Merewether High School

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2025 **Trial HSC Examination**

Year 12 Mathematics Extension 2

General Instructions

- Reading time – 10 minutes.
- Working time – 3 hours.
- Write using black pen.
- NESA approved calculators may be used.
- A reference sheet is provided.
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations.
- Submit ALL work at the end of the examination.
- Diagrams are NOT drawn to scale.

Total marks: 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10.
- Allow about 15 minutes for this section.

Section II – 90 marks (pages 7 – 15)

- Attempt Questions 11 – 16.
- Allow about 2 hour and 45 minutes for this section.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1 Consider the statement

‘for any integers m and n, if $m+n \geq 9$ then $m \geq 5$ or $n \geq 5$’.

The contrapositive of this statement is:

A. If $m<5$ or $n<5$, then $m+n<9$

B. If $m \geq 5$ or $n \geq 5$, then $m+n \geq 9$

C. If $m<5$ and $n<5$, then $m+n<9$

D. If $m \leq 5$ and $n \leq 5$, then $m+n \leq 9$

2 A particle starts from rest, 2 metres to the right of the origin, and moves along the x – axis in simple harmonic motion with a period of 2 seconds.

Which equation could represent the motion of the particle?

A. $x=2\cos \pi t$

B. $x=2\cos 2t$

C. $x=2+2\sin \pi t$

D. $x=2+2\sin 2t$

3 If $z=-(2a+1)+2ai$, where a is a non-zero constant, then $\frac{4a}{1+\bar{z}}$ is equal to:

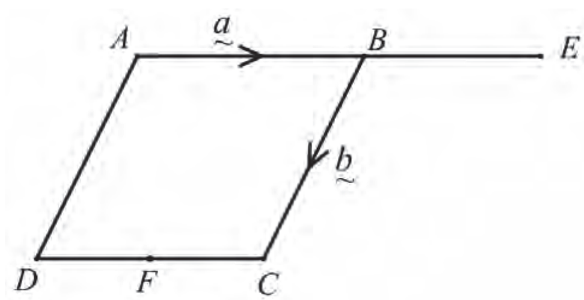
A. $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

B. $\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$

C. $\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right)$

D. $\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)$

4 *ABCD* is a rhombus. $\overrightarrow{AB}=\underset{\sim}{a}$ and $\overrightarrow{BC}=\underset{\sim}{b}$.



$\overrightarrow{BE}$ is an extension of $\overrightarrow{AB}$, such that $\overrightarrow{AB}:\overrightarrow{BE}=4:3$.

The point F is the midpoint of CD.

Which expression gives the vector $\overrightarrow{FE}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$?

A. $\frac{5}{4}\underset{\sim}{a}-\underset{\sim}{b}$

B. $\frac{5}{4}\underset{\sim}{a}+\underset{\sim}{b}$

C. $\frac{11}{6}\underset{\sim}{a}-\underset{\sim}{b}$

D. $\frac{11}{6}\underset{\sim}{a}+\underset{\sim}{b}$

5 Using the substitution $u = 1 + e^x$, $\int_0^{\log_e 2} \frac{1}{1+e^x}\,dx$ can be expressed as:

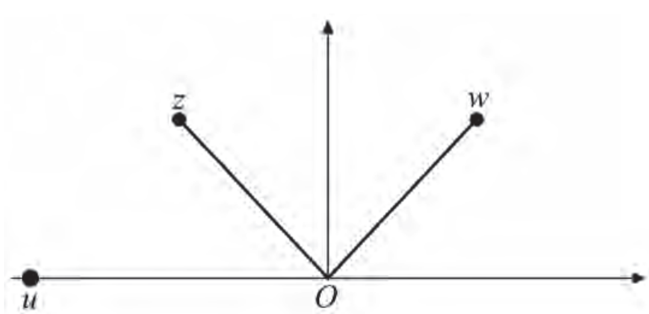
A. $\int_0^{\log_e 2} \left(\frac{1}{u-1} - \frac{1}{u}\right) du$

B. $\int_2^3 \left(\frac{1}{u} - \frac{1}{u-1}\right) du$

C. $\int_1^3 \left(\frac{1}{u} - \frac{1}{u-1}\right) du$

D. $\int_2^3 \left(\frac{1}{u-1} - \frac{1}{u}\right) du$

6 The Argand diagram shows the complex numbers w, z and u, where w lies in the first quadrant, z lies in the second quadrant and u lies on the negative real axis.



Which statement could be true?

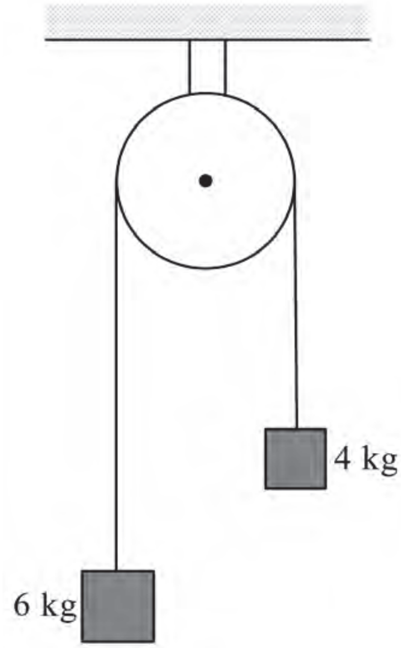
A. $u = zw$ and $u = z + w$

B. $u = zw$ and $u = z - w$

C. $z = uw$ and $u = z + w$

D. $z = uw$ and $u = z - w$

7 A light string passes over a smooth pulley. Attached to the ends of the string are masses of 6 kg and 4 kg, as shown.



The acceleration due to gravity is $g\ \mathrm{ms}^{-2}$.
What is the acceleration of the 6 kg mass?

A. $\frac{1}{5}g$

B. $\frac{1}{2}g$

C. $\frac{2}{5}g$

D. g

8 The complex number z satisfies $|z-1|=1$.
What is the greatest distance that z can be from the point i on the Argand diagram?

A. 1

B. $\sqrt{5}$

C. $2\sqrt{2}$

D. $\sqrt{2}+1$

9 Consider the line segments given by the vector equations below.

$$\underset{\sim}{r} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \quad -2 \le \lambda \le 2;$$

$$\underset{\sim}{l} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 6 \end{pmatrix}, \quad 0 \le \mu \le 2;$$

$$\underset{\sim}{v} = \gamma \begin{pmatrix} 9 \\ -6 \end{pmatrix}, \quad 1 \le \gamma \le 3$$

Which of the following statements about the line segments is NOT true?

A. At least two of them are parallel

B. At least two of them are perpendicular

C. At least two of them have the same intercept

D. At least two of them have the same length

10 Without evaluating the integrals, which one of the following integrals is greater than zero?

A. $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x}{2+\cos x}\,dx$

B. $\displaystyle\int_{-\pi}^{\pi} x^3 \sin x dx$

C. $\displaystyle\int_{-1}^{1} \left(e^{-x^2} - 1\right) dx$

D. $\displaystyle\int_{-2}^{2} \tan^{-1}\left(x^3\right) dx$

End of Section I

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations

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Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) Points A, B and C in three-dimensional space have position vectors:

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \ \overrightarrow{OB} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} \text{ and } \overrightarrow{OC} = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}.$$

(i) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **1**

(ii) Hence, find the acute angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$, correct to the nearest degree. **2**

(b) Find $\displaystyle\int \frac{1}{x^2+2x+3}\,dx$. **2**

(c) Consider the complex number $z = (b-i)^3$, where $b \in \mathbb{R}^+$.

Find b given that $\arg(z) = -\dfrac{\pi}{2}$. **3**

(d) Sketch the region in the Argand diagram defined by **2**

$6 \le \mathrm{Re}\left[(2-3i)z\right] < 12$ and $Re(z)Im(z) \ge 0$.

Question 11 continues page 8

Question 11 (continued)

(e) (i) Find A, B and C such that: **3**

$$\frac{x}{(x-2)^2(x-1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x-1}.$$

(ii) Hence show that: **2**

$$\int_3^4 \frac{x}{(x-2)^2(x-1)}\,dx = \ln\left(\frac{3}{4}\right) + 1.$$

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a) Prove that, if n is an even integer, then $n^3 - n$ is divisible by 6. **2**

(b) (i) Show that the derivative of $\sec\theta$ is $\sec\theta\tan\theta$. **1**

(ii) Use the substitution $x = \sec\theta - 1$ to find the exact value of **4**

$$\int_{\sqrt{2}-1}^{1} \frac{2}{(x+1)\sqrt{x^2+2x}}\,dx.$$

(c) Find a vector perpendicular to both $\underset{\sim}{u} = -5\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$. **3**

(d) The points P and Q have position vectors:

$$\overrightarrow{OP} = -2\underset{\sim}{i} + 14\underset{\sim}{j} - 5\underset{\sim}{k} \text{ and } \overrightarrow{OQ} = -\underset{\sim}{i} + 12\underset{\sim}{j} - 2\underset{\sim}{k}$$

(i) Show that the equation of the line l_1 that passes through P and Q is **2**

$$\underset{\sim}{r_1} = (-2+\lambda)\underset{\sim}{i} + (14-2\lambda)\underset{\sim}{j} + (-5+3\lambda)\underset{\sim}{k}$$

where λ is a real number.

