Student Name :________________________ Teacher Name : ______________________

Saint Ignatius' College, Riverview
Mathematics Assessment Task
2025

Year 12
Extension 2 Mathematics
Task 4
Trial Higher School Certificate Examination
Date: 27th August 2025, 1:00 pm

General Instructions:

- **Reading time: 10 mins**
- **Time Allowed: 3 hours**
- Write using blue or black pen only
- Board approved calculators may be used
- Attempt all questions in the space provided on the paper
- Write **your name** and your teachers initial on the cover of each booklet
- Marks may not be awarded for missing or carelessly arranged working
- A reference sheet is provided as a separate booklet.
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

Teachers:

- Mr R Maxwell REM
- Mr D Reidy DPR

Section 1 **10 Marks**
Multiple Choice

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section 2 **90 Marks**
Extended Response

- Attempt Questions 11-16
- Allow about 2 hours and 45 minutes for this section

Total **100 Marks**

SECTION I
10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

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1. The points A, B, C are collinear where $\overrightarrow{OA} = \underset{\sim}{i} + \underset{\sim}{j}$, $\overrightarrow{OB} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$, $\overrightarrow{OC} = 3\underset{\sim}{i} + a\underset{\sim}{j} + b\underset{\sim}{k}$
What are the values of a and b ?

A. $a = -3,\ b = -2$

B. $a = 3,\ b = -2$

C. $a = -3,\ b = 2$

D. $a = 3,\ b = 2$

2. The negation of the statement $\forall x \in \mathbb{R}, \exists\, y \in \mathbb{R}$ such that $2x + 3y = 12$ is :

A. $\forall x \in \mathbb{R}, \exists\, y \in \mathbb{R}$ such that $2x + 3y \neq 12$;

B. $\exists\, x \in \mathbb{R}, \forall\, y \in \mathbb{R}\quad 2x + 3y \neq 12$;

C. $\forall x \in \mathbb{R}, \forall\, y \in \mathbb{R}\quad 2x + 3y \neq 12$;

D. $\exists\, x \in \mathbb{R}, \exists\, y \in \mathbb{R}$ such that $2x + 3y \neq 12$.

3. Which of the following is the converse of $\sim P \Rightarrow Q$?

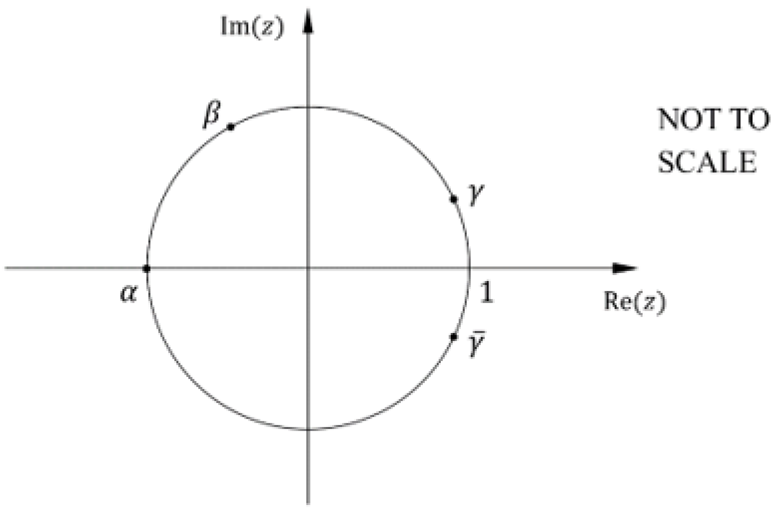
A. $\sim Q \Rightarrow P$

B. $Q \Rightarrow \sim P$

C. $P \Rightarrow Q$

D. $\sim P \Rightarrow \sim Q$

4. Four roots of the polynomial $P(x)$ lie on the unit circle on a complex plane, as shown below. There may be other roots.



Which statement about $P(x)$ MUST be True.

A. If $P(x)$ is a quartic then all its coefficients are real.

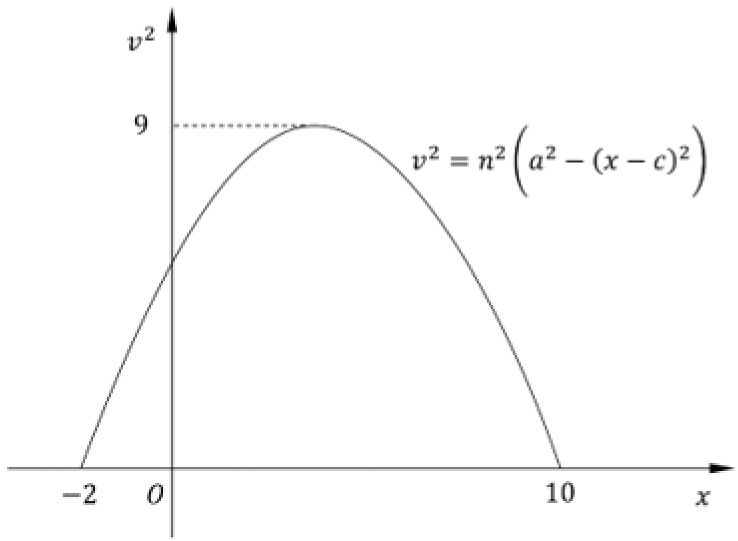
B. If $P(x)$ is a quartic then at least one of its coefficients is real.

C. If all of the coefficients of $P(x)$ are real then its degree is at least 5.

D. $P(x)$ is in the form $z^n + 1 = 0$ where n is odd and at least 5.

5. A particle is moving along the x -axis in simple harmonic motion. The displacement of the particle is x metres and its velocity is $v\ ms^{-1}$

The graph of $v^2 = n^2\left(a^2 - (x-c)^2\right)$, where a , c and n are positive constants, is shown below.



NOT TO SCALE

The values of a , c and n are :-

A. $a = 4,\ c = 6$ and $n = \frac{1}{2}$

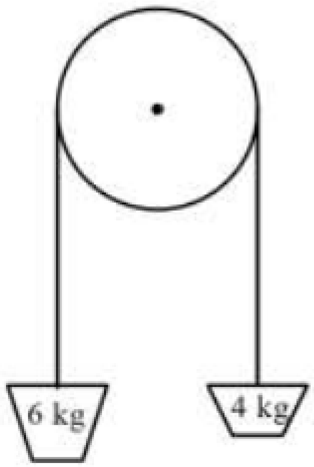
B. $a = 4,\ c = 6$ and $n = \frac{3}{2}$

C. $a = 6,\ c = 4$ and $n = \frac{1}{2}$

D. $a = 6,\ c = 4$ and $n = \frac{3}{2}$

.

6. Particles of mass 6 kg and 4 kg are attached to each end of a light inextensible string. The string passes over a pulley as shown in the diagram below.



Which expression gives the acceleration of the 6 kg mass as it moves downwards ?

