Killara HIGH SCHOOL

MATHEMATICS DEPARTMENT

Student Number

2025 **YEAR 12**

Mathematics Extension 2

Trial Examination

Q	Marks
MC	/10
11	/15
12	/16
13	/16
14	/15
15	/13
16	/15
Total	**/100**

General Instructions:

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 100

Section I - 10 marks

- Allow about 15 minutes for this section

Section II - 90 marks

Allow about 2 hour 45 minutes for this section

This assessment task constitutes 30% of the course.

Section I

10 marks
Allow about 15 minutes for this section

Use the multiple-choice sheet for Question 1–10

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1 Which of the follow is the contrapositive of the statement

"If Gandalf has come to the Shire, then there will be fireworks."

(A) If there are fireworks, then Gandalf has come to the Shire.

(B) If there are not fireworks, then Gandalf has not come to the Shire.

(C) If Gandalf does not come to the Shire there will be no fireworks.

(D) Gandalf has come to the Shire and there are no fireworks

2 Which one of the following is NOT the primitive of

$$\int \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}\, dx$$

(A) $$x + \frac{\cos 2x}{4} + c$$

(B) $$x - \frac{\sin 2x}{4} + c$$

(C) $$x + \frac{\cos^2 x}{2} + c$$

(D) $$x - \frac{\sin^2 x}{2} + c$$

3 What is the negation of the statement $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, y \geq x$

(A) $\forall x \in \mathbb{Z}, \nexists\, y \in \mathbb{Z}, y > x$

(B) $\forall x \in \mathbb{Z}, \nexists\, y \in \mathbb{Z}, y < x$

(C) $\exists x \in \mathbb{Z}, \exists\, y \in \mathbb{Z}, y < x$

(D) $\exists x \in \mathbb{Z}, \forall\, y \in \mathbb{Z}, y < x$

4 Given the vector $\overrightarrow{OA} = \sqrt{2}i - j + k$, then this vector

(A) makes an angle of $120°$ with the positive y-axis and $30°$ with the positive z-axis.

(B) makes an angle of $45°$ with the positive x-axis and $120°$ with the positive y-axis.

(C) makes an angle of $45°$ with the positive x-axis and $150°$ with the positive y-axis.

(D) makes an angle of $135°$ with the positive x-axis and $150°$ with the positive y-axis.

5 Evaluate the following complex number

$$\left(-2 + 2i\sqrt{3}\right)^3$$

(A) $-2 + 2i\sqrt{3}$

(B) $8 + 6i\sqrt{3}$

(C) 8

(D) 64

6 A particle of mass m kg is on a smooth inclined plane that makes an angle θ with the horizontal. The particle is attached to a light inextensible string that passes over a smooth pulley at the top of the plane. The other end of the string is connected to a hanging particle of mass M kg. The system is released from rest and moves without friction.
Assume the string remains taut and the acceleration of both masses is a, and gravity is g. Which of the following pairs of equations correctly represent the motion of the two masses?

(A)
$$mg\sin\theta - T = ma$$
$$Mg - T = Ma$$

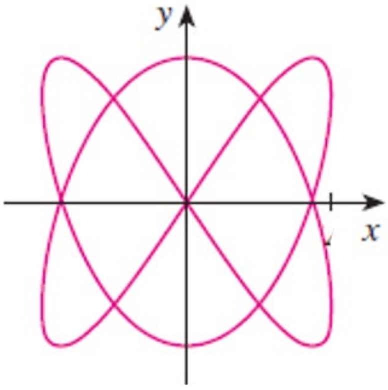
(B)
$$mg\cos\theta - T = ma$$
$$Mg - T = -Ma$$

(C)
$$T - mg\cos\theta = ma$$
$$Mg - T = Ma$$

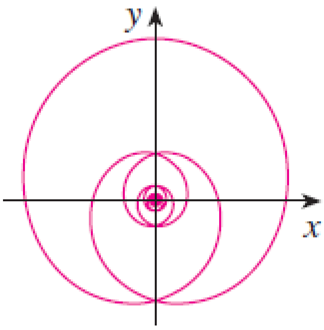
(D)
$$T - mg\sin\theta = ma$$
$$Mg - T = Ma$$

7 Which graph matches the parametric equation $x = 2\cos 5t, \; y = 2\sin 2t$

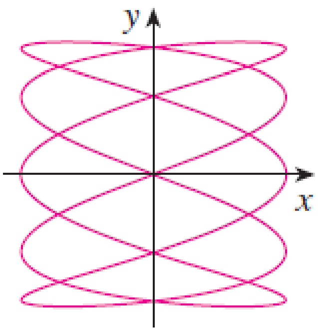
(A)



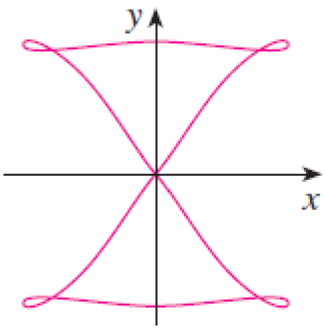
(B)



(C)



(D)



8 The polynomial equation $P(z)$ has one complex coefficient. Three of the roots of this equation are $z = 4 + 2i$, $z = -9 - 2i$ and $z = 0$.
The minimum degree of $P(z)$ is

(A) 3

(B) 4

(C) 5

(D) 6

9 There are 4 trolls on the bridge.

Troll 1: Every troll can see at least 1 Knave.
Troll 2: I can see at least 1 troll that can only see Knaves.
Troll 3: All trolls are afraid of goats.
Troll 4: Some trolls are afraid of goats.

Knowing that all Trolls are either knights (always tell the truth) or Knaves (always Lie). What can be deduced?

(A) Three Trolls are Knaves

(B) No trolls are afraid of goats

(C) Some trolls are afraid of goats

(D) All trolls are afraid of goats

10 $\vec{r_1}(t) = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} + t\begin{pmatrix} \lambda \\ 1 \\ 2 \end{pmatrix}$ intersects $\left|\vec{r_2}(t) - \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}\right| = 3$ at exactly one point. Which one of the following is correct?

(A) $\lambda = -\frac{1}{4}$

(B) $\lambda = \frac{1}{4}$

(C) $t = -\frac{1}{4}$

(D) $t = \frac{1}{4}$

End of Section I

Section II

In Questions 11 – 14, your response should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Answer each question in a NEW booklet.

(a) Let $z = 1 + 3i$ and $w = 2 - 6i$

(i) Find $\overline{z} + w$ in modulus-argument form. **2**

(ii) Write $\frac{z}{w}$ in the form $x + iy$, where x and y are real numbers. **2**

(b) If the points $A(0, 1, 1)$, $B(2, 5, 9)$ and $C(3, b, c)$ are collinear, find the values of b and c. **2**

(c) Prove that if $n \in \mathbb{Z}$, then $n^2 + 2$ is not divisible by 4 **2**

(d) Find **3**

$$\int \frac{-x - 17}{x^2 - x - 6}\, dx$$

(e) Given $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$, find the angle between the two vectors to the nearest degree. **2**

(f) By using the contrapositive prove that if a and b are integers and $a + b \geq 15$, then either $a \geq 7$ or $b \geq 8$. **2**

Question 12 (16 marks) Answer each question in a NEW booklet.

