BAULKHAM HILLS HIGH SCHOOL

2019

HSC Assessment Task 1

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 50 minutes
- Write using non-erasable black or blue pen.
- Calculators approved by NESA may be used.
- Marks may be deducted for careless or badly arranged work.
- Relevant formulae from the NESA Reference Sheet are provided on page 2.
- Answer all questions in the space provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.
- If extra paper is used, it must be clearly labelled with your name and the question being answered, and stapled inside your answer booklet.

Total marks – 45
Exam consists of 12 pages.

Section I: Pages 3-4
5 marks
Attempt Questions 1-5

Section II: Pages 4-12
40 marks
Attempt Questions 6-15.

Formulae from NESA Reference Sheet:

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mathematics Extension 2
Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

Start Here → **1.** A ○ B ○ C ○ D ○

2. A ○ B ○ C ○ D ○

3. A ○ B ○ C ○ D ○

4. A ○ B ○ C ○ D ○

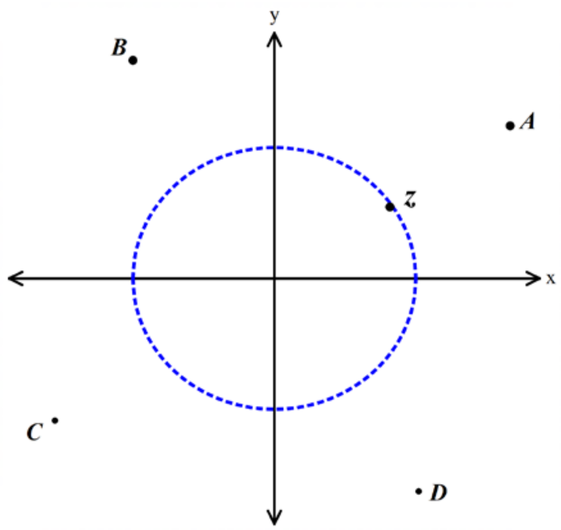
5. A ○ B ○ C ○ D ○

Section I:
5 marks
Attempt Questions 1-5

Use the multiple choice answer sheet on the previous page.

		Marks
1	Let $z = 2 - 3i$ and $w = 3 + 2i$. What is the value of $z - \overline{w}$? (A) $5 - i$ (B) $-1 - 5i$ (C) $-1 - i$ (D) 0	**1**
2	If z is a complex number, which of the following is equivalent to $z\bar{z} - \|z\|^2$? (A) 1 (B) 0 (C) $(z + \bar{z})(z - \bar{z})$ (D) $(z + iz)(z - iz)$	**1**
3	Let $z = a + bi$, where a and b are non-zero real numbers. If $z + \frac{1}{z}$ is purely real, which of the following statements must be true? (A) $a = b$ (B) $a = -b$ (C) $\|z\| = 1$ (D) $z^2 = 1$	**1**
4	When the complex number $-1 + 5i$ is written in the form $\sqrt{26}e^{i\theta}$, which of the following is the approximate value of θ ? (A) -1.77 (B) -1.37 (C) 1.37 (D) 1.77	**1**

5 The complex number z is shown on the Argand diagram below.



Which of the following best represents $-2iz$?

(A) Point A (B) Point B (C) Point C (D) Point D

End of Section I

Section II: 40 marks
Attempt Questions 6-15 in the space provided.

Question 6 (5 marks)

Let $z = 1 - 2i$ and $w = 3 + i$. Find in the form $x + iy$ (where x and y are real numbers):

