2\theta-\sin^2\theta}{1-\cos^2\theta}}\right] \\
&= \tan^{-1}\left[\frac{2\sin\theta}{0}\right] \\
&= \pm\frac{\pi}{2} \qquad (1) \qquad \boxed{B}
\end{aligned}$$

Arg( ) ≠ tan⁻¹

OK

$$\begin{aligned}
\left|\frac{1+w}{1-w}\right| &= \left|\frac{1+\cos\theta + i\sin\theta}{1-\cos\theta - i\sin\theta}\right| \\
&= \frac{\sqrt{(1+\cos\theta)^2+\sin^2\theta}}{\sqrt{(1-\cos\theta)^2+\sin^2\theta}} \\
&= \frac{\sqrt{1+2\cos\theta+\cos^2\theta+\sin^2\theta}}{\sqrt{1-2\cos\theta+\cos^2\theta+\sin^2\theta}} \\
&= \frac{\sqrt{2+2\cos\theta}}{\sqrt{2-2\cos\theta}} \\
&= \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} \\
&= \sqrt{\frac{1+\frac{1-t^2}{1+t^2}}{1-\frac{1-t^2}{1+t^2}}} \\
&= \sqrt{\frac{1+t^2+1-t^2}{1+t^2-1+t^2}} \\
&= \sqrt{\frac{1}{t^2}} \\
&= \left|\frac{1}{\tan\frac{\theta}{2}}\right| \\
&= \left|\cot\frac{\theta}{2}\right| \qquad (2)
\end{aligned}$$

wow but Not used

From (1) and (2):

$$w = i\cot\frac{\theta}{2} \qquad \boxed{C}$$

Question 16 (b) (ii)

Criteria	Marks
• Provides correct solution	3
• Shows $z = \frac{1+w}{1-w}$ where $\lvert w\rvert = 1$ and $\text{Arg}(w) = \theta$, or shows the solutions satisfy $\lvert z-1\rvert = \lvert z+1\rvert$ and $\arg\left[\frac{z-1}{z+1}\right] = \frac{(2k+1)\pi}{8}$, or equivalent merit	2
• Shows $\frac{z-1}{z+1}$ is equal to one the eighth roots of -1, or rearranges the equation to $\left(\frac{1+z}{1-z}\right)^8 = -1$, or shows the solutions satisfy $\lvert z-1\rvert = \lvert z+1\rvert$ or $\arg\left[\frac{z-1}{z+1}\right] = \frac{(2k+1)\pi}{8}$, or equivalent merit	1

Suggested solution(s)	**comments**

$$\left(\frac{z-1}{z+1}\right)^8 = -1$$

$$= \text{cis}\,(2k+1)\pi \quad = e^{i(\pi + 2k\pi)} \qquad k \in \mathbb{Z}$$

$$\therefore \frac{z-1}{z+1} = \text{cis}\frac{(2k+1)\pi}{8} \quad \text{for } k = -4, -3, \ldots, 3 \qquad \boxed{A}$$

$$\therefore z - 1 = z\,\text{cis}\frac{(2k+1)\pi}{8} + \text{cis}\frac{(2k+1)\pi}{8} - z$$

$$z\left(1 - \text{cis}\frac{(2k+1)\pi}{8}\right) = \left(1 + \text{cis}\frac{(2k+1)\pi}{8}\right)$$

Note
$-\pi < (2k+1)\frac{\pi}{8} \le \pi$
$-4\frac{1}{2} < k \le 3\frac{1}{2}$
$\therefore k = -4, -3, \ldots, 2, 3$

$$z = \frac{1 \cancel{+\text{cis}}\frac{(2k+1)\pi}{8}}{1 - \text{cis}\frac{(2k+1)\pi}{8}} \qquad \boxed{B}$$

$$= i\cot\frac{(2k+1)\pi}{16} \quad \text{with } w = \text{cis}\frac{(2k+1)\pi}{8} \text{ in part (i)}$$

$$= i\cot\left(\pm\frac{\pi}{16}\right),\ i\cot\left(\pm\frac{3\pi}{16}\right), \ldots, i\cot\left(\pm\frac{7\pi}{16}\right) \qquad \boxed{C}$$

$$= \pm i\cot\left(\frac{\pi}{16}\right), \pm i\cot\left(\frac{3\pi}{16}\right), \ldots, \pm i\cot\left(\frac{7\pi}{16}\right)$$