(a) Let a line L be defined by the vector equation:

$$\vec{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \lambda \in R$$

Let A have a position vector $\vec{a} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}$

(i) Find the projection of $\vec{a} - \overrightarrow{r_0}$ on to the line where $\overrightarrow{r_0} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. **2**

(ii) Hence, find the point at which the perpendicular from the point A to the line L intersects line L. **1**

(iii) Find the shortest distance from the point A to the line L. **1**

(b) Prove or disprove there exists an integer k for which the following expression is purely real or purely imaginary **2**

$$\frac{(1+i)^k}{(1-i)^{k-1}}$$

(c) Find **2**

$$\int \frac{\tan 2x}{1 + \cos 2x}\, dx.$$

(d) The velocity $v\ m/s$ of a particle moving in simple harmonic motion along the $x-axis$ is given by $v^2 = -4x^2 + 96x + 20$

(i) Find the acceleration of the particle in term of x. **1**

(ii) State the period and the centre of the motion. **2**

(iii) What is the maximum speed of the particle. **1**

Question 12 continues on the next page

(e) (i) Sketch on the Argand diagram the set of all points satisfying **2**

$\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$. Showing all important features.

(ii) On your diagram in part (i), **1**

Sketch on the Argand diagram the set of all points satisfying

$\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{2}$. Showing all important features.

(iii) Hence, or otherwise, shade the region where $\frac{\pi}{4} \leq \arg\left(\frac{z-2}{z+2}\right) \leq \frac{\pi}{2}$ **1**

Question 13 (16 marks)

(a) Consider **1**

$$\begin{aligned} e^{i\pi} &= -1 \\ e^{2i\pi} &= 1 \\ 2i\pi &= \ln 1 \\ (2i\pi)^2 &= (\ln 1)^2 \\ -4\pi^2 &= 0 \end{aligned}$$

Explain what is wrong with this argument.

(b) Consider $y = \ln(x),\ x \in [1, e]$.

(i) Find the exact value of **2**

$$\int_1^e [\ln(x)]^2\, dx$$

(ii) $y = \ln(x)$ on the interval $[1, e]$, is rotated about the x-axis. Find the exact volume of the solid generated. **2**

(c) A monic 4th degree polynomial has integer coefficients; it also has 2 real integer roots. Which of the following could be a root of the polynomial. Justify your answer. **2**

$$\frac{1+\sqrt{11}i}{2}, \qquad 1+\frac{i}{2}, \quad \frac{1}{2}+i, \qquad \frac{1+\sqrt{13}i}{2}$$

(d) Determine the coordinate point which divides the line segment joining (1, -2, 3) and (3, 4, -5) in the ratio of 2:3 externally. **2**

(e) Let x, y, z be positive real numbers and $xy + yz + zx = 1$. **2**

Prove $10x^2 + 10y^2 + z^2 \geq 4$

Question 13 continues on the next page

(f) Two birds are in flight. Relative to the origin the flight path of each bird is modelled by a straight line.
In the model the equation of the flight path of the first bird is

$$r_1 = \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ a \\ 0 \end{pmatrix}$$

The equation of the flight path of the second bird is

$$r_2 = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

Where λ and μ are scalar parameters.

(i) In this model, the angle between the birds' flight paths is $120°$. Find the value of a. **2**

(ii) Determine whether the fight paths will meet, and if so, where? **3**

Question 14 (15 marks)

(a) Let z be a solution to $z^7 - 1 = 0$ where $z \neq 1$. Compute the numerical value of **3**

$$z^{10} + \frac{1}{z^{10}} + z^{30} + \frac{1}{z^{30}} + z^{50} + \frac{1}{z^{50}}$$

(b) Find the greatest value of the moduli of the complex numbers z satisfying the equation **3**

$$\left|z - \frac{4}{z}\right| = 2.$$

(c) A particle with mass m and positive charge q initially has a velocity of $v = \sqrt{\frac{qA}{2m}}$ and starts $\frac{\pi}{6}$ m to the right. It subsequently moves along the $x - axis$ under the influence of a spatially varying electric potential, $P(x)$.

$P(x) = -A\sin^2 x$ where $A > 0$ and $\frac{\pi}{6} \leq x \leq \frac{\pi}{2}$.

It is given that the electric force that the positively charged particle experiences is

$$\vec{F} = q\vec{E}$$

The electric field is the negative derivative of the electric potential with respect to x.

$$\vec{E} = -\frac{dP}{dx}$$

(i) Assume the electric force is the only force experienced by the positively charged particle, show that the acceleration of the particle is: **1**

$$a(x) = \frac{qA}{m}\sin(2x)$$

(ii) Hence show that the velocity of the particle is: **3**

$$V(t) = \sqrt{\frac{2qA}{m}}\sin(x)$$

(iii) Hence find how much time it takes to reach $x = \frac{\pi}{2}$ from $x = \frac{\pi}{6}$ **2**

Question 14 continues on the next page

(d) (i) Show that **1**

$$\int_0^{\frac{\pi}{2}} (\sin x)^{2k} \cos x \, dx = \frac{1}{2k+1}$$

Where k is a positive integer.

(ii) Hence, or otherwise show that **2**

$$\int_0^{\frac{\pi}{2}} (\cos x)^{2n+1} \, dx = \sum_{k=0}^{n} \frac{(-1)^k}{2k+1} \binom{n}{k}$$

Question 15 (13 marks)

(a) Find **3**

$$\int \frac{\sin x}{\sin x + \cos x + e^x} dx$$

(b) Two stones are thrown simultaneously from the same point, in the same direction and with the same non-zero angle of projection, upwards from the horizontal, α, but with different velocities U, V metres per second, $U < V$.
The slower stone hits the ground a point P, at the same level as the point of projection. At that exact instant that the faster stone cleared a wall height h above the level of projection, and its downwards path makes an angle β with the horizontal.

(i) Show that while both stones are in flight the line joining the two stones has a constant inclination. Hence, express the horizontal distance from P to the wall in terms of h and α. **2**

(ii) Show that $V(\tan\alpha + \tan\beta) = 2U\tan\alpha$. **3**

(c) Let

$$I_n = \int_1^e x(\ln x)^n dx,$$

where $n \geq 1$ and $n \in \mathbb{Z}$

(i) Show that **2**

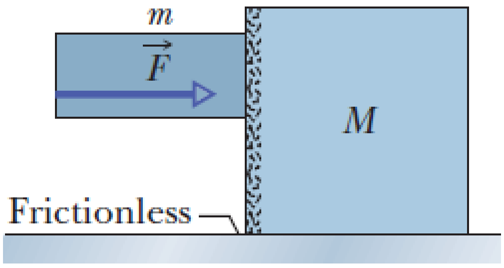
$$I_{n+1} = \frac{1}{2}e^2 - \frac{1}{2}(n+1)I_n$$

(ii) Hence, prove by induction that for all positive integers n, I_n is of the form $A_n e^2 + B_n$, where A_n and B_n are rational numbers **3**

Question 16 (15 marks)

(a) Two blocks are not attached to each other. The larger block has a mass, $M = 88\ kg$ and the smaller one has a mass, $m = 16\ kg$ **4**

The coefficient of static friction between the blocks is $\mu = 0.38$, but the surface beneath the larger block is frictionless. What is the minimum magnitude of the horizontal force required to keep the smaller block from slipping down the larger block?
(You can assume $g = 9.8ms^{-2}$ and leave your final answer to 2 significant figures).



(b) It is known that the arithmetic mean of three positive numbers is greater than the geometric mean of those numbers. That is:

$$\frac{a+b+c}{3} \geq \sqrt[3]{(abc)}$$

(i) Given $a+b+c = t$ and $(a, b, c > 0)$, prove that **2**

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq \frac{9}{t}.$$

(ii) Hence, given $a, b, c > 0$ and $a+b+c = 1$, prove **2**

$$\left\{\frac{1}{a} - 1\right\} \cdot \left\{\frac{1}{b} - 1\right\} \cdot \left\{\frac{1}{c} - 1\right\} \geq 8.$$

Question 15 continues on the next page

(c) Two drones, A and B, both of mass 1 kg, are in motion under gravity and air resistance.

The resistive force acting on each drone is proportional to its velocity, $F = -kv$, where $k > 0$. The acceleration due to gravity is g where $g > 0$

Initially, drone A is launched vertically upward from ground level with an initial speed $U\ m/s$.