(i) zw **1**

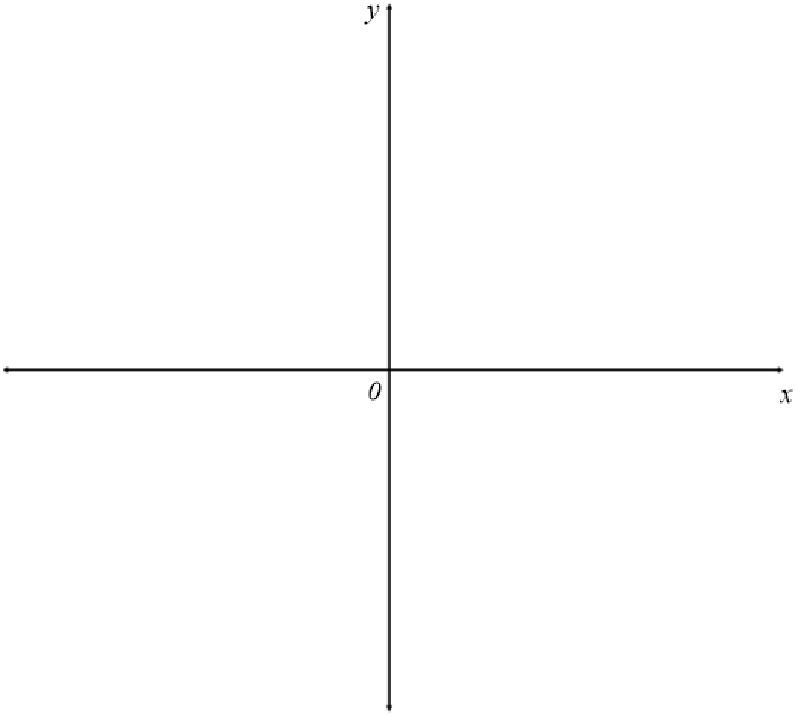
(ii) $z^2 - w^2$ **2**

(iii) $\overline{\left(\frac{10}{z}\right)}$ **2**

Question 7 (3 marks)

If $|z-2| = |z-i|$:

(i) Sketch the locus of z on an Argand diagram. **2**

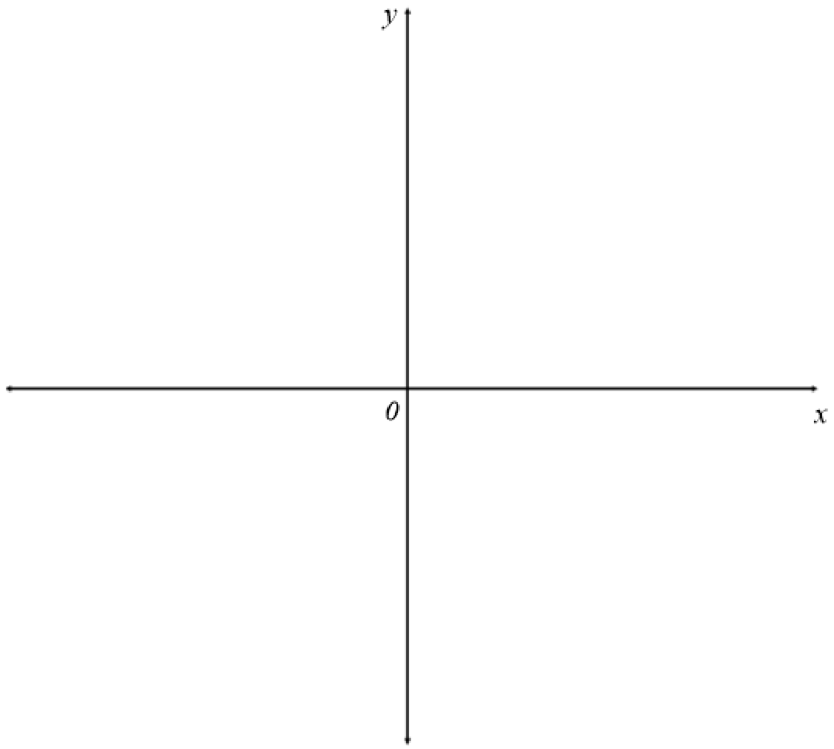


(ii) Find the equation of the locus of z in Cartesian form. **1**

Question 8 (3 marks)

Given that $|z + 2 - 3i| = 2$:

(i) Draw the locus of z on the Argand diagram. **2**



(ii) Find the maximum value of $|z|$. **1**

Question 9 (5 marks)

Given $z = \sqrt{3} - i$

(i) Express z in modulus-argument form. **2**

(ii) Hence or otherwise, evaluate z^{10}, giving your answer in Cartesian form. **2**

(iii) What is the value of the least positive integer n for which z^n is purely imaginary? **1**

Question 10 (4 marks)

(i) Find the complex numbers z such that $z^2 = 5 + 12i$. Answer in Cartesian form. **2**

(ii) Hence or otherwise, solve the equation $z^2 + 3z + 1 - 3i = 0$. **2**

Question 11 (3 marks)

(i) Expand $(\cos\theta + i\sin\theta)^3$. **1**

(ii) Hence express $\sin 3\theta$ in terms of $\sin\theta$. **2**

Question 12 (5 marks)

The polynomial $P(z) = 3z^4 - 4z^3 - 6z^2 + 20z - 16$ has a zero at $z = 1 + i$.