A. $\frac{1}{5}g$

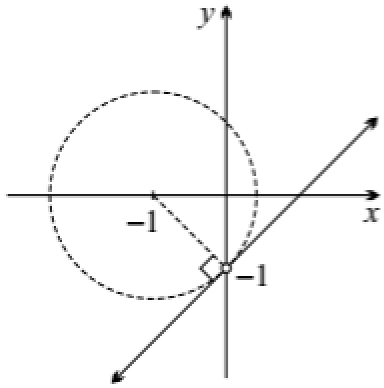
B. $\frac{2}{5}g$

C. $2.5\,g$

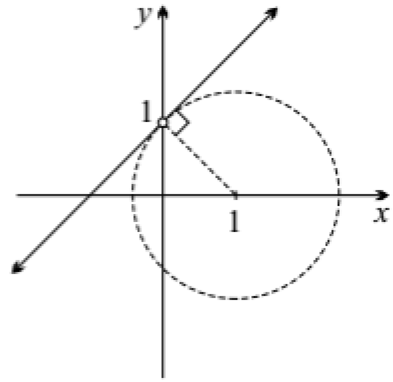
D. $5\,g$

7. If $\omega = \dfrac{z+1}{z+i}$ and ω is purely imaginary, what is the locus of z ?

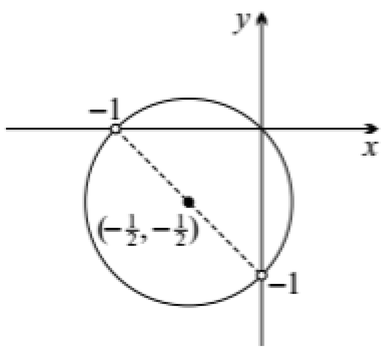
(A)



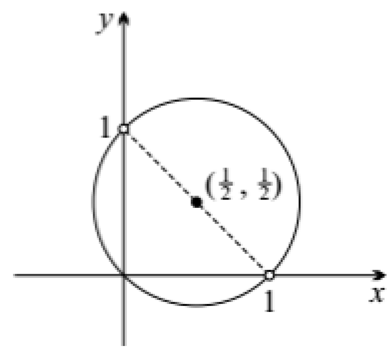
(B)



(C)



(D)



8. Which of the following is equivalent to $\dfrac{e^{-\frac{i\pi}{2}}}{e^{\frac{i\pi}{6}}}$?

A. $-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$

B. $-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

C. $\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$

D. $\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

9. The complex number $z = a + ib$, where $0 < a < b$.

Which of the following best describes the complex number z^4 ?

A. $Re\left(z^4\right) < 0$

B. $Re\left(z^4\right) \leq 0$

C. $Im\left(z^4\right) < 0$

D. $Im\left(z^4\right) \leq 0$

10. Suppose there exist vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{r}$ such that $\underset{\sim}{r} = \underset{\sim}{a} + \left(\underset{\sim}{a} \cdot \underset{\sim}{b}\right)\underset{\sim}{b}$.

If $\underset{\sim}{r}$ is perpendicular to $\left(\underset{\sim}{a} - \underset{\sim}{b}\right)$, the minimum value of $1 + \left|\underset{\sim}{b}\right|^2$ is ?

A. $\left|\underset{\sim}{a}\right| + 1$

B. $\dfrac{\left|\underset{\sim}{a}\right| + 1}{2}$

C. $2\left|\underset{\sim}{a}\right|$

D. $\left|\underset{\sim}{a}\right|$

END OF SECTON 1

SECTION II
90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section
Answer each question in a **SEPARATE** writing booklet. Extra writing booklets are available.
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations

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Marks

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Find $\int xe^{-2x}\,dx$. **2**

(b) If $z = 2+i$ and $\omega = 3-2i$, express in the form $x+iy$

(i) $2z+\overline{\omega}$ **1**

(ii) $\dfrac{1}{z-w}$ **1**

(c) Consider the lines

$$l_1 : \quad x = y = z$$

$$l_2 = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

where t is a parameter. **2**

Find the angle between the two lines.

(d) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \frac{d\theta}{4+2\sin 2\theta}$, using the substitution, $t = \tan\theta$. **3**

Question 11 continued on the next page

Marks

(e) Consider the complex number $z = -1+\sqrt{3}\,i$.

(i) Write z in modulus-argument form. **2**

(ii) Find the exact value of z^{10} in exact cartesian form. **1**

(f) The locus of a point P which moves in the complex plane is represented by the equation

$|z-3-4i| = 5$.

(i) Sketch the locus of the point P, noting intercepts. **2**

(ii) Find the exact value of $arg\ z$ when P is in the position of maximum modulus. **1**

End of Question 11

Marks

Question 12 (14 marks) Use a SEPARATE writing booklet.

(a) Find the point(s) of intersection of the line with parametric equation

$\underset{\sim}{r} = \underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k} + t\left(\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}\right)$, with the sphere with equation

$(x-1)^2 + (y-3)^2 + (z+4)^2 = 81$ **3**

(b) Use partial fraction to find

$$\int \frac{x^2 + 8x + 11}{(x-3)(x^2+2)}\,dx$$ **3**

(c) For s, a real variable, the complex number z moves on the Argand plane according to the formulae $z = \dfrac{1}{1+i\,s}$ as s varies from $-\infty$ to $+\infty$.

(i) Find the value of s for which $z = \dfrac{1}{2} + \dfrac{1}{2}i$. **1**

(ii) The locus of z is, in fact, a circle. Find the cartesian equation of the locus, writing down its centre and radius. **3**

(d) The equation of two lines are

$$\underset{\sim}{r_1} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} \quad \text{and} \quad \underset{\sim}{r_2} = \begin{pmatrix} -4 \\ 6 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -11 \\ 3 \end{pmatrix}$$

These lines are not parallel so determine if they skew or whether they intersect, showing all necessary working. **2**

Question 12 continued on the next page

Marks

(e) Show that for $x \in \mathbb{Z}$, if $x^2 - 6x + 5$ is even, then x is odd. **2**

End of Question 12

Question 13 (16 marks) Use a SEPARATE writing booklet **Marks**

(a) Find the point on the line with vector equation $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ that is closest to the origin. **3**

(b) A particle is moving along the x axis in simple harmonic motion centred at $x = 2$ and with amplitude 5.

The velocity of the particle is given by $v^2 = p + 8x - qx^2$, where p and q constants.

Find the exact numerical value for the period of the motion. **3**

(c) A particle is moving in the positive direction along a straight line in a medium that exerts a resistance to motion proportional to the cube of the velocity.

No other forces act on the particle, that is $\ddot{x} = -kv^3$, where k is a positive constant.

At time $t = 0$, the particle is at the origin and has velocity U. At time $t = T$, the particle is at $x = D$ and has a velocity V.

(i) Show that $\dfrac{1}{V^2} - \dfrac{1}{U^2} = 2kT$. **3**

(ii) Also, show that $\dfrac{1}{V} - \dfrac{1}{U} = kD$. **3**

(iii) Hence show that $\dfrac{D}{T} = \dfrac{2UV}{U+V}$. **1**

(d) (i) Prove that $a^2 + b^2 + c^2 \geq ab + ac + bc$. **2**

(ii) Hence, or otherwise prove that $a^2b^2 + a^2c^2 + b^2c^2 \geq abc\,(a + b + c)$ **1**

End of Question 13

Marks

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) Suppose that n and $n+1$ are positive integers, neither of which is divisible by 3. **2**

Prove that $n^3 + (n+1)^3$ is divisible by 9.

(b) The complex number ω is given as $\omega = \dfrac{z-1}{z-2i}$.

Describe the locus of z geometrically if ω is real. **2**

(c) Suppose that p is a prime number and a is a positive integer.

(i) Use the expansion of $(1+a)^p$ to prove that $(1+a)^p - 1 - a^p$ is divisible by p.