At the same time, drone B begins to glide downward from height H meters above the ground with initial horizontal speed $V\ m/s$ and no initial vertical velocity. Drone B is located a horizontal distance d meters away from Drone A.

You may consider the initial position of A is $(0, 0)$.

(i) Show that the position of A is **2**

$$y_A = \left(\frac{kU + g}{k^2}\right)(1 - e^{-kt}) - \frac{gt}{k}$$

(iii) Hence express d in terms of U, V, k and H so that two drones will collide. Explain why $V > kd$ **5**

End of Examination

Killara 2025 Maths Extension 2 Trial August 2025 SOLUTIONS

Multiple Choice Answers

1. B
2. B
3. D
4. B
5. D
6. D
7. D
8. A
9. C
10. A

Q11

(a) $\bar{z} + w$

(i) $= 1 - 3i + 2 - 6i \quad = 3 - 9i$

modulus $= \sqrt{9+81} \quad = 3\sqrt{10} \quad = 3\sqrt{10}\left[\cos(\tan^{-1}(-3)) + \sin(\tan^{-1}(-3))\, i\right]$

argument $\tan^{-1}(-3)$

(ii) $\dfrac{z}{w} = \dfrac{(1+3i)(2+6i)}{(2-6i)(2+6i)} = \dfrac{\cancel{2}+6i+6i-18}{40} = \dfrac{-16+\cancel{32}\,12\,i}{40}$

$= -\dfrac{16}{40} + \dfrac{\cancel{12}}{40}i \quad = \cancel{-\dfrac{2}{5} + \dfrac{4}{5}i} \quad -\dfrac{2}{5} + \dfrac{3}{10}i$

(b) $\vec{AB} = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix} \qquad \vec{BC} = \begin{pmatrix} 1 \\ b-5 \\ c-9 \end{pmatrix}$

$b - 5 = 2 \qquad c - 9 = 4$

$b = 7 \qquad C = 13$

(c) If $n = 2k \quad k \in \mathbb{Z}$

$n^2 + 2 = 4k^2 + 2 \qquad 4k^2$ is divisible by 4

2 is not $\therefore 4k^2 + 2$ is not divisible by 4

If $n = 2k+1 \quad k \in \mathbb{Z}$

$n^2 + 2 = 4k^2 + 4k + 3$

$\Rightarrow n^2 + 2$ has a remainder 3 when divided by 4

Q11

(d) $\int \frac{-x-17}{x^2-x-6}\,dx$

$= \int \frac{A}{x-3} + \frac{B}{x+2}\,dx$

$A(x+2) + B(x-3) = -x-17$

$x=3 \qquad 5A = -20 \qquad A=-4$

$x=-2 \qquad -5B = -15 \qquad B=3$

$\int \frac{-4}{x-3} + \frac{3}{x+2}\,dx$

$= 3\ln|x+2| - 4\ln|x-3| + C$

Q11 (e) $\vec{a} = (+1, -1, 2) \qquad \vec{b} = (1, -2, 3)$

$\vec{a}\cdot\vec{b} = 1+2+6 = 9$

$|\vec{a}| = \sqrt{4+1+1} = \sqrt{6} \qquad |\vec{b}| = \sqrt{1+4+9} = \sqrt{14}$

$\cos\theta = \frac{9}{\sqrt{6}\sqrt{14}}$

$\theta = \cos^{-1}\frac{9}{\sqrt{84}} \simeq 10.89° \simeq 11°$

Q11

(f) If $a < 7$ and $B < 8$

~~Ca~~ then $a+b < 7+8 < 15$.

This is logically equivalent to the original statement.

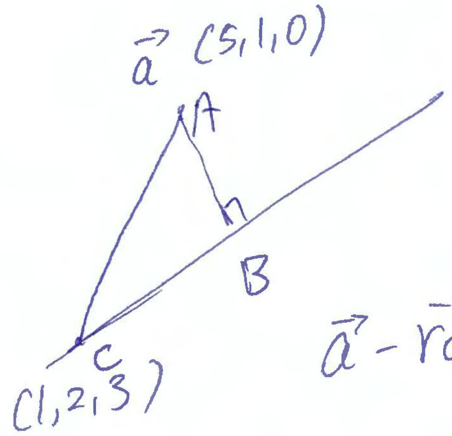
If $a<7$ and $b<8$, then $a+b<15$

is logically equivalent to if $a+b \geq 15$, then

either $a \geq 7$ or $b \geq 8$.

Q12(a)

(i)



directional vector $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$\vec{a} - \vec{r_0} = \begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix}$

$$\text{Proj vector} = \frac{4 \times 2 + (-1) \times (-1) + (-3) \times 1}{2^2 + (-1)^2 + 1^2} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

(ii) let the point to be B

$\vec{CB} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$. $\quad \vec{OB} - \vec{OC} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$\vec{OB} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ - B (3,1,4)

(iii) Shortest distance AB =

$$\sqrt{(5-3)^2 + (1-1)^2 + (0-4)^2}$$

$$= \sqrt{4 + 16} \quad = 2\sqrt{5} \text{ units}$$

Q12

(b) $$\frac{(1+i)^k}{(1-i)^{k-1}} = \sqrt{2}\, e^{\frac{k\pi + 8k\pi m}{4} i} \div e^{\frac{-\pi(k-1)+8m(k-1)\pi}{4} i}$$

$m \in \mathbb{Z}$

$$= \sqrt{2}\, e^{\frac{2k\pi - \pi + 8\pi m}{4} i} = \sqrt{2}\, e^{\frac{(2k-1)\pi}{4} i}$$

If $k=1$ we have $\sqrt{2} e^{\frac{\pi}{4} i} = (1+i)$

not real, not purely imaginary.

If $k=2$, we have $\sqrt{2} e^{\frac{3}{4}\pi i} = \sqrt{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)$

$= -1 + i$

not real, not purely imaginary

If $k=3$ we have $\sqrt{2} e^{\frac{5}{4}\pi i} = \sqrt{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i\right)$

neither $= -1 - i$

If $k=4$, we have $\sqrt{2} e^{\frac{7}{4}\pi i} = \sqrt{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i\right)$

$= 1 - i$

Then the patten repeates itself.

$\therefore$ There is no integer k such that $\frac{(1+i)^k}{(1-i)^{k-1}}$ is real or purely imaginary

Q12

(c)

$$\int \frac{\tan 2x}{1+\cos 2x}\,dx$$

method 1: $\tan x = t$

$$\frac{dt}{dx} = \sec^2 x \qquad dx = \frac{dt}{\sec^2 x} = \frac{1}{1+t^2}\,dt$$

$$\int \frac{\frac{2t}{1-t^2}}{1+\frac{1-t^2}{1+t^2}} \cdot \frac{1}{1+t^2}\,dt$$

$$\int \frac{\frac{2t}{1-t^2}}{2}\,dt = \int \frac{t}{1-t^2}\,dt$$

$$= -\frac{1}{2}\int \frac{-2t}{1-t^2}\,dt$$

$$= -\frac{1}{2}\ln|1-t^2| + c$$

$$= -\frac{1}{2}\ln|1-\tan^2 x| + c$$

Q_{12}

(c) method 2

$$\int \frac{\tan 2x}{1+\cos 2x}\,dx$$

$$= \int \frac{\cancel{3\sin x\cos x}\ \sin(2x)}{\cos 2x(1+\cos 2x)}\,dx$$

$$= -\frac{1}{2}\int \frac{-2\sin(2x)}{\cos 2x(1+\cos 2x)}\,dx$$

$$= -\frac{1}{2}\int \frac{1}{\cos 2x(1+\cos 2x)}\,d\cos(2x)$$

$$= -\frac{1}{2}\int \frac{1}{\cos 2x} - \frac{1}{\cos(2x)+1}\,d\cos(2x)$$

$$= -\frac{1}{2}\left[\ln\left|\frac{\cos 2x}{\cos 2x+1}\right|\right] + C$$

$= \cancel{C}$

Note $\ln\left|\frac{\cos 2x}{\cos 2x+1}\right| = \ln\left|\frac{\cos^2 x - \sin^2 x}{2\cos^2 x}\right|$

$$= \ln\left|\frac{1}{2} - \frac{1}{2}\tan^2 x\right| = \ln|1-\tan^2 x| - \ln 2$$

which differs from method 1 by a constant

Q12

(a) $v^2 = -4x^2 + 96x + 20$

(i) $\frac{1}{2}v^2 = -2x^2 + 48x + 10$

acceleration $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -\cancel{x^2 2x} \; -4x + 48$