(ii) Consider the line l_2 with the equation **3**

$$\underset{\sim}{r_2} = (2+a\mu)\underset{\sim}{i} + \left(27+(a+1)\mu\right)\underset{\sim}{j} + \left(1+(a+2)\mu\right)\underset{\sim}{k}$$

where a is a constant and μ is a real number.

The line l_2 intersects l_1 at the point R. Find the coordinates of R.

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a) Given that w is a complex cube root of unity, determine the exact value of **2**

$$1 + w + w^2 + w^3 + \ldots + w^{1001}.$$

(b) Let $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\,dx$, where n is an integer, and $n \geq 3$.

(i) Show that $I_n + I_{n-2} = \dfrac{1}{n-1}$. **3**

(ii) Hence evaluate I_7. **4**

(c) A particle is moving horizontally. Initially the particle is at the origin O moving with velocity 1 ms^{-1}.

The acceleration of the particle is given by $\ddot{x} = x - 1$, where x is its displacement at time t.

(i) Show that the velocity of the particle is given by $\dot{x} = 1 - x$. **3**

(ii) Find an expression for x as a function of t. **2**

(iii) Find the limiting position of the particle. **1**

End of Question 13

Question 14 (14 marks) Use the Question 14 Writing Booklet

(a) (i) Let x and y be real numbers such that $x \geq 0$ and $y \geq 0$.

Prove that $\dfrac{x+y}{2} \geq \sqrt{xy}$. **1**

(ii) Suppose that a, b, c are real numbers.
Prove that $a^4 + b^4 + c^4 \geq a^2b^2 + a^2c^2 + b^2c^2$. **2**

(iii) Show that $a^2b^2 + a^2c^2 + b^2c^2 \geq a^2bc + b^2ac + c^2ab$. **2**

(iv) Deduce that if $a + b + c = d$, then $a^4 + b^4 + c^4 \geq abcd$. **1**

(b) A submarine of mass m is travelling underwater at maximum power. At maximum power, its engines deliver a force F on the submarine. The water exerts a resistive force proportional to the square of the submarine's speed v.

(i) Explain why **1**

$$\frac{dv}{dt} = \frac{1}{m}\left(F - kv^2\right)$$

where k is a positive constant.

(ii) The submarine increases it speed from v_1 to v_2. Show that the distance travelled during this time is **3**

$$\frac{m}{2k}\log_e\left(\frac{F - kv_1^{\,2}}{F - kv_2^{\,2}}\right).$$

Question 14 continues page 12

(c) (i) Prove that $1-x \le e^{-x}$ for $x \ge 0$. **2**

(ii) $X_1, X_2, X_3, ..., X_n$ are independent Bernoulli random variables with parameters $p_1, p_2, p_3, ..., p_n$ respectively. The random variable T is given by $T = X_1 + X_2 + X_3 + ... + X_n$ and $E(T) = \mu$ denotes the mean of T. Prove that $P(T=0) \le e^{-\mu}$. **2**

End of Question 14

Question 15 (16 marks) Use the Question 15 Writing Booklet

(a) (i) Solve $x^2 > 2x+1$. **1**

(ii) Prove by mathematical induction that $2^n > n^2$ for all integers $n \geq 5$. **3**

(b) Prove by contradiction that if x is a rational number $(x \in \mathbb{Q})$, then $x^2 \neq 2$. **3**

(c) Let $P(z) = z^8 - \frac{5}{2}z^4 + 1$. The complex number w is a root of $P(z)=0$.

(i) Show that iw and $\frac{1}{w}$ are also roots of $P(z)=0$. **2**

(ii) Find one of the roots of $P(z)=0$ in exact form. **2**

(iii) Hence find all the roots of $P(z)=0$. **2**

(d) For three 3-dimensional vectors, $\underset{\sim}{p} = (a,b,c)$, $\underset{\sim}{q} = (e,f,g)$ and $\underset{\sim}{r} = (x,y,z)$, **3**
prove that the scalar product is distributive over vector addition,
i.e. prove that:

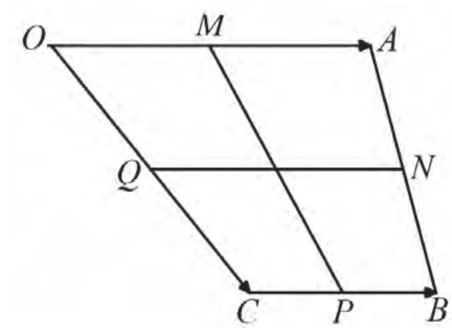
$$\underset{\sim}{p} \bullet (\underset{\sim}{q} + \underset{\sim}{r}) = \underset{\sim}{p} \bullet \underset{\sim}{q} + \underset{\sim}{p} \bullet \underset{\sim}{r}$$

End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet

(a) A body of mass m kg is projected from point O with initial speed $V\,\text{ms}^{-1}$ at an angle α above the horizontal. It moves in a vertical plane under gravity in a medium which the resistance to motion has magnitude mkv Newtons where $v\,\text{ms}^{-1}$ is its speed and $k > 0$ is a constant. What is an expression for the horizontal distance x metres travelled by the body in the first t seconds of its motion? **3**

(b)



In the diagram, $OABC$ is a trapezium in which $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OC} = \underset{\sim}{c}$ and $\overrightarrow{CB} = k\overrightarrow{OA}$ for some constant $0 < k < 1$. The points M, N, P and Q are the midpoints of OA, AB, BC and CO respectively.

(i) Use vector methods to show that MP and QN bisect each other. **2**

(ii) If $MP \perp QN$, use vector methods to show that $\left|\overrightarrow{OC}\right|^2 = \left|\overrightarrow{AB}\right|^2$. **4**

Question 16 continues page 15

Question 16 (continued)

(c) (i) Use the formula for the sum of a geometric series to show that

$$\sum_{k=1}^{n}\left(z+z^2+\ldots+z^k\right)=\frac{nz}{1-z}-\frac{z^2}{(1-z)^2}\left(1-z^n\right),\ z\neq 1.$$ **2**

(ii) Let $z=\cos\theta+i\sin\theta$, where $0<\theta<2\pi$.

By considering the imaginary part of the left-hand side of the equation of part (i), deduce that

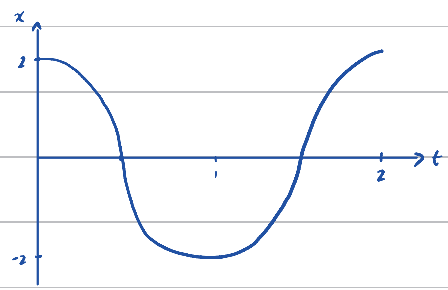
$$\sum_{k=1}^{n}\left(\sin\theta+\sin 2\theta+\ldots+\sin k\theta\right)=\frac{(n+1)\sin\theta-\sin(n+1)\theta}{4\sin^2\dfrac{\theta}{2}}.$$ **4**

(You may assume that $\dfrac{z}{1-z}=\dfrac{i}{2\sin\dfrac{\theta}{2}}\left(\cos\dfrac{\theta}{2}+i\sin\dfrac{\theta}{2}\right)$.)

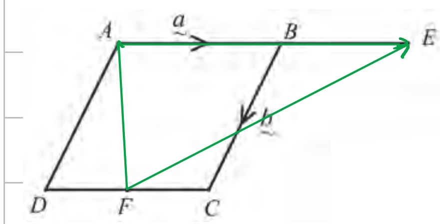
End of Examination

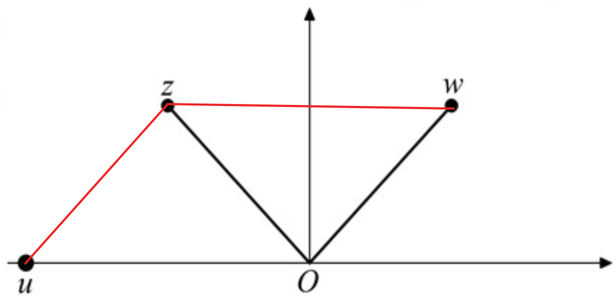
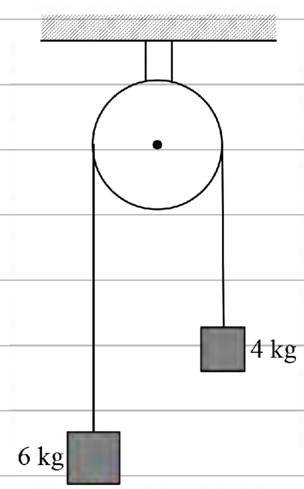
Merewether High School