(You may use the fact that $\begin{pmatrix} p \\ k \end{pmatrix}$ is divisible by p for $k = 1,2,3,\ldots\ldots,p-1$.) **1**

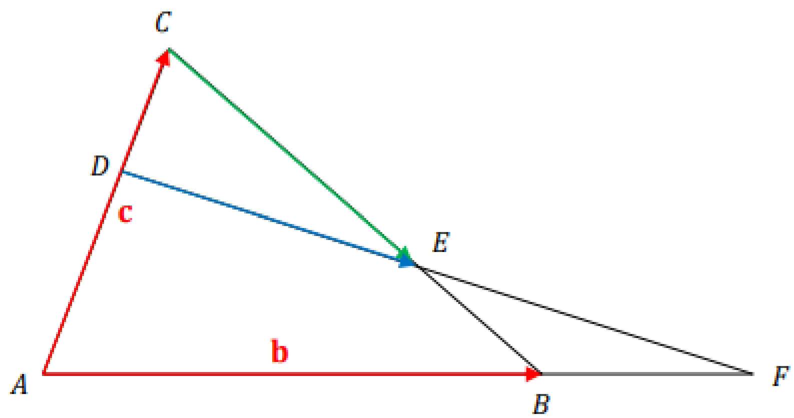
(ii) Hence prove by mathematical induction that $a^p - a$ is divisible by p for all positive integers values of a. **3**

(d) Find $\displaystyle\int \frac{x^2}{x+1}\,dx$. **2**

Question 14 continued on the next page

Marks

(e)



In ΔABC above, D is the point on AC such that $AD : DC = 2 : 1$ and E is the point on BC such that $BE : EC = 1 : 2$. When DE is extended, it meets AB extended at F.

Let $\overrightarrow{AB} = b$ and $\overrightarrow{AC} = c$.

(i) Express $\overrightarrow{CE}$ in terms of b and c. **1**

(ii) Show that $\overrightarrow{DE} = \frac{2}{3} b - \frac{1}{3} c$. **1**

(iii) Show that $AF : FB = 4 : 1$ **3**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Let $I_n = \int_0^1 (1-x^2)^{\frac{n-1}{2}}\, dx$. where $n = 1,2,3,\ldots\ldots\ldots$ show that

Show that $n\, I_n = (n-1)\, I_{n-2}$ for $n = 2,3,4,\ldots\ldots\ldots$ **3**

(b) A particle of mass m is moving in a straight line under the action of a force F where $F = \dfrac{m}{x^3}(6-10x)$ and x is its displacement from the origin.

(i) Find its velocity if the particle starts from rest at $x = 1$. **3**

(ii) At what other position does the particle come to rest. **1**

(c) Find $\displaystyle\int_1^2 \frac{x+1}{\sqrt{-2+3x-x^2}}\, dx$ **3**

Question 15 continued on the next page

Marks

(d) Consider the line $r = \lambda \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

(i) Show that the position vector of the point on the line $\underset{\sim}{r}$ closest to the point (x_0, y_0, z_0) is : **2**

$$\frac{x_0 + 2y_0 + z_0}{6} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

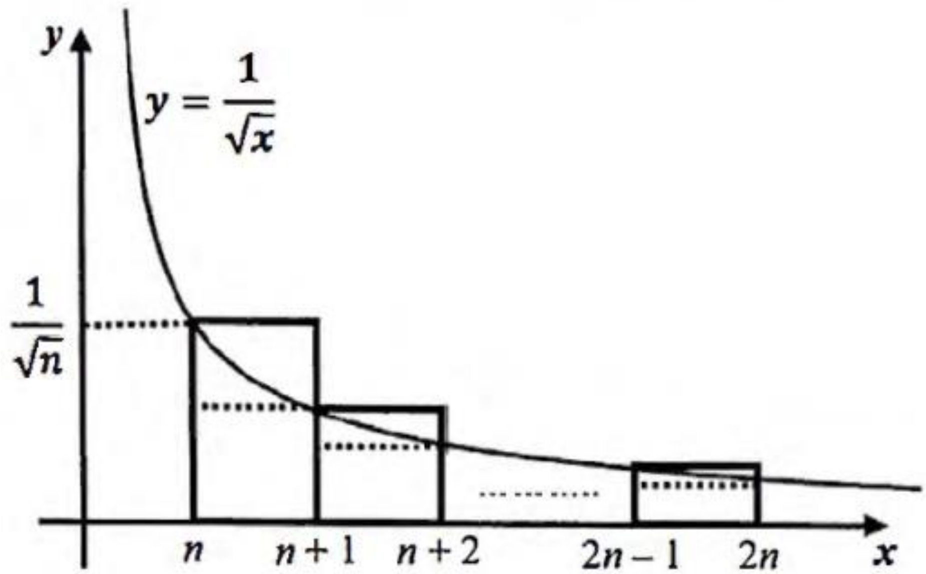
(ii) Hence, or otherwise, find the point on the line $\underset{\sim}{r}$ that is closest to a second line **3**

$$\underset{\sim}{s} = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}, \text{ where } t \in (-\infty, \infty).$$

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) In the diagram below, the graph of $y = \dfrac{1}{\sqrt{x}}$ has been drawn, and n rectangles have been constructed between $x = n$ and $x = 2n$, each of width 1 unit.



(i) Show that $\displaystyle\int_n^{2n} \frac{dx}{\sqrt{x}} = 2\sqrt{n}\left(\sqrt{2}-1\right)$. **1**

(ii) Let $S_n = \dfrac{1}{\sqrt{n+1}} + \dfrac{1}{\sqrt{n+2}} + \dots\dots\dots\dots + \dfrac{1}{\sqrt{2n}}$

Show that ;

$$2\sqrt{n}\left(\sqrt{2}-1\right) + \frac{1-\sqrt{2}}{\sqrt{2n}} < S_n < 2\sqrt{n}\left(\sqrt{2}-1\right)$$ **4**

Question 16 continued on the next page

Marks

(b) A particle's movement is represented by the vector $\underset{\sim}{r} = \begin{pmatrix} t + t^{-1} \\ t^3 + t^{-3} \end{pmatrix}$, where $t \in \mathbb{R}^+$

Sketch the Cartesian graph that shows the path the particle can move along. **3**

(c) A particle A of mass m falls vertically from rest at a height h above the ground. It is Subject to a resistance mkv where v is its speed and k is a positive constant. The acceleration due to gravity is g.

(i) Show that $\ddot{x} = k(U - v)$, where U is the terminal speed of the particle. **1**

(ii) Prove that the speed of particle A at any time t is given by $v = U\left(1 - e^{-kt}\right)$ **3**

(iii) At the same time as particle A falls from rest another particle B of mass m is projected upwards in the same vertical line as particle A with a speed of kU, where U is the terminal speed of the particle. Hence it can be shown that for particle B its speed at any time t is given by

$v = kU\, e^{-kt} - \; U\left(1 - e^{-kt}\right)$. **(DO NOT PROVE THIS)**

If the two particles collide after time T seconds when particle A has fallen a distance of X, prove that **3**

$$T = \frac{1}{k}\log_e\left(\frac{U}{U-h}\right)$$

End of Exam

Year 12 - 2025 Term 3 Mathematics (Ext 2) Assessment Task 4 Trial HSC Exam Solutions

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

1. $\overrightarrow{OA} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\overrightarrow{OB} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

$\overrightarrow{OC} = \begin{bmatrix} 3 \\ a \\ b \end{bmatrix}$

$\overrightarrow{AB} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$\overrightarrow{BC} = \begin{bmatrix} 3 \\ a \\ b \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ a+1 \\ b-1 \end{bmatrix}$

$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ a+1 \\ b-1 \end{bmatrix}$

$1 = \lambda$

$-2 = a\lambda + \lambda \Rightarrow a = -3$

$1 = b\lambda - \lambda \Rightarrow b = 2$

(C)

2. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} :$

$2x + 3y = 12$

Negation

$\exists x \in \mathbb{R}, \forall y \in \mathbb{R}$

$2x + 3y \neq 12$

(B)

3. $\sim P \Rightarrow Q$

Converse is

$Q \Rightarrow \sim P$

(B)

4.