• Fairly well attempted

Alternatively:

$$z = i\cot\left(\frac{\pi}{16}\right),\ i\cot\left(\frac{3\pi}{16}\right), \ldots\ i\cot\left(\frac{15\pi}{16}\right)$$

od

• Best solve $w^8 = -1 = e^{i(\pi + 2k\pi)}$

Alternative solutions next two pages

Alternative solution I: solve $w^8 = -1$

$$z - 1 = e^{\frac{(2k+1)\pi}{8}i} z + e^{\frac{(2k+1)\pi}{8}i}$$

$$z\left(1 - e^{\frac{(2k+1)\pi}{8}i}\right) = 1 + e^{\frac{(2k+1)\pi}{8}i}$$

$$z = \frac{1 + e^{\frac{(2k+1)\pi}{8}i}}{1 - e^{\frac{(2k+1)\pi}{8}i}} \qquad \boxed{B}$$

$$= i\cot\left(\frac{(2k+1)\pi}{16}\right) \quad \text{with } w = e^{\frac{(2k+1)\pi}{8}i} \text{ in part (i)}$$

$$= i\cot\left(\pm\frac{\pi}{16}\right),\ i\cot\left(\pm\frac{3\pi}{16}\right),\ \ldots,\ i\cot\left(\pm\frac{7\pi}{16}\right) \qquad \boxed{C}$$

$$= \pm i\cot\left(\frac{\pi}{16}\right), \pm i\cot\left(\frac{3\pi}{16}\right), \ldots, \pm i\cot\left(\frac{7\pi}{16}\right)$$

Alternative solution II: Not use this Alternative

$$\left|\left(\frac{z-1}{z+1}\right)^8\right| = |-1|$$

$$\left|\frac{z-1}{z+1}\right|^8 = 1$$

$$\left|\frac{z-1}{z+1}\right| = 1$$

$$|z-1| = |z+1| \qquad \boxed{A}$$

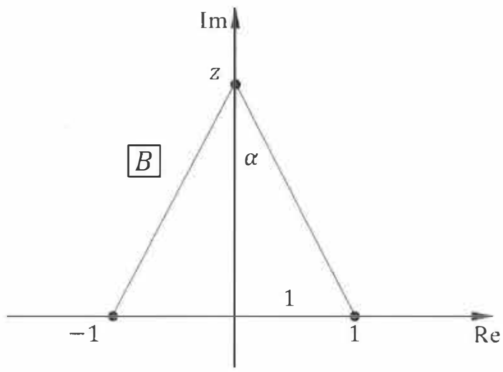
The solutions lie on the perpendicular bisector of the interval joining $(1,0)$ and $(-1,0)$, which is the imaginary axis. The solutions are all purely real.

$$\arg\left[\left(\frac{z-1}{z+1}\right)^8\right] = \arg(-1)$$

$$8\arg\left[\frac{z-1}{z+1}\right] = (2k+1)\pi$$

$$\arg\left[\frac{z-1}{z+1}\right] = \frac{(2k+1)\pi}{8}$$

$\boxed{B}$



In the diagram $\alpha = \frac{(2k+1)\pi}{16}$

$$\tan\alpha = \frac{1}{|z|}$$

$$|z| = \cot\alpha$$

$$= \cot\left(\frac{(2k+1)\pi}{16}\right)$$

Since z lies on the imaginary axis above or below zero,

$$z = i\cot\left(\pm\frac{\pi}{16}\right),\ i\cot\left(\pm\frac{3\pi}{16}\right),\ \ldots,\ i\cot\left(\pm\frac{7\pi}{16}\right) \qquad \boxed{C}$$

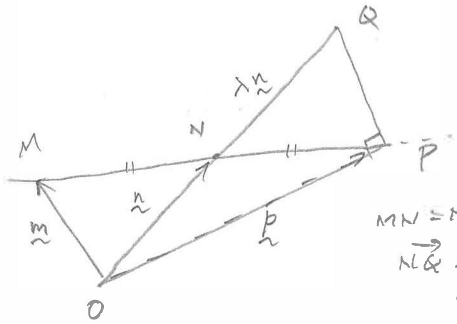
$$= \pm i\cot\left(\frac{\pi}{16}\right), \pm i\cot\left(\frac{3\pi}{16}\right), \ldots, \pm i\cot\left(\frac{7\pi}{16}\right)$$