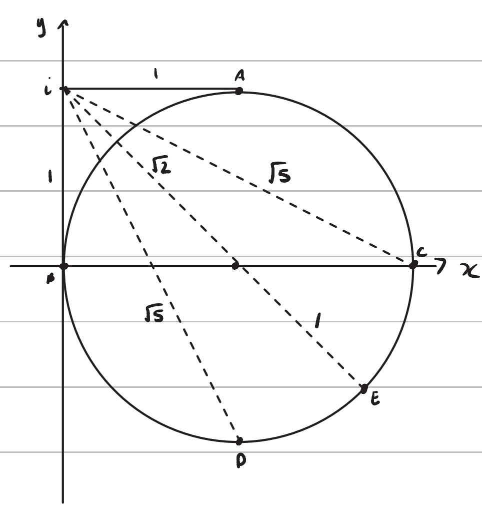
Year 12 Mathematics Extension 2 Trial HSC Solutions 2025

Question	Suggested solution	Marking guidelines	Markers Notes
1.	'for any integers m and n, if $m+n \geq 9$ then $m \geq 5$ or $n \geq 5$ Contrapositive ∴ switch the hypothesis and the conclusion and negate both ∴ If $m<5$ and $n<5$, then $m+n<9$ C		
2.	 amplitude $= 2$ period $= \frac{2\pi}{n} = 2$ $\frac{2\pi}{2} = n$ $n = \pi$ $\therefore x = a\cos(nt)$ $= 2\cos(\pi t)$ A		
3.	$z = -(2a+1) + 2ai$ $\therefore \bar{z} = -(2a+1) - 2ai$ $\frac{4a}{1+\bar{z}} = \frac{4a}{1 + -(2a+1) - 2ai}$ $= \frac{4\cancel{a}}{-2\cancel{a} - 2\cancel{a}i} \times \frac{-2\cancel{a} + 2\cancel{a}i}{-2\cancel{a} + 2\cancel{a}i}$ $= \frac{4(-2+2i)}{(-2)^2 - (2i)^2}$ $= \frac{-8+8i}{4+4}$ $= \frac{-8+8i}{8}$ $= -1+i$ $\lvert -1+i \rvert = \sqrt{1^2+1^2}$ $= \sqrt{2}$ $\arg(-1+i) = \tan^{-1}\left(\frac{1}{-1}\right)$ $= \frac{3\pi}{4}$ B		

$= \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

Question	Suggested solution	Marking guidelines	Markers Notes
4.	 $\frac{\overrightarrow{AE}}{\overrightarrow{AB}} = \frac{7}{4}$ $\therefore \overrightarrow{AE} = \frac{7}{4}\underset{\sim}{a}$ $\overrightarrow{FA} = \overrightarrow{FD} + \overrightarrow{DA}$ $= -\frac{1}{2}\underset{\sim}{a} - \underset{\sim}{b}$ $\overrightarrow{FE} = \overrightarrow{FA} + \overrightarrow{AE}$ $= \frac{7}{4}\underset{\sim}{a} - \frac{1}{2}\underset{\sim}{a} - \underset{\sim}{b}$ $= \frac{5}{4}\underset{\sim}{a} - \underset{\sim}{b}$ A		
5.	$u = 1 + e^x$ $x = \ln 2 \quad u = 1 + e^{\ln 2} = 1 + 2 = 3$ $\frac{du}{dx} = e^x$ $\frac{du}{e^x} = dx$ $x = 0 \quad u = 1 + e^0 = 1+1 = 2$ $\therefore \int_0^{\ln 2} \frac{1}{1+e^x}\,dx = \int_2^3 \frac{1}{u} \cdot \frac{du}{e^x}$ $= \int_2^3 \frac{1}{u(u-1)}\,du$ $\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1}$ $1 = A(u-1) + Bu$ $u = 1 \quad 1 = B$ $u = 0 \quad 1 = A(-1)$ $A = -1$ $= \int_2^3 \frac{1}{u-1} - \frac{1}{u}\,du$ D		

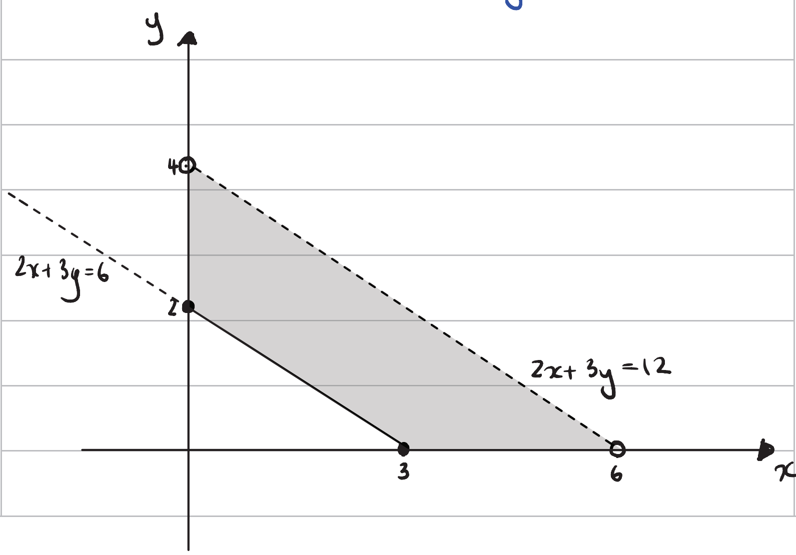
Question	Suggested solution	Marking guidelines	Markers Notes
6.			
	$\arg(u) = \arg(z) + \arg(w)$ $= \arg(zw)$ $\therefore u = zw$ $\therefore$ Either A or B $w + u = z$ (parallelogram method) $\therefore u = z - w$ B		
7.	Let: $m_1 = 6$ kg (heavier mass - accelerates downward) $m_2 = 4$kg (lighter mass - accelerates upwards) T = tension in string a = acceleration		
			

Question	Suggested solution	Marking guidelines	Markers Notes
	For 6 kg mass: $m_1 g - T = m_1 a$ ①		$m_1 g - m_1 a = T$
	For 4kg mass $T - m_2 g = m_2 a$ ②		$T = m_2 a + m_2 g$
	① + ② $m_1 g - m_2 g = m_1 a + m_2 a$ $g(m_1 - m_2) = a(m_1 + m_2)$ $g(6 - 4) = a(6+4)$ $2g = 10a$ $a = \frac{2g}{10}$ $= \frac{g}{5}$ A		
8.			
	$1 + \sqrt{2} > \sqrt{5}$ $\therefore$ E is the greatest distance away from i on the locus of z D		

Question	Suggested solution	Marking guidelines	Markers Notes
9.	Line segment 1: $\underset{\sim}{r} = \begin{pmatrix}-3\\2\end{pmatrix} + \lambda\begin{pmatrix}-3\\2\end{pmatrix}, \quad -2 \le \lambda \le 2$ • Direction vector: $\underset{\sim}{d_1} = \begin{pmatrix}-3\\2\end{pmatrix}$ • Length: $\lvert\lambda\rvert \le 2 \Rightarrow$ Total length $= 4 \times \lvert\underset{\sim}{d_1}\rvert$ $\lvert\underset{\sim}{d_1}\rvert = \sqrt{(-3)^2+2^2}$ $= \sqrt{9+4}$ $= \sqrt{13}$ $\therefore$ length $= 4\sqrt{13}$ Line segment 2: $\underset{\sim}{l} = \begin{pmatrix}-2\\-1\end{pmatrix} + \mu\begin{pmatrix}4\\6\end{pmatrix}, \quad 0 \le \mu \le 2$ • Direction vector: $\underset{\sim}{d_2} = \begin{pmatrix}4\\6\end{pmatrix}$ • Length: $2 \times \underset{\sim}{d_2}$ $= 2 \times \sqrt{4^2+6^2}$ $= 2 \times \sqrt{16+36}$ $= 2\sqrt{52}$ $= 4\sqrt{13}$ Line segment 3: $\underset{\sim}{v} = \gamma\begin{pmatrix}9\\-6\end{pmatrix}, \quad 1 \le \gamma \le 3$ • Direction vector: $\underset{\sim}{d_3} = \begin{pmatrix}9\\-6\end{pmatrix} = -3\begin{pmatrix}-3\\2\end{pmatrix}$ • Length: $(3-1) \times \lvert\underset{\sim}{d_3}\rvert = 2 \times \sqrt{9^2+(-6)^2}$ $= 2 \times \sqrt{81+36}$ $= 2\sqrt{117}$		

