Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for questions 1-10

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1 Consider the statement:

If an animal can fly, then it is a bird.

Which of the following is the contrapositive of this statement?

A. If an animal is not a bird, then it cannot fly.

B. If an animal is a bird, then it can fly.

C. If an animal can fly, then it is not a bird.

D. If an animal cannot fly, then it is not a bird.

2 What is the equation of the line which passes through $A(4, 6, 5)$ and $B(-2, 3, -1)$?

A. $\underset{\sim}{r} = \begin{pmatrix} 4 \\ 6 \\ 5 \end{pmatrix} - \lambda \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}$

B. $\underset{\sim}{r} = \begin{pmatrix} 4 \\ 6 \\ 5 \end{pmatrix} - \lambda \begin{pmatrix} 2 \\ 9 \\ 4 \end{pmatrix}$

C. $\underset{\sim}{r} = \begin{pmatrix} 4 \\ 6 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 9 \\ 4 \end{pmatrix}$

D. $\underset{\sim}{r} = \begin{pmatrix} 4 \\ 6 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -6 \\ -3 \\ -6 \end{pmatrix}$

3 Consider the following proposition:

If a positive integer is not a perfect square, then it is not prime.

A counter example to this proposition is to be found. A number with which of the following properties would suffice?

A. A positive integer that is not a perfect square and is prime.

B. A positive integer that is not a perfect square and is not prime.

C. A positive integer that is a perfect square and is prime.

D. A positive integer that is a perfect square and is not prime.

4 A particle is moving in SHM in a straight line with an amplitude of 3 metres.

When the particle is 2 metres from the centre of motion, its speed is $5m/s$.

What is the period of the motion?

A. $\frac{\sqrt{5}\pi}{2}$

B. $\frac{2\sqrt{5}\pi}{5}$

C. $\sqrt{5}\pi$

D. $2\sqrt{5}\pi$

5 The roots of the cubic equation $f(x) = 0$ are α, β and γ.

Which of the following cubic equations has roots $2\alpha + 1, 2\beta + 1$ and $2\gamma + 1$?

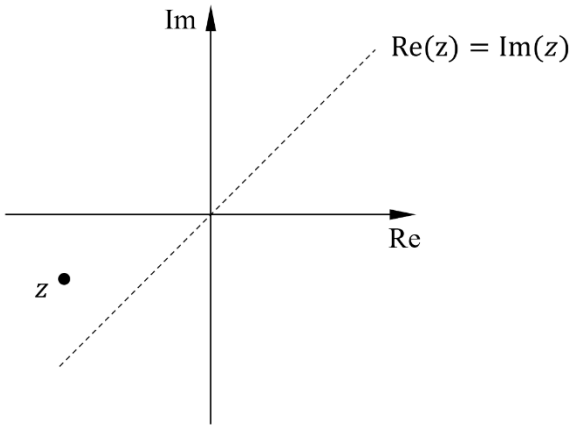
A. $f(2x + 1) = 0$

B. $f\left(\frac{x}{2} + 1\right) = 0$

C. $f\left(\frac{x-1}{2}\right) = 0$

D. $f\left(\frac{x+1}{2}\right) = 0$

6 The diagram below shows the complex number $z = x + iy$, which lies in the third quadrant such that $x < y$.



Given that $z^2 = a + ib$, which of the following statements is true?

A. $a < 0,\ b < 0$

B. $a < 0,\ b > 0$

C. $a > 0,\ b < 0$

D. $a > 0,\ b > 0$

7 Consider the position vector of a particle $\underset{\sim}{r} = t^2\underset{\sim}{i} + 2cost\underset{\sim}{j} + 2sint\underset{\sim}{k}$.

Which of the following best describes the motion of the particle?

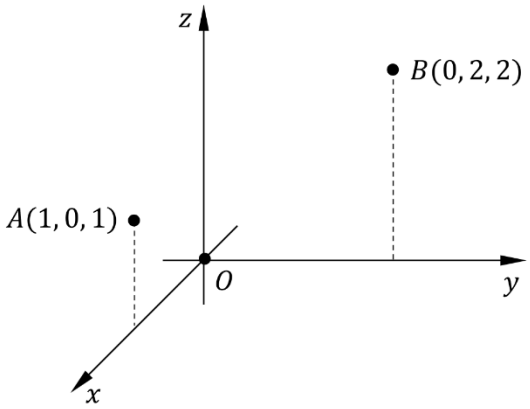
A. Spiralling at an increasing rate about the z axis in an anticlockwise direction.

B. Spiralling at an increasing rate about the x axis in an anticlockwise direction.

C. Spiralling at an increasing rate about the x axis in a clockwise direction.

D. Spiralling at a constant rate about the x axis in an anticlockwise direction.

8 Consider triangle AOB, where A is the point $(1, 0, 1)$ and B is the point $(0, 2, 2)$.



NOT TO SCALE

Which of the following is true?

A. $\angle AOB$ is a right angle

B. ΔAOB is isosceles

C. $\angle OAB$ is twice the size of $\angle OBA$

D. $\angle AOB$ is twice the size of $\angle OBA$

9 Consider the complex equation $z^3 = -p^3 i$, where p is a positive, real number.

What is the sum of all of the imaginary parts of all of the solutions that have a negative principal argument?

A. pi

B. $-pi$

C. p

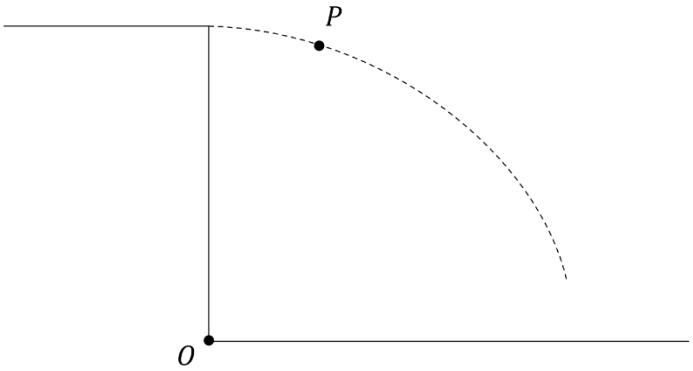
D. $-p$

10 A unit mass is projected horizontally from the top of a tall building. Air resistance is R and the gravitational force is G.

The position vector of the mass is $\underset{\sim}{r}$, with the origin being the base of the building.

The velocity vector of the mass is $\underset{\sim}{v}$.

At the point P, $\underset{\sim}{r}.\underset{\sim}{v} = 0$.



Which of the following is consistent with the motion of the particle at P?

A. $\dfrac{\mathbf{R}\cdot\mathbf{v}}{|\mathbf{R}||\mathbf{v}|} < \dfrac{\mathbf{G}\cdot\mathbf{r}}{|\mathbf{G}||\mathbf{r}|} < \dfrac{\mathbf{R}\cdot\mathbf{r}}{|\mathbf{R}||\mathbf{r}|} < \dfrac{\mathbf{G}\cdot\mathbf{v}}{|\mathbf{G}||\mathbf{v}|}$

B. $\dfrac{\mathbf{G}\cdot\mathbf{r}}{|\mathbf{G}||\mathbf{r}|} < \dfrac{\mathbf{G}\cdot\mathbf{v}}{|\mathbf{G}||\mathbf{v}|} < \dfrac{\mathbf{v}\cdot\mathbf{R}}{|\mathbf{v}||\mathbf{R}|} < \dfrac{\mathbf{R}\cdot\mathbf{r}}{|\mathbf{R}||\mathbf{r}|}$

C. $\dfrac{\mathbf{r}\cdot\mathbf{R}}{|\mathbf{r}||\mathbf{R}|} < \dfrac{\mathbf{v}\cdot\mathbf{G}}{|\mathbf{v}||\mathbf{G}|} < \dfrac{\mathbf{r}\cdot\mathbf{G}}{|\mathbf{r}||\mathbf{G}|} < \dfrac{\mathbf{v}\cdot\mathbf{R}}{|\mathbf{v}||\mathbf{R}|}$