Question 16 (c)

Criteria	Marks
• Provides correct solution	3
• Finds an equation for the dot product of $\overrightarrow{QP}$ and $\overrightarrow{MN}$ in terms of $\underset{\sim}{m}$ and $\underset{\sim}{n}$, or equivalent merit	2
• Finds expressions for $\underset{\sim}{p}$ and $\underset{\sim}{q}$ in terms of $\underset{\sim}{m}$ and $\underset{\sim}{n}$, or equivalent merit	1

Suggested solution(s)	comments

$$\overrightarrow{OP} = \overrightarrow{ON} + \overrightarrow{NP}$$
$$\underset{\sim}{p} = \overrightarrow{ON} + \overrightarrow{MN}$$
$$= \underset{\sim}{n} + \underset{\sim}{n} - \underset{\sim}{m}$$
$$= 2\underset{\sim}{n} - \underset{\sim}{m}$$
$$\overrightarrow{OQ} = \overrightarrow{ON} + \overrightarrow{NQ}$$
$$\underset{\sim}{q} = \overrightarrow{ON} + \lambda\overrightarrow{ON} \checkmark$$
$$= (1+\lambda)\overrightarrow{ON}$$
$$= (1+\lambda)\underset{\sim}{n}$$



$\overrightarrow{MP} = \underset{\sim}{p} - \underset{\sim}{m}$

$\overrightarrow{MN} = \frac{1}{2}(\underset{\sim}{p} - \underset{\sim}{m})$

$MN = NP$

$\overrightarrow{NQ} = \lambda\overrightarrow{ON}$ $\lambda \in \mathbb{R}$

$= \lambda\underset{\sim}{n}$

$\boxed{A}$

$\overrightarrow{PQ} = -\overrightarrow{QP}$

$$\overrightarrow{QP} = \underset{\sim}{p} - \underset{\sim}{q}$$
$$= 2\underset{\sim}{n} - \underset{\sim}{m} - (1+\lambda)\underset{\sim}{n} \quad = \underset{\sim}{m} - (1-\lambda)\underset{\sim}{n}$$
$$= (1-\lambda)\underset{\sim}{n} - \underset{\sim}{m}$$

$\overrightarrow{PQ} \cdot \overrightarrow{PN} = 0$

$\overrightarrow{PQ} \cdot \overrightarrow{NM}$

$$\overrightarrow{PQ} \cdot \overrightarrow{MN} = 0$$
$$\left(\underset{\sim}{m} - (1-\lambda)\underset{\sim}{n}\right) \cdot \left(\underset{\sim}{n} - \underset{\sim}{m}\right) = 0 \qquad \boxed{B}$$
$$\underset{\sim}{m} \cdot \underset{\sim}{n} - \underset{\sim}{m} \cdot \underset{\sim}{m} - (1-\lambda)\underset{\sim}{n} \cdot \underset{\sim}{n} + (1-\lambda)\underset{\sim}{n} \cdot \underset{\sim}{m} = 0$$
$$(\lambda - 1)\left|\underset{\sim}{n}\right|^2 + (2-\lambda)\underset{\sim}{m} \cdot \underset{\sim}{n} - \left|\underset{\sim}{m}\right|^2 = 0$$
$$\lambda\left(\left|\underset{\sim}{n}\right|^2 - \underset{\sim}{m} \cdot \underset{\sim}{n}\right) + 2\underset{\sim}{m} \cdot \underset{\sim}{n} - \left|\underset{\sim}{m}\right|^2 - \left|\underset{\sim}{n}\right|^2 = 0$$
$$\lambda\left(\left|\underset{\sim}{n}\right|^2 - \underset{\sim}{m} \cdot \underset{\sim}{n}\right) = \left|\underset{\sim}{m}\right|^2 + \left|\underset{\sim}{n}\right|^2 - 2\underset{\sim}{m} \cdot \underset{\sim}{n}$$
$$\lambda = \frac{\left|\underset{\sim}{m}\right|^2 + \left|\underset{\sim}{n}\right|^2 - 2\underset{\sim}{m} \cdot \underset{\sim}{n}}{\left|\underset{\sim}{n}\right|^2 - \underset{\sim}{m} \cdot \underset{\sim}{n}} \qquad \boxed{C}$$

Comments:

• Fairly well attempted

• Best to Re Draw Diagram to aid work

like Q11 c)

15 c)

$\perp$

Question 16 (d)

Criteria	Marks
• Provides correct solution	3
• Simplifies the denominator, or equivalent merit	2
• Uses $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\,f(x)\,dx = \int_0^{\frac{\pi}{4}} \left(\frac{\pi}{2}-x\right) f\left(\frac{\pi}{2}-x\right) dx$, shift/King's Prop or $A_2 = \int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2}-x\right) f\left(\frac{\pi}{2}-x\right) dx$, or equivalent merit	1

Suggested solution(s)	comments

$A_1 = \ln 2$

$$A_2 = \int_0^{\frac{\pi}{2}} x\,f(x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{\pi}{2}-x\right) f\left(\frac{\pi}{2}-x\right) dx$$

shift or do subst $u = \frac{\pi}{2} - x$ $\boxed{A}$

since 0 and $\frac{\pi}{2}$ are equidistant from $\frac{\pi}{4}$ and the two functions are reflections about x

do they know?

$= \frac{\pi}{4}$

$$= \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}-x}{1+\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}-x}{1+\sin x+\cos x}\,dx \qquad \boxed{B}$$

$$= \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos x+\sin x} - \int_0^{\frac{\pi}{2}} \frac{x\,dx}{1+\cos x+\sin x}$$

$$A_2 = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} f(x)\,dx - A_2$$

$\therefore 2A_2 = \frac{\pi}{2}A_1$

$\pi A_1 = 4A_2$

$\therefore\ A_1 : A_2 = 4 : \pi$

$\pi \ln 2 : 4A_2$

$A_2 = \frac{\pi}{4}\ln 2$

$\frac{A_1}{A_2} = \frac{4}{\pi}$

$\ln 2 : \frac{\pi}{4}\ln 2$

$\therefore$

$\boxed{C}$

Comments:

• Not well attempted

• Need to think of subst $u = \frac{\pi}{2} - x$ due to "$\int_0^a$"

Alternative solution I:

$$= \int_0^{\frac{\pi}{4}} x\, f(x)\, dx + \int_0^{\frac{\pi}{4}} \left(\frac{\pi}{2} - x\right) f\left(\frac{\pi}{2} - x\right) dx \qquad \boxed{A}$$

since $\left(\frac{\pi}{2} - x\right) f\left(\frac{\pi}{2} - x\right)$ is a reflection of $xf(x)$ about $x = \frac{\pi}{4}$

Do they know this property of $\int$?

$$= \int_0^{\frac{\pi}{4}} \frac{x}{1 + \cos x + \sin x} dx + \int_0^{\frac{\pi}{4}} \frac{\frac{\pi}{2} - x}{1 + \cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{x}{1 + \cos x + \sin x} dx + \int_0^{\frac{\pi}{4}} \frac{\frac{\pi}{2} - x}{1 + \sin x + \cos x} dx \qquad \boxed{B}$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{4}} \frac{dx}{1 + \cos x + \sin x}$$

$$= \frac{\pi}{2} \times \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cos x + \sin x} \qquad \text{since } f(x) \text{ is symmetrical about } x = \frac{\pi}{4}$$

$$= \frac{\pi}{2} \times \frac{1}{2} \ln 2$$

$$= \frac{\pi}{4} \ln 2$$

$$\therefore\ A_1 : A_2 = \ln 2 : \frac{\pi}{4} \ln 2$$

$$= 4 : \pi \qquad \boxed{C}$$

IBPs on $\int_0^{\pi/2} x\, f(x)\, dx$ — Did NOT go well!!