(ii) $\ddot{x} = -4(x-12)$ centre $x = 12$

$n^2 = 4$ $n = 2$ Period $\frac{2\pi}{2} = \pi$

(iii) max speed at centre $x = 12$

$v^2 = -4 \times 12 \times 12 + 96 \times 12 + 20$

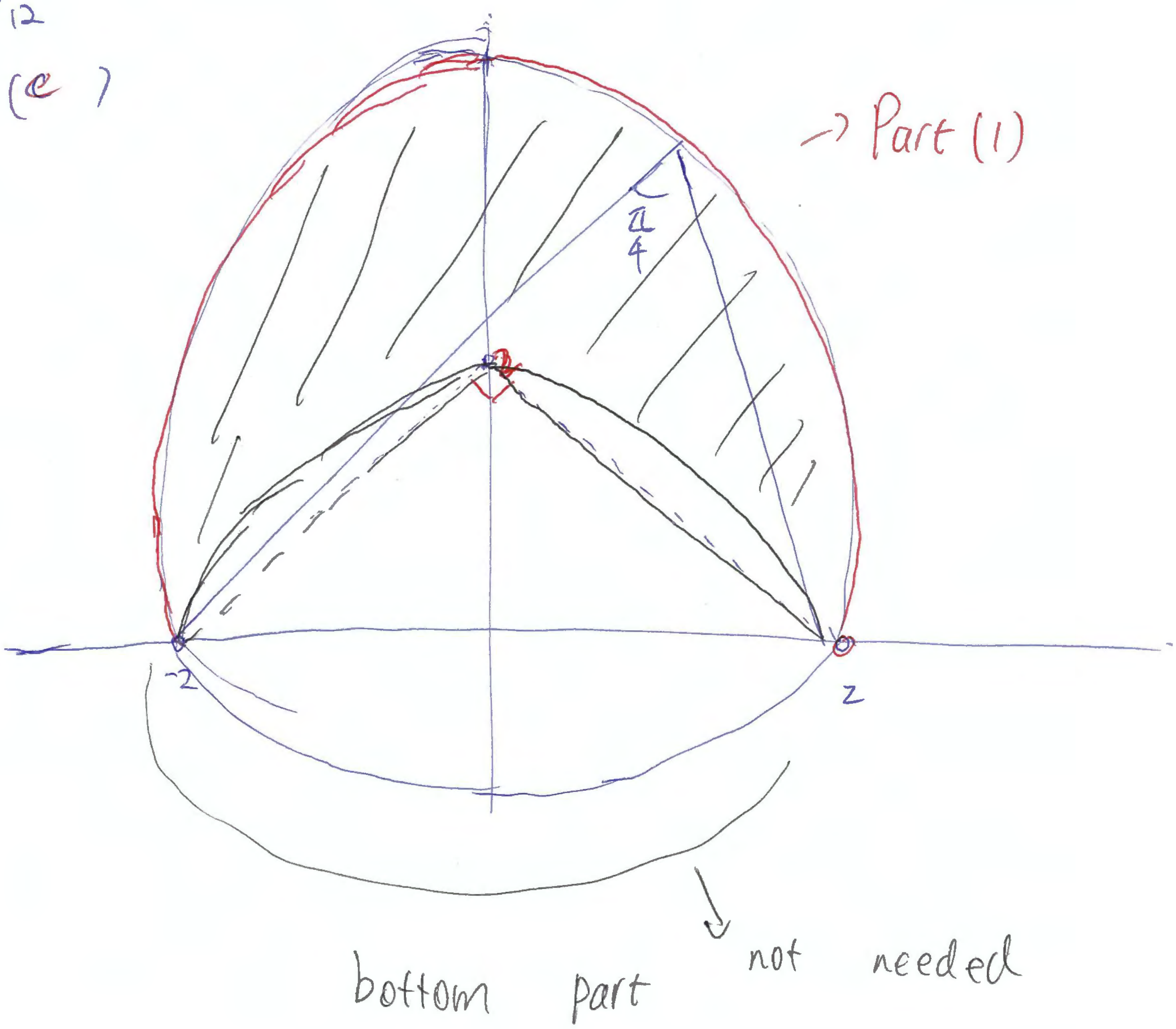
$= 596 = 2\sqrt{149}$

max speed $= \sqrt{596}$ m/s

$2\sqrt{149}$

Q_{12}

(c)



Q13

a) $e^{i\pi} = -1$

$(e^{i\pi})^2 = (-1)^2 = 1$

Also

$e^{2k\pi i} = 1$ not just $e^{2i\pi} = 1$

The mistake is $\ln 1 = 2i\pi$. $\Rightarrow$ It is not one-to-one

If $k=0$ then $e^{2\times 0\times \pi\times i} = 1$

$\Rightarrow \ln 1 = 0$ otherwise $\ln 1 = 2k\pi i$

b) $y = \ln x$

$\int (\ln x)^2 dx = x(\ln x)^2 - \int x \cdot 2\ln(x) \cdot \frac{1}{x}\, dx$

$= x \cdot (\ln x)^2 - 2\int \ln x\, dx$

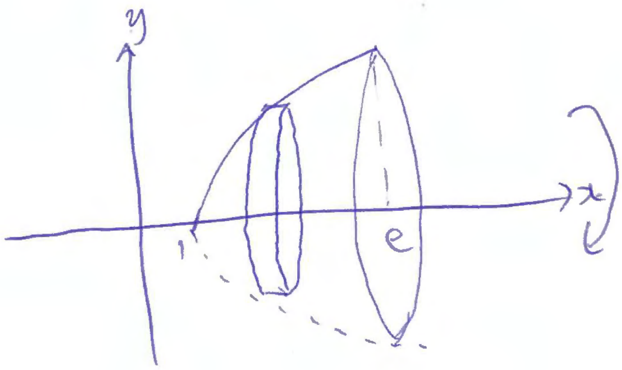
$= x(\ln x)^2 - 2\left[x\ln x - \int x \cdot \frac{1}{x}\, dx\right]$

$= x(\ln x)^2 - 2\left[x\ln x - x\right]$

$= x(\ln x)^2 - 2x\ln x + 2x + C$

$\int_1^e (\ln x)^2 dx = e - 2e + 2e - 2 = e - 2$

(b)