D. $\dfrac{\mathbf{v}\cdot\mathbf{G}}{|\mathbf{v}||\mathbf{G}|} < \dfrac{\mathbf{r}\cdot\mathbf{G}}{|\mathbf{r}||\mathbf{G}|} < \dfrac{\mathbf{R}\cdot\mathbf{v}}{|\mathbf{R}||\mathbf{v}|} < \dfrac{\mathbf{r}\cdot\mathbf{R}}{|\mathbf{r}||\mathbf{R}|}$

End of Section I

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) (Start a new writing booklet.) **Marks**

a. Consider the vector $\underset{\sim}{r} = p\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k}$ 2

If $\underset{\sim}{r}$ has a magnitude of $5\sqrt{2}$ units, what is the exact value/s of p?

b. Find the size of the angle between the vectors 2

$$\underset{\sim}{a} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \text{ and } \underset{\sim}{b} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$$

giving your answer correct to the nearest minute.

c. Sketch the region in the complex plane where the inequalities 3

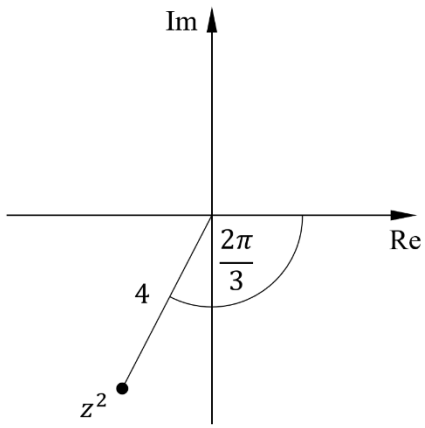
$|z - i| \leq 1$ and $0 \leq \arg(z - i) \leq \frac{\pi}{3}$ hold simultaneously.

d. Show that, if a and b are positive integers, then $\frac{a}{b} + \frac{b}{a} \geq 2$ 2

Question 11 continued on next page.

Question 11 continued.

e. The complex number z^2 is shown on the diagram below. 2



NOT TO SCALE

Given that z lies in the fourth quadrant, find z in mod-arg form.

f. Consider the equation $(p+5)^{36}+(p+3)^{35}=7$. 2

Give reasons as to why there is no integer p root for the equation.

g. Find: 2

$$\int x \ln x \, dx$$

End of Question 11

Question 12 (15 marks) (Start a new writing booklet.) **Marks**

a. i) Find *A, B,* and *C* such that: 2

$$\frac{x}{(x-2)^2(x-1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x-1}.$$

ii) Hence, show that: 3

$$\int_3^4 \frac{x}{(x-2)^2(x-1)}\,dx = \ln\left(\frac{3}{4}\right) + 1.$$

b. Consider the statement *P*: 2

"If $2n^2 + 3n + 1$ is even, then n is odd."

Prove that the converse of *P* is true.

c. Factorise the equation $P(z) = z^4 - 6z^3 + 18z^2 - 14z - 39$, given that 3

one of the zeros of $P(z)$ is $(2 - \ 3i)$.

d. Consider the complex number $z = re^{i\theta}$: 2

Show that $\phi = \dfrac{2z}{z^2 + |z|^2}$ is purely real.

e. Find: 3

$$\int \frac{e^{2x} - e^x}{e^x + 1}\,dx$$

End of Question 12

Question 13 (15 marks) (Start a new writing booklet.) **Marks**

a. For the positive real numbers a and b, $a \neq b$, it is given that

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2) \text{ and } (a-b)^2 > 0.$$

i) Using these facts, or otherwise, prove that: 1

$$a^3 + b^3 > ab(a+b)$$

ii) Hence, or otherwise, show that if c is also a positive real number 2

that is not equal to a or b, then:

$$2(a^3 + b^3 + c^3) > a^2(b+c) + b^2(a+c) + c^2(a+b)$$

b. A particle moves along the x axis in a simple harmonic motion.

Its displacement, x cm, from the origin at time t seconds is given by:

$$x = 5 - 4\cos^2 2\pi t$$

i) Find the centre and the amplitude of the motion. 3

ii) Find the distance travelled by the particle in the first 5 seconds. 2

Question 13 continued on next page.

c. Prove by contradiction that 3

$$P(x) = 120 + 120x + 60\,x^2 + 20x^3 + 5x^4 + x^5$$

has no double roots.

d. Let $z = 1 + i$ and $w = -\sqrt{3} + i$

i) Find zw in modulus argument form. 2

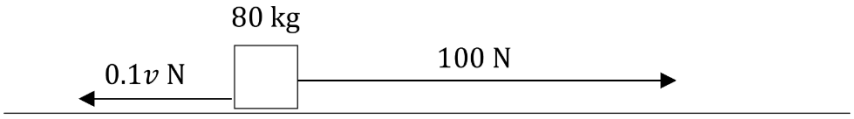
ii) Hence, show that $cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$ 2

Question 14 (15 marks) (Start a new writing booklet.) **Marks**

a. Find: 3

$$\int \frac{2x-5}{x^2+2x+2}\, dx$$

b. An 80-kilogram mass is being dragged along a horizontal surface by a force of 100 newtons.



The mass is subject to friction of $0.1v$ newtons, where v is the speed of the mass in metres per second. The mass is initially at rest.

The equation of motion is

$$\ddot{x} = \frac{1000 - v}{800}. \qquad \text{(DO NOT PROVE THIS.)}$$

i) Show that the equation for time as a function of velocity is 3

$$t = 800 \ln\left(\frac{1000}{1000 - v}\right).$$

ii) Find the velocity of the mass after 10 seconds, correct to 1 decimal place. 3

Question 14 continued on next page.

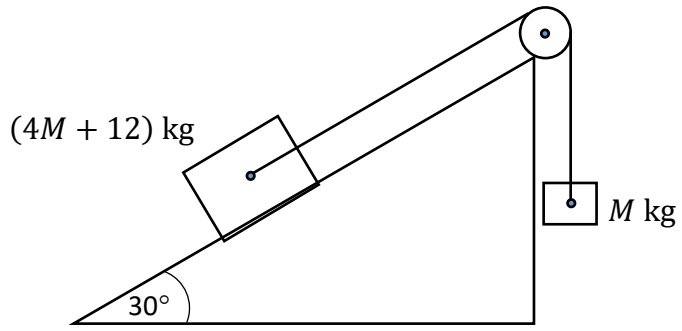
Question 14 continued.

c. The vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are defined as $\underset{\sim}{u} = \underset{\sim}{a} + m\underset{\sim}{b}$ and $\underset{\sim}{v} = m\underset{\sim}{a} - \underset{\sim}{b}$, where $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are unit vectors and m is a real constant. 3

Given that the dot product of $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is 1, and that the acute angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is $\frac{\pi}{3}$, find the value/s of m.

d. Two masses M kg and $(4M + 12)$ kg are attached to ends of a light inextensible string passing through a pulley. 3

The mass M kg is accelerating upwards at 3 ms^{-2} while the $(4M + 12)$ kg mass is moving downwards on a smooth inclined plane at angle $30°$ as shown.



The acceleration due to gravity is $10ms^{-2}$.

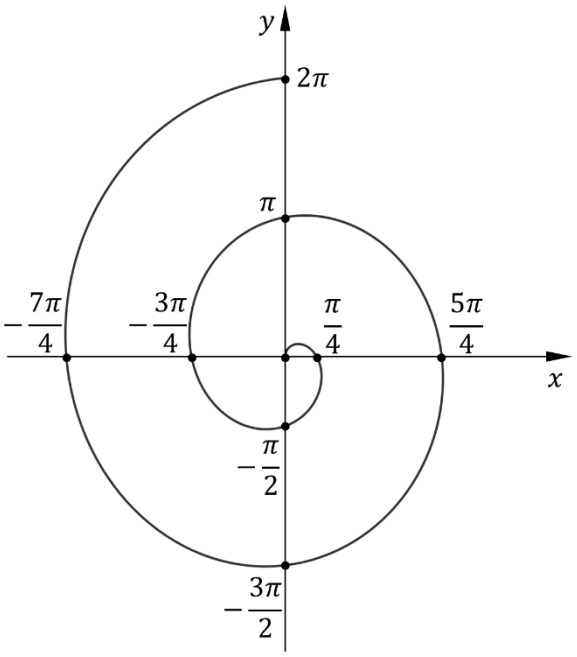
What is the size of the mass M?

