Student Name: ______________________________

KINROSS WOLAROI SCHOOL

2025

Year 12 Mathematics Extension 2

Trial HSC Examination

Teacher Setting Paper: Mr Bowman
Head of Department: Mr Doyle

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculator may be used
- Write your answers for Section I on the multiple-choice answer sheet provided
- Write your answers for Section II in the booklets provided. Extra writing booklets are available.
- A reference sheet is provided with this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations.

Total marks – 100

Section I – Multiple-Choice

10 marks

Attempt Questions 1-10

Allow 15 minutes for this section

Section II – Free Response

90 marks

Attempt questions 11 – 16

Allow 2 hours and 45 minutes for this section

This examination paper does not necessarily reflect the content or format of the Higher School Certificate Examination in this subject.

Student Name:______________________________

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 - 10.

QUESTION 1

ω is a complex fifth root of unity. What is the value of ω^{2025} ?

(A) 1

(B) – 1

(C) i

(D) $-i$

QUESTION 2

What is the contrapositive of this statement?

"If it is snowing, then the ground is white."

(A) If the ground is white, then it is snowing.

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QUESTION 3

Which equation describes the line joining A (4, 6, 5) to B $(-2, 3, -1)$?

(A) $r = \begin{pmatrix} 4 \\ 6 \\ 5 \end{pmatrix} - \lambda \begin{pmatrix} -2 \\ 3 \\ -5 \end{pmatrix}$

(B) $r = \begin{pmatrix} 4 \\ 6 \\ 5 \end{pmatrix} - \lambda \begin{pmatrix} 2 \\ 9 \\ 4 \end{pmatrix}$

(C) $r = \begin{pmatrix} 4 \\ 6 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -6 \\ -3 \\ -6 \end{pmatrix}$

(D) $r = \begin{pmatrix} 4 \\ 6 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 9 \\ 4 \end{pmatrix}$

QUESTION 4

A particle is describing Simple Harmonic Motion in a straight line with an amplitude of 3 metres.
When the particle is 2 metres from the centre of the motion, its speed is 5 m/s.
What is the period of the motion?

(A) $\frac{\sqrt{5}\pi}{2}$

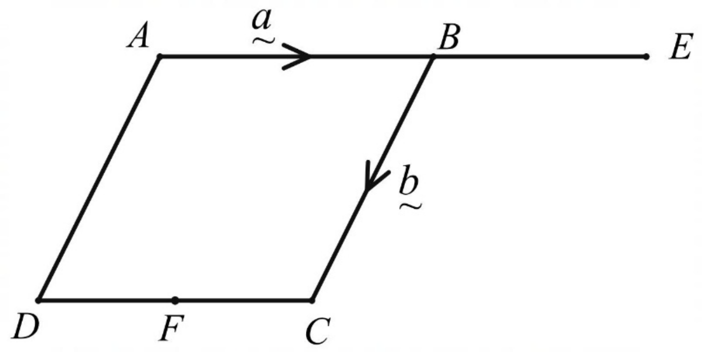
(B) $\frac{2\sqrt{5}\pi}{5}$

(C) $\sqrt{5}\pi$

(D) $2\sqrt{5}\pi$

QUESTION 5

$ABCD$ is a rhombus. $\overrightarrow{AB} = \underset{\sim}{a}$ and $\overrightarrow{BC} = \underset{\sim}{b}$.



$\overrightarrow{BE}$ is an extension of $\overrightarrow{AB}$, such that $\overline{AB} : \overline{BE} = 4 : 3$.
The point F is the midpoint of CD.

Which expression gives the vector $\overrightarrow{FE}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$?

(A) $\frac{5}{4}\underset{\sim}{a} - \underset{\sim}{b}$

(B) $\frac{5}{4}\underset{\sim}{a} + \underset{\sim}{b}$

(C) $\frac{11}{6}\underset{\sim}{a} - \underset{\sim}{b}$

(D) $\frac{11}{6}\underset{\sim}{a} + \underset{\sim}{b}$

QUESTION 6

A cubic polynomial $P(z)$ has all real coefficients.

$z = 3$ and $z = 2 + i$ are two roots of $P(z) = 0$.

Which equation describes $P(z)$?

(A) $P(z) = z^3 - 7z^2 - 17z + 15$

(B) $P(z) = z^3 - 7z^2 + 17z - 15$

(C) $P(z) = z^3 + 7z^2 - 35z + 15$

(D) $P(z) = z^3 + 7z^2 + 35z - 15$

QUESTION 7

Issy tries to find $\displaystyle\int_{-2}^{0} \frac{1}{x^2 - 1}\,dx$ but she can't seem to get a simple answer.

Which of the following could be the reason she is having trouble?

(A) Because the function is not differentiable at $x = 0$.

(B) Because there is no appropriate substitution to begin calculating the integral.

(C) Because the function has a horizontal asymptote in the domain $[-2,0]$.

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QUESTION 8

Ollie is trying to find $\int \sqrt{4x^2 - 9}\,dx$. Which substitution should he use?

(A) $x = \sec\theta$

(B) $x = \frac{3}{2}\sec\theta$

(C) $x = \frac{2}{3}\sec\theta$

(D) $x = \frac{3}{2}\sin\theta$

QUESTION 9

If a, b, c and d are consecutive integers, then which of the following statements may be false ?

(A) $a + b + c + d$ is divisible by 2

(B) $a + b + c + d$ is divisible by 4

(C) $a \times b \times c \times d$ is divisible by 3

(D) $a \times b \times c \times d$ is divisible by 8

QUESTION 10

A particle starts moving along a straight line from rest at the origin. At time t, its velocity is v and its displacement from the origin is x.

If $\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{2v}$, which of the following best describes the particle's acceleration and velocity?

(A) Constant acceleration and constant velocity

(B) Constant acceleration and decreasing velocity

(C) Constant acceleration and increasing velocity

(D) Increasing acceleration and increasing velocity

END OF SECTION I

Student Name:______________________________

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours 45 minutes for this section

Answer each question in the booklets provided. Extra writing booklets are available.