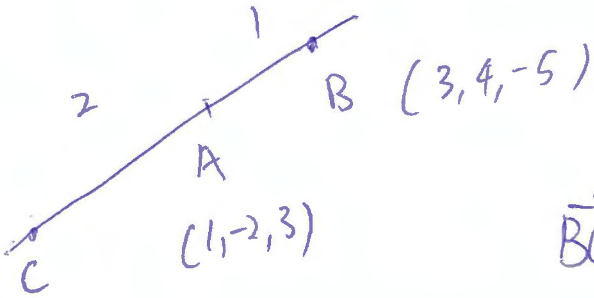
volume

$$\pi \int_1^e (\ln x)^2 \, dx$$

$$= \pi \left[x(\ln x)^2 - 2x\ln(x) + 2x \right]_1^e$$

$$= \pi \left[e - 2 \right] \text{ units}^3$$

(d)



$$\vec{AC} = -\frac{2}{3}\vec{CB}$$

$$\vec{BC} = 3\vec{BA}$$

$$\vec{BC} = 3\begin{pmatrix} -2 \\ -6 \\ 8 \end{pmatrix} = \begin{pmatrix} -6 \\ -18 \\ 24 \end{pmatrix}$$

$$\vec{OC} = \vec{OB} + \vec{BC} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} + \begin{pmatrix} -6 \\ -18 \\ 24 \end{pmatrix} = \begin{pmatrix} -3 \\ -14 \\ 19 \end{pmatrix}$$

The coordinates of the external point is $(-3, 14, 19)$

13

(c)

$$\left(x-\frac{1}{2}-\frac{\sqrt{11}}{2}i\right)\left(x-\frac{1}{2}+\frac{\sqrt{11}}{2}i\right)$$

$$=\left(x-\frac{1}{2}\right)^2+\frac{11}{4}$$

$$=x^2-x+\frac{1}{4}+\frac{11}{4}=x^2-x+3$$ ← All integers

$$\left(x-1-\frac{i}{2}\right)\left(x-1+\frac{i}{2}\right)$$

$$=x^2-2x+1+\frac{1}{4}=x^2-2x+\frac{5}{4}$$ → fraction

$$\left(x-\frac{1}{2}-i\right)\left(x-\frac{1}{2}+i\right)$$

$$=x^2-x+\frac{1}{4}+1=x^2-x+\frac{5}{4}$$ → fraction

$$\left(x-\frac{1}{2}-\frac{\sqrt{13}}{2}i\right)\left(x-\frac{1}{2}+\frac{\sqrt{13}}{2}i\right)$$

$$=x^2-x+\frac{1}{4}+\frac{13}{4}=x^2-x+\frac{7}{2}$$ fraction

$\therefore$ $\frac{1}{2}+\frac{\sqrt{11}}{2}i$ could be a root

Using the relationship between roots and coefficients should be nicer.

For example:

$1+\frac{i}{2}+1-\frac{i}{2}=2$ $\quad\left(1+\frac{i}{2}\right)\left(1-\frac{i}{2}\right)=\frac{5}{4}$ not an integer

13

(e) $xy+yz+zx=1$

$10x^2+10y^2+z^2-4$

$=10x^2+10y^2+z^2-4xy-4yz-4zx$

$=(2x+2y-z)^2+6x^2-12xy+6y^2$

$=(2x+2y-4)^2+6(x-y)^2 \geqslant 0$

$\therefore 10x^2+10y^2+z^2 \geqslant 4$

13

(f) $(2,a,0)\cdot(0,1,-1)$

i) $a = \sqrt{4+a^2}\cdot\sqrt{2}\cos 120^\circ$

$\therefore a<0$

$a^2 = (4+a^2)\times 2\times\left(\frac{1}{4}\right)$

$4a^2 = 2a^2+8$ $\quad a^2=4 \quad a=-2$

Reject $a=2$! $\quad \cos 120^\circ<0 \quad \therefore a=-2$

(iii) r_1 $\quad x=2\lambda-1$, $y=2\lambda+5$, $z=2$ $\qquad r_2$ $\quad x=4$, $y=\mu-1$, $z=-\mu+3$

① $2\lambda-1=4 \Rightarrow \lambda=\frac{5}{2}$ $\qquad$ check ②

② $-2\lambda+5=\mu-1$ $\qquad -2\times\frac{5}{2}+5 = 0 \quad 1-\cancel{1}=0$

$\mu-1=0$

③ $2=3-\mu \quad \mu=1$

13 (f) ii cont:

The three equations are consistent

$x = 2\lambda - 1 = 2 \times \frac{5}{2} - 1 = 4$

$y = -2\lambda + 5 = -2 \times \frac{5}{2} + 5 = 0$

$z = 2$

Check $x = 4$

$y = \mu - 1 = 1 - 1 = 0$

$z = -\mu + 3 = -1 + 3 = 2$

$\therefore$ The paths meet at $\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

Q14

(a) $z^7 = 1$ $\qquad z^7 = e^{7i\theta} = e^{2k\pi i}$

$z = e^{\frac{2k\pi}{7} i}$ $\qquad k \in \mathbb{Z}$

$z^{10} + \frac{1}{z^{10}} + z^{30} + \frac{1}{z^{30}} + z^{50} + \frac{1}{z^{50}}$

$= z + z^2 + z^3 + z^4 + z^5 + z^6$

$z = e^{\frac{2k\pi}{7} i}$ and $z^6 = e^{\frac{12k\pi}{7} i}$ are conjugate

z^2 and z^5 are conjugate

z^3 and z^4 are conjugate

z, z^2, z^3, z^4, z^5 and z^6 are the roots of $z^7 = 1$

We know $1 + z + z^2 + z^3 + z^4 + z^5 + z^6 = 0$

$\therefore z + z^2 + z^3 + z^4 + z^5 + z^6 = -1$

Hence $z^{10} + \frac{1}{z^{10}} + z^{30} + \frac{1}{z^{30}} + z^{50} + \frac{1}{z^{50}} = -1$

If $z = 1$, then the sum will be 6

Q14 (a)

Alternatively,

$$\frac{z^{60}+z^{40}+z^{80}+z^{20}+z^{100}+1}{z^{50}} \qquad z^7=1$$

$$= \frac{z^4+z^5+z^3+z^6+z^2+z^7}{z}$$

$$= \frac{z^2+z^3+z^4+z^5+z^6+z^7}{(\cancel{z-1})\ z} \rightarrow \text{GP}$$

$$\cancel{= \frac{z^2(z^6-1)}{z-z^7} = z^2}$$

$$\frac{z^2(z^6-1)}{(z-1)z} = \frac{z(z^6-1)}{z-1} = \frac{z^7-z}{z-1}$$

$$= \frac{1-z}{z-1} = -1$$

Q14

(b) method 1.

$$\left(z - \frac{4}{z}\right)\left(\bar{z} - \frac{4}{\bar{z}}\right) = 4$$

$z = re^{i\theta}$ $\quad z\bar{z} = r^2 \quad z^2 + \bar{z}^2 = 2r^2\cos 2\theta$

$$z\bar{z} - \frac{4z}{\bar{z}} - \frac{4\bar{z}}{z} + \frac{16}{z\bar{z}} = 4$$

$$r^2 - \frac{8r^2\cos 2\theta}{r^2} + \frac{16}{r^2} = 4$$

$$r^4 - (8\cos 2\theta + 4)r^2 + 16 = 0$$

$$r^2 = 4\cos 2\theta + 2 \pm 2\sqrt{4\cos^2 2\theta + 4\cos 2\theta - 3}$$

$r^2_{max} = 4 + 2 + 2\sqrt{5}$ where $\cos 2\theta = 1$

$$r^2_{max} = 6 + 2\sqrt{5}$$

$$r_{max} = \sqrt{5} + 1$$

This is not very pleasant. see method 2.