End of Question 14

Question 15 (15 marks) (Start a new writing booklet.) **Marks**

a. A spiral starts at the origin and completes two clockwise spirals to end at $(0, 2\pi)$.

Its position from $t = 0$ to $t = T$ is shown in the diagram.



The parametric equations of the spiral are:

$$\begin{cases} x = f(t) \\ y = g(t) \end{cases} \quad \text{for} \quad \left[0, T\right].$$

i) Find $f(t)$, $g(t)$ and T. 3

ii) The parametric equations of a second spiral are: 1

$$\begin{cases} x = g(t) \\ y = f(t) \end{cases} \quad \text{for} \quad \left[0, \frac{T}{2}\right].$$

If the second spiral was drawn on the axes above, how many points of intersection would it have with the first spiral?

Question 15 continued on next page.

Question 15 continued.

b. Consider the integral

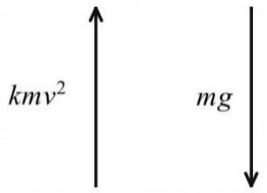
$$I_n = \int_0^1 \sqrt{x}(1-x)^n \, dx, \qquad n = 0, 1, 2, 3, \ldots$$

i) Show that 3

$$I_n = \frac{2n}{2n+3} I_{n-1}, \qquad n = 1, 2, 3, \ldots$$

ii) Evaluate I_1. 1

c. A particle of mass *m* is released and allowed to fall vertically in a medium in which the resistance is *mk* times the square of the velocity (*v*) of the particle.



i) Write an equation for the acceleration of the particle in terms of *v*. 1

ii) Show that the terminal velocity (V_T) is given by $V_T = \sqrt{\frac{g}{k}}$. 1

iii) Show that the distance fallen (*x*) in terms of *v* is: 3

$$x = \frac{1}{2k} ln\left(\frac{g}{g - kv^2}\right)$$

iv) Hence, find an expression for the distance that the particle has fallen when the velocity reaches $\frac{1}{2}V_T$. 2

End of Question 15

Question 16 (15 marks) (Start a new writing booklet.) **Marks**

a. The line L with equation $\underset{\sim}{r} = \begin{pmatrix} -1 \\ 2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ intersects with the sphere with equation $(x+1)^2 + (y-2)^2 + (z-3)^2 = 45$ at the two points, A and B. 4

Find the shortest distance between the centre of the sphere C and the chord AB.

b. Given $a_{n+1} = \sqrt{2a_n}$ and $a_1 = 1$, prove by mathematical induction that for all integers $n \geq 1$ 3

$$a_n = 2^{\left(1-2^{(1-n)}\right)}$$

c. i) Use De Moivre's Theorem to express $\cos 4\theta$ in terms of $\cos\theta$ 2

ii) Use the result above to solve the equation $8x^4 - 8x^2 + 1 = 0$ 2

iii) Hence, show that $\cos\frac{\pi}{8}\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\cos\frac{7\pi}{8} = \frac{1}{8}$. 1

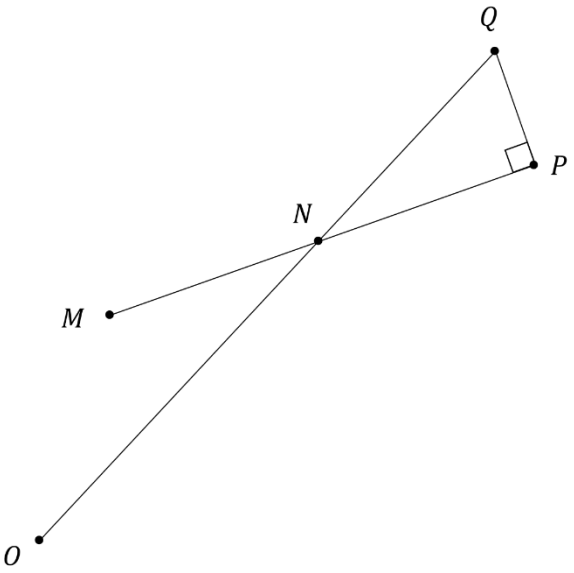
Question 16 continued on next page.

Question 16 continued.

d. M and P are two points with position vectors $\overrightarrow{OM} = \underset{\sim}{m}$ and $\overrightarrow{OP} = \underset{\sim}{p}$ 3
respectively.

N is the midpoint of MP and has position vector $\overrightarrow{ON} = \underset{\sim}{n}$.

PQ is perpendicular to MN and $\overrightarrow{NQ} = \lambda\overrightarrow{ON}$.



NOT TO SCALE

Show that

$$\lambda = \frac{\left|\underset{\sim}{m}\right|^2 + \left|\underset{\sim}{n}\right|^2 - 2\underset{\sim}{m} \cdot \underset{\sim}{n}}{\left|\underset{\sim}{n}\right|^2 - \underset{\sim}{m} \cdot \underset{\sim}{n}}.$$

End of Question 16

End of Paper

Student Number

Starting from the left, please write your student number NEATLY in the boxes above.

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Centre Number

NSW GOVERNMENT

NSW Education Standards Authority

2024

HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

Writing Booklet

~~Question 11~~ Multiple Choice

00 067706

Instructions

- Use this booklet for Question 11. → **11**
- Write the number of this booklet and the total number of booklets that you have used for this question (eg: 1 of 3). → ☐ of ☐ (this booklet / number of booklets for this question)
- Write your Student Number at the top of this page.
- Write in black or blue pen (black is recommended).
- You may ask for an extra writing booklet if you need more space.
- If you have not attempted the question(s), you must still hand in the writing booklet, with 'NOT ATTEMPTED' written clearly on the front cover.
- You may NOT take any writing booklets, used or unused, from the examination room.

Additional blank writing booklets set: 1 of 2

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Start here for Question Number: 11

Ext 2 Trial 2025 Soln's

Multichoice :

1. By obs. A

ADABC

DBDDA

2. $A \to B \quad \begin{pmatrix} -6 \\ -3 \\ -6 \end{pmatrix} \Rightarrow D$

3. By obs. A

4. $v^2 = n^2(a^2 - x^2)$

$25 = n^2(9-4)$

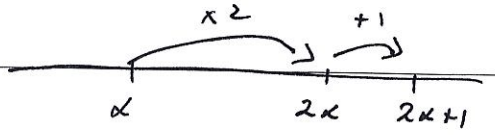
$n^2 = 5$

$T = \frac{2\pi}{n}$

$= \frac{2\pi}{\sqrt{5}}$

$= \frac{2\sqrt{5}\pi}{5} \Rightarrow B$

5.



To get these roots, $f(x)$ has been :

- horizontally dilated by sf of 2 $\Rightarrow f\left(\frac{x}{2}\right)$
- horizontally translated to right by 1 $\Rightarrow f\left(\frac{x-1}{2}\right) \Rightarrow C$

6. $z = x + iy$

$z^2 = (x^2 - y^2) + 2xy\,i$

$= a + bi$

$x < y$

$\therefore x^2 > y^2$ since both neg.

$x^2 - y^2 > 0$

ie $a > 0$

$x < 0,\ y < 0$

$\therefore xy > 0$

ie $b > 0$

$\Rightarrow$ D

7. spiral about x-axis $\Rightarrow$ not A

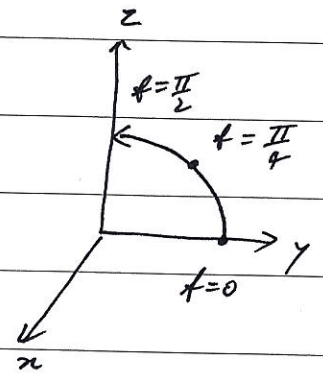
rate will increase $\Rightarrow$ not D

$t = 0,\ y = 2,\ z = 0$

$t = \frac{\pi}{4},\ y = \sqrt{2},\ z = \sqrt{2}$

$t = \frac{\pi}{2},\ y = 0,\ z = 2$

anticlockwise $\Rightarrow$ B

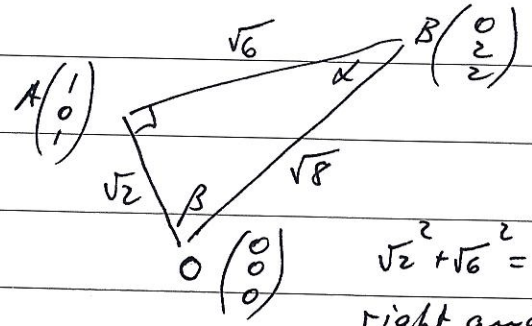


8.