Marks

QUESTION 11 (14 marks) **Start a new writing booklet**

(a) (i) Write $1 - i\sqrt{3}$ in mod-arg form. **1**

(ii) Hence write $\frac{(1-i\sqrt{3})^4}{1+i\sqrt{3}}$ in exact Cartesian form. **3**

(b) Find a vector equation for the line that passes through $A(1, -1, 0)$ and $B(2, -1, 4)$. **2**

(c) Evaluate **3**

$$\int_0^{\frac{\pi}{2}} \frac{1}{1 + \cos x} \, dx.$$

(d) Prove that $\sqrt{2}$ is irrational by contradiction. **3**

(e) Let $\underset{\sim}{u} = 5\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{v} = -2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$.

(i) Find the dot product $\underset{\sim}{u} \cdot \underset{\sim}{v}$ **1**

(ii) Find the unit vector $\hat{v}$. **1**

Student Name:______________________________

QUESTION 12 (16 marks) **Start a new writing booklet**

(a) (i) Show that $(1-3i)^2 = -8-6i$. **1**

(ii) Hence, or otherwise, solve the equation $2z^2 - 8z + (12+3i) = 0$. **2**

(b) The points D, E and F have position vectors relative to the origin O given by **4**

$\overrightarrow{OD} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$, $\overrightarrow{OE} = \begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix}$, $\overrightarrow{OF} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}$.

Find $\angle DEF$ to the nearest minute.

(c) Sketch, on an Argand Diagram, the region that satisfies **2**

$$|z-1| \leq 3.$$

(d) A light string passes over a smooth pulley. **3**

Attached to one end is a 9 kg mass, and to the other end, a 7 kg mass.

Find the acceleration of the 9 kg mass in terms of g.

(e) (i) Express $\dfrac{x^3+3x^2-11x+12}{(x-2)^2(x^2+1)}$ in the form $\dfrac{a}{(x-2)^2} + \dfrac{b}{x-2} + \dfrac{cx+d}{x^2+1}$. **2**

(ii) Hence evaluate **2**

$$\int_0^1 \frac{x^3+3x^2-11x+12}{(x-2)^2(x^2+1)}\, dx.$$

Leave your answer in exact form.

Student Name:________________________________

QUESTION 13 (14 marks) **Start a new writing booklet**

(a) A particle travels in a straight line so that its velocity v cm/s and displacement x cm are related by the equation $v = -0.2x$.

(i) Determine the acceleration a in terms of its displacement x. **2**

(ii) Is the particle moving in simple harmonic motion? Justify your answer. **1**

(iii) It is known that the initial displacement of the particle is $x = 4$ cm. Determine, correct to 2 decimal places, when the particle has a displacement of 2 cm. **2**

(b) The area enclosed by the graph of $y = \sqrt{x}\sin x$, the x-axis and the lines $x = 0, x = \pi$ is rotated around the x-axis. **4**

Find the exact volume of the solid formed.

(c) Consider the polynomial $P(z) = 2z^5 - 13z^4 + 52z^3 - 91z^2 + 86z - 26$. **5**

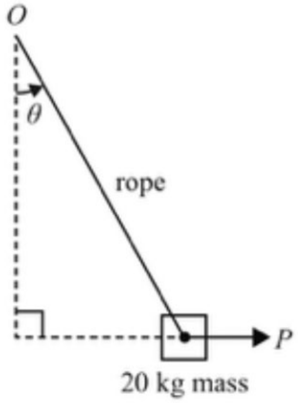
Given $z = 1 - i$ is a zero of $P(z)$, find all the roots of $P(z) = 0$.

QUESTION 14 (15 marks) **Start a new writing booklet**

(a) A taut (tightly pulled) rope of length $1\frac{2}{3}$ m suspends a mass of 20 kg from a fixed point O.

A horizontal force of P newtons displaces the mass by 1 metre horizontally so that the taut rope is then at an angle θ to the vertical.

(i) Copy the diagram below into your writing booklet and show clearly all the forces acting on the mass. **1**



(ii) Show that $\sin\theta = \frac{3}{5}$. **1**

(iii) Find the magnitude of the tension force in the rope in newtons (Leave your answer in terms of g). **3**

(b) Prove that, if $a^2 - 2a + 2025$ is even, then a is odd. **3**

(c) Use integration by parts to find an expression for **3**

$$\int e^x \sin x \ dx.$$

(d) Use the t-formulas to find an expression for **4**

$$\int \frac{1}{5 + 4\cos x + 3\sin x} \ dx.$$

Student Name:________________________________

QUESTION 15 (16 marks) **Start a new writing booklet**

(a) $OABC$ is a kite, with OB as its line of symmetry. $|OA| = |OC|$. $|AB| = |BC|$.

Let $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$, $\underset{\sim}{c} = \overrightarrow{OC}$.

(i) Prove that $\underset{\sim}{a}.\underset{\sim}{b} = \underset{\sim}{b}.\underset{\sim}{c}$. **1**

(ii) Hence or otherwise, prove that the diagonals of a kite are perpendicular. **3**

(b) Use the principle of mathematical induction to prove that **3**

$$(1+x)^n > 1 + nx \text{ for } n > 1 \text{ and } x > 0$$

(c) Kajan decides to go bungy-jumping. This involves being tied to a bridge at point O by an elastic cable of length L metres, and then falling vertically from rest from this point.

After Kajan free falls L metres, he is slowed down by the cable, which exerts a force (in newtons) of mgk times the distance greater than L that he has fallen (where m is his mass in kg and k is a constant).

Let x metres be the distance Kajan has fallen, and let v m/s be his speed at x.