Question	Suggested solution	Marking guidelines	Markers Notes
	$\therefore \underset{\sim}{r} \parallel \underset{\sim}{v}$, $\lvert \underset{\sim}{v} \rvert = \lvert \underset{\sim}{\ell} \rvert$,		
	$\underset{\sim}{d_1} \cdot \underset{\sim}{d_2} = -3\times4 + 2\times6$ $= 0$ or $\underset{\sim}{d_2} \cdot \underset{\sim}{d_3} = 4\times9 + (-6)\times6$ $= 0$		
	$\therefore \underset{\sim}{v} \perp \underset{\sim}{\ell}$		
	$\therefore$ C is not true		
10.	$\dfrac{x}{2\cos x}$ and $\tan^{-1}(x^3)$ are odd functions, $\therefore$ definite integrals involving limits indicated will be 0.		
	$x^3\sin x$ and $e^{-x^2}-1$ are even functions, $\therefore$ definite integrals involving indicated limits will be non-zero.		
	$e^{-x^2}-1 \leq 0$ in the domain of the integral $\therefore$ value of integral will be <0		
	$x^3\sin x \geq 0$ in the domain of the integral $\therefore$ value of integral will be >0		
	B		

Question	Suggested solution	Marking guidelines	Markers Notes
11. a)	(i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \begin{pmatrix}-1\\-2\\1\end{pmatrix} + \begin{pmatrix}3\\0\\2\end{pmatrix} = \begin{pmatrix}2\\-2\\3\end{pmatrix}$ $\quad \overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = \begin{pmatrix}-1\\-2\\1\end{pmatrix} + \begin{pmatrix}-1\\4\\1\end{pmatrix} = \begin{pmatrix}-2\\2\\2\end{pmatrix}$	1	
	(ii) $\overrightarrow{AB} \cdot \overrightarrow{AC} = 2(-2) + (-2)(2) + 3(2) = -2$		
	$\lvert\overrightarrow{AB}\rvert = \sqrt{2^2+(-2)^2+3^2} = \sqrt{17}$ $\quad \lvert\overrightarrow{AC}\rvert = \sqrt{(-2)^2+2^2+2^2} = \sqrt{12}$		
	$\cos\theta = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AC}\rvert} = \dfrac{-2}{\sqrt{17} \times \sqrt{12}}$	1	
	$= -0.14\ldots$		
	$\theta = \cos^{-1}(-0.14\ldots) = 98.04\ldots = 98°$		
	$\therefore$ acute $\angle$ between $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is $82°$	1	
b)	$\displaystyle\int \frac{1}{x^2+2x+3}\,dx = \int \frac{1}{(x+1)^2+2}\,dx$	1	
	$= \dfrac{1}{\sqrt{2}} \tan^{-1}\left(\dfrac{x+1}{\sqrt{2}}\right) + c$	1	

Question	Suggested solution	Marking guidelines	Markers Notes
11 c)	$\arg[(b-i)^3] = -\frac{\pi}{2}$		
	$\therefore \arg(b-i) = -\frac{\pi}{2} \div 3$		
	$= -\frac{\pi}{6}$	1	
	$\therefore \tan\left(-\frac{\pi}{6}\right) = -\frac{1}{b}$	1	
	$-\frac{1}{\sqrt{3}} = -\frac{1}{b}$		
	$\therefore b = \sqrt{3}$	1	
	OR		
	$(b-i)^3 = b^3 - 3b^2 i + 3bi^2 - i^3$		
	$= b^3 - 3b^2 i - 3b + i$		
	$= b^3 - 3b + (1-3b^2)i$	1	
	If $\arg(z) = -\frac{\pi}{2}$ then $b^3 - 3b = 0$	1	
	$b(b^2-3) = 0$		
	$\therefore b = \sqrt{3}$ since $b > 0$	1	
d)	Let $z = x+iy$		
	Then $(2-3i)(x+iy) = (2x+3y) + (2y-3x)i$		
	$6 \le \mathrm{Re}[(2-3i)z] < 12 \;=\; 6 \le 2x+3y < 12$	2	correct region provided
	and $\mathrm{Re}\, z \times \mathrm{Im}\, z \ge 0 \implies xy \ge 0$		
		1	1 error
		0	more than 1 error

Question	Suggested solution	Marking guidelines	Markers Notes
11 e)	(i) $\dfrac{x}{(x-2)^2(x-1)} = \dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{C}{(x-1)}$		
	$\therefore \quad x \equiv A(x-1)(x-2) + B(x-1) + C(x-2)^2$		
	$= A(x^2-3x+2) + B(x-1) + C(x^2-4x+4)$		
	$= (A+C)x^2 + (B-3A-4C)x + 2A - B + 4C$		
	let: $x=1$, $1 = C(-1)^2$		
	$\therefore C = 1$	1	
	$x = 2$, $2 = B(2-1)$		
	$\therefore B = 2$	1	
	Equating coefficients of x^2:		
	$A + C = 0$		
	$A + 1 = 0$		
	$A = -1$	1	
	$\therefore \dfrac{x}{(x-2)^2(x-1)} = \dfrac{-1}{x-2} + \dfrac{2}{(x-2)^2} + \dfrac{1}{x-1}$		
	(ii) $\displaystyle\int_3^4 \frac{x}{(x-2)^2(x-1)}\,dx = \int_3^4 \left[\frac{-1}{x-2} + \frac{2}{(x-2)^2} + \frac{1}{x-1}\right] dx$		
	$= \left[-\ln\lvert x-2\rvert - \dfrac{2}{x-2} + \ln\lvert x-1\rvert\right]_3^4$	1	
	$= \left[\ln\left\lvert\dfrac{x-1}{x-2}\right\rvert - \dfrac{2}{x-2}\right]_3^4$		
	$= \ln\left\lvert\dfrac{4-1}{4-2}\right\rvert - \dfrac{2}{4-2} - \left(\ln\left\lvert\dfrac{3-1}{3-2}\right\rvert - \dfrac{2}{3-2}\right)$		
	$= \ln\left(\tfrac{3}{2}\right) - 1 - \ln(2) + 2$		
	$= \ln\left(\tfrac{3}{4}\right) + 1$	1	

Question	Suggested solution	Marking guidelines	Markers Notes
12. a)	Let $n = 2k$, where $k \in \mathbb{Z}$		
	Then $n^3 - n = (2k)^3 - 2k$		
	$= 8k^3 - 2k$		
	$= 2k(4k^2 - 1)$, which is divisible by 2	1	
	To show divisibility by 6, also need to show it's divisible by 3.		
	$= 2k(2k-1)(2k+1)$		
	$= (2k-1)\,2k\,(2k+1)$		
	This is a product of 3 consecutive integers: $2k-1$, $2k$, $2k+1$		
	Among any three consecutive integers, one is divisible by 3	1	
	$\therefore$ Expression is also divisible by 3		
	Hence divisible by LCM$(2,3) = 6$		
b)	(i) $\frac{d}{dx}(\sec\theta) = \frac{d}{dx}\left(\frac{1}{\cos\theta}\right)$		
	$= \frac{d}{dx}\left[(\cos\theta)^{-1}\right]$		
	$= -1(\cos\theta)^{-2} \times (-\sin\theta)$		
	$= \frac{\sin\theta}{\cos^2\theta}$		
	$= \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}$	1	
	$= \tan\theta\sec\theta$ as required		
	(ii) $x = \sec\theta - 1$ $\frac{dx}{d\theta} = \sec\theta\tan\theta$ $dx = \sec\theta\tan\theta\, d\theta$ $x = 1 \rightarrow 1 = \sec\theta - 1$, $2 = \sec\theta$, $\frac{1}{2} = \cos\theta$, $\theta = \frac{\pi}{3}$ $x = \sqrt{2} - 1 \rightarrow \sqrt{2} - 1 = \sec\theta$, $\frac{1}{\sqrt{2}} = \cos\theta$, $\theta = \frac{\pi}{4}$	1	

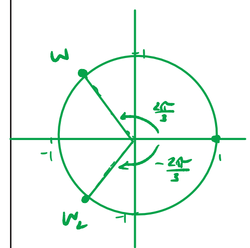
Question	Suggested solution	Marking guidelines	Markers Notes
	$\displaystyle\int_{\sqrt{2}-1}^{1} \frac{2}{(x+1)\sqrt{x^2+2x}}\,dx$		
	$\displaystyle= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\cancel{\sec\theta}\tan\theta\,d\theta}{\cancel{\sec\theta}\sqrt{(\sec\theta-1)^2+2(\sec\theta-1)}}$	1	
	$\displaystyle= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\tan\theta\,d\theta}{\sqrt{\sec^2\theta-\cancel{2\sec\theta}+1+\cancel{2\sec\theta}-2}}$		
	$\displaystyle= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{2\tan\theta\,d\theta}{\sqrt{\tan^2\theta}}$		
	$\displaystyle= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 2\,d\theta$	1	
	$\displaystyle= 2\Big[\theta\Big]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$		
	$\displaystyle= 2\left[\frac{\pi}{3}-\frac{\pi}{4}\right]$	1	
	$\displaystyle= \frac{\pi}{6}$		
12 c)	Let $\underset{\sim}{w} = x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$ be a unit vector such that:		
	$\underset{\sim}{u}\cdot\underset{\sim}{w} = -5x+2y+3z = 0$ ①		
	$\underset{\sim}{v}\cdot\underset{\sim}{w} = 2x-y-2z = 0$ ②		
	and $x^2+y^2+z^2 = 1$ ③		