$\tan\alpha = \frac{\sqrt{2}}{\sqrt{6}}$

$\alpha = \frac{\pi}{6}$ $\therefore \beta = \frac{\pi}{3}$

ie $\angle AOB = 2\angle ABO$ $\Rightarrow$ D



$\sqrt{2}^2 + \sqrt{6}^2 = \sqrt{8}^2$

right angle at A

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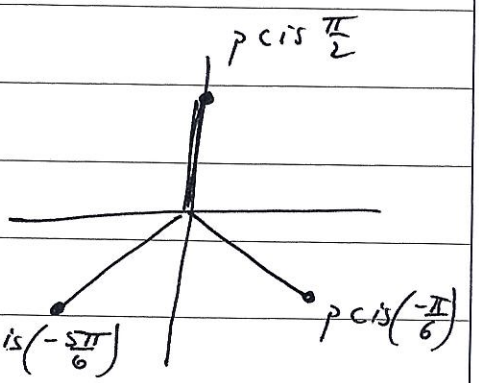
9. $r^3 \text{cis}\, 3\theta = -p^3 \text{cis}\left(\frac{\pi}{2}\right)$

$= p^3 \text{cis}\left(\frac{3\pi}{2}\right)$

$\therefore r = p, \quad 3\theta = \frac{3\pi}{2} + 2k\pi$

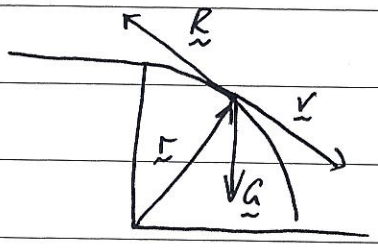
$k = 0, \quad \theta = \frac{\pi}{2}$

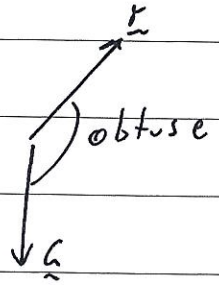


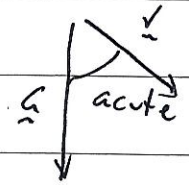
Im parts $= 2 \times p \times \sin\left(\frac{-\pi}{6}\right)$

$= -p \Rightarrow D$

10.







$\underset{\sim}{r} \cdot \underset{\sim}{v} = 0 \qquad \underset{\sim}{R} \cdot \underset{\sim}{r} = 0 \qquad \frac{\underset{\sim}{R} \cdot \underset{\sim}{v}}{|\underset{\sim}{R}||\underset{\sim}{v}|} = -1$

$\frac{\underset{\sim}{G} \cdot \underset{\sim}{r}}{|\underset{\sim}{G}||\underset{\sim}{r}|} < 0$ since obtuse

$\frac{\underset{\sim}{G} \cdot \underset{\sim}{v}}{|\underset{\sim}{G}||\underset{\sim}{v}|} > 0$ since acute

A is only option to fit all. $\Rightarrow$ A

☐ ⇦ Tick this box if you have continued this answer in another writing booklet.

Student Number

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2024

HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

Writing Booklet

Question 11

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Instructions

- Use this booklet for Question 11. → 11
- Write the number of this booklet and the total number of booklets that you have used for this question (eg: 1 of 3). → / of / (this booklet; number of booklets for this question)
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Additional blank writing booklets set: 2 of 2

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Start here for Question Number: **11**

a. $|\underset{\sim}{v}| = \sqrt{p^2+(-4)^2+5^2}$ ①

$5\sqrt{2} = \sqrt{p^2+16+25}$

$50 = p^2+41$

$p^2 = 9$

$p = \pm 3$ ①

b. $\underset{\sim}{a}\cdot\underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta$

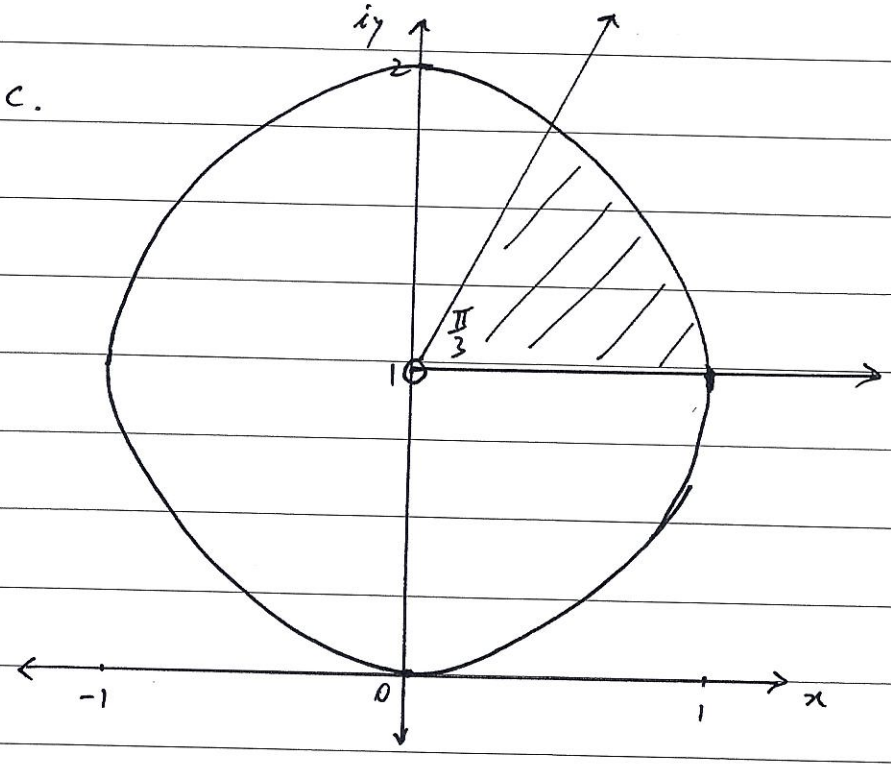
$\cos\theta = \dfrac{1\times 3+(-2)(-2)+2(-1)}{\sqrt{1^2+(-2)^2+2^2}\times\sqrt{3^2+(-2)^2+(-1)^2}}$ ①

$= \dfrac{5}{3\times\sqrt{14}}$

$\theta = 63^\circ\ 32'\ 55.68''$

$= 63^\circ\ 33'$ (nearest min) ①

c.



① mod

① arg

① open circle solid lines shading.

d. Consider $(a-b)^2 \geq 0$ (1)

$$a^2 - 2ab + b^2 \geq 0$$

$$a^2 + b^2 \geq 2ab$$

$$\frac{a^2}{ab} + \frac{b^2}{ab} \geq 2$$

$$\frac{a}{b} + \frac{b}{a} \geq 2 \quad (1)$$

or, by contradiction, or other methods.

e. $z^2 = 4\,\text{cis}\left(\frac{-2\pi}{3}\right)$

$$= 4e^{-\frac{2\pi}{3}i}$$

$$z = \pm\left(4e^{-\frac{2\pi}{3}i}\right)^{\frac{1}{2}} \quad (1)$$

$$= \pm 2e^{-\frac{\pi}{3}i}$$

but only $z = 2e^{-\frac{\pi}{3}i}$ lies in the 4th quadrant.

$$= 2\,\text{cis}\left(\frac{-\pi}{3}\right) \quad (1)$$

f. If p is an integer, then it could be odd or even.