2025 Year 12 Trial HSC Mathematics Extension 2

Mapping Grid

Section I

Question	Marks	Content	Syllabus Outcome	Targeted Bands
1	1	MEX-P1 The Nature of Proof	MEX12-2	E2-E3
• use the formal language of proof, including the terms statement, implication, converse, negation and contrapositive (ACMSM024)				
2	1	MEX-N2 Using Complex Numbers	MEX12-4	E2-E3
• identify subsets of the complex plane determined by relations, for example $\lvert z-3i\rvert \leq 4$, $\pi/4 \leq Arg(z) \leq \frac{3\pi}{4}$, $Re(z) > Im(z)$ and $\lvert z-1\rvert = 2\lvert z-i\rvert$ (ACMSM086)				
3	1	MEX-P1 The Nature of Proof	MEX12-2	E2-E3
• use examples and counter-examples (ACMSM028)				
4	1	MEX-N2 Using Complex Numbers	MEX12-4	E2-E3
• represent and use complex numbers in the complex plane (ACMSM071)				
5	1	MEX-C1 Further Integration	MEX12-5	E2-E3
• decompose rational functions whose denominators have simple linear or quadratic factors, or a combination of both, into partial fractions.				
6	1	MEX-N1 Introduction to Complex Numbers	MEX12-4	E3-E4
• represent and use complex numbers in Cartesian form AAM				
7	1	MEX-V1 Further Work with Vectors	MEX12–3	E3-E4
• define and use the scalar (dot) product of two vectors in three dimensions AAM				
8	1	MEX-V1 Further Work with Vectors	MEX12–3	E3-E4
• recognise and find the equations of spheres				
9	1	MEX-M1 Applications of Calculus to Mechanics	MEX 12–6	E3-E4
• derive equations for displacement, velocity and acceleration in terms of time, given that a motion is simple harmonic and describe the motion modelled by these equations AAM				
10	1	MEX-M1 Applications of Calculus to Mechanics	MEX 12–6	E3-E4
• solve problems involving projectiles in a variety of contexts AAM				

Section II

Question	Marks	Content	Syllabus Outcome	Targeted Bands
11 (a)	2	MEX-V1 Further Work with Vectors	MEX12-3	E2-E3
• perform addition and subtraction of three-dimensional vectors and multiplication of three-dimensional vectors by a scalar algebraically and geometrically, and interpret these operations in geometric terms				
11 (b)	2	MEX-N1 Introduction to Complex Numbers	MEX12-1	E2-E3
• use multiplication, division and powers of complex numbers in polar form and interpret these geometrically				
11 (c) (i)	1	MEX-V1 Further Work with Vectors	MEX12-3	E2-E3
• determine when a given point lies on a given line in vector form				
11 (c) (ii)	2	MEX-V1 Further Work with Vectors	MEX12-3	E2-E3
• determine a vector equation of a straight line or straight-line segment, given the position of two points or equivalent information, in two and three dimensions (ACMSM105)				
11 (d)	2	MEX-N2 Using Complex Numbers	MEX12-4	E2-E3
• solve quadratic equations of the form $ax^2 + bx + c = 0$, where a, b, c are complex numbers AAM				
11 (e)	2	MEX-M1 Applications of Calculus to Mechanics	MEX 12–6	E2-E3
• consider and solve problems involving motion in a straight line with both constant and non-constant acceleration and derive and use the expressions $\frac{dv}{dt}$, $v\frac{dv}{dx}$ and $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ for acceleration (ACMSM136) AAM				
11 (f) (i)	2	MEX-C1 Further Integration	MEX12-5	E2-E3
• decompose rational functions whose denominators have simple linear or quadratic factors, or a combination of both, into partial fractions				
11 (f) (i)	2	MEX-C1 Further Integration	MEX12-5	E2-E3
• use partial fractions to integrate functions				
12 (a)	2	MEX-N1 Introduction to Complex Numbers	MEX12-4	E2-E3
• prove and use the basic identities involving modulus and argument (ACMSM080) AAM				
12 (b)	2	MEX-C1 Further Integration	MEX12-5	E2-E3
• evaluate integrals using the method of integration by parts (ACMSM123)				
12 (c)	3	MEX-P1 The Nature of Proof	MEX12-2	E2-E3
• prove simple results involving numbers (ACMSM061)				
12 (d)	2	MEX-N1 Introduction to Complex Numbers	MEX12-4	E2-E3
• represent and use complex numbers in exponential form, $z = re^{i\theta}$, where r is the modulus of z and θ is the argument of z				