Roots are $\gamma, \bar{\gamma}$,

α (real) $= -1$, β

If coefficients of $P(x)$ are real then

it degree must be at least 5 as $\bar{\beta}$ is needed

(C)

NOTE: D could be true but not must be true.

5. $v^2 = n^2(a^2 - (x-c)^2)$

$v = 0 \quad x = -2$ or 10

$\therefore$ centre is $x = 4$

amplitude $a = 6$

max velocity at $x = c$

ie $v^2 = n^2 a^2$

$n^2 = \frac{9}{36} = \frac{1}{4} \quad \therefore n = \frac{1}{2}$

$a = 6, \; c = 4, \; n = \frac{1}{2}$

(C)

Year 12 - 2025 Term 3 Mathematics (Ext 2) Assessment Task 4 Trial HSC Exam Solutions

Suggested Solution (s) | **Comments**

6.

At $m = 6\text{kg}$

$F_{\text{net down}} = Mg - T$

$= 6g - T$

$\ddot{x} = \frac{6g - T}{6}$

At $m = 4\text{kg}$

$F_{\text{net up}} = T - Mg$

$= T - 4g$

$\ddot{x} = \frac{T - 4g}{4}$

$\frac{6g - T}{6} = \frac{T - 4g}{4}$

$24g - 4T = 6T - 24g$

$10T = 48g$

$T = \frac{48g}{10}$

$\ddot{x}_{6kg} = \frac{6g - \frac{48g}{10}}{6}$

$= g - \frac{4}{5}g$

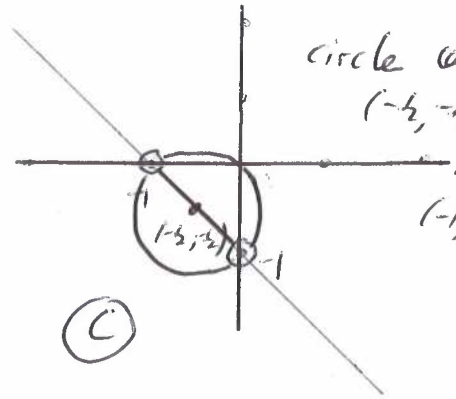
$= \frac{1}{5}g$

(A)

7.

$w = \frac{z+1}{z+i}$ is purely imaginary

$\therefore \arg(w) = \arg(z+1) - \arg(z+i) = \pm\frac{\pi}{2}$



circle centre $(-\frac{1}{2}, -\frac{1}{2})$ but not including $(-1, 0)$ or $(0, -1)$

(C)

8.

$\frac{e^{-i\frac{\pi}{2}}}{e^{i\frac{\pi}{6}}}$

$= e^{-i\frac{\pi}{2} - i\frac{\pi}{6}} = e^{-\frac{4i\pi}{6}}$

$= e^{-2i\frac{\pi}{3}}$

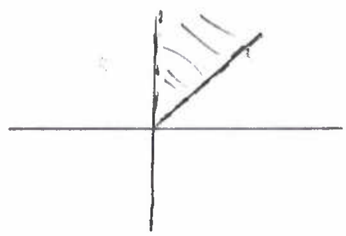
$= \cos(-\frac{2\pi}{3}) + i\sin(-\frac{2\pi}{3})$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

(A)

Suggested Solution (s)	Comments

9. $z = a + ib \quad 0 \leq a < b$



$\frac{\pi}{4} < \arg(z) < \frac{\pi}{2}$

$\therefore \pi < \arg(z^4) < 2\pi$

$\therefore \operatorname{Im}(z^4) < 0$

(C)

10. $\mathbf{r} \perp (\mathbf{a} - \mathbf{b})$

$\therefore \mathbf{r} \cdot (\mathbf{a} - \mathbf{b}) = 0$

$\mathbf{r} \cdot \mathbf{a} = \mathbf{r} \cdot \mathbf{b}$

Subing $\mathbf{r} = \mathbf{a} + (\mathbf{a} \cdot \mathbf{b})\mathbf{b}$

$\mathbf{a} \cdot [\mathbf{a} + (\mathbf{a} \cdot \mathbf{b})\mathbf{b}] = \mathbf{b} \cdot [\mathbf{a} + (\mathbf{a} \cdot \mathbf{b})\mathbf{b}]$

$|\mathbf{a}|^2 + (\mathbf{a} \cdot \mathbf{b})^2 = \mathbf{a} \cdot \mathbf{b} + (\mathbf{a} \cdot \mathbf{b})|\mathbf{b}|^2$

$= \mathbf{a} \cdot \mathbf{b}\,(1 + |\mathbf{b}|^2)$

$\therefore (\mathbf{a} \cdot \mathbf{b})^2 - (\mathbf{a} \cdot \mathbf{b})(1 + |\mathbf{b}|^2) + |\mathbf{a}|^2 = 0$

quadratic in $\mathbf{a} \cdot \mathbf{b}$

$\therefore$ real solutions

$\Delta = (1 + |\mathbf{b}|^2)^2 - 4|\mathbf{a}|^2 > 0$

$1 + |\mathbf{b}|^2 > 2|\mathbf{a}|$

min value $2|\mathbf{a}|$

(C)

11.

(a) $\int x e^{-2x}\,dx$

$= x \cdot -\frac{1}{2} e^{-2x} - \int -\frac{1}{2} e^{-2x}\,dx$

$= -\frac{1}{2} x e^{-2x} + \frac{1}{2} \int e^{-2x}\,dx$

$= -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C$

OR

$= -\frac{1}{4} e^{-2x}(2x + 1) + C$

(b) $z = 2 + i \quad w = 3 - 2i$

(i) $2z + \bar{w}$

$= 2(2 + i) + (3 + 2i)$

$= 7 + 4i$

Year 12 - 2025 Term 3 Mathematics (Ext 2) Assessment Task 4 Trial HSC Exam Solutions

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

(ii) $\dfrac{1}{z-w}$

$= \dfrac{1}{2+i-(3-2i)}$

$= \dfrac{1}{-1+3i} \times \dfrac{-1-3i}{-1-3i}$

$= \dfrac{-1-3i}{(-1)^2-(3i)^2} = \dfrac{-1-3i}{1+9}$

$= -\frac{1}{10} - \frac{3}{10}i$ ✓

(c)

$\ell_1 : x = y = z$

$\ell_2 : \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

$\cos\alpha = \dfrac{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}}{\sqrt{3}\,\sqrt{6}} = 0$ ✓

$\therefore$ lines perpendicular

$\alpha = 90^0$ ✓

(d) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{d\theta}{4+2\sin 2\theta}$

$t = \tan\theta$

$dt = \sec^2\theta \, d\theta$

$= (1+\tan^2\theta)\, d\theta$

$= (1+t^2)\, d\theta$

$d\theta = \dfrac{dt}{1+t^2}$

$\theta = \frac{\pi}{4} \quad t = 1$

$\theta = 0 \quad t = 0$

$I = \displaystyle\int_0^1 \frac{1}{4+2\left(\frac{2t}{1+t^2}\right)} \frac{dt}{1+t^2}$ ✓