If p is odd, then both $(p+5)$ and $(p+3)$ are even

$\therefore (p+5)^{36}$ and $(p+3)^{35}$ are even (1)

and the sum of two even numbers cannot be odd

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$\therefore$ there is no odd p solution.

If p is even, then both $(p+5)$ and $(p+3)$ are odd

$\therefore$ $(p+5)^{36}$ and $(p+3)^{35}$ are both odd (1)

and the sum of two odd numbers is even

$\therefore$ there is no integer solution

$\therefore$ there is no integer root for the eqn.

g. $\int x \ln x \, dx = \int \frac{d\left(\frac{x^2}{2}\right)}{dx} \ln x \, dx$ (1)

$= \frac{x^2 \ln x}{2} - \frac{1}{2} \int x^2 \times \frac{1}{x} \, dx$

$= \frac{x^2 \ln x}{2} - \frac{1}{2} \int x \, dx$

$= \frac{x^2 \ln x}{2} - \frac{1}{2} \times \frac{x^2}{2} + c$

$= \frac{x^2 \ln x}{2} - \frac{x^2}{4} + c$ (1)

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Mathematics Extension 2

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Question 12

00 0677 05 06

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Start here for Question Number: 12

a i) $\frac{x}{(x-2)^2(x-1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x-1}$

$x = A(x-2)(x-1) + B(x-1) + C(x-2)^2$

$x=1, \quad 1 = 0 + 0 + C$

$\therefore C = 1$

$x=2, \quad 2 = 0 + B + 0$

$\therefore B = 2$

$x=0, \quad 0 = A(-2)(-1) + 2(-1) + 1(-2)^2$

$0 = 2A - 2 + 4$

$2A = -2$ (1) some

$A = -1$ (1) all

$\therefore \frac{x}{(x-2)^2(x-1)} = \frac{-1}{x-2} + \frac{2}{(x-2)^2} + \frac{1}{x-1}$

ii) $\int_3^4 \frac{x}{(x-2)^2(x-1)}\,dx = \int_3^4 \frac{-1}{x-2} + \frac{2}{(x-2)^2} + \frac{1}{x-1}\,dx$

$= \int_3^4 \frac{-1}{x-2} + 2(x-2)^{-2} + \frac{1}{x-1}\,dx$

$= \left[-\ln|x-2| - 2(x-2)^{-1} + \ln|x-1|\right]_3^4$ (1)

$= -\ln|4-2| - \frac{2}{4-2} + \ln|4-1| - \left(-\ln|3-2| - \frac{2}{3-2} + \ln|3-1|\right)$ (1)

$= -\ln 2 - 1 + \ln 3 + \ln 1 + 2 - \ln 2$

$= \ln 3 - 2\ln 2 + 1$

$= \ln 3 - \ln 4 + 1$

$= \ln\left(\frac{3}{4}\right) + 1$ (1)

b. Converse is: if n is odd, then $2n^2+3n+1$ is even.

Let $n = 2k+1$ where $k \in \mathbb{Z}$ (1)

$\therefore 2n^2+3n+1 = 2(2k+1)^2 + 3(2k+1) + 1$

$= 2(4k^2+4k+1) + 6k + 3 + 1$

$= 8k^2 + 8k + 2 + 6k + 4$

$= 8k^2 + 14k + 6$

$= 2(4k^2+7k+3)$ (1)

$= 2P$ where $P = 4k^2+7k+3$ and must be an integer since $k \in \mathbb{Z}$

which is even

$\therefore$ the converse is true.

c. Since $P(z)$ is real, $2+3i$ must also be a zero (1)

Let the zeroes be $\alpha, \beta, 2-3i, 2+3i$

$\therefore \alpha + \beta + 2 - 3i + 2 + 3i = 6$

$\alpha + \beta = 2$

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$\therefore \beta = 2 - \alpha$

$\alpha\beta(2+3i)(2-3i) = -39$

$\alpha\beta(4+9) = -39$

$\alpha\beta = -3$ (1)

$\alpha(2-\alpha) = -3$

$2\alpha - \alpha^2 = -3$

$\alpha^2 - 2\alpha - 3 = 0$

$(\alpha+1)(\alpha-3) = 0$

$\alpha = -1$ or $\alpha = 3$

ie $\beta = 3$ $\beta = -1$

$\therefore P(z) = (z+1)(z-3)(z-(2-3i))(z-(2+3i))$ (1)

$= (z+1)(z-3)(z-2+3i)(z-2-3i)$

d. $\phi = \dfrac{2re^{i\theta}}{(re^{i\theta})^2 + r^2}$

$= \dfrac{2re^{i\theta}}{r^2(e^{2i\theta}+1)}$ (1)

$= \dfrac{2e^{i\theta}}{re^{i\theta}(e^{i\theta}+e^{-i\theta})}$

$= \dfrac{2}{r(z+\bar{z})}$

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Mathematics Extension 2

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Question 12

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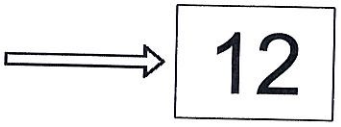

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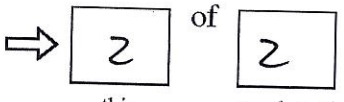

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d. (cont) $= \dfrac{2}{r \times 2\,\mathrm{Re}(z)}$ (1)

which must be real.

e. $\displaystyle\int \frac{e^{2x} - e^{x}}{e^{x}+1}\,dx = \int \frac{e^{x}(e^{x}-1)}{e^{x}+1}\,dx$

$\displaystyle= \int \frac{e^{x}(e^{x}+1-2)}{e^{x}+1}\,dx$ (1)

$\displaystyle= \int \frac{e^{x}(e^{x}+1)}{e^{x}+1} - \frac{2e^{x}}{e^{x}+1}\,dx$

$\displaystyle= \int e^{x}\,dx - 2\int \frac{e^{x}}{e^{x}+1}\,dx$ (1)

$= e^{x} - 2\ln|e^{x}+1| + c$

$= e^{x} - 2\ln(e^{x}+1) + c$ (1)

since $e^{x} > 0$

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Question 13

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Start here for Question Number: 13

a. i) $(a-b)^2 > 0$

$a^2 - 2ab + b^2 > 0$

$a^2 - ab + b^2 > ab$

Now $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$> (a+b)\,ab$

$> ab(a+b)$ (1)

ii) Similarly,

$b^3 + c^3 > bc(b+c)$

$c^3 + a^3 > ca(c+a)$

$\therefore 2(a^3+b^3+c^3) > ab(a+b) + bc(b+c) + ca(c+a)$ (1)

$> a^2b + ab^2 + b^2c + bc^2 + ac^2 + a^2c$

$> a^2b + a^2c + b^2a + b^2c + c^2a + c^2b$

$> a^2(b+c) + b^2(a+c) + c^2(a+b)$ (1)

b. i) $x = 5 - 4\cos^2 2\pi t$

$= 5 - 4\left(\frac{1 + \cos 4\pi t}{2}\right)$ (1)

$= 5 - 2(1 + \cos 4\pi t)$

$= 3 - 2\cos 4\pi t$

$\therefore$ centre of motion: $x = 3$ (1)

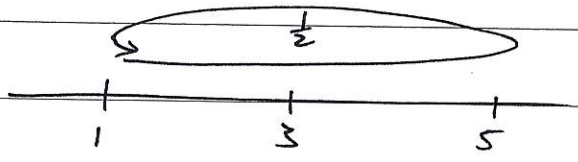
amplitude is 2 (1)

ii) $T = \frac{2\pi}{a}$

$= \frac{2\pi}{4\pi}$

$= \frac{1}{2}$ sec

$\therefore$ in 5 seconds, will complete 10 complete oscillations (1)



one oscillation is 8cm

$\therefore$ travels 80 cm in first 5 seconds. (1)

c. Assume that $P(x)$ has a double root at $x = \alpha$

$\therefore P(\alpha) = P'(\alpha) = 0$ (1)

or $P(\alpha) - P'(\alpha) = 0$

ie. $\alpha^5 + 5\alpha^4 + 20\alpha^3 + 60\alpha^2 + 120\alpha + 120 - (5\alpha^4 + 20\alpha^3 + 60\alpha^2 + 120\alpha + 120) = 0$

ie $\alpha^5 = 0$

$\alpha = 0$ (1)

but, if $\alpha = 0$ then $P(\alpha) = 120$ (1)

and, hence, $\alpha = 0$ cannot be a root

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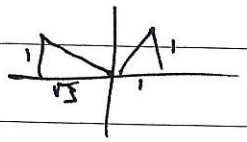
which is a contradiction