Question	Suggested solution	Marking guidelines	Markers Notes
	① + 2×② : $-x - z = 0$ $\therefore z = -x$ ④ ①×2 + 3×② : $-4x + y = 0$ $y = 4x$ ⑤ sub into ③ $x^2 + (4x)^2 + (-x)^2 = 1$ $x^2 + 16x^2 + x^2 = 1$ $18x^2 = 1$ $x^2 = \frac{1}{18}$ $x = \pm\frac{1}{3\sqrt{2}}$ $y = \pm\frac{4}{3\sqrt{2}}$ $z = \mp\frac{1}{3\sqrt{2}}$ $\therefore \frac{1}{3\sqrt{2}}(\underset{\sim}{i} + 4\underset{\sim}{j} - \underset{\sim}{k})$ is a vector perpendicular to both $\underset{\sim}{u}$ and $\underset{\sim}{v}$	1 1 1	

Question	Suggested solution	Marking guidelines	Markers Notes
12 d) (i)	$\vec{PQ} = \vec{OQ} - \vec{OP}$		
	$= \begin{pmatrix} -1 \\ 12 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ 14 \\ -5 \end{pmatrix}$		
	$= \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \Rightarrow$ direction vector	1	
	$\therefore \underset{\sim}{r_1} = \begin{pmatrix} -2 \\ 14 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$		
	$= (-2+\lambda)\underset{\sim}{i} + (14-2\lambda)\underset{\sim}{j} + (-5+3\lambda)\underset{\sim}{k}$	1	
(ii)	$-2+\lambda = 2+a\mu$ (1) $14-2\lambda = 27+(a+1)\mu$ (2) $-5+3\lambda = 1+(a+2)\mu$ (3)	1	
	From (1) $a\mu = \lambda - 4$		
	From (2) $14-2\lambda = 27+\lambda-4+\mu$ $-9 = 3\lambda+\mu$ (5)		
	From (3) $-5+3\lambda = 1+\lambda-4+2\mu$ $2\lambda-2\mu = 2$ $\lambda-\mu = 1$ (6)		
	(5)+(6) $4\lambda = -8$ $\lambda = -2$	1	
	$\therefore \underset{\sim}{r_1} = (-2-2)\underset{\sim}{i} + (14+4)\underset{\sim}{j} + (-5-6)\underset{\sim}{k}$		

$= -4\underset{\sim}{i} + 18\underset{\sim}{j} - 11\underset{\sim}{k}$ 1

$\therefore R = (-4, 18, -11)$

Question	Suggested solution	Marking guidelines	Markers Notes
13. a)	$w^3 = 1$ but $w \neq 1$		
	$\therefore w^3 - 1 = 0$		
	$(w-1)(w^2+w+1) = 0$		
	$\therefore 1 + w + w^2 = 0$ since $w \neq 1$	1	
	$\sum_{k=0}^{1001} w^k = 1 + w + w^2 + \dots + w^{1001}$		
	In this sum there are 1002 terms, which is divisible by 3		
	$1002 = 334$		
	$\therefore \sum_{k=0}^{1001} w^k = (1+w+w^2) + (w^3+w^4+w^5) + \dots + (w^{996}+w^{997}+w^{998}) + (w^{999}+w^{1000}+w^{1001})$		
	$= (1+w+w^2) + w^3(1+w+w^2) + \dots w^{996}(1+w+w^2) + w^{999}(1+w+w^2)$		
	$= 0 + 0 + \dots + 0 + 0$		
	$= 334 \times 0$	1	
	$= 0$		
b)	(i) $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$		
	$= \int_0^{\frac{\pi}{4}} \tan^{n-2} x \tan^2 x \, dx$		
	$= \int_0^{\frac{\pi}{4}} \tan^{n-2} x (\sec^2 x - 1) \, dx$		
	$= \int_0^{\frac{\pi}{4}} \tan^{n-2} x \sec^2 x - \int_0^{\frac{\pi}{4}} \tan^{n-2} x \, dx$		$\int \sec^2 x [\tan x]^n \, dx$ $= \int f'(x)[f(x)]^n \, dx$
	$= \frac{1}{n-1}\left[\tan^{n-1} x\right]_0^{\frac{\pi}{4}} - I_{n-2}$		
	$= \frac{1}{n-1}\left[\tan^{n-1}\frac{\pi}{4} - \tan^{n-1} 0\right] - I_{n-2}$		

Question	Suggested solution	Marking guidelines	Markers Notes
13 (b)	$= \frac{1}{n-1}\left[1-0\right] - I_{n-2}$		
	$I_n = \frac{1}{n-1} - I_{n-2}$		
	$\therefore I_n + I_{n-2} = \frac{1}{n-1}$		
	(ii) When $n=7$ $\quad I_7 + I_5 = \frac{1}{7-1}$ $\quad \therefore I_7 + I_5 = \frac{1}{6}$ ①		
	$n=5$ $\quad I_5 + I_3 = \frac{1}{5-1}$ $\quad \therefore I_5 + I_3 = \frac{1}{4}$ ②		1
	$n=3$ $\quad I_3 + I_1 = \frac{1}{3-1}$ $\quad \therefore I_3 + I_1 = \frac{1}{2}$ ③		
	$I_1 = \int_0^{\frac{\pi}{4}} \tan x \, dx$		
	$= \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} \, dx$		
	$= -\left[\ln\lvert\cos x\rvert\right]_0^{\frac{\pi}{4}}$	1	
	$= -\left[\ln\lvert\cos\frac{\pi}{4}\rvert - \ln\lvert\cos 0\rvert\right]$		
	$= -\left[\ln\frac{1}{\sqrt{2}} - \ln 1\right]$		
	$= -\ln\frac{1}{\sqrt{2}}$		
	$= -\ln 2^{-\frac{1}{2}}$		
	$= \frac{1}{2}\ln 2$	1	
	From ③ $\quad I_3 = \frac{1}{2} - I_1 = \frac{1}{2} - \frac{1}{2}\ln 2$		
	From ② $\quad I_5 = \frac{1}{4} - I_3 = \frac{1}{4} - \left(\frac{1}{2} - \frac{1}{2}\ln 2\right)$		
	$= -\frac{1}{4} + \frac{1}{2}\ln 2$		
	From ① $\quad I_7 = \frac{1}{6} - I_5 = \frac{1}{6} - \left(-\frac{1}{4} + \frac{1}{2}\ln 2\right)$		
	$= \frac{5}{12} - \frac{1}{2}\ln 2$	1	

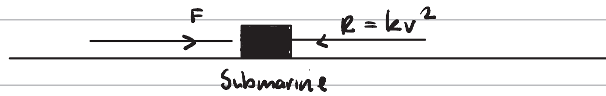
Question	Suggested solution	Marking guidelines	Markers Notes
13 (c)	$\ddot{x} = x - 1 \qquad t=0, \ x=0, \ v=1$		
	(i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = x - 1$	1	
	$\frac{1}{2}v^2 = \frac{x^2}{2} - x + C_1$		
	when $x=0 \quad v=1$		
	$\frac{1}{2} = 0 - 0 + C_1$		
	$\frac{1}{2} = C_1$		
	$\therefore \frac{1}{2}v^2 = \frac{x^2}{2} - x + \frac{1}{2}$		
	$v^2 = x^2 - 2x + 1$		
	$v = \pm\sqrt{x^2 - 2x + 1}$		
	$= \pm\sqrt{(x-1)^2}$	1	
	$= \pm(x-1)$		
	But when $x = 0 \quad v = 1$		
	$\therefore v = -(x-1)$		
	$= 1 - x$ as required	1	
	(ii) $v = 1 - x$		
	$\frac{dx}{dt} = 1 - x$		
	$\frac{dt}{dx} = \frac{1}{1-x}$	1	
	$t = -\ln\|1-x\| + C_2$		
	when $t = 0 \quad x = 0$		
	$0 = -\ln\|1-0\| + C_2$		
	$= C_2$		
	$\therefore t = -\ln\|1-x\|$		
	$-t = \ln\|1-x\|$		
	$\therefore e^{-t} = 1 - x$		

$x = 1 - e^{-t}$ (1)

Question	Suggested solution	Marking guidelines	Markers Notes
13 c)	(iii) $x = 1 - e^{-t}$ As $t \to \infty$, $e^{-t} \to 0$ $\Rightarrow x \to 1 - 0$ $= 1$ $\therefore$ limiting position is $x = 1$	1	