Question	Marks	Content	Syllabus Outcome	Targeted Bands
12 (e) (i)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6	E2-E3
• solve problems involving resisted motion of a particle moving along a horizontal line AAM				
12 (e) (ii)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6	E2-E3
• solve problems involving resisted motion of a particle moving along a horizontal line AAM				
13 (a)	3	MEX-C1 Further Integration	MEX12-5	E2-E3
• integrate rational functions involving a quadratic denominator by completing the square or otherwise				
13 (b)	2	MEX-P1 The Nature of Proof	MEX12-2	E2-E3
• prove results involving inequalities.				
13 (c)	3	MEX-P2 Further Mathematical Induction	MEX12-2	E2-E4
• use mathematical induction to prove first-order recursive formulae				
13 (d)	3	MEX-N2 Using Complex Numbers	MEX12-4	E2-E3
• solve problems involving real polynomials with conjugate roots				
13 (e) (i)	3	MEX-M1 Applications of Calculus to Mechanics	MEX12-6	E2-E4
• describe mathematically the motion of particles in situations other than projectile motion and simple harmonic motion				
13 (e) (ii)	1	MEX-M1 Applications of Calculus to Mechanics	MEX12-6	E2-E4
• describe mathematically the motion of particles in situations other than projectile motion and simple harmonic motion				
14 (a)	2	MEX-C1 Further Integration	MEX12-5	E3-E4
• find and evaluate indefinite and definite integrals using the method of integration by substitution, where the substitution may or may not be given				
14 (b)	4	MEX-M1 Applications of Calculus to Mechanics	MEX12-6	E2-E4
• derive $v^2 = g(x)$ and the equations for velocity and displacement in terms of time when given $\ddot{x} = f(x)$ and initial conditions, and describe the resulting motion				
14 (c) (i)	2	MEX-N2 Using Complex Numbers	MEX12-4	E2-E3
• use De Moivre's theorem with complex numbers in both polar and exponential form AAM				
14 (c) (ii)	3	MEX-N2 Using Complex Numbers	MEX12-4	E2-E4
• use De Moivre's theorem with complex numbers in both polar and exponential form AAM				

Question	Marks	Content	Syllabus Outcome	Targeted Bands
14 (d) (i)	3	MEX-V1 Further Work with Vectors	MEX12-3	E3-E4
• use vector equations of curves in two or three dimensions involving a parameter, and determine a corresponding Cartesian equation in the two-dimensional case, where possible (ACMSM104) AAM				
14 (d) (ii)	1	MEX-V1 Further Work with Vectors	MEX12-3	E3-E4
• use vector equations of curves in two or three dimensions involving a parameter, and determine a corresponding Cartesian equation in the two-dimensional case, where possible (ACMSM104) AAM				
15 (a) (i)	2	MEX-P1 The Nature of Proof	MEX12-2	E2-E3
• prove further results involving inequalities by logical use of previously obtained inequalities				
15 (a) (ii)	2	MEX-P1 The Nature of Proof	MEX12-2	E3-E4
• prove further results involving inequalities by logical use of previously obtained inequalities				
15 (b)	3	MEX-P1 The Nature of Proof	MEX12-2	E2-E4
• use proof by contradiction including proving the irrationality for numbers such as $\sqrt{2}$ and $\log_2 5$ (ACMSM025, ACMSM063)				
15 (c)	3	MEX-V1 Further Work with Vectors	MEX12–3	E2-E4
• define and use the scalar (dot) product of two vectors in three dimensions AAM				
15 (d)	4	MEX-M1 Applications of Calculus to Mechanics	MEX12-6	E2-E4
• solve problems involving the motion of a particle moving vertically (upwards or downwards) in a resisting medium and under the influence of gravity AAM				
16 (a) (i)	3	MEX-C1 Further Integration	MEX12-5	E2-E4
• derive and use recurrence relationships				
16 (a) (ii)	1	MEX-C1 Further Integration	MEX12-5	E2-E3
• derive and use recurrence relationships				
16 (b) (i)	3	MEX-N1 Introduction to Complex Numbers	MEX12-4	E3-E4
• represent and use complex numbers in polar or modulus-argument form, $z = r(\cos\theta + i\sin\theta)$, where r is the modulus of z and θ is the argument of z AAM				
16 (b) (ii)	3	MEX-N1 Introduction to Complex Numbers	MEX12-4	E3-E4
• represent and use complex numbers in polar or modulus-argument form, $z = r(\cos\theta + i\sin\theta)$, where r is the modulus of z and θ is the argument of z AAM				
16 (c)	3	MEX-V1 Further Work with Vectors	MEX12-3	E3-E4
• define and use the scalar (dot) product of two vectors in three dimensions AAM				
16 (d)	3	MEX-C1 Further Integration	MEX12-2	E2-E4
• apply these techniques of integration to practical and theoretical situations AAM				