(i) Show that $\ddot{x} = \begin{cases} g & x \le L \\ g - gk(x-L) & x > L \end{cases}$ **2**

(ii) Show that $v^2 = 2gL$ at the instant Kajan is at $x = L$. **1**

(iii) Show that $v^2 = 2gx - kg(x-L)^2$ for $x > L$. **1**

(iv) Show that Kajan stops for the first time at $x = L + \dfrac{1}{k} + \sqrt{\dfrac{2L}{k} + \left(\dfrac{1}{k}\right)^2}$. **3**

(v) Given that $\dfrac{1}{k} = \dfrac{L}{4}$, show that O must be at least $2L$ metres above any obstruction on Kajan's path. **2**

Student Name:________________________________

QUESTION 16 (15 marks) **Start a new writing booklet**

(a) Evaluate **4**

$$\int_1^2 \frac{x+1 \;\, dx}{\sqrt{3x-2-x^2}}.$$

(b) Find $|\underset{\sim}{u}+\underset{\sim}{v}|$ if $|\underset{\sim}{u}|=6$, $|\underset{\sim}{v}|=5$ and $\underset{\sim}{u}.\underset{\sim}{v} = -4$. **3**

(c) Prove that **2**

$$\frac{n!}{r!\,(n-r)!}+\frac{n!}{(r-1)!\,(n-r+1)!}=\frac{(n+1)!}{r!\,(n-r+1)!}.$$

(d) It is known that the arithmetic mean of three numbers is greater than the geometric mean of those numbers, i.e.

$$\frac{a+b+c}{3} \geq \sqrt[3]{abc}.$$

(i) Given $a+b+c=t$ and $a,b,c>0$, prove that **3**

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq \frac{9}{t}.$$

(ii) Hence, given $a,b,c>0$ and $a+b+c=1$, prove that **3**

$$\left(\frac{1}{a}-1\right)\left(\frac{1}{b}-1\right)\left(\frac{1}{c}-1\right) \geq 8.$$

END OF PAPER

Student Name:______________________________

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2025

Year 12 Mathematics Extension 2

Trial HSC Examination

MULTIPLE-CHOICE ANSWER SHEET

For multiple choice questions, choose the best answer A, B, C or D and fill in the correct circle.

1. A B C D
2. A B C D
3. A B C D
4. A B C D
5. A B C D
6. A B C D
7. A B C D
8. A B C D
9. A B C D
10. A B C D

Student Name: ______________________________

KINROSS WOLAROI SCHOOL

SOLUTIONS

2025

Year 12 Mathematics Extension 2

Trial HSC Examination

Teacher Setting Paper: Mr Bowman
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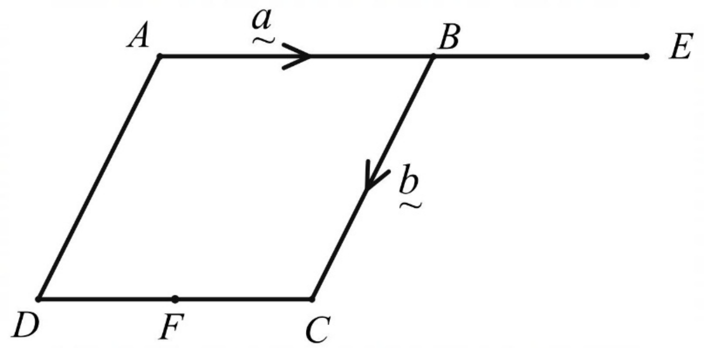

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If a, b, c and d are consecutive integers, then which of the following statements may be false ?

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QUESTION 10

A particle starts moving along a straight line from rest at the origin. At time t, its velocity is v and its displacement from the origin is x.

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END OF SECTION I

Student Name:______________________________

Section II

90 marks
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Marks

QUESTION 11 (15 marks) **Start a new writing booklet**

(a) (i) Write $1 - i\sqrt{3}$ in mod-arg form. **1**

$$\begin{aligned}1 - i\sqrt{3} &= 2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\\ &= 2(\cos\left(-\frac{\pi}{3}\right) + isin\left(-\frac{\pi}{3}\right))\\ &= 2cis\left(-\frac{\pi}{3}\right)\end{aligned}$$

(ii) Hence write $\frac{(1-i\sqrt{3})^4}{1+i\sqrt{3}}$ in exact Cartesian form. **3**

$$\begin{aligned}\frac{(1-i\sqrt{3})^4}{1+i\sqrt{3}} &= \frac{\left(2cis\left(-\frac{\pi}{3}\right)\right)^4}{2cis\frac{\pi}{3}}\\ &= \frac{16cis\left(-\frac{4\pi}{3}\right)}{2cis\left(\frac{\pi}{3}\right)}\\ &= \frac{16cis\left(\frac{2\pi}{3}\right)}{2cis\left(\frac{\pi}{3}\right)}\\ &= 8cis\left(\frac{\pi}{3}\right)\\ &= 8\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\\ &= 4 + 4i\sqrt{3}\end{aligned}$$

(b) Find the vector equation of a line that passes through $A(1, -1, 0)$ and $B(2, -1, 4)$. **2**

$$r = \begin{pmatrix}1\\-1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\0\\4\end{pmatrix}$$

(c) Evaluate **3**

$$\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos x}\,dx.$$

$$= \int_0^1 \frac{\frac{2dt}{1+t^2}}{1+\frac{1-t^2}{1+t^2}}$$

$$= \int_0^1 \frac{2dt}{1+t^2+1-t^2}$$

$$= \int_0^1 1\,dt$$

$$= [t]_0^1$$

$$= 1$$

(d) Prove that $\sqrt{2}$ is irrational by contradiction. **3**

Assume rational, so

$$\sqrt{2} = \frac{p}{q} \ (p, q \text{ in lowest terms})$$

$$2 = p^2/q^2$$

$$p^2 = 2q^2$$

$$\therefore\ p^2 \text{is even}$$

$$\therefore\ \boldsymbol{p} \textbf{ is even}$$

$$p = 2r$$

$$p^2 = 4r^2$$

$$2q^2 = 4r^2$$

$$q^2 = 2r^2$$

$$\therefore\ q^2 \text{is even}$$

$$\therefore\ \boldsymbol{q} \textbf{ is even}$$

$$\text{Both p, q are even}$$

$$\therefore \frac{p}{q} \text{ not in lowest terms.}$$

$$\therefore \sqrt{2} \text{ is irrational}$$

(e) Let $\boldsymbol{u} = 5\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{v} = -2\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$.