$\therefore P(x)$ has no double roots.

d. $z = 1+i \qquad w = -\sqrt{3} + i$



i) $zw = \sqrt{2}\,\text{cis}\,\frac{\pi}{4} \times 2\,\text{cis}\,\frac{5\pi}{6}$ (1)

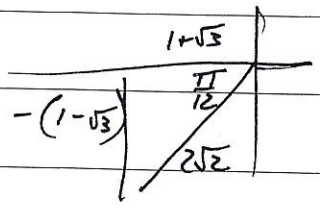
$= 2\sqrt{2}\,\text{cis}\left(\frac{13\pi}{12}\right)$

$= 2\sqrt{2}\,\text{cis}\left(\frac{-11\pi}{12}\right)$ (1)

ii) $zw = (1+i)(-\sqrt{3}+i)$

$= (-\sqrt{3}-1) - \sqrt{3}i + i$

$= -(1+\sqrt{3}) + i(1-\sqrt{3})$ (1)



$\therefore \cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}+\sqrt{6}}{4}$ (1)

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Mathematics Extension 2

Writing Booklet

Question 14

00 0677 03 06

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Start here for Question Number: **14**

a. $\int \frac{2x-5}{x^2+2x+2}\,dx = \int \frac{2x+2-7}{x^2+2x+2}\,dx$ (1)

$= \int \frac{2x+2}{x^2+2x+2}\,dx - 7\int \frac{1}{(x^2+2x+1)+1}\,dx$ (1)

$= \ln|x^2+2x+2| - 7\int \frac{1}{1+(x+1)^2}\,dx$

$= \ln|x^2+2x+2| - 7\tan^{-1}(x+1) + C$ (1)

b. i) $\ddot{x} = \frac{1000-v}{800}$

$\frac{dv}{dt} = \frac{1000-v}{800}$

$\frac{800}{1000-v}\,dv = dt$

$-800\int_0^v \frac{-1}{1000-v}\,dv = \int_0^t dt$ (1)

$-800\left[\ln|1000-v|\right]_0^v = t$ (1)

$t = -800\left[\ln|1000-v| - \ln 1000\right]$

$= 800\left(\ln 1000 - \ln(1000-v)\right]$ (1)

since $v < 1000$

$= 800\ln\left(\frac{1000}{1000-v}\right)$

11) when $t = 10$

$$10 = 800 \ln\left(\frac{1000}{1000 - v}\right) \quad (1)$$

$$e^{\frac{1}{80}} = \frac{1000}{1000 - v} \quad (1)$$

$$1000 - v = 1000\, e^{-\frac{1}{80}}$$

$$v = 1000 - 1000\, e^{-\frac{1}{80}}$$

$$= 12.422199\,51$$

$$= 12.4 \text{ m/s (1dp)} \quad (1)$$

c. $\underset{\sim}{u} = \underset{\sim}{a} + m\underset{\sim}{b}$ $\underset{\sim}{v} = m\underset{\sim}{a} - \underset{\sim}{b}$

$|\underset{\sim}{a}| = |\underset{\sim}{b}| = 1$ $\underset{\sim}{u} \cdot \underset{\sim}{v} = 1$

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}| \cos\frac{\pi}{3}$$

$$= 1 \times 1 \times \frac{1}{2}$$

$$= \frac{1}{2} \quad (1)$$

$$(\underset{\sim}{a} + m\underset{\sim}{b}) \cdot (m\underset{\sim}{a} - \underset{\sim}{b}) = 1 \quad (1)$$

$$m\,\underset{\sim}{a} \cdot \underset{\sim}{a} - \underset{\sim}{a} \cdot \underset{\sim}{b} + m^2\,\underset{\sim}{a} \cdot \underset{\sim}{b} - m\,\underset{\sim}{b} \cdot \underset{\sim}{b} = 1$$

$$m|\underset{\sim}{a}|^2 + (m^2 - 1)\,\underset{\sim}{a} \cdot \underset{\sim}{b} - m|\underset{\sim}{b}|^2 = 1$$

$$m \times 1 + (m^2 - 1) \times \frac{1}{2} - m \times 1 = 1$$

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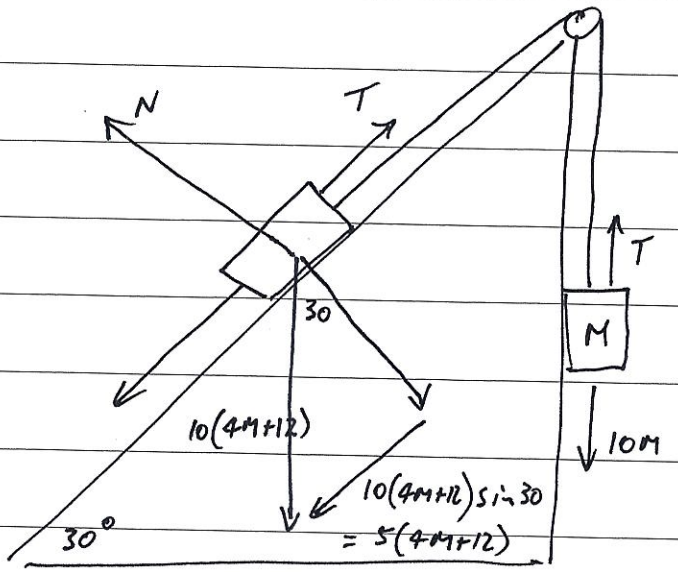
$m^2 - 1 = 2$

$m^2 = 3$

$m = \pm\sqrt{3}$ (1)

d.



(1)

For the system:

$\Sigma F = \Sigma m\ddot{x}$

ie $(5M+12)3 = 5(4M+12) - T + T - 10M$ (1)

$15M + 36 = 20M + 60 - 10M$

$15M + 36 = 10M + 60$

$5M = 24$

$M = \frac{24}{5}$

$= 4.8\ kg$ (1)

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Question 15

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Start here for Question Number: 15

a. i) $f(t) = \frac{t \sin t}{2}$ (1)

$g(t) = \frac{t \cos t}{2}$ (1)

dist. from O is always $\frac{t}{2}$

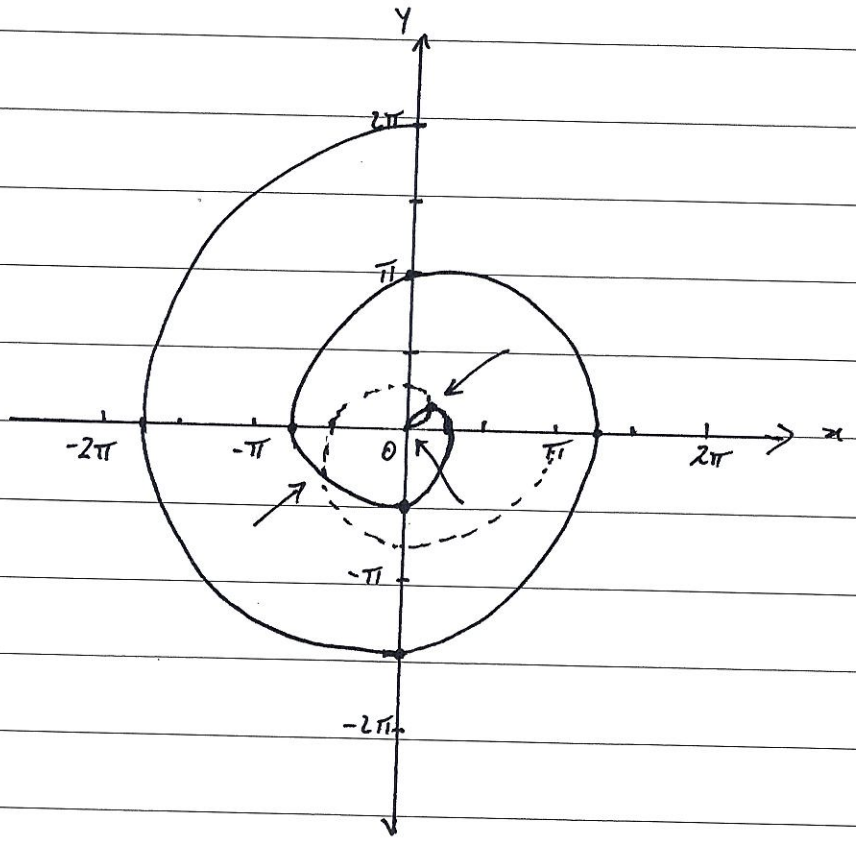
$\therefore T = 4\pi$ (1)