Q14

(b) Method 2

$$\left|z - \frac{4}{z}\right| \le |z| + \frac{4}{|z|}$$

$$|z| + \frac{4}{|z|} \geq 2$$

$|z|^2 - 2|z| + 4 \geq 0$ → always true

Let us try

$$\left|z - \frac{4}{z}\right| \geq \left|\,|z| - \frac{4}{|z|}\,\right|$$ let $|z| = r$

Hence $\left|r - \frac{4}{r}\right| \le 2$

$$-2 \le r - \frac{4}{r} \le 2$$

① $r^2 - 2r - 4 \le 0$

$0 \le r \le \sqrt{5} + 1$

$\therefore \sqrt{5} - 1 \le r \le \sqrt{5} + 1$

② $r^2 + 2r - 4 \geq 0$

$r \geq \sqrt{5} - 1$

$\therefore r_{max} = \sqrt{5} + 1$

OR

$$|z| = \left|\left(z - \frac{4}{z}\right) + \frac{4}{z}\right|$$

$$\leq \left|z - \frac{4}{z}\right| + \frac{4}{|z|}$$

$$|z| \leq 2 + \frac{4}{|z|}$$

$$|z|^2 - 2|z| - 4 \leq 0$$

$$\boxed{|z| \leq \frac{2 \pm \sqrt{4+16}}{2}}$$

$$1-\sqrt{5} \leq |z| \leq 1+\sqrt{5}$$

$$\Downarrow$$

$$0 \leq |z| \leq 1+\sqrt{5}$$

$$|z|_{max} = 1+\sqrt{5}$$

14 b) $\left|z - \frac{4}{z}\right| = 2$

$$\left(z - \frac{4}{z}\right)^2 = 4$$

$$z^2 - 8 + \frac{16}{z^2} = 4$$

$$z^4 - 12z^2 + 16 = 0$$

$$z^2 = \frac{12 \pm \sqrt{80}}{2}$$

$$= \frac{12 \pm 2\sqrt{20}}{2}$$

$$= 6 \pm 2\sqrt{5}$$

$$z = \sqrt{6 + 2\sqrt{5}}$$ (max value not min)

Full marks ↑

$$z = \sqrt{1 + 2\sqrt{5} + (\sqrt{5})^2}$$

$$= \sqrt{(1+\sqrt{5})^2}$$

$$= 1 + \sqrt{5}$$

Just proof it is the same

Q14 (~~b~~ c) $F = ma$

i) $E = -\frac{dP}{dx} = +2A\sin x\cos x = +A\sin 2x$

$F = qE = qA\sin 2x$

$qA\sin 2x = ma \Rightarrow a = \frac{qA}{m}\sin(2x)$

(ii) $\frac{d}{dx}v^2 = \frac{2qA}{m}\sin 2x$

$v^2 = \frac{-qA}{m}\cos 2x + C$

$t=0$, $x = \frac{\pi}{6}$, $v = \sqrt{\frac{qA}{2m}}$

$\frac{qA}{2m} = \frac{-qA}{m}\cdot\frac{1}{2} + C \qquad C = \frac{qA}{m}$

$\rightarrow$ 1 mark

$\therefore v^2 = \frac{qA}{m}(1 - \cos 2x)$

$v^2 = \frac{2qA}{m}\sin^2 x \qquad v = \sqrt{\frac{2qA}{m}}\cdot\sin x$

$\rightarrow$ 1 mark

$a = \frac{qA}{m}\sin(2x) \geq 0 \quad \forall x \in (0, \frac{\pi}{2}]$

$v = \sqrt{\frac{qA}{2m}}$ initially $\therefore$ The particle moves to the right. Reject $v = -\sqrt{\frac{2qA}{m}}\sin x$

$v = \sqrt{\frac{2qA}{m}}\cdot\sin x$

$\Downarrow$

1 mark for rejecting the negative case.

Q14 b (iii)

$$\frac{dx}{dt} = \sqrt{\frac{2qA}{m}} \sin x$$

$$\frac{dx}{\sin x} = \sqrt{\frac{2qA}{m}}\, dt$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1 \times \sin x}{\sin x \cdot \sin x}\, dx = \int_0^T \sqrt{\frac{2qA}{m}}\, dt$$

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \frac{1}{1-\cos^2 x}\, d\cos x = \sqrt{\frac{2qA}{m}} \cdot T$$

$$\frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{\pi}{6}} \frac{1}{1+\cos x} + \frac{1}{1-\cos x}\, d\cos x = \sqrt{\frac{2qA}{m}} \cdot T$$

$$\frac{1}{2} \ln \frac{1+\cos x}{1-\cos x} \Big|_{x=\frac{\pi}{2}}^{x=\frac{\pi}{6}} = \sqrt{\frac{2qA}{m}} \cdot T$$

$$\frac{1}{2} \ln \frac{2\cos^2\frac{x}{2}}{2\sin^2\frac{x}{2}} \Big|_{x=\frac{\pi}{2}}^{x=\frac{\pi}{6}} = \sqrt{\frac{2qA}{m}} \cdot T$$

$$\left\{ \ln \left|\cot \frac{x}{2}\right| \right\} \Big|_{x=\frac{\pi}{2}}^{x=\frac{\pi}{6}} = \sqrt{\frac{2qA}{m}} \cdot T$$

$$\ln\left(\cot \frac{\pi}{12}\right) - \ln\left(\cot \frac{\pi}{4}\right) = \sqrt{\frac{2qA}{m}}\, T$$

$$\therefore\ T = \sqrt{\frac{m}{2qA}} \cdot \ln\left[\cot\left(\frac{\pi}{12}\right)\right]$$

$$T = \sqrt{\frac{m}{2qA}} \cdot \ln\left(2+\sqrt{3}\right)$$

Q14 (b) ii

For Your Reference. Do NOT USE in HSC

Part ii) can be understood from conservation of energy

$$\underbrace{\tfrac{1}{2}mv^2}_{\text{kinetic energy}} + \underbrace{qP}_{\text{potential energy}} = \text{fixed}.$$

$$\underbrace{\tfrac{1}{2}m\cdot\left(\sqrt{\frac{qA}{2m}}\right)^2}_{\text{initial kinetic}} + \underbrace{\left(-A\sin^2 x\right)\cdot q}_{\text{initial potential } x=\frac{\pi}{6}} = \tfrac{1}{2}mv^2 + q\left(-A\sin^2 x\right)$$

At any time!

$$\frac{qA}{4} - A\cdot\frac{1}{4}q = \tfrac{1}{2}mv^2 - qA\sin^2 x$$

$$\tfrac{1}{2}mv^2 = qA\sin^2 x$$

$$\therefore v^2 = \frac{2qA}{m}\cdot\sin^2 x.$$ YES!!!

Feedback :

Q14 B

(i) $\displaystyle\int_0^{\frac{\pi}{2}} (\sin x)^{2k} \cos x \, dx$

$\displaystyle= \int_0^{\frac{\pi}{2}} (\sin x)^{2k} \, d\sin x$

$\displaystyle= \frac{(\sin x)^{2k+1}}{2k+1} \Bigg|_{x=0}^{x=\frac{\pi}{2}} = \frac{1}{2k+1}$

ii) $\displaystyle\int_0^{2\pi} (\cos x)^{2n+1} \, dx = \int_0^{2\pi} (\cos x)^{2n} \cos x \, dx$

$\displaystyle= \int_0^{2\pi} (1-\sin^2 x)^n \cos x \, dx$

$\displaystyle= \int_0^{2\pi} \sum_{k=0}^{n} \binom{n}{k} (1)^{n-k} (-\sin^2 x)^k \cdot \cos x \, dx$

$\displaystyle= \int_0^{2\pi} \sum_{k=0}^{n} \binom{n}{k} (-1)^k (\sin x)^{2k} \cos x \, dx$

$\displaystyle= \sum_{k=0}^{n} \binom{n}{k} (-1)^k \int_0^{2\pi} (\sin x)^{2k} \cos x \, dx$

$\displaystyle= \sum_{k=0}^{n} (-1)^k \binom{n}{k} \cdot \frac{1}{2k+1}$

Q15

(a)

$$\int \frac{\sin x}{\sin x+\cos x+e^x}\,dx$$

$$= -\int \frac{\cos x-\sin x+e^x}{\sin x+\cos x+e^x}\,dx + \int \frac{\cos x+e^x}{\sin x+\cos x+e^x}\,dx$$