(i) Find the dot product $\boldsymbol{u}.\boldsymbol{v}$ **1**

$$u.v = 5(-2) + 1(-2) + 3(1)$$
$$= -9$$

(ii) Find the unit vector $\hat{\boldsymbol{v}}$. **1**

$$|v| = \sqrt{9} = 3$$

$$\therefore \hat{v} = -\frac{2}{3}i - \frac{2}{3}j + \frac{1}{3}k$$

QUESTION 12 (16 marks) **Start a new writing booklet**

(a) (i) Show that $(1-3i)^2 = -8-6i$. **1**

$$\begin{aligned}(1-3i)^2 &= 1-6i-9\\ &= -8-6i \text{ as required}\end{aligned}$$

(ii) Hence or otherwise solve the equation $2z^2 - 8z + (12+3i) = 0$. **2**

$$\begin{aligned}z &= \frac{8 \pm \sqrt{64-8(12+3i)}}{4}\\ &= \frac{8 \pm \sqrt{-32-24i}}{4}\\ &= \frac{8 \pm 2\sqrt{-8-6i}}{4}\\ &= \frac{8 \pm 2(1-3i)}{4} \text{ using part (i)}\\ &= \frac{8+2-6i}{4} \ or \ \frac{8-2+6i}{4}\\ &= \frac{5}{2} - \frac{3}{2}i \ or \ \frac{3}{2} + \frac{3}{2}i\end{aligned}$$

(b) The points D, E and F have position vectors relative to the origin O given by **4**

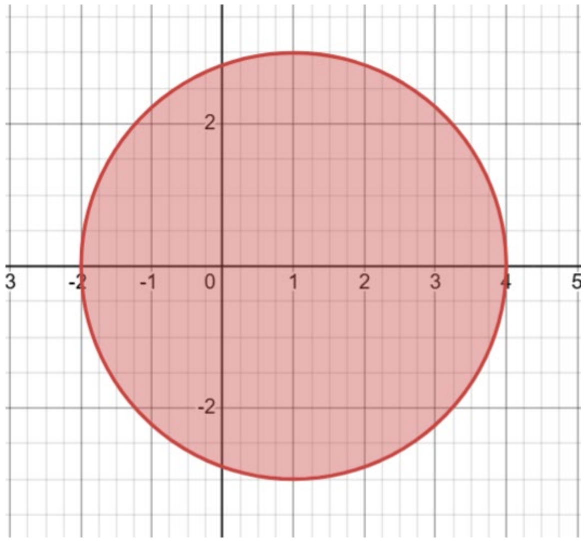
$$\overrightarrow{OD} = \begin{pmatrix}4\\2\\-3\end{pmatrix},\ \overrightarrow{OE} = \begin{pmatrix}6\\3\\-1\end{pmatrix},\ \overrightarrow{OF} = \begin{pmatrix}5\\1\\0\end{pmatrix}.$$

Find $\angle DEF$ to the nearest minute.

$$\begin{aligned}\overrightarrow{ED} &= \begin{pmatrix}2\\1\\2\end{pmatrix}\\ \overrightarrow{EF} &= \begin{pmatrix}-1\\-2\\1\end{pmatrix}\\ |ED| &= 3\\ |EF| &= \sqrt{6}\\ cos\theta &= \frac{ED.EF}{|ED||EF|}\\ &= -\frac{2}{3\sqrt{6}}\\ \theta &= 105°48'\end{aligned}$$

(c) Sketch, on an Argand Diagram, the region that satisfies **2**

$$|z-1| \le 3.$$



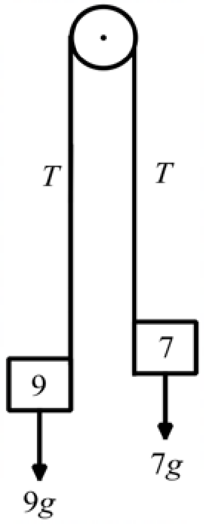
(d) A light string passes over a smooth pulley. **3**

Attached to one end is a 9 kg mass, and to the other end, a 7kg mass.

Find the acceleration of the 9 kg mass in terms of g.

9 kg mass: $9\ddot{x} = 9g - T \ldots\ldots(i)$

7 kg mass: $7\ddot{x} = T - 7g \ldots\ldots(ii)$



$$(i) + (ii) \rightarrow 16\ddot{x} = 2g$$

$$\ddot{x} = \frac{g}{8} \ (accepting\ -\frac{g}{8})$$

(e) Evaluate **4**

$$\int_0^1 \frac{x^3+3x^2-11x+12}{(x-2)^2(x^2+1)}\,dx.$$

leaving your answer in exact form.

$$Let\ \frac{x^3+3x^2-11x+12}{(x-2)^2(x^2+1)} = \frac{a}{(x-2)^2} + \frac{b}{x-2} + \frac{cx+d}{x^2+1}$$
$$x^3+3x^2-11x+12 = a(x^2+1) + b(x-2)(x^2+1) + (cx+d)(x-2)^2$$

$$Let\ x = 2 \to 10 = 5a \to \boldsymbol{a = 2}$$

Equate coefficients of x^3

$$1 = b + c \ldots..(i)$$

Equate coefficients of x^2

$$3 = a - 2b + d - 4c$$
$$3 = 2 - 2b + d - 4c$$
$$d - 2b - 4c = 1 \ldots.(ii)$$

Equate constants

$$12 = a - 2b + 4d$$
$$4d - 2b = 10 \ldots..(iii)$$

From (i),

$$c = 1 - b$$

(ii):

$$d - 2b - 4(1-b) = 1$$
$$2b + d = 5 \ldots(iv)$$

(iii):

$$2d - b = 5$$
$$4d - 2b = 10 \ldots(v)$$

(iv)+(v):

$$\boldsymbol{d = 3}$$
$$\boldsymbol{b = 1}$$
$$\boldsymbol{c = 0}$$

$$\therefore \frac{x^3+3x^2-11x+12}{(x-2)^2(x^2+1)} = \frac{2}{(x-2)^2} + \frac{1}{x-2} + \frac{3}{x^2+1}$$

$$\int_0^1 \frac{x^3+3x^2-11x+12}{(x-2)^2(x^2+1)}\,dx = \int_0^1 \frac{2}{(x-2)^2} + \frac{1}{x-2} + \frac{3}{x^2+1}\,dx$$
$$= \left[\frac{-2}{x-2} + \ln|x-2| + 3\tan^{-1}x\right]_0^1$$
$$= \left(\frac{-2}{-1} + 0 + \frac{3\pi}{4}\right) - \left(\frac{-2}{-2} + ln2 + 0\right)$$
$$= 1 + \frac{3\pi}{4} - ln2$$