ii) $x = g(t) = \frac{t \cos t}{2}$

$y = f(t) = \frac{t \sin t}{2}$

$[0, 2\pi]$

t	x	y
0	0	0
$\frac{\pi}{4}$	$\frac{\pi}{8} \times \frac{\sqrt{2}}{2}$ (+ve)	$\frac{\pi}{8} \times \frac{\sqrt{2}}{2}$ (+ve)
$\frac{\pi}{2}$	0	$\frac{\pi}{4}$
π	$-\frac{\pi}{2}$	0
$\frac{3\pi}{2}$	0	$-\frac{3\pi}{4}$
2π	π	0



There are 3 pts of intersection. (1)

b. i) $I_n = \int_0^1 \sqrt{x}\,(1-x)^n\,dx$

$= \int_0^1 \frac{d\left(\frac{2x^{\frac{3}{2}}}{3}\right)}{dx}\,(1-x)^n\,dx$ (1)

$= \left[\frac{2x^{\frac{3}{2}}}{3}(1-x)^n\right]_0^1 - \frac{2}{3}\int_0^1 x^{\frac{3}{2}} \times n(1-x)^{n-1} \times -1\,dx$

$= 0 - \frac{2n}{3}\int_0^1 -x\sqrt{x}\,(1-x)^{n-1}\,dx$

$= -\frac{2n}{3}\int_0^1 (1-x-1)\sqrt{x}\,(1-x)^{n-1}\,dx$ (1)

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$$= \frac{-2n}{3}\int_0^1 (1-x)\sqrt{x}\,(1-x)^{n-1} - 1\sqrt{x}\,(1-x)^{n-1}\,dx$$

$$= \frac{-2n}{3}\int_0^1 \sqrt{x}\,(1-x)^{n} - \sqrt{x}\,(1-x)^{n-1}\,dx$$

ie $$I_n = -\frac{2n}{3} I_n + \frac{2n}{3} I_{n-1}$$

$$I_n\left(1 + \frac{2n}{3}\right) = \frac{2n}{3} I_{n-1}$$

$$I_n\left(\frac{3+2n}{3}\right) = \frac{2n}{3} I_{n-1} \qquad (1)$$

$$I_n = \frac{2n}{2n+3} I_{n-1}$$

ii) $$I_1 = \frac{2}{2+3} I_0$$

$$= \frac{2 I_0}{5}$$

$$I_0 = \int_0^1 \sqrt{x}\,dx$$

$$= \left[\frac{2x^{\frac{3}{2}}}{3}\right]_0^1$$

$$= \frac{2}{3}$$

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Question 15

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b. ii) (cont) $\therefore I_1 = \frac{2}{5} \times \frac{2}{3}$

$= \frac{4}{15}$ (1)

c. i) $\sum F = \sum m\ddot{x}$

$m\ddot{x} = mg - mkv^2$

$\ddot{x} = g - kv^2$ (1)

ii) achieves V_T when $\ddot{x} = 0$

ie $g = kv^2$

$v^2 = \frac{g}{k}$

$v = \sqrt{\frac{g}{k}}$ since $v > 0$ (1)

iii) $v\frac{dv}{dx} = g - kv^2$

$\frac{v}{g - kv^2} dv = dx$

$\frac{-1}{2k} \times \frac{-2kv}{g - kv^2} dv = dx$

$\frac{-1}{2k} \int_0^v \frac{-2kv}{g - kv^2} dv = \int_0^x dx$ (1)

$\frac{-1}{2k} \left[\ln |g - kv^2| \right]_0^v = x$ (1)

$$x = \frac{-1}{2k}\left[\ln|g - kv^2| - \ln g\right]$$

$$= \frac{1}{2k}\left[\ln g - \ln|g - kv^2|\right] \quad (1)$$

and, since $g - kv^2 > 0$

$$x = \frac{1}{2k}\ln\left(\frac{g}{g - kv^2}\right)$$

iv) when $v = \frac{1}{2}\sqrt{\frac{g}{k}}$

$$v^2 = \frac{g}{4k} \quad (1)$$

$$\therefore x = \frac{1}{2k}\ln\left(\frac{g}{g - k \times \frac{g}{4k}}\right)$$

$$= \frac{1}{2k}\ln\left(\frac{g}{\frac{3g}{4}}\right)$$

$$= \frac{1}{2k}\ln\left(\frac{4}{3}\right) \quad (1)$$

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Mathematics Extension 2

Writing Booklet

Question 16

00 0677 01 06

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Start here for Question Number: **16**

a. $\underset{\sim}{r} = \begin{pmatrix} -1 \\ 2 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$\therefore \quad x = -1+\lambda, \quad y = 2+2\lambda, \quad z = 6+\lambda$

At pts of intersection:

$$(-1+\lambda+1)^2 + (2+2\lambda-2)^2 + (6+\lambda-3)^2 = 45 \quad (1)$$

$$\lambda^2 + 4\lambda^2 + (3+\lambda)^2 = 45$$

$$5\lambda^2 + 9 + 6\lambda + \lambda^2 = 45$$

$$6\lambda^2 + 6\lambda - 36 = 0$$

$$\lambda^2 + \lambda - 6 = 0$$

$$(\lambda - 2)(\lambda + 3) = 0$$

$$\lambda = 2 \quad \text{or} \quad \lambda = -3$$

$\therefore$ Pts of int at:

$A\begin{pmatrix} 1 \\ 6 \\ 8 \end{pmatrix}$ and $B\begin{pmatrix} -4 \\ -4 \\ 3 \end{pmatrix}$ (1)

Shortest distance from C will be to M, midpoint of AB

M is $\begin{pmatrix} -\frac{3}{2} \\ 1 \\ \frac{11}{2} \end{pmatrix}$ C is $\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ (1)

$$MC = \sqrt{\left(\frac{-3}{2}+1\right)^2 + (1-2)^2 + \left(\frac{11}{2}-3\right)^2}$$

$$= \sqrt{\frac{1}{4} + 1 + \frac{25}{4}}$$

$$= \sqrt{\frac{30}{4}}$$

$$= \frac{\sqrt{30}}{2} \quad (1)$$

b. $a_{n+1} = \sqrt{2a_n}$ $\quad a_1 = 1$

RTP: $a_n = 2^{(1-2^{(1-n)})}$ for $n \geq 1$

Prove true for $n = 1$

$$a_1 = 2^{(1-2^{(1-1)})}$$

$$= 2^{1-1}$$

$$= 2^0$$

$$= 1 \quad (1)$$

$\therefore$ true for $n = 1$

Assume true for $n = k$

ie $a_k = 2^{(1-2^{(1-k)})}$

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Prove true for $n = k+1$

ie $a_{k+1} = 2^{\left(1-2^{(1-(k+1))}\right)}$

$= 2^{\left(1-2^{-k}\right)}$

Now, $a_{k+1} = \sqrt{2\,a_k}$

$= \sqrt{2 \times 2^{\left(1-2^{(1-k)}\right)}}$ (1) from assumption

$= \sqrt{2^{\left(1-2^{(1-k)}+1\right)}}$

$= \sqrt{2^{\left(2-2^{(1-k)}\right)}}$

$= \sqrt{2^{\left(2^1-2^1\times 2^{-k}\right)}}$

$= \sqrt{2^{2\left(1-2^{-k}\right)}}$

$= \left(2^{2\left(1-2^{-k}\right)}\right)^{\frac{1}{2}}$

$= 2^{1-2^{-k}}$ (1)