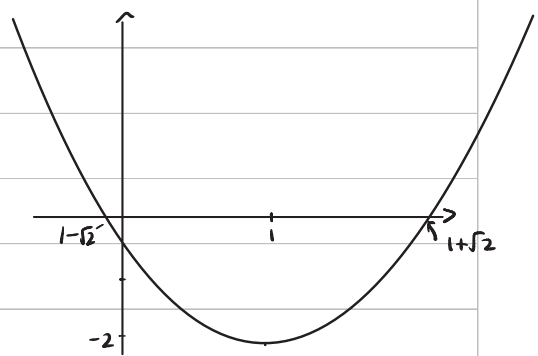
Question	Suggested solution	Marking guidelines	Markers Notes
14 a)	(i) Prove $\frac{x+y}{2} \geq \sqrt{xy}$ Consider $\frac{x+y}{2} - \sqrt{xy} \geq 0$ LHS $= \frac{x+y}{2} - \sqrt{xy}$ $= \frac{x - 2\sqrt{xy} + y}{2}$ $= \frac{(\sqrt{x} - \sqrt{y})^2}{2}$ ≥ 0 since $(\sqrt{x} - \sqrt{y})^2 > 0$ if $x \neq y$ and $(\sqrt{x} - \sqrt{y})^2 = 0$ if $x = y$ $\therefore \frac{x+y}{2} \geq \sqrt{xy}$	1	
	(ii) using result from part (i) $\frac{a^4+b^4}{2} \geq \sqrt{a^4b^4} = a^2b^2$ ① $\frac{b^4+c^4}{2} \geq \sqrt{b^4c^4} = b^2c^2$ ② $\frac{a^4+c^4}{2} \geq \sqrt{a^4c^4} = a^2c^2$ ③	1	
	①+②+③: $\frac{(a^4+b^4)+(b^4+c^4)+(a^4+c^4)}{2} \geq a^2b^2+b^2c^2+a^2c^2$ $\frac{\cancel{2}(a^4+b^4+c^4)}{\cancel{2}} \geq a^2b^2+b^2c^2+a^2c^2$ $\therefore a^4+b^4+c^4 \geq a^2b^2+b^2c^2+a^2c^2$	1	

Question	Suggested solution	Marking guidelines	Markers Notes
14 a)	(iii) Using the result in part (i) again, then		
	$\dfrac{a^2b^2+a^2c^2}{2} = \dfrac{a^2(b^2+c^2)}{2} \geq \sqrt{a^2b^2 \cdot a^2c^2} = a^2bc$ (1)		
	$\dfrac{b^2c^2+b^2a^2}{2} = \dfrac{b^2(c^2+a^2)}{2} \geq \sqrt{b^2c^2 \cdot b^2a^2} = b^2ca$ (2)		1
	$\dfrac{c^2a^2+c^2b^2}{2} = \dfrac{c^2(a^2+b^2)}{2} = \sqrt{c^2a^2 \cdot c^2b^2} = c^2ab$ (3)		
	(1) + (2) + (3) :		
	$\dfrac{a^2(b^2+c^2)+b^2(c^2+a^2)+c^2(a^2+b^2)}{2} \geq a^2bc+b^2ca+c^2ab$		
	i.e $\dfrac{\cancel{2}(a^2b^2+b^2c^2+c^2a^2)}{\cancel{2}} \geq a^2bc+b^2ca+c^2ab$	1	
	$\therefore\ a^2b^2+b^2c^2+c^2a^2 \geq a^2bc+b^2ca+c^2ab$		
	(iv) From (ii) and (iii)		
	$a^4+b^4+c^4 \geq a^2b^2+b^2c^2+a^2c^2$ and		
	$a^2b^2+b^2c^2+c^2a^2 \geq a^2bc+b^2ca+c^2ab$		
	$\therefore\ a^4+b^4+c^4 \geq a^2bc+b^2ca+c^2ab$		
	$= abc(a+b+c)$		
	$= abcd$ since $a+b+c=d$	1	

Question	Suggested solution	Marking guidelines	Markers Notes
14 b)	(i)  Resultant force in $\longrightarrow$ direction is $F - R = F - kv^2$ By Newton's Law of Motion, if the acceleration of submarine (of mass m) in $\longrightarrow$ direction is 'a' where $a = \frac{dv}{dt}$, then the force acting on the submarine in $\longrightarrow$ direction is $ma = m \cdot \frac{dv}{dt}$ Thus $m \cdot \frac{dv}{dt} = F - kv^2$ i.e. $\frac{dv}{dt} = \frac{1}{m}(F - kv^2)$	1	
	(ii) $\frac{dv}{dt} = v\frac{dv}{dx}$ $\therefore \; v\frac{dv}{dx} = \frac{1}{m}(F - kv^2)$ $\frac{dv}{dx} = \frac{1}{m} \cdot \frac{(F - kv^2)}{v}$ $\frac{dx}{dv} = m \cdot \frac{v}{F - kv^2}$ $x = m\int \frac{v}{F - kv^2}\,dv$ The distance x travelled during the period when submarine increases its speed from v_1 to v_2 is given by	1	

Question	Suggested solution	Marking guidelines	Markers Notes
	$X = \frac{m}{-2k}\int_{v_1}^{v_2} \frac{-2kv\,dv}{F-kv^2}$		
	$= -\frac{m}{2k}\left[\ln\lvert F-kv^2\rvert\right]_{v_1}^{v_2}$	1	
	$= -\frac{m}{2k}\left[\ln\lvert F-kv_2^2\rvert - \ln\lvert F-kv_1^2\rvert\right]$		
	$= -\frac{m}{2k}\log_e\left\lvert\frac{F-kv_2^2}{F-kv_1^2}\right\rvert$		
	$= \frac{m}{2k}\log_e\left\lvert\frac{F-kv_1^2}{F-kv_2^2}\right\rvert$	1	
c)	(i) Prove $1-x \le e^{-x}$ for $x \ge 0$		
	Consider $1-x-e^{-x} \le 0$		
	Let $f(x) = 1-x-e^{-x}$		
	then $f'(x) = -1+e^{-x}$		
	≤ 0 for $x \ge 0$		
	$\therefore$ $f(x)$ is decreasing or stationary as x increases for $x \ge 0$	1	
	Also $f(0) = 0$		
	$\therefore$ $f(x) \le 0$ for $x \ge 0$	1	
	Hence $1-x-e^{-x} \le 0$		
	$\Rightarrow$ $1-x \le e^{-x}$		
	(ii) $P(T=0) = P(X_k = 0 \ \forall k = 1, 2, \ldots, n)$		
	$= (1-p_1)(1-p_2)\ldots(1-p_n)$ since the X_k are independent Bernoulli random variables taking the value 1 with probability p_k and 0 with probability $1-p_k$	1	

Question	Suggested solution	Marking guidelines	Markers Notes
14 c)	Then using part (i), $p_k \geq 0 \ \forall k$ $\Rightarrow P(T=0) \leq e^{-p_1} e^{-p_2} \dots e^{-p_n}$ $= e^{-(p_1+p_2+\dots p_n)}$ But $\mu = \sum_{k=1}^{n} E(X_k)$ $= \sum_{k=1}^{n} p_k$ $\therefore P(T=0) \leq e^{-\mu}$	1	

Question	Suggested solution	Marking guidelines	Markers Notes
15. a)	(i) $x^2 > 2x+1$		
	$x^2 - 2x - 1 > 0$		
	$(x-1)^2 - 2 > 0$		
	$(x-1-\sqrt{2})(x-1+\sqrt{2}) > 0$		
			
	∴ $x^2 - 2x - 1 > 0$ for $x < 1-\sqrt{2}$ or $x > 1+\sqrt{2}$		
	i.e $x^2 > 2x+1$ for $x < 1-\sqrt{2}$ or $x > 1+\sqrt{2}$		1
	(ii) Prove $2^n > n^2$ for $n \geq 5$		
	Step 1 Prove true for $n=5$		
	LHS $= 2^5$ RHS $= 5^2$		
	$= 32$ $= 25$		
	LHS > RHS		
	∴ True for $n=5$	1	
	Step 2: Assume true for $n=k$, $k \geq 5$		
	i.e $2^k > k^2$		
	∴ $2^k - k^2 > 0$		
	Step 3: Prove true for $n=k+1$		
	i.e $2^{k+1} - (k+1)^2 > 0$		
	LHS $= 2^{k+1} - (k+1)^2$		
	$= 2.2^k - k^2 - 2k - 1$		
	$= 2.2^k - 2k^2 + k^2 - 2k - 1$		
	$= 2(2^k - k^2) + (k^2 - 2k - 1)$		
	> 0		