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QUESTION 13 (14 marks) **Start a new writing booklet**

(a) A particle travels in a straight line so that its velocity v cm/s and displacement x cm are related by the equation $v = -0.2x$.

(i) Determine the acceleration a in terms of its displacement x. **2**

$$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$
$$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}(-0.2x)^2\right) = \frac{d}{dx}(0.04x^2)$$

$$\ddot{x} = 0.04x$$

(ii) Is the particle moving in simple harmonic motion? Justify your answer. **1**

NO because equation is not of format $\ddot{x} = -n^2x$

(iii) It is known that the initial displacement of the particle is $x = 4$ cm. Determine, correct to 2 decimal places, when the particle has a displacement of 2 cm. **2**

$$\frac{dx}{dt} = -0.2 = -\frac{x}{5}$$
$$\frac{dt}{dx} = -\frac{5}{x}$$
$$t = -5lnx + C$$
$$0 = -5ln4 + C$$
$$C = 5ln4$$
$$t = 5\ln\left(\frac{4}{x}\right)$$
When $x = 2$,
$$t = 5ln2 = 3.47$$

Particle has displacement of 2 cm after 3.47 sec.

(b) The area enclosed by the graph of $y = \sqrt{x}\sin x$, the x-axis and the lines $x = 0, x = \pi$ is rotated around the x-axis. **4**

Find the exact volume of the solid formed.

$$V = \pi\int_0^\pi y^2 dx$$
$$= \pi\int_0^\pi x\sin^2 x\, dx$$

$$\text{By parts: } u = x, v' = \sin^2 x$$
$$u' = 1, v = \frac{x}{2} - \frac{\sin 2x}{4}$$

$$V = \pi\left[\frac{x^2}{2} - \frac{x\sin 2x}{4}\right]_0^\pi - \pi\int_0^\pi \frac{x}{2} - \frac{1}{4}\sin 2x\, dx$$
$$= \pi\left[\frac{x^2}{2} - \frac{x\sin 2x}{4}\right]_0^\pi - \pi\left[\frac{x^2}{4} + \frac{1}{8}\cos 2x\right]_0^\pi$$
$$= \pi\left(\frac{\pi^2}{2} - 0 - 0 + 0\right) - \pi\left(\frac{\pi^2}{4} + \frac{1}{8} - 0 - \frac{1}{8}\right)$$
$$= \frac{\pi^3}{2} - \frac{\pi^3}{4}$$
$$= \frac{\pi^3}{4}\ cu\ units.$$

(c) Consider the polynomial $P(z) = 2z^5 - 13z^4 + 52z^3 - 91z^2 + 86z - 26$. **5**

Given $z = 1 - i$ is a zero of $P(z)$, find all the roots of $P(z) = 0$.

$$z = 1 - i \text{ is a zero, so } z = 1 + i \text{ is a zero as well.}$$

$$\text{Hence the quadratic } z^2 - 2z + 2 \text{ is a factor of } P(z).$$

Let $2z^5 - 13z^4 + 52z^3 - 91z^2 + 86z - 26 = (z^2 - 2z + 2)(2z^3 + az^2 + bz - 13)$

Equating coefficients of z^4:

$$a - 4 = -13 \rightarrow \boldsymbol{a = -9}$$

Equating coefficients of z^3:

$$b - 2a + 4 = 52$$
$$b + 22 = 52 \rightarrow \boldsymbol{b = 30}$$

$$\therefore 2z^5 - 13z^4 + 52z^3 - 91z^2 + 86z - 26 = (z^2 - 2z + 2)(2z^3 - 9z^2 + 30z - 13)$$

Now for factors of $2z^3 - 9z^2 + 30z - 13$:

$$P\left(\frac{1}{2}\right) = 0, \text{hence } 2z - 1 \text{ is a factor of } 2z^3 - 9z^2 + 30z - 13$$

Let $2z^3 - 9z^2 + 30z - 13 = (2z - 1)(z^2 + cz + 13)$

Equating coefficients of z^2:

$$2c - 1 = -9 \rightarrow c = -4$$

$$\therefore 2z^3 - 9z^2 + 30z - 13 = (2z - 1)(z^2 - 4z + 13)$$

The roots of $z^2 - 4z + 13$ *are* $z = \frac{4 \pm \sqrt{-36}}{2} = z = 2 \pm 3i$

So the 5 roots of $P(z) = 0$ are:

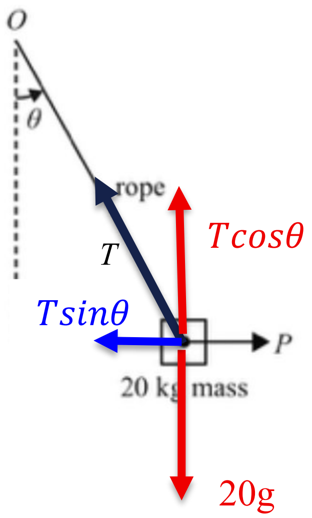
$$\boldsymbol{z = 1 - i, \quad 1 + i, \quad \frac{1}{2}, \quad 2 + 3i, \quad 2 - 3i}$$

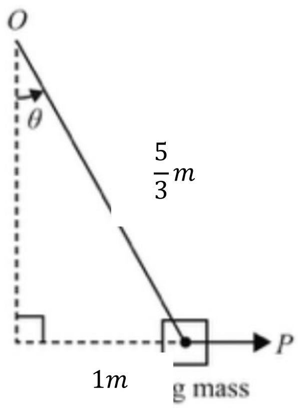
QUESTION 14 (15 marks) **Start a new writing booklet**

(a) A taut (tightly pulled) rope of length $1\frac{2}{3}$ m suspends a mass of 20 kg from a fixed point O.