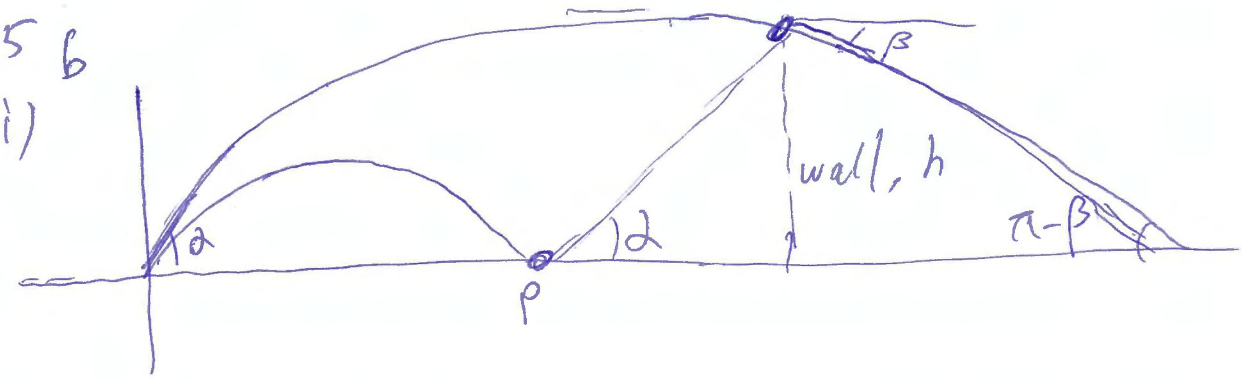
$$= -\ln|\sin x+\cos x+e^x| + \int 1 - \frac{\sin x}{\sin x+\cos x+e^x}\,dx$$

$$2\int \frac{\sin x}{\sin x+\cos x+e^x}\,dx = -\ln|\sin x+\cos x+e^x| + x$$

$$\therefore \int \frac{\sin x}{\sin x+\cos x+e^x}\,dx = \frac{x}{2} - \frac{1}{2}\ln|\sin x+\cos x+e^x| + C$$

Q15 b

i)



slow stone.

$\ddot{x}=0 \qquad \dot{x}=u\cos\alpha \qquad x=u\cos\alpha t+C \qquad t=0 \quad x=0$

$C=0$

$\therefore x=u\cos\alpha\cdot t$

$\ddot{y}=-g \qquad \dot{y}=-gt+u\sin\alpha \qquad y=u\sin\alpha\cdot t-\frac{1}{2}gt^2+C$

$t=0 \quad y=0 \quad C=0$

coordinates of the slow stone

$(u\cos\alpha\cdot t,\ u\sin\alpha\cdot t-\frac{1}{2}gt^2)$

Similarly coordinates of the fast stone

$(v\cos\alpha t,\ v\sin\alpha t-\frac{1}{2}gt^2)$

$$\text{slope}=\frac{v\sin\alpha t-u\sin\alpha\cdot t}{v\cos\alpha\cdot t-u\cos\alpha\cdot t}=\tan\alpha$$

$\therefore$ The line joining the two stones has a constant angle of inclination, α,

The horizontal distance from P to the wall is $h\cot\alpha$ or $\frac{h}{\tan\alpha}$

Q15(ii) At that instant;

$$u\sin\alpha \cdot t - \frac{1}{2}gt^2 = 0 \qquad t = \frac{2u\sin\alpha}{g}$$

$t \neq 0$

The slow stone hits the ground.

Fast stone.

$(u\cos\alpha t,\ u\sin\alpha \cdot t - \frac{1}{2}gt^2)$

$\tan(\alpha - \beta)$

$$\text{slope of tangent} = \frac{u\sin\alpha - gt}{u\cos\alpha} = -\tan\beta$$

$$t = \frac{2u\sin\alpha}{g} \qquad \frac{v\sin\alpha - 2u\sin\alpha}{v\cos\alpha} = -\tan\beta$$

$$v\sin\alpha - 2u\sin\alpha = -v\cos\alpha\tan\beta$$

$\div\cos\alpha$ $\qquad\qquad$ $\div\cos\alpha$

$$v\tan\alpha - 2u\tan\alpha = -v\tan\beta$$

$$\therefore v(\tan\alpha + \tan\beta) = 2u\tan\alpha$$

Q15 (c)

(i) $I_n = \int_1^e x(\ln x)^n \, dx$

$$= \left[\frac{x^2}{2}(\ln x)^n\right]_1^e - n\int_1^e \frac{x^2}{2}(\ln x)^{n-1} \cdot \frac{1}{x} \, dx$$

$$= \frac{e^2}{2} - \frac{n}{2}\int x(\ln x)^{n-1} \, dx$$

$$= \frac{e^2}{2} - \frac{n}{2} I_{n-1}$$

$I_1 = \frac{e^2}{2} - \frac{1}{2} I_0 = \frac{1}{4}e^2 + \frac{1}{4}$

(ii) base ~~$I_0 = \frac{1}{2}$~~

$I_0 = \int_1^e x(\ln x)^0 \, dx = \frac{e^2}{2} - \frac{1}{2}$ $\quad A_0 = \frac{1}{2} \quad B_0 = -\frac{1}{2}$

$A_1 = \frac{1}{4} \quad B_1 = \frac{1}{4}$

Both are rational

Assume $I_n = \int_1^e x(\ln x)^n \, dx = A_n e^2 + B_n$

A_n and B_n are ~~real~~ Rational

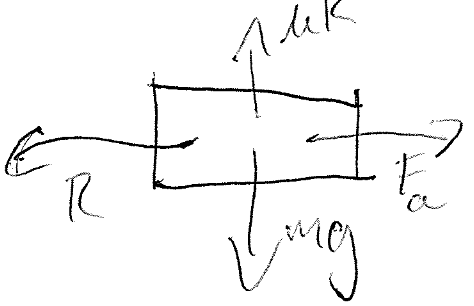
$$I_{n+1} = \frac{e^2}{2} - \frac{1}{2}(n+1)\left[A_n e^2 + B_n\right]$$

$$= \left[\frac{1}{2} - \frac{1}{2}(n+1)A_n\right]e^2 - \frac{1}{2}(n+1)B_n$$

$$= \underbrace{\frac{1}{2}\left[1-(n+1)A_n\right]}_{A_{n+1}}e^2 - \frac{1}{2}\underbrace{(n+1)B_n}$$

A_{n+1} and B_{n+1} are rational.

(6a) small block



large block

small block vertical

$$F = ma = mg - \mu R = 0$$

$$mg = \mu R \quad (1)$$

$$\frac{mg}{\mu} = R$$

small block Horizontal

$$F_H = ma = F_a - R \quad (2)$$

Big block Horizontal

$$F_H = Ma = R \quad (3)$$

(2) + 3

$$(m+M)a = F_a$$

$$a = \frac{F_a}{M+m} \quad (4)$$

sub ① & ④ into ③

$$\frac{M F_a}{(M+m)} = \frac{mg}{\mu}$$

$$F_a = \frac{mg(m+M)}{\mu}$$

$$= \frac{16 \times 9.8 \times (16+88)}{88 \times 0.38}$$

$$\approx 490\ N$$

1 mark → Free body diagrams

2 mark → at least one equation established

3 marks → 3 equations (Small vert, small horizontal & Big Horizontal) established

4 marks → correct answer from correct working

Q16

(b) $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$ $\quad \sqrt[3]{abc} \leq \frac{t}{3} \Rightarrow$ 1 mark

i)

$\frac{1}{\sqrt[3]{abc}} \geq \frac{3}{t}$

$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 3 \cdot \sqrt[3]{\frac{1}{a}\cdot\frac{1}{b}\cdot\frac{1}{c}} = \frac{3}{\sqrt[3]{abc}} \geq 3 \times \frac{3}{t} = \frac{9}{t}$