Question	Suggested solution	Marking guidelines	Markers Notes
	Since for $k \geq 5$: $2^2 - k^2 > 0$ using assumption		
	and from (i) $k^2 - 2k - 1 > 0$ for $k > 1+\sqrt{2} \approx 2.4$		
	$\therefore$ Also true for $k \geq 5$		
	Step 4: Conclusion		
	True for $n = k+1$ if true for $n = k$		
	True for $n = 5$		
	$\therefore$ by induction true for all integers $n \geq 5$.		
b)	Assume that there exists a rational number		
	$x \in \mathbb{Q}$ such that $x^2 = 2$		
	Since $x \in \mathbb{Q}$, $x = \frac{p}{q}$, where $p, q \in \mathbb{Z}, q \neq 0$ and p, q have no common factors		
	$\therefore \left(\frac{p}{q}\right)^2 = 2$		
	$\frac{p^2}{q^2} = 2$		
	$p^2 = 2q^2$	1	
	$\therefore p^2$ is even $\Rightarrow p$ must be even (since the square of an odd number is odd)		
	Let $p = 2k$, $k \in \mathbb{Z}$		
	$(2k)^2 = 2q^2$		
	$4k^2 = 2q^2$		
	$2k^2 = q^2$	1	
	$\therefore q^2$ is even $\Rightarrow q$ is also even		
	$\therefore p$ and q are both even meaning		

the have a common factor of 2 1

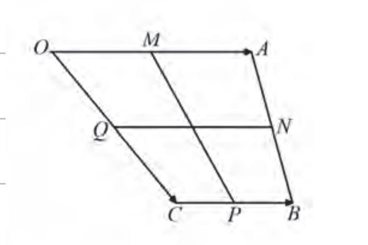
This contradicts the assumption that $\frac{p}{q}$ is in simplest form

$\therefore$ by contradiction if $x \in \mathbb{Q}$, $x^2 \neq 2$

Question	Suggested solution	Marking guidelines	Markers Notes
15. c)	$P(z) = z^8 - \frac{5}{2}z^4 + 1$ w is complex root of $P(z)=0$		
	$\therefore w^8 - \frac{5}{2}w^4 + 1 = 0$		
	(i) $P(iw) = (iw)^8 - \frac{5}{2}(iw)^4 + 1$		
	$= (i^2)^4 w^8 - \frac{5}{2}(i^2)^2 w^4 + 1$		
	$= w^8 - \frac{5}{2}w^4 + 1$		
	$= 0$	1	
	$\therefore iw$ is a root of $P(z)=0$		
	$P\left(\frac{1}{w}\right) = \left(\frac{1}{w}\right)^8 - \frac{5}{2}\left(\frac{1}{w}\right)^4 + 1$		
	$= \frac{1}{w^8}\left(1 - \frac{5}{2}w^4 + w^8\right)$	1	
	$= \frac{1}{w^8} \times 0$		
	$= 0$		
	$\therefore \frac{1}{w}$ is a root of $P(z)=0$		
	(ii) $P(z) = z^8 - \frac{5}{2}z^4 + 1$		
	$= \frac{1}{2}(2z^8 - 5z^4 + 2)$		
	$= \frac{1}{2}(2z^4 - 1)(z^4 - 2)$		
	$= \frac{1}{2}(\sqrt{2}z^2 - 1)(\sqrt{2}z^2 + 1)(z^2 - \sqrt{2})(z^2 + \sqrt{2})$		
	$= \frac{1}{2}(2^{\frac{1}{4}}z - 1)(2^{\frac{1}{4}}z + 1)(\sqrt{2}z^2 + 1)(z - 2^{\frac{1}{4}})(z + 2^{\frac{1}{4}})(z^2 + \sqrt{2})$		1
	The roots of $P(z)=0$ in exact form are		
	those when		
	$(2^{\frac{1}{4}}z - 1)(2^{\frac{1}{4}}z + 1)(z - 2^{\frac{1}{4}})(z + 2^{\frac{1}{4}}) = 0$	1	
	i.e. $z = 2^{-\frac{1}{4}}, -2^{-\frac{1}{4}}, 2^{\frac{1}{4}}, -2^{\frac{1}{4}}$		
	Any one of these will be an exact root of		
	$P(z)=0$		

Question	Suggested solution	Marking guidelines	Markers Notes
	(iii) Noting $i^2=-1$, then the complex roots of $P(z)=0$ are those when		
	$(\sqrt{2}z^2+1)(z^2+\sqrt{2})=0$		
	i.e. $(\sqrt{2}z^2-i^2)(z^2-\sqrt{2}i^2)=0$		
	i.e. $(2^{\frac{1}{4}}z-i)(2^{\frac{1}{4}}z+i)(z-2^{\frac{1}{4}}i)(z+2^{\frac{1}{4}}i)=0$	1	
	i.e $z=i2^{-\frac{1}{4}}, -i2^{-\frac{1}{4}}, i2^{\frac{1}{4}}, -i2^{\frac{1}{4}}$	1	
	$\therefore$ All the roots of $P(z)=0$ are		
	$z=\pm 2^{-\frac{1}{4}}, \pm 2^{\frac{1}{4}}, \pm i2^{-\frac{1}{4}}, \pm i2^{\frac{1}{4}}$		
	Note: If $w=i2^{\frac{1}{4}}$, then		
	$iw=-2^{\frac{1}{4}}$, $\frac{1}{w}=\frac{2^{-\frac{1}{4}}}{i}=-i2^{-\frac{1}{4}}$ which are		
	also roots of $P(z)=0$, from part (i)		
d)	Let $\underset{\sim}{p}=(a,b,c)$, $\underset{\sim}{q}=(e,f,g)$ and $\underset{\sim}{r}=(x,y,z)$	3m –	correctly worked proof with conclusion
	LHS $=\underset{\sim}{p}\cdot(\underset{\sim}{q}+\underset{\sim}{r})$	2m –	for worked proof with minor error in logic or algebraic manipulation
	$=(a,b,c)\cdot(e+x, f+y, g+z)$		
	$=a(e+x)+b(f+y)+c(g+z)$		
	$=ae+ax+bf+by+cg+cz$	1m –	for some relevant algebraic working
	RHS $=\underset{\sim}{p}\cdot\underset{\sim}{q}+\underset{\sim}{p}\cdot\underset{\sim}{r}$		
	$=(a,b,c)\cdot(e,f,g)+(a,b,c)\cdot(x,y,z)$		
	$=ae+bf+cg+ax+by+cz$		
	$=ae+ax+bf+by+cg+cz$		
	$=$ LHS		
	$\therefore\ \underset{\sim}{p}\cdot(\underset{\sim}{q}+\underset{\sim}{r})=\underset{\sim}{p}\cdot\underset{\sim}{q}+\underset{\sim}{p}\cdot\underset{\sim}{r}$		

Question	Suggested solution	Marking guidelines	Markers Notes
16. a)	$m\ddot{x} = -mk\dot{x}$		
	$\dot{x}\frac{d}{dx}(\dot{x}) = -k\dot{x}$		
	$\frac{d}{dx}(\dot{x}) = -k$		
	$\dot{x} = -kx + c$	1	
	when $t=0$		
	$x=0$		
	$\dot{x} = V\cos\alpha$		
	$\therefore \quad V\cos\alpha = -k(0) + c$		
	$= c$		
	$\therefore \quad \dot{x} = V\cos\alpha - kx$		
	$\frac{dx}{dt} = V\cos\alpha - kx$		
	$\frac{dt}{dx} = \frac{1}{V\cos\alpha - kx}$		
	$-k\frac{dt}{dx} = \frac{-k}{V\cos\alpha - kx}$		
	$\int_0^t -k\,dt = \int_0^x \frac{-k}{V\cos\alpha - kx}\,dx$	1	
	$-k\left[t\right]_0^t = \left[\ln\lvert V\cos\alpha - kx\rvert\right]_0^x$		
	$-k(t-0) = \left[\ln\lvert V\cos\alpha - kx\rvert - \ln\lvert V\cos\alpha - 0\rvert\right]$		
	$-kt = \ln\left\lvert\frac{V\cos\alpha - kx}{V\cos\alpha}\right\rvert$		
	$e^{-kt} = \frac{V\cos\alpha - kx}{V\cos\alpha}$		
	$= \frac{V\cos\alpha}{V\cos\alpha} - \frac{kx}{V\cos\alpha}$		
	$= 1 - \frac{kx}{V\cos\alpha}$		
	$\frac{kx}{V\cos\alpha} = 1 - e^{-kt}$		
	$x = \frac{V\cos\alpha}{k}(1 - e^{-kt})$	1	