A horizontal force of P newtons displaces the mass by 1 metre horizontally so that the taut rope is then at an angle θ to the vertical.

(i) Copy the diagram below into your writing booklet and show clearly all the forces acting on the mass. **1**





(ii) Show that $\sin\theta = \frac{3}{5}$. **1**

From diagram at right,

$$\sin\theta = \frac{1}{\left(\frac{5}{3}\right)} = \frac{3}{5}$$

(iii) Find the magnitude of the tension force in the rope in newtons (Leave your answer in terms of g). **3**

$$Horizontal\ forces:\ P = T\sin\theta$$
$$Vertical\ forces:\ 20g = T\cos\theta$$
$$\tan\theta = \frac{P}{20g}$$
$$\frac{3}{4} = \frac{P}{20g}$$
$$P = 15g$$
$$15g = T\sin\theta$$
$$15g = \frac{3T}{5}$$
$$T = 25g\ newtons$$

(b) Prove that, if $a^2 - 2a + 2025$ is even, then a is odd. **3**

By contrapositive, or this is quicker:

If $a^2 - 2a + 2025$ is even, then
$a^2 - 2a$ is odd.
$a(a-2)$ is odd.
$\therefore$ both a and $a-2$ are odd.
a is odd.

(c) Use integration by parts to find an expression for **3**

$$\int e^x \sin x \; dx.$$

$$\boxed{\begin{array}{c} u = e^x, v' = \sin x \\ u' = e^x, v = -\cos x \end{array}}$$

$$I = uv - \int u'v$$

$$= -e^x \cos x - \int -e^x \cos x \, dx$$

$$= -e^x \cos x + \int e^x \cos x \, dx$$

$$\boxed{\begin{array}{c} u = e^x, v' = \cos x \\ u' = e^x, v = \sin x \end{array}}$$

$$= -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$$

$$2I = e^x(\sin x - \cos x)$$

$$I = \frac{e^x}{2}(\sin x - \cos x) + C$$

(d) Use the t-formulas to find an expression for **4**

$$\int \frac{1}{5 + 4\cos x + 3\sin x} \; dx.$$

$$\boxed{\begin{array}{c} dx = \dfrac{2dt}{1+t^2} \\ \cos x = \dfrac{1-t^2}{1+t^2} \\ \sin x = \dfrac{2t}{1+t^2} \end{array}}$$

$$= \int \frac{\frac{2dt}{1+t^2}}{5 + \frac{4-4t^2}{1+t^2} + \frac{6t}{1+t^2}} \times \frac{1+t^2}{1+t^2}$$

$$= \int \frac{2dt}{5 + 5t^2 + 4 - 4t^2 + 6t}$$

$$= \int \frac{2dt}{t^2 + 6 + 9}$$

$$= \int \frac{2dt}{(t+3)^2}$$

$$= -\frac{2}{t+3} + C$$

$$= -\frac{2}{\tan\left(\frac{x}{2}\right) + 3} + C$$

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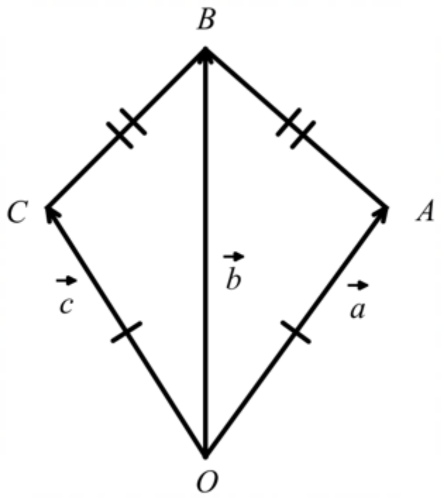
Type equation here.

QUESTION 15 (16 marks) **Start a new writing booklet**

(a) $OABC$ is a kite, with OB as its line of symmetry. $|OA| = |OC|$. $|AB| = |BC|$.

Let $\underset{\sim}{a} = \overrightarrow{OA},\ \underset{\sim}{b} = \overrightarrow{OB},\ \underset{\sim}{c} = \overrightarrow{OC}$.

(i) Prove that $\underset{\sim}{a}.\underset{\sim}{b} = \underset{\sim}{b}.\underset{\sim}{c}$. **1**



As the kite has line of symmetry OB, let $\angle AOB = \angle COB = \theta$.

Also, $|\underset{\sim}{a}| = |\underset{\sim}{c}|$ (properties of a kite).

From $\Delta AOB,\ cos\theta = \frac{\underset{\sim}{a}.\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}$

From $\Delta AOC,\ cos\theta = \frac{\underset{\sim}{c}.\underset{\sim}{b}}{|\underset{\sim}{c}||\underset{\sim}{b}|} = \frac{\underset{\sim}{c}.\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}$

As these are equal, $\underset{\sim}{a}.\underset{\sim}{b} = \underset{\sim}{b}.\underset{\sim}{c}$

(ii) Hence or otherwise, prove that the diagonals of a kite are perpendicular. **3**

From $\Delta OCA,\ \overrightarrow{CA} = \underset{\sim}{a} - \underset{\sim}{c}$.

Dot product of diagonals:

$$\begin{aligned} &\overrightarrow{CA}.\overrightarrow{OB} \\ &= \left(\underset{\sim}{a} - \underset{\sim}{c}\right).\underset{\sim}{b} \\ &= \underset{\sim}{a}.\underset{\sim}{b} - \underset{\sim}{b}.\underset{\sim}{c} \\ &= \underset{\sim}{a}.\underset{\sim}{b} - \underset{\sim}{a}.\underset{\sim}{b}\ \left(from\ part\ (i)\right) \\ &= 0 \end{aligned}$$

Therefore, the diagonals are perpendicular.