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16 b. (cont) $= RHS$

$\therefore$ true for $n = k+1$

$\therefore$ by induction, the statement is true for all integers $n \geq 1$

c. i) $cis\,4\theta = (\cos\theta + i\sin\theta)^4$

$= \cos^4\theta + 4\cos^3\theta i\sin\theta + 6\cos^2\theta i^2\sin^2\theta + 4\cos\theta i^3\sin^3\theta + i^4\sin^4\theta$ (1)

Equating reals:

$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

$= \cos^4\theta - 6\cos^2\theta(1-\cos^2\theta) + (1-\cos^2\theta)^2$

$= \cos^4\theta - 6\cos^2\theta + 6\cos^4\theta + 1 - 2\cos^2\theta + \cos^4\theta$

$= 8\cos^4\theta - 8\cos^2\theta + 1$ (1)

ii) $8x^4 - 8x^2 + 1 = 0$

let $x = \cos\theta$

$\therefore 8\cos^4\theta - 8\cos^2\theta + 1 = 0$

ie $\cos 4\theta = 0$

$\therefore 4\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \ldots$ (1)

$\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \ldots$

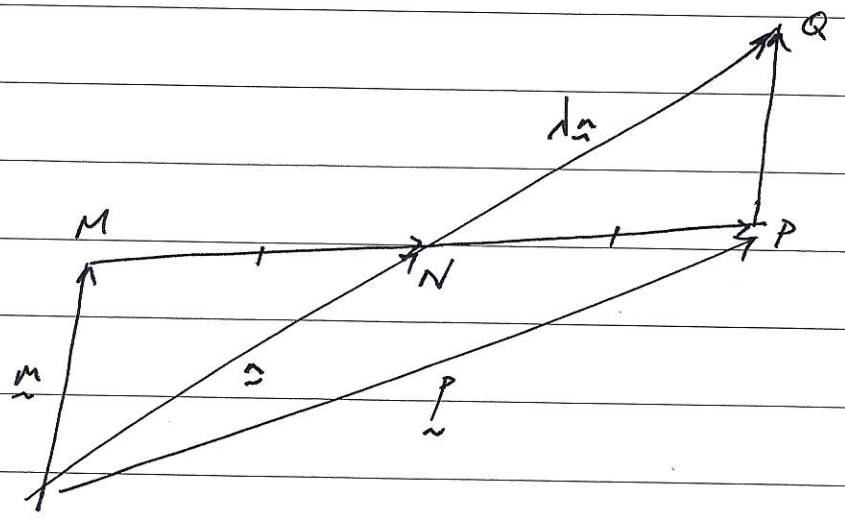
$\therefore$ The 4 distinct solutions are :

$x = \cos\frac{\pi}{8}, \quad x = \cos\frac{3\pi}{8}, \quad x = \cos\frac{5\pi}{8}, \quad x = \cos\frac{7\pi}{8}$ (1)

iii) Product of roots : $\frac{e}{a} = \frac{1}{8}$ (1)

i.e $\cos\frac{\pi}{8}\cos\frac{3\pi}{8}\cos\frac{5\pi}{8}\cos\frac{7\pi}{8} = \frac{1}{8}$

d.



$\vec{OM} + \vec{MP} = \vec{OP}$

$\vec{MP} = \vec{OP} - \vec{OM}$

$= \underset{\sim}{p} - \underset{\sim}{m}$

(1) some prep

$\vec{OP} + \vec{PQ} = \vec{OQ}$

$\vec{PQ} = \vec{OQ} - \vec{OP}$

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$$\vec{PQ} = (1+\lambda)\underset{\sim}{n} - \underset{\sim}{p}$$

Now, $\vec{MP} \perp \vec{PQ}$

$\therefore \vec{MP} \cdot \vec{PQ} = 0$

ie $(\underset{\sim}{p} - \underset{\sim}{m}) \cdot ((1+\lambda)\underset{\sim}{n} - \underset{\sim}{p}) = 0$ (1)

$$(1+\lambda)\underset{\sim}{p}\cdot\underset{\sim}{n} - (1+\lambda)\underset{\sim}{n}\cdot\underset{\sim}{n} + \underset{\sim}{p}\cdot\underset{\sim}{m} - \underset{\sim}{p}\cdot\underset{\sim}{p} = 0$$

$$(1+\lambda)[\underset{\sim}{p}\cdot\underset{\sim}{n} - \underset{\sim}{m}\cdot\underset{\sim}{n}] = \underset{\sim}{p}\cdot\underset{\sim}{p} - \underset{\sim}{p}\cdot\underset{\sim}{m}$$

$$(1+\lambda) = \frac{\underset{\sim}{p}\cdot\underset{\sim}{p} - \underset{\sim}{p}\cdot\underset{\sim}{m}}{\underset{\sim}{p}\cdot\underset{\sim}{n} - \underset{\sim}{m}\cdot\underset{\sim}{n}}$$

Now, $\vec{OM} + \vec{MN} = \vec{ON}$

$$\underset{\sim}{m} + \frac{1}{2}(\underset{\sim}{p} - \underset{\sim}{m}) = \underset{\sim}{n}$$

$$\frac{1}{2}\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{m} = \underset{\sim}{n}$$

$$\underset{\sim}{p} + \underset{\sim}{m} = 2\underset{\sim}{n}$$

$$\underset{\sim}{p} = 2\underset{\sim}{n} - \underset{\sim}{m}$$

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d. (cont)

$$\therefore \quad 1+\lambda = \frac{(2\underset{\sim}{n}-\underset{\sim}{m})\cdot(2\underset{\sim}{n}-\underset{\sim}{m}) - (2\underset{\sim}{n}-\underset{\sim}{m})\cdot\underset{\sim}{m}}{(2\underset{\sim}{n}-\underset{\sim}{m})\cdot\underset{\sim}{n} - \underset{\sim}{m}\cdot\underset{\sim}{n}}$$

$$= \frac{4\underset{\sim}{n}\cdot\underset{\sim}{n} - 4\underset{\sim}{m}\cdot\underset{\sim}{n} + \underset{\sim}{m}\cdot\underset{\sim}{m} - 2\underset{\sim}{m}\cdot\underset{\sim}{n} + \underset{\sim}{m}\cdot\underset{\sim}{m}}{2\underset{\sim}{n}\cdot\underset{\sim}{n} - 2\underset{\sim}{m}\cdot\underset{\sim}{n}}$$

$$= \frac{4|\underset{\sim}{n}|^2 - 6\underset{\sim}{m}\cdot\underset{\sim}{n} + 2|\underset{\sim}{m}|^2}{2|\underset{\sim}{n}|^2 - 2\underset{\sim}{m}\cdot\underset{\sim}{n}}$$

$$1+\lambda = \frac{2|\underset{\sim}{n}|^2 + |\underset{\sim}{m}|^2 - 3\underset{\sim}{m}\cdot\underset{\sim}{n}}{|\underset{\sim}{n}|^2 - \underset{\sim}{m}\cdot\underset{\sim}{n}}$$

$$\therefore \quad \lambda = \frac{2|\underset{\sim}{n}|^2 + |\underset{\sim}{m}|^2 - 3\underset{\sim}{m}\cdot\underset{\sim}{n} - |\underset{\sim}{n}|^2 + \underset{\sim}{m}\cdot\underset{\sim}{n}}{|\underset{\sim}{n}|^2 - \underset{\sim}{m}\cdot\underset{\sim}{n}}$$

$$= \frac{|\underset{\sim}{n}|^2 + |\underset{\sim}{m}|^2 - 2\underset{\sim}{m}\cdot\underset{\sim}{n}}{|\underset{\sim}{n}|^2 - \underset{\sim}{m}\cdot\underset{\sim}{n}} \qquad (1)$$