$= \displaystyle\int_0^1 \frac{dt}{4t^2+4t+4}$

$= \frac{1}{4}\displaystyle\int_0^1 \frac{dt}{t^2+t+1}$

$= \frac{1}{4}\displaystyle\int_0^1 \frac{dt}{(t+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$

$= \frac{1}{4}\left[\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)\right]_0^1$

$= \frac{1}{2\sqrt{3}}\left(\tan^{-1}\sqrt{3} - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$

$= \frac{1}{2\sqrt{3}}\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \dfrac{\pi}{12\sqrt{3}}$ or $\dfrac{\sqrt{3}\pi}{36}$

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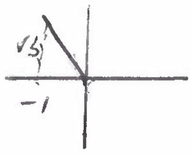
Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

e) $z = -1+\sqrt{3}i$

(i) $|z| = \sqrt{(-1)^2+(\sqrt{3})^2} = 2$ ✓

arg(z)



$= \pi - \tan^{-1}\sqrt{3}$

$= \frac{2\pi}{3}$ ✓

$z = 2\operatorname{cis}\frac{2\pi}{3}$

(ii) $z^{10} = \left(2\operatorname{cis}\frac{2\pi}{3}\right)^{10}$

$= 2^{10}\left(\cos\frac{20\pi}{3} + i\sin\frac{20\pi}{3}\right)$

$= 2^{10}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ ✓

$= 2^{10}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

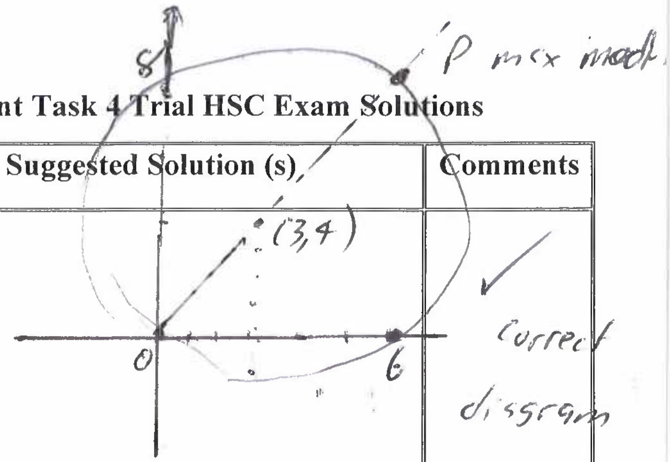
$= 2^{9}(-1+\sqrt{3}i)$ ✓

(f) $|z-3-4i|$

$= |z-(3+4i)| = 5$

circle centre (3,4) radius 5

(i)



✓ correct diagram

$(x-3)^2+(y-4)^2 = 25$ ✓ correct intercepts

$x=0 \quad y-4 = \pm 4$

$y = 0$ or 8

$y=0 \quad x = 0, 6$

(ii) when P has max modulus it will be on diameter opposite origin

$\arg(z) = \tan^{-1}\left(\frac{4}{3}\right)$ ✓

12.

(a) $\underset{\sim}{r} = \underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k} + t(\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k})$

$\underset{\sim}{r} = \begin{pmatrix}1\\3\\-4\end{pmatrix} + t\begin{pmatrix}1\\2\\2\end{pmatrix}$

Sphere $\left|\begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}1\\3\\-4\end{pmatrix}\right| = 9$

from $\underset{\sim}{r}$ $\quad x = 1+t$

$y = 3+2t$

$z = -4+2t$

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Sub into equat of sphere $(1+t-1)^2+(3+2t-3)^2+(-4+2t+4)^2=81$ ✓ $t^2+4t^2+4t^2=81$ $t^2=9$ $t=\pm 3$ ✓ $\therefore$ Points of intersection are $t=3$ $\begin{bmatrix}4\\9\\2\end{bmatrix}$ $t=-3$ $\begin{bmatrix}-2\\-3\\-10\end{bmatrix}$ ✓		$\underline{x=1}$ $20=12-2B+2$ $B=-3$ $I=\int \frac{4}{x-3}+\frac{-3x-1}{x^2+2}$ $=4\ln(x-3)-\frac{3}{2}\int\frac{2x}{x^2+2}-\int\frac{1}{x^2+2}$ $=4\ln(x-3)-\frac{3}{2}\ln(x^2+2)-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+C$	✓ ✓
(b) $\frac{x^2+8x+11}{(x-3)(x^2+2)}=\frac{A}{x-3}+\frac{Bx+C}{x^2+2}$ $x^2+8x+11=A(x^2+2)+(Bx+C)(x-3)$ $\underline{x=3}$ $11A=44$ $A=4$ $\underline{x=0}$ $11=2A-3C$ $\therefore 3C=8-11$ $C=-1$ ✓		(c) $z=\frac{1}{1+is}$ $-\infty<s<\infty$ (i) $\frac{1}{1+is}=\frac{1}{2}+\frac{1}{2}i$ $\frac{2}{1+is}=1+i$ $2=(1+i)(1+is)$ $2=(1-s)+i(1+s)$ $\therefore 2=1-s \Rightarrow \underline{s=-1}$ (ii) Let $z=x+iy$	✓

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
$\therefore x+iy = \frac{1}{1+is}$ $(x+iy)(1+is) = 1$ $x - ys + i(y+xs) = 1$ $x - ys = 1 \qquad y + xs = 0$ $s = \frac{x-1}{y}$ $\therefore y + \frac{x(x-1)}{y} = 0$ $x^2 - x + y^2 = 0$ $(x-\frac{1}{2})^2 + y^2 = \frac{1}{4}$ $(x-\frac{1}{2})^2 + y^2 = (\frac{1}{2})^2$ circle centre $(\frac{1}{2}, 0)$ radius $r = \frac{1}{2}$.	✓ ✓ ✓	$x = 2+\lambda = -4+4\mu \ldots$ $y = -3+4\lambda = 6-11\mu \ldots$ $z = 1+\lambda = -2+3\mu \ldots$ from (3) $\lambda = -3+3\mu$ sub into (1) $2+(-3+3\mu) = -4+4\mu$ $\mu = 3 \quad \& \quad \lambda = 6$ ✓ Check in (2) $LHS = -3+4(6) = 21$ ✓ $RHS = 6-11(3) = -27$ $LHS \neq RHS$ $\therefore$ lines are skew	(1) (2) (3)
(d) $\underset{\sim}{r_1} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$ $\underset{\sim}{r_2} = \begin{pmatrix} -4 \\ 6 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -11 \\ 3 \end{pmatrix}$		(e) $x \in \mathbb{R}$: if x^2-6x+5 is even then x is odd. Using contrapositive let x be even ie $x = 2k \qquad k \in \mathbb{Z}$ $\therefore x^2-6x+5 = (2k)^2-6(2k)+5$	