(b) Use the principle of mathematical induction to prove that **3**

$$(1+x)^n > 1+nx \text{ for } n>1 \text{ and } x>0$$

$$Let\ n=2:$$
$$LHS=(1+x)^2=1+2x+x^2$$
$$RHS=1+2x$$
$$LHS>RHS \therefore true\ for\ n=2$$

$$Assume\ true\ for\ n=k, i.e.$$

$$(1+x)^k > 1+kx \ldots\ldots..(i)$$

$$To\ prove\ true\ for\ n=k+1\ \ i.e.$$

$$(1+x)^{k+1} > 1+(k+1)x \ldots.(ii)$$

$$LHS\ of\ (ii)$$

$$\begin{aligned}
&=(1+x)^{k+1}\\
&=(1+x)(1+x)^k\\
&>(1+x)(1+kx)\ \big(using\ (i)\big)\\
&=1+kx+x+kx^2\\
&=1+(k+1)x+kx^2\\
&>1+(k+1)x
\end{aligned}$$

Therefore true for $n=k+1$, if true for $n=k$.

True for $n=2$, hence by principle of mathematical induction, true for $n=3,4,\ldots\forall n$.

(c) Kajan decides to go bungy-jumping. This involves being tied to a bridge at point O by an elastic cable of length L metres, and then falling vertically from rest from this point.

After Kajan free falls L metres, he is slowed down by the cable, which exerts a force (in newtons) of mgk times the distance greater than L that he has fallen (where m is his mass in kg and k is a constant).

Let x metres be the distance Kajan has fallen, and let v m/s be his speed at x.

(i) Show that $\ddot{x} = \begin{cases} g & x \le L \\ g - gk(x-L) & x > L \end{cases}$ **2**

When $x \le L$, he is free falling, i.e.

$$m\ddot{x} = mg$$
$$\ddot{x} = g$$

When $x \ge L$, after free falling,

$$m\ddot{x} = mg - mgk(x-L)$$
$$\ddot{x} = g - gk(x-L)$$

(ii) Show that $v^2 = 2gL$ at the instant Kajan is at $x = L$. **1**

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = g$$
$$\frac{1}{2}v^2 = gx + C$$
$$at\ x = 0, v = 0\ \rightarrow C = 0$$
$$v^2 = 2gx$$
$$at\ x = L, v^2 = 2gL$$

(iii) Show that $v^2 = 2gx - kg(x-L)^2$ for $x > L$. **1**

$$\ddot{x} = g - gk(x - L$$
$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = g - gk(x-L)$$
$$\frac{1}{2}v^2 = gx - \frac{gk(x-L)^2}{2} + C$$
$$at\ x = L, v = \sqrt{2gL}\ \rightarrow C = 0$$
$$v^2 = 2gx - gk(x-L)^2$$

(iv) Show that Kajan stops for the first time at $x = L + \frac{1}{k} + \sqrt{\frac{2L}{k} + \left(\frac{1}{k}\right)^2}$. **3**

we want x when $v = 0$

$$2gx - gk(x-L)^2 = 0$$
$$2gx - gk(x^2 - 2xL + L^2) = 0$$
$$2gx - gkx^2 + 2gkLx - gkL^2 = 0$$
$$kx^2 - 2(1+kL)x + kL^2 = 0$$
$$x^2 - \frac{2(1+kL)}{k} + L^2 = 0$$
$$x^2 - \frac{2(1+kL)}{k} + \frac{(1+kL)^2}{k^2} = \frac{(1+kL)^2}{k^2} - L^2$$
$$\left(x - \frac{(1+kL)}{k}\right)^2 = \frac{(1+kL)^2 - k^2L^2}{k^2}$$
$$\left(x - \frac{(1+kL)}{k}\right)^2 = \frac{1+2kL}{k^2}$$
$$x - \frac{(1+kL)}{k} = \frac{\sqrt{1+2kL}}{k}$$
$$x = \frac{1+kL}{k} + \frac{\sqrt{1+2kL}}{k}$$
$$x = \frac{1}{k} + L + \sqrt{\frac{1+2kl}{k^2}}$$
$$x = \frac{1}{k} + L + \sqrt{\frac{2L}{k} + \frac{1}{k^2}}$$

(v) Given that $\frac{1}{k} = \frac{L}{4}$, show that O must be at least $2L$ metres above any obstruction on Kajan's path. **2**

$$\frac{1}{k} = \frac{L}{4}$$
$$\therefore x = \frac{L}{4} + L + \sqrt{\frac{L}{4}(2L) + \frac{L^2}{16}}$$
$$x = \frac{L}{4} + L + \sqrt{\frac{L^2}{2} + \frac{L^2}{16}}$$
$$x = \frac{L}{4} + L + \sqrt{\frac{9L^2}{16}}$$
$$x = \frac{L}{4} + L + \frac{3L}{4}$$
$$x = 2L$$

QUESTION 16 (15 marks) **Start a new writing booklet**

(a) Evaluate **4**

$$\int_1^2 \frac{x+1\ dx}{\sqrt{3x-2-x^2}}.$$

$$= -\frac{1}{2}\int_1^2 \frac{-2x+3\ dx}{\sqrt{3x-2-x^2}} + \frac{5}{2}\int_1^2 \frac{1}{\sqrt{3x-2-x^2}}\,dx$$

$$= -\frac{1}{2}\int_1^2 \frac{-2x+3\ dx}{\sqrt{3x-2-x^2}} + \frac{5}{2}\int_1^2 \frac{1}{\sqrt{\frac{1}{4}-\left(x^2-\frac{3}{2}\right)^2}}\,dx$$

$$= \left[-\frac{1}{2}\times 2\sqrt{3x-2-x^2}\right]_1^2 + \left[\frac{5}{2}\sin^{-1}(2x-3)\right]_1^2$$