Question	Suggested solution	Marking guidelines	Markers Notes
16 b)	 		
	(i) N is the midpoint of AB		
	$\therefore\ \overrightarrow{ON} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$		since diagonals of a parallelogram bisect each other
	$\therefore\ \overrightarrow{ON} = \frac{1}{2}(\underset{\sim}{a} + (\underset{\sim}{c} + k\underset{\sim}{a}))$		
	$= \frac{1}{2}\underset{\sim}{c} + \frac{1}{2}(k+1)\underset{\sim}{a}$		
	$\therefore\ \frac{1}{2}(\overrightarrow{OQ} + \overrightarrow{ON}) = \frac{1}{2}\left(\frac{1}{2}\underset{\sim}{c} + \frac{1}{2}\underset{\sim}{c} + \frac{1}{2}(k+1)\underset{\sim}{a}\right)$		
	$= \frac{1}{2}\underset{\sim}{c} + \frac{1}{4}(k+1)\underset{\sim}{a}$	1	
	But $\frac{1}{2}(\overrightarrow{OM} + \overrightarrow{OP}) = \frac{1}{2}\left(\frac{1}{2}\underset{\sim}{a} + (\underset{\sim}{c} + \frac{1}{2}k\underset{\sim}{a})\right)$		
	$= \frac{1}{2}\underset{\sim}{c} + \frac{1}{4}(k+1)\underset{\sim}{a})$	1	
	Hence QN and MP have a common midpoint T where $\overrightarrow{OT} = \frac{1}{2}\underset{\sim}{c} + \frac{1}{4}(k+1)\underset{\sim}{a}$		
	Hence QN and MP bisect each other in their intersection point T.		
	(ii		
	$\overrightarrow{MP} = \overrightarrow{MO} + \overrightarrow{OC} + \overrightarrow{CP}$		
	$= -\frac{1}{2}\underset{\sim}{a} + \underset{\sim}{c} + \frac{1}{2}k\underset{\sim}{a}$		
	$= \underset{\sim}{c} + \frac{1}{2}(k-1)\underset{\sim}{a}$		
	$\overrightarrow{QN} = \overrightarrow{QO} + \overrightarrow{ON}$		
	$= -\frac{1}{2}\underset{\sim}{c} + \frac{1}{2}\underset{\sim}{c} + \frac{1}{2}(k+1)\underset{\sim}{a}$		
	$= \frac{1}{2}(k+1)\underset{\sim}{a}$ using expression for $\overrightarrow{ON}$ from (i)	1	

Question	Suggested solution	Marking guidelines	Markers Notes
16 b)	(ii) Hence if $MP \perp QN$		
	$\left(\underset{\sim}{c} + \frac{1}{2}(k-1)\underset{\sim}{a}\right) \cdot \left(\frac{1}{2}(k+1)\underset{\sim}{a}\right) = 0$		
	$\frac{1}{2}(k+1)\underset{\sim}{a} \cdot \underset{\sim}{c} + \frac{1}{4}(k-1)(k+1)\underset{\sim}{a} \cdot \underset{\sim}{a} = 0$		
	$\frac{1}{2}\cancel{(k+1)}\underset{\sim}{a} \cdot \underset{\sim}{c} = -\frac{1}{4}(k-1)\cancel{(k+1)}\underset{\sim}{a} \cdot \underset{\sim}{a}$		
	$\underset{\sim}{a} \cdot \underset{\sim}{c} = -\frac{1}{2}(k-1)\underset{\sim}{a} \cdot \underset{\sim}{a}$	1	
	$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OC} + \overrightarrow{CB}$		
	$= -\underset{\sim}{a} + \underset{\sim}{c} + k\underset{\sim}{a}$		
	$= \underset{\sim}{c} + (k-1)\underset{\sim}{a}$	1	
	$\therefore \lvert\overrightarrow{AB}\rvert^2 = \left(\underset{\sim}{c} + (k-1)\underset{\sim}{a}\right) \cdot \left(\underset{\sim}{c} + (k-1)\underset{\sim}{a}\right)$		
	$= \underset{\sim}{c} \cdot \underset{\sim}{c} + 2(k-1)\underset{\sim}{a} \cdot \underset{\sim}{c} + (k-1)^2\underset{\sim}{a} \cdot \underset{\sim}{a}$		
	$= \underset{\sim}{c} \cdot \underset{\sim}{c} + (k-1)^2\underset{\sim}{a} \cdot \underset{\sim}{a} + 2(k-1)\left(-\frac{1}{2}(k-1)\right)\underset{\sim}{a} \cdot \underset{\sim}{a}$	if $MP \perp QN$	
	$= \underset{\sim}{c} \cdot \underset{\sim}{c} + \left\{(k-1)^2 - (k-1)^2\right\}\underset{\sim}{a} \cdot \underset{\sim}{a}$		
	$= \underset{\sim}{c} \cdot \underset{\sim}{c}$		
	$= \lvert\overrightarrow{OC}\rvert^2$	1	
c)	(i) $z + z^2 + \dots + z^k$ is a geometric series		
	with $a = z$ and $r = z$ $\quad z \neq 1$		
	$\therefore S_n = \dfrac{z(1-z^k)}{1-z}$		
	$\therefore \displaystyle\sum_{k=1}^{n}(z + z^2 + \dots + z^k) = \sum_{k=1}^{n}\frac{z(1-z^k)}{1-z}$		
	$= \dfrac{z}{1-z}\displaystyle\sum_{k=1}^{n}(1-z^k)$		Note: $\frac{z}{1-z}$ is independent of k
	$= \dfrac{z}{1-z}\left[(1-z) + (1-z^2) + (1-z^3) + \dots + (1-z^n)\right]$		
	$= \dfrac{z}{1-z}\left[(1+1+1+\dots 1) - (z + z^2 + z^3 + \dots + z^n)\right]$		
	$= \dfrac{z}{1-z}\left[n - \dfrac{z(1-z^n)}{1-z}\right]$		

$$= \frac{nz}{1-z} - \frac{z^2(1-z^n)}{(1-z)^2}, \quad z \neq 1$$

Question	Suggested solution	Marking guidelines	Markers Notes
16 b)	(ii) If $z = \cos\theta + i\sin\theta$, then by de Moivre		
	$z^2 = \cos 2\theta + i\sin 2\theta$		
	$z^3 = \cos 3\theta + i\sin 3\theta$ and so on		
	Now $\sum_{k=1}^{n}(z + z^2 + z^3 + \dots z^k)$		
	$= \sum_{k=1}^{n}(\text{cis}\,\theta + \text{cis}\,2\theta + \text{cis}\,3\theta + \dots \text{cis}\,k\theta)$		
	$= \sum_{k=1}^{n}[\cos\theta + \cos 2\theta + \dots \cos k\theta + i(\sin\theta + \sin 2\theta + \dots + \sin 3\theta)]$		
	Now $\text{Im}(z + z^2 + \dots z^k)$		
	$= \sin\theta + \sin 2\theta + \dots + \sin k\theta$		
	$\therefore \text{Im}\sum_{k=1}^{n}(z + z^2 + \dots z^k) = \sum_{k=1}^{n}(\sin\theta + \sin 2\theta + \dots \sin k\theta)$	1	
	Given that $\frac{z}{1-z} = \frac{i}{2\sin\frac{\theta}{2}}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)$		
	$\therefore \text{Im}\,\frac{nz}{1-z} = \text{Im}\,\frac{n}{2\sin\frac{\theta}{2}}\left(i\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)$		
	$= \frac{n}{2\sin\frac{\theta}{2}} \times \cos\frac{\theta}{2}$	1	
	Also $\frac{z^2}{(1-z)^2}(1-z^n) = \left[\frac{i}{2\sin\frac{\theta}{2}}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)\right]^2 \times [1 - (\cos\theta + i\sin\theta)^n]$		
	$= \frac{-1}{4\sin^2\frac{\theta}{2}}(\cos\theta + i\sin\theta)[(1-\cos n\theta) - i\sin(n\theta)]$		
	$\therefore \text{Im}\,\frac{z^2}{(1-z)^2}(1-z^n)$		
	$= \frac{-1}{4\sin^2\frac{\theta}{2}} \times \text{Im}\{(\cos\theta + i\sin\theta)[(1-\cos n\theta) - i\sin(n\theta)]\}$		
	$= \frac{-1}{4\sin^2\frac{\theta}{2}}\{\cos\theta(-\sin(n\theta)) + \sin\theta(1-\cos n\theta)\}$		
	$= \frac{-1}{4\sin^2\frac{\theta}{2}}\{\sin\theta - (\sin(n\theta)\cos\theta + \sin\theta\cos(n\theta))\}$		
	$= \frac{-1}{4\sin^2\frac{\theta}{2}}\{\sin\theta - \sin(n\theta + \theta)\}$		
	$= \frac{-1}{4\sin^2\frac{\theta}{2}}\{\sin\theta - \sin(n+1)\theta\}$	1	

Question	Suggested solution	Marking guidelines	Markers Notes
	$\therefore \ \text{Im}\left\{\frac{nz}{1-z} - \frac{z^2}{(1-z)^2}(1-z^2)\right\}$		
	$= \frac{n\cos\frac{\theta}{2}}{2\sin\frac{\theta}{2}} + \frac{1}{4\sin^2\frac{\theta}{2}}\left\{\sin\theta - \sin(n+1)\theta\right\}$		
	$= \frac{2n\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \sin\theta - \sin(n+1)\theta}{4\sin^2\frac{\theta}{2}}$		
	$= \frac{n\sin\theta + \sin\theta - \sin(n+1)\theta}{4\sin^2\frac{\theta}{2}}$		
	$= \frac{(n+1)\sin\theta - \sin(n+1)\theta}{4\sin^2\frac{\theta}{2}}$	1	