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
$= 4k^2 - 12k + 4 + 1$ $= 2(2k^2 - 6k + 2) + 1$ $= 2p + 1 \quad p \in \mathbb{Z}$ $\therefore x^2 - 6x + 5$ is odd $\therefore$ by contrapositive if $x^2 - 6x + 5$ even then x is odd.		$\lambda = -2$ point $= \begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix} + (-2)\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}$ closest to origin	
13. (a) $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ point on r $x = 1 + 2\lambda$ $y = 4 + \lambda$ $z = 6 + \lambda$ distance from origin $d^2 = (1+2\lambda)^2 + (4+\lambda)^2 + (6+\lambda)^2$ $= 1 + 4\lambda + 4\lambda^2 + 16 + 8\lambda + \lambda^2 + 36 + 12\lambda + \lambda^2$ $= 6\lambda^2 + 24\lambda + 53$ $\frac{d\,d^2}{d\lambda} = 12\lambda + 24 = 0$ at max/min $\lambda = -2$		(b)  $v^2 = p + 8x - qx^2$ $\ddot{x} = \frac{d}{dx}(\frac{1}{2}v^2)$ $= \frac{d}{dx}(\frac{1}{2}p + 4x - \frac{q}{2}x^2)$ $= 4 - qx$ $= -q(x - \frac{4}{q})$ Centre at $x = 2$ $\therefore \frac{4}{q} = 2$ $q = 2$ $n^2 = q = 2$ $n = \sqrt{2}$ period $= \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$ OR	

$\frac{d\,d^2}{d\lambda^2} = 12 > 0 \quad \therefore$ min

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Suggested Solution (s) | **Comments**

$$v^2 = n^2(a^2 - (x-c)^2)$$

$$v^2 = n^2(25 - (x-2)^2)$$

$$= n^2(25 - x^2 + 4x - 4)$$

$$= 21n^2 + 4n^2x - n^2x$$

$$p = 21n^2 \quad q = n^2$$

$$4n^2 = 8$$

$$n = \pm\sqrt{2}$$

$$P = \sqrt{2}\pi$$

(c) $\ddot{x} = -kv^3$

$t=0 \quad x=0 \quad v=U$

$t=T \quad x=D \quad v=V$

(i) $\dfrac{dv}{dt} = -kv^3$

$$\frac{1}{v^3}\,dv = -k$$

$$\int_U^V v^{-3}\,dv = \int_0^T -k\,dt \quad ✓$$

$$\left[\frac{v^{-2}}{-2}\right]_U^V = \left[-kt\right]_0^T \quad ✓$$

$$-\frac{1}{2V^2} + \frac{1}{2U^2} = -kT \quad ✓$$

$$\frac{1}{V^2} - \frac{1}{U^2} = 2kT$$

(ii)

$$v\frac{dv}{dx} = -kv^3$$

$$\int_U^V \frac{1}{v^2}\,dv = \int_0^D -k\,dx \quad ✓$$

$$\left[-\frac{1}{v}\right]_U^V = \left[-kx\right]_0^D \quad ✓$$

$$-\frac{1}{V} + \frac{1}{U} = -kD \quad ✓$$

$$\frac{1}{V} - \frac{1}{U} = kD$$

(iii) from (i)

$$\frac{1}{V^2} - \frac{1}{U^2} = 2kT$$

$$\left(\frac{1}{V} - \frac{1}{U}\right)\left(\frac{1}{V} + \frac{1}{U}\right) = 2kT$$

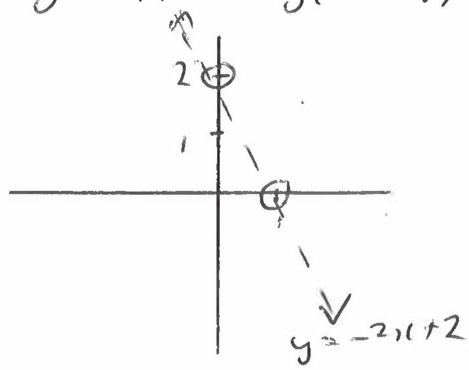
$$kD\left(\frac{V+U}{VU}\right) = 2kT \quad ✓$$

$$\frac{D}{T} = \frac{2VU}{U+V}$$

(d) (i)

$$(a-b)^2 = a^2 + b^2 - 2ab \geq 0$$

$$a^2 + b^2 \geq 2ab$$

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Similarly $a^2+c^2 \geq 2ac$ $b^2+c^2 \geq 2bc$ $\therefore 2(a^2+b^2+c^2) \geq 2(ab+ac+bc)$ $a^2+b^2+c^2 \geq ab+ac+bc$ (ii) from (i) $x^2+y^2+z^2 \geq xy+xz+yz$ let $x=ab$ $y=ac$ $z=bc$ $(ab)^2+(ac)^2+(bc)^2 \geq a^2bc+ab^2c$ $+abc^2$ $a^2b^2+a^2c^2+b^2c^2 \geq abc(a+b+c)$		$n^3+(n+1)^3=(3k+1)^3+(3k+2)^3$ $=27k^3+27k^2+9k+1+27k^3+54k^2$ $+36k+8$ $=54k^3+81k^2+45k+9$ $=9[6k^3+9k^2+5k+1]$ $=9Q \quad Q\in\mathbb{Z}$ $\therefore n^3+(n+1)^3$ is divisible by 9	3
Q14. (a) If $n, n+1$ are not divisible by 3 then let $n=3k+1$ $n=3k+2$ $k\in\mathbb{Z}$		(b) $w=\dfrac{z-1}{z-2i}$ if w real $\Rightarrow$ $\arg(w)=0$ or π $\arg(z-1)-\arg(z-2i)=0, \pi$  $\therefore$ locus is the line $y=-2x+2$ excluding $x=0$ and $x=1$	

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Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

(c) p is prime

(i) a +ve integer ie $\in \mathbb{Z}^+$

$(1+a)^p = 1 + {}^pC_1 a + {}^pC_2 a^2 + \dots + {}^pC_p a^p$

$\therefore (1+a)^p - 1 - a^p$

$= {}^pC_1 a + {}^pC_2 a^2 + \dots + {}^pC_{p-1} a^{p-1}$ ✓

${}^pC_k$ is divisible by p

$\therefore (1+a)^p - 1 - a^p$ is divisible by p

(ii) $a^p - a$

let $a = 1$

$a^p - a = 1^p - 1 = 0 = p \times 0$

$\therefore$ true for $a = 1$ ✓

Assume true for $a = k$

$k^p - k = pM \quad M \in \mathbb{Z}$

let $a = k+1$

LHS $= (k+1)^p - (k+1)$

$= (k+1)^p - 1 - k$

$= (k+1)^p - 1 - k^p + k^p - k$ ✓

$= p\alpha + k^p - k$ from (i) $\alpha \in \mathbb{Z}$ ✓

$= p\alpha + pM$ from assumption

$= pN; \quad N \in \mathbb{Z}$

$\therefore$ true for $a = k+1$

$\therefore$ true by Mathematical Induction

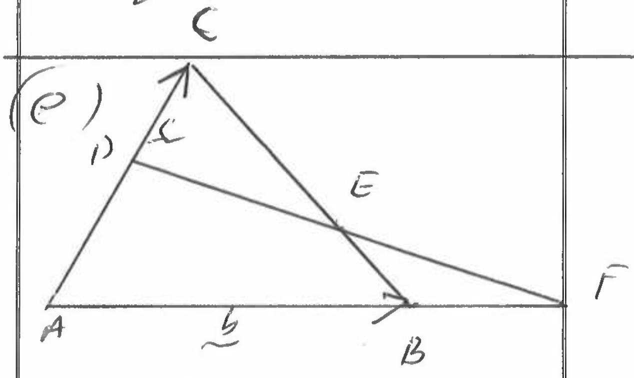
ie $a^p - a$ is divisible by p, $a \in \mathbb{Z}^+$

(d) $\displaystyle\int \frac{x^2}{x+1}\,dx$

$= \displaystyle\int x - 1 + \frac{1}{x+1}\,dx$ ✓

$= \dfrac{x^2}{2} - x + \ln(x+1) + C$ ✓

(e)