$$= 0 + \frac{5}{2}\left(\sin^{-1}1 - \sin^{-1}(-1)\right)$$

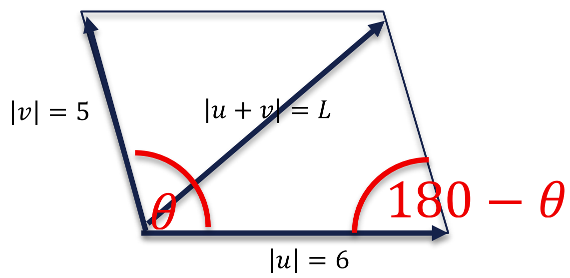
$$= \frac{5}{2}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)$$

$$= \frac{5\pi}{2}$$

(b) Find $|\underset{\sim}{u} + \underset{\sim}{v}|$ if $|\underset{\sim}{u}| = 6,\ y\ |\underset{\sim}{v}| = 5$ and $\underset{\sim}{u}.\underset{\sim}{v} = -4$. **3**

$$(u+v).(u+v) = |u+v|^2 = |u|^2 + 2u.v + |v|^2$$
$$|u+v|^2 = |u|^2 + 2u.v + |v|^2$$
$$|u+v|^2 = 36 - 8 + 25$$
$$|u+v|^2 = 53$$
$$|u+v| = \sqrt{53}$$

Alternatively, this is can be modelled by a parallelogram with adjacent sides $|u| = 6, |v| = 5$, and long diagonal.
$|u+v|$.



Let the included angle θ be given by

$$cos\theta = \frac{u.v}{|u||v|}$$
$$= -\frac{4}{30}$$

Hence the other angle is $180 - \theta$ and its cosine is $+\frac{2}{15}$

By the cosine rule,

$$L^2 = |u|^2 + |v|^2 - 2|u||v|\cos(180-\theta)$$
$$= 36 + 25 - 2 \times 6 \times 5 \times \frac{4}{30}$$
$$= 53$$
$$|u+v| = \sqrt{53}$$

(c) Prove that **2**

$$\frac{n!}{r!\,(n-r)!}+\frac{n!}{(r-1)!\,(n-r+1)!}=\frac{(n+1)!}{r!\,(n-r+1)!}.$$

$$LHS\ has\ an\ LCD\ of\ r!\,(n-r+1)!$$

$$\begin{aligned}LHS &= \frac{n!}{r!\,(n-r)!}+\frac{n!}{(r-1)!\,(n-r+1)!}\\ &= \frac{n!\,(n-r+1)+n!\,r}{r!\,(n-r+1)!}\\ &= \frac{n!\,(n+1)}{r!\,(n-r+1)!}\\ &= \frac{(n+1)!}{r!\,(n-r+1)!}\\ &= RHS\ \text{ as required}\end{aligned}$$

(d) It is known that the arithmetic mean of three numbers is greater than the geometric mean of those numbers, i.e.

$$\frac{a+b+c}{3} \geq \sqrt[3]{abc}.$$

(i) Given $a+b+c=t$ and $a,b,c>0$, prove that **3**

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \geq \frac{9}{t}.$$

$\frac{a+b+c}{3} \geq (abc)^{\frac{1}{3}}$ (by AM ≥ GM)

$a+b+c \geq 3(abc)^{\frac{1}{3}}$

$t \geq 3(abc)^{\frac{1}{3}}$

$\frac{t}{3} \geq (abc)^{\frac{1}{3}}$

$\frac{3}{t} \leq \frac{1}{(abc)^{\frac{1}{3}}}$

$\frac{1}{(abc)^{\frac{1}{3}}} \geq \frac{3}{t}$

Now,

$\frac{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)}{3} \geq \left\{\frac{1}{a}\cdot\frac{1}{b}\cdot\frac{1}{c}\right\}^{\frac{1}{3}}$ (by AM ≥ GM)

$$\begin{aligned}\frac{1}{a}+\frac{1}{b}+\frac{1}{c} &\geq 3\left\{\frac{1}{a}\cdot\frac{1}{b}\cdot\frac{1}{c}\right\}^{\frac{1}{3}}\\ &\geq 3\left\{\frac{1}{abc}\right\}^{\frac{1}{3}}\\ &\geq 3\frac{1}{(abc)^{\frac{1}{3}}}\\ &\geq 3\cdot\frac{3}{t}\\ &\geq \frac{9}{t}\end{aligned}$$

(ii) Hence, given $a, b, c > 0$ and $a + b + c = 1$, prove that **3**

$$\left(\frac{1}{a} - 1\right)\left(\frac{1}{b} - 1\right)\left(\frac{1}{c} - 1\right) \geq 8.$$

$$\left\{\frac{1}{a} - 1\right\} \cdot \left\{\frac{1}{b} - 1\right\} \cdot \left\{\frac{1}{c} - 1\right\} = \frac{(1 - a)(1 - b)(1 - c)}{abc}$$

$$= \frac{1 - (a + b + c) + (ab + ac + bc) - (abc)}{abc}$$

Since $a + b + c = 1$

$$= \frac{ab + ac + bc - abc}{abc}$$

$$= \frac{1}{a} + \frac{1}{b} + \frac{1}{c} - 1$$

But $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 9$ from (i)

$\therefore \left\{\frac{1}{a} - 1\right\} \cdot \left\{\frac{1}{b} - 1\right\} \cdot \left\{\frac{1}{c} - 1\right\} \geq 8.$

END OF PAPER

Student Name:______________________________

2025

Year 12 Mathematics Extension 2

Trial HSC Examination

MULTIPLE-CHOICE ANSWER SHEET

For multiple choice questions, choose the best answer A, B, C or D and fill in the correct circle.

1. Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓐ Ⓑ Ⓒ Ⓓ
3. Ⓐ Ⓑ Ⓒ Ⓓ
4. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓐ Ⓑ Ⓒ Ⓓ
6. Ⓐ Ⓑ Ⓒ Ⓓ
7. Ⓐ Ⓑ Ⓒ Ⓓ
8. Ⓐ Ⓑ Ⓒ Ⓓ
9. Ⓐ Ⓑ Ⓒ Ⓓ
10. Ⓐ Ⓑ Ⓒ Ⓓ