$\Downarrow$

2 marks

ii) $\left\{\frac{1}{a} - 1\right\}\left\{\frac{1}{b} - 1\right\}\left\{\frac{1}{c} - 1\right\}$

$= \frac{1}{abc} - \frac{1}{ab} - \frac{1}{bc} - \frac{1}{ac} + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} - 1$

$= \frac{1}{abc} - \frac{a+b+c}{abc} + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} - 1$

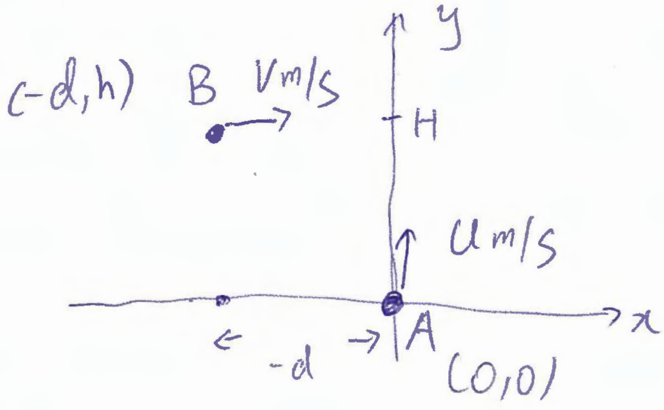
$\because a+b+c = 1 \quad \Downarrow$ $\quad \Rightarrow$ 1 mark

$= 0 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} - 1$

From i) $\quad \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq \frac{9}{t} \quad t = 1$

$\therefore \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 9 - 1 = 8 \Rightarrow$ 2 marks

16 (c)



For A

$a_A = -g - kv$ $\quad$ $t = 0 \quad v = u$ m/s

$\frac{dv}{dt} = -g - kv$ $\quad$ $\boxed{v(t) = (u + \frac{g}{k})e^{-kt} - \frac{g}{k}}$

$$\frac{dy_A}{dt} = (u + \frac{g}{k})e^{-kt} - \frac{g}{k}$$

$$y_A = -\frac{1}{k}(u + \frac{g}{k})e^{-kt} - \frac{g}{k}t + C$$

$t = 0 \quad y_A = 0$

$C = -\frac{1}{k}(u + \frac{g}{k}) + C \qquad C = \frac{ku + g}{k^2}$

$$\therefore y_A = \frac{ku + g}{k^2}(1 - e^{-kt}) - \frac{gt}{k}$$

$\rightarrow$ 1 mark

16

c

(ii) For B in y direction

$$a_y = -g - kv$$

$$\frac{dv_y}{dt} = -g - kv_y$$

$$\frac{dv_y}{g + kv_y} = -dt$$

$$\frac{1}{k}\ln(g + kv_y) = -t + C_1$$

$$\ln(g + kv_y) = -kt + C_2$$

$$g + kv_y = ce^{-kt}$$

$$kv_y = ce^{-kt} - g$$

$t = 0 \quad v_y = 0 \quad c = g$

$$kv_y = g(e^{-kt} - 1) \qquad v_y = \frac{g}{k}(e^{-kt} - 1)$$

$$\frac{dy}{dt} = \frac{g}{k}e^{-kt} - \frac{g}{k}$$

$$y_B = \frac{g}{-k^2}e^{-kt} - \frac{g}{k}t + c$$

$t = 0 \qquad y = H$

$$y = \frac{g}{-k^2} + c = H \qquad c = H + \frac{g}{k^2}$$

$$y_B = \frac{-g}{k^2}e^{-kt} - \frac{g}{k}t + H + \frac{g}{k^2}$$

$$= H - \frac{g}{k}t + \frac{g}{k^2}(1 - e^{-kt})$$

a_B in x direction

$$a_x = -kv_x \qquad \frac{dv}{v} = -k\,dt$$

$$\frac{dv_x}{dt} = -kv_x$$

$$v = Ce^{-kt}$$

$$\ln v = -kt + C_1$$

$$t=0 \quad v_x = V \qquad C = V$$

$$v_{(t)} = Ve^{-kt}$$

$$\frac{dx}{dt} = Ve^{-kt}$$

$$x = \frac{V}{-k}e^{-kt} + c \qquad t=0 \quad x=-d$$

$$x = -\frac{V}{k} + c = -d$$

$$c = \frac{V}{k} - d$$

$$\therefore \quad x = \frac{V}{k} - \frac{V}{k}e^{-kt} - d$$

$$x = \frac{V}{k}\left(1 - e^{-kt}\right) - d$$

In Summary

$$A\left(0,\ \frac{ku+g}{k^2}(1-e^{-kt})-\frac{gt}{k}\right)$$

$$B\left(\frac{V}{k}(1-e^{-kt})-d,\ H-\frac{g}{k}t+\frac{g}{k^2}(1-e^{-kt})\right)$$

When A and B collide

$$\frac{V}{k}(1-e^{-kt})-d=0 \qquad e^{-kt}=1-\frac{kd}{V}$$

$$\frac{V}{k}(1-e^{-kt})=d \qquad =\frac{V-kd}{V}$$

$$1-e^{-kt}=\frac{kd}{V} \qquad \text{Here } e^{-kt}>0$$

$$\therefore\ V-kd>0 \quad V>kd.$$ ~~the~~

It means that the initial ~~velo~~ speed of A has to be more than kd, otherwise A is not fast enough to travel to $x=0$.

When they collide,

$$y_A=y_B$$

$$\frac{kU+g}{k^2}\left(1-e^{-kt}\right)-\frac{g}{\cancel{k}}t$$

$$= H+\frac{g}{k^2}\left(1-e^{-kt}\right)-\frac{g}{\cancel{k}}t$$

$$\cancel{\frac{kU+g}{k^2}}\ \frac{U}{k}\left(1-e^{-kt}\right)=H$$

$$\frac{U}{k}\times\frac{kd}{V}=H$$

$$\frac{Ud}{V}=H \qquad \boxed{d=\frac{VH}{U}}$$

Also $d=\frac{V}{k}\left(1-e^{-kt}\right)$

$$\frac{V}{k}>d$$

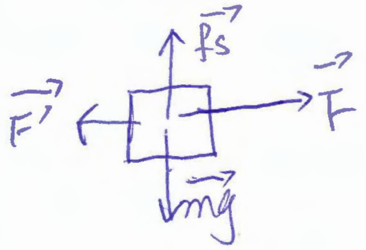
$x_B=0$

From our ~~pre viso~~ previous discussion

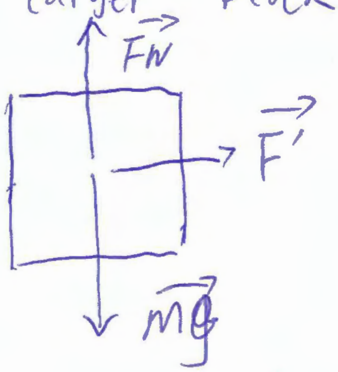
Q16 Let F' to be the ~~contact~~ normal force between blocks

a)

small block



larger block



unknown: a, F, F'

$\therefore$ 3 equations are needed.

system as a whole

① $(m+M)a = F$

small block ② $f_s = \mu F' = mg$

large block ③ $F' = Ma$

From equation ① $a = \dfrac{F}{m+M}$ from ② $F' = \dfrac{mg}{\mu}$

Sub into ③

$$\frac{mg}{\mu} = \frac{M \cdot F}{m+M} \qquad F = \frac{mg(m+M)}{M\mu}$$

$$F = \frac{16 \times 9.8 \times (16+88)}{88 \times 0.38} \approx 490N$$

1 mark → free body diagram

2 marks → At least 1 equation established and above

3 marks → 3 equations and right substitution and above

4 marks → correct answer from correct working.