$AD : DC = 2 : 1$

$BE : EC = 1 : 2$

(i) $\overrightarrow{CB} = \underset{\sim}{b} - \underset{\sim}{c}$

$\overrightarrow{CE} = \frac{2}{3}\overrightarrow{CB}$

$= \frac{2}{3}(\underset{\sim}{b} - \underset{\sim}{c})$ ✓

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Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

(ii)

$\overrightarrow{DE} = \overrightarrow{DC} + \overrightarrow{CE}$

$= \frac{1}{3}\underset{\sim}{c} + \frac{2}{3}(\underset{\sim}{b} - \underset{\sim}{c})$

$= \frac{2}{3}\underset{\sim}{b} - \frac{1}{3}\underset{\sim}{c}$

(iii) $\overrightarrow{EF}$ is a scalar multiple of $\overrightarrow{DE}$ and $\overrightarrow{BF}$ is a scalar multiple of $\overrightarrow{AB}$

So $\overrightarrow{EF} = k\,\overrightarrow{DE} = k(\frac{2}{3}\underset{\sim}{b} - \frac{1}{3}\underset{\sim}{c})$

$\overrightarrow{BF} = l\,\overrightarrow{AB} = l\,\underset{\sim}{b}$; $k, l \in \mathbb{Z}$

$\overrightarrow{EB} + \overrightarrow{BF} = \overrightarrow{EF}$

$\therefore \frac{1}{3}(\underset{\sim}{b} - \underset{\sim}{c}) + l\underset{\sim}{b} = k(\frac{2}{3}\underset{\sim}{b} - \frac{1}{3}\underset{\sim}{c})$

$\frac{1}{3}\underset{\sim}{b} - \frac{1}{3}\underset{\sim}{c} + l\underset{\sim}{b} = \frac{2k}{3}\underset{\sim}{b} - \frac{k}{3}\underset{\sim}{c}$

as $\underset{\sim}{b}$ & $\underset{\sim}{c}$ not parallel

$\frac{1}{3} + l = \frac{2k}{3}$

$-\frac{1}{3} = -\frac{k}{3} \Rightarrow k = 1$

$l = \frac{1}{3}$

$\therefore \overrightarrow{BF} = \frac{1}{3}\overrightarrow{AB}$

so $AF : FB = 4 : 1$

15(a)

$I_n = \int_0^1 (1-x^2)^{\frac{n-1}{2}}\,dx$

$I_n = \left[x(1-x^2)^{\frac{n-1}{2}}\right]_0^1 - \frac{n-1}{2}\int_0^1 x(1-x^2)^{\frac{n-3}{2}}(-2x)\,dx$

$= 0 - (n-1)\int_0^1 (-x^2)(1-x^2)^{\frac{n-3}{2}}\,dx$

$= 0 - (n-1)\int ((1-x^2-1))(1-x^2)^{\frac{n-3}{2}}\,dx$

$= 0 - (n-1)\int (1-x^2)^{\frac{n-1}{2}} + (n-1)\int (1-x^2)^{\frac{n-3}{2}}$

$= 0 - (n-1)I_n + (n-1)I_{n-2}$

$nI_n = x(1-x^2)^{n-1} + (n-1)I_{n-2}$

(b) $F = \frac{m}{x^3}(6-10x)$

(i) $t=0 \quad v=0 \quad x=1$

$v\frac{dv}{dx} = \frac{1}{x^3}(6-10x)$

$\int_0^v v\,dv = \int_1^x \frac{6-10x}{x^3}\,dx$

$\frac{v^2}{2} = \int_1^x 6x^{-3} - 10x^{-2}\,dx$

$= \left[-\frac{3}{x^2} + \frac{10}{x}\right]_1^x$

$= -\frac{3}{x^2} + \frac{10}{x} - 7$

$v^2 = -\frac{6}{x^2} + \frac{20}{x} - 14$ or $2\left(\frac{-3+10x-7x}{x^2}\right.$

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Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

$v = \pm\sqrt{\dfrac{20x - 14x^2 - 6}{x^2}}$

$= -\dfrac{\sqrt{20x - 14x^2 - 6}}{x}$

as $x = 1$ $v = 0$ and $\ddot{x} < 0$ $\therefore$ it moves in -ve direction

(ii) comes to rest when $v = 0$

$\therefore 20x - 14x^2 - 6 = 0$

$7x^2 - 10x + 3 = 0$

$(7x - 3)(x - 1) = 0$

$x = 1$ or $\frac{3}{7}$

ie at $x = 3/7$

(c) $\displaystyle\int_1^2 \frac{x+1}{\sqrt{-2+3x-x^2}}\,dx$

$= -\frac{1}{2}\displaystyle\int_1^2 \frac{-2x+3-5}{\sqrt{-2+3x-x^2}}$

$= -\frac{1}{2}\displaystyle\int_1^2 \frac{-2x+3}{\sqrt{-2+3x-x^2}} + \frac{5}{2}\int \frac{1}{\sqrt{(\frac{1}{2})^2 - (x-\frac{3}{2})^2}}$

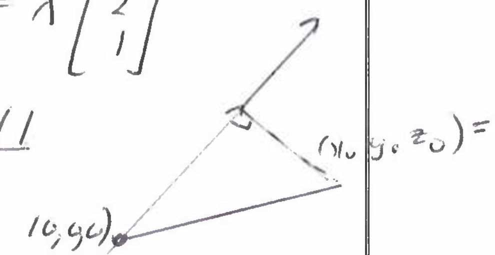
$= \left[\sqrt{-2+3x-x^2}\right]_1^2 + \frac{5}{2}\left[\sin^{-1}(2x-3)\right]_1^2$

$= \frac{5}{2}\left(\sin^{-1}1 - \sin^{-1}(-1)\right)$

$= \frac{5\pi}{2}$

(d) $r = \lambda\begin{bmatrix}1\\2\\1\end{bmatrix}$

Method 1



let $\underset{\sim}{s} = (x_0 y_0 z_0)$

$\text{proj}_{\underset{\sim}{r}}\underset{\sim}{s} = \dfrac{(x_0 y_0 z_0)\cdot(1,2,1)}{\sqrt{1+4+1}\,\sqrt{1+4+1}}\begin{bmatrix}1\\2\\1\end{bmatrix}$

$= \dfrac{x_0 + 2y_0 + z_0}{6}\begin{bmatrix}1\\2\\1\end{bmatrix}$

Method 2

Point on $\underset{\sim}{r} = \begin{bmatrix}\lambda\\2\lambda\\\lambda\end{bmatrix}$

distance to $\underset{\sim}{s} = d$

$d^2 = (x_0 - \lambda)^2 + (y_0 - 2\lambda)^2 + (z_0 - \lambda)^2$

$= x_0^2 - 2x_0\lambda + \lambda^2 + y_0^2 - 4y_0\lambda + 4\lambda^2 + z_0^2 - 2z_0\lambda + \lambda^2$

$\dfrac{dd^2}{d\lambda} = -2x_0 + 2\lambda - 4y_0 + 8\lambda - 2z_0 + 2\lambda = 0$

$12\lambda = 2x_0 + 4y_0 + 2z_0$

$\lambda = x_0 + 2y_0 + z_0 / 6$

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Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
∴ point on $\underset{\sim}{r}$ closest to $(x_0 y_0 z_0)$ $= \frac{x_0+2y_0+z_0}{6}\begin{bmatrix}1\\2\\1\end{bmatrix}$ (ii) $\underset{\sim}{s} = \begin{bmatrix}4\\1\\0\end{bmatrix} + t\begin{bmatrix}-1\\1\\5\end{bmatrix}$ $= \begin{bmatrix}4-t\\1+t\\5t\end{bmatrix}$ ∴ from (i) above closest point ~~to~~ on $\underset{\sim}{r}$ will be $\frac{4-t+2(1+t)+5t}{6}\begin{bmatrix}1\\2\\1\end{bmatrix}$ $= (1+t)\begin{bmatrix}1\\2\\1\end{bmatrix}$ ✓ So point on $\underset{\sim}{r}$ that has a perpendicular		distance to a point on $\underset{\sim}{s}$ is $(t+1, 2t+2, t+1)$ Point on $\underset{\sim}{s}$ is $(4-t, 1+t, 5t)$ distance between points l $l^2 = (4-t-t-1)^2 + (1+t-2t-2)^2 + (5t-t-t)^2$ $= 21t^2 - 18t + 11$ ✓ $\frac{dl^2}{dt} = 42t - 18 = 0$ at min $t = \frac{3}{7}$ ✓ $\frac{d^2l^2}{dt^2} = 42 > 0$ ∴ min ∴ position vector is $\left(1+\frac{3}{7}\right)\begin{bmatrix}1\\2\\1\end{bmatrix}$ $= \begin{bmatrix}\frac{10}{7}\\\frac{20}{7}\\\frac{10}{7}\end{bmatrix}$ or $\frac{10}{7}\begin{bmatrix}1\\2\\1\end{bmatrix}$	

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Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Q16

(a) (i)

$$\int_n^{2n} \frac{dx}{\sqrt{x}}$$

$$= \left[2\sqrt{x}\right]_n^{2n}$$

$$= 2\sqrt{2n} - 2\sqrt{n}$$

$$= 2\sqrt{n}\left(\sqrt{2}-1\right)$$

(ii)

Area of lower rectangle < Area under curve < Area of upper rectangles

$$\frac{1}{\sqrt{n+1}} + \frac{1}{\sqrt{n+2}} + \cdots \frac{1}{\sqrt{n}} < \int_n^{2n} \frac{dx}{\sqrt{x}} <$$

$$\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n+1}} + \cdots + \frac{1}{\sqrt{2n-1}}$$

$$S_n < 2\sqrt{n}(\sqrt{2}-1) < \frac{1}{\sqrt{n}} + \left(\frac{1}{\sqrt{n+1}} + \frac{1}{\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{2n-1}} + \frac{1}{\sqrt{2n}}\right) - \frac{1}{\sqrt{2n}}$$

$$S_n < 2\sqrt{n}(\sqrt{2}-1) < \frac{1}{\sqrt{n}} + S_n - \frac{1}{\sqrt{2n}}$$

$$0 < 2\sqrt{n}(\sqrt{2}-1) - S_n < \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{2n}}$$

$$-2\sqrt{n}(\sqrt{2}-1) < -S_n < \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{2n}} - 2\sqrt{n}(\sqrt{2}-1)$$

$$2\sqrt{n}(\sqrt{2}-1) > S_n > -\frac{1}{\sqrt{n}} + \frac{1}{\sqrt{2n}} + 2\sqrt{n}(\sqrt{2}-1)$$

ie $2\sqrt{n}(\sqrt{2}-1) + \frac{1-\sqrt{2}}{\sqrt{2n}} < S_n < 2\sqrt{n}(\sqrt{2}-1)$

(b)

$$\underset{\sim}{r} = \begin{pmatrix} t+\frac{1}{t} \\ t^3+\frac{1}{t^3} \end{pmatrix} \quad t \in \mathbb{R}^+$$

$$x = t + \frac{1}{t}$$

$$y = t^3 + \frac{1}{t^3}$$

now $\left(t+\frac{1}{t}\right)^3 = t^3 + 3t^2\frac{1}{t} + 3t\frac{1}{t^2} + \frac{1}{t^3}$

$$= \left(t^3 + \frac{1}{t^3}\right) + 3\left(t+\frac{1}{t}\right)$$

$$= y + 3x$$

$$\therefore x^3 = y + 3x$$

$$y = x^3 - 3x = x(x-\sqrt{3})(x+\sqrt{3})$$

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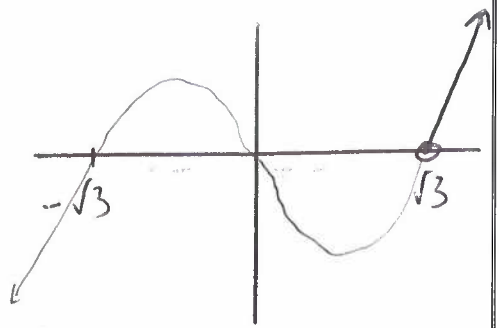
Suggested Solution (s)	Comments	Suggested Solution (s)	Comments



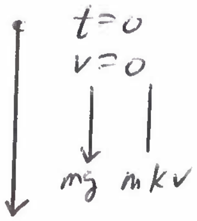
NOTE: $t \in \mathbb{R}^+$

$\therefore x > 0$ and $y > 0$

$\therefore y = x^3 - 3x = x(x-\sqrt{3})(x+\sqrt{3})$
with $x > 0, y > 0$

(c)

(i)



$$F_{\text{net down}} = mg - mkv$$

$$m\ddot{x} = mg - mkv$$

$$\ddot{x} = g - kv$$

terminal velocity $\Rightarrow$
$\ddot{x} = 0$

$$\therefore v_T = \frac{g}{k} = u$$

$$\ddot{x} = k\left(\frac{g}{k} - v\right)$$

$$\ddot{x} = k(u - v)$$

(ii)

$$\frac{dv}{dt} = g - kv$$
$$= k(u - v)$$

$$\frac{1}{u-v}\,dv = k\,dt$$

$$\int_0^v \frac{1}{u-v}\,dv = \int_0^t k\,dt$$

$$\Big[-\ln(u-v)\Big]_0^v = \Big[kt\Big]_0^t$$

$$-\ln(u-v) + \ln u = kt$$

$$\ln\left(\frac{u}{u-v}\right) = kt$$

$$-\ln\left(\frac{u-v}{u}\right) = kt$$

$$\frac{u-v}{u} = e^{-kt}$$

$$u - v = ue^{-kt}$$

$$v = u(1 - e^{-kt})$$

(iii) For particle A

$$\frac{dx}{dt} = u(1 - e^{-kt}) \text{ from (ii)}$$

$$x_A = \int u(1 - e^{-kt})\,dt$$

$t = T \quad x = X$

$$X = \int_0^T u(1 - e^{-kt})\,dt$$

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Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
For particle B $v = kue^{-kt} - u(1-e^{-kt})$ $\frac{dx}{dt} = kue^{-kt} - u(1-e^{-kt})$ $x_B = \int kue^{-kt} - u(1-e^{-kt})\,dt$ $t = T \quad x_B = h - X$ $\therefore h - X = \int_0^T kue^{-kt}\,dt - \int_0^T u(1-e^{-kt})\,dt$ $h - X = u\int_0^T ke^{-kt}\,dt - X$ ✓ $h = \left[-ue^{-kt}\right]_0^T$ $h = -ue^{-kT} + u$ ✓ $ue^{-kT} = u - h$ $e^{-kT} = \frac{u-h}{u}$ $T = -\frac{1}{k}\ln\left(\frac{u-h}{h}\right)$ $= \frac{1}{k}\ln\left(\frac{u}{u-h}\right)$			