(i) Find a quadratic factor of $P(z)$. **2**

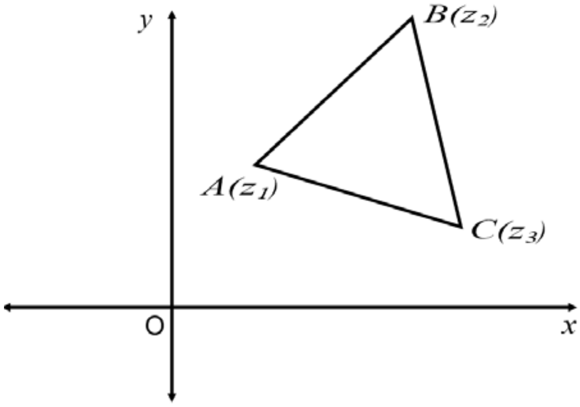
(ii) Express $P(z)$ as a product of linear factors. **3**

Question 13 (2 marks)

Simplify $\left(e^{i\theta} - e^{-i\theta}\right)^4$, expressing your answer as a trigonometric function. **2**

Question 14 (4 marks)

In the diagram below, ΔABC is an equilateral triangle with $\overrightarrow{OA} = z_1,\ \overrightarrow{OB} = z_2,\ \overrightarrow{OC} = z_3$ and $\arg(z_2 - z_1) = \alpha$:



(i) Find $\arg(z_2 - z_3)$ in terms of α. **1**

(ii) Find the modulus and argument of $\frac{z_2 - z_1}{z_3 - z_2}$. **3**

Question 15 (6 marks)

Given $P(z) = z^5 - 1$ and $z \in C$:

(i) Find the roots of $P(z) = 0$. **2**

(ii) Express $P(z)$ as a product of real linear and real irreducible quadratic factors. **2**

(iii) Using the identity:

$$z^5 - 1 = (z - 1)(z^4 + z^3 + z^2 + z + 1)$$

Find the value of : $$(1 + \cos\frac{2\pi}{5})\,(1 + \cos\frac{4\pi}{5})$$ **2**

END OF EXAMINATION

X2 : 2019 HSC Task 1 (December)
SOLUTIONS.

Section I:
5 marks
Attempt Questions 1-5

Marks

Use the multiple choice answer sheet on the previous page.

1 Let $z = 2 - 3i$ and $w = 3 + 2i$. **1**
What is the value of $z - \overline{w}$?

(A) $5 - i$
(B) $-1 - 5i$
(C) $-1 - i$
(D) 0

$2 - 3i - (3 - 2i)$
$= -1 - i$

2 If z is a complex number, which of the following is equivalent to $z\bar{z} - |z|^2$? **1**

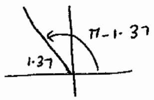
(A) 1
(B) 0
(C) $(z + \bar{z})(z - \bar{z})$
(D) $(z + iz)(z - iz)$

$z\bar{z} = |z|^2 \quad \therefore \quad 0$

3 Let $z = a + bi$, where a and b are non-zero real numbers. If $z + \frac{1}{z}$ is purely real, which of the following statements must be true? **1**

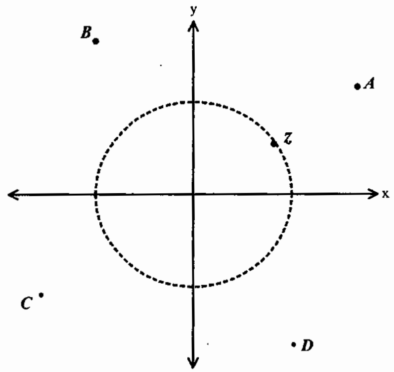
(A) $a = b$
(B) $a = -b$
(C) $|z| = 1$
(D) $z^2 = 1$

4 When the complex number $-1 + 5i$ is written in the form $\sqrt{26}e^{i\theta}$, which of the following is the approximate value of θ ? **1**



(A) -1.77
(B) -1.37
(C) 1.37
(D) 1.77

5 The complex number z is shown on the Argand diagram below.



Which of the following best represents $-2iz$?

(A) Point A (B) Point B (C) Point C (D) Point D

End of Section I

Section II: 40 marks
Attempt Questions 6-15 in the space provided.

Question 6 (5 marks)

Let $z = 1 - 2i$ and $w = 3 + i$. Find in the form $x + iy$ (where x and y are real numbers):

(i) zw **1**

$= (1-2i)(3+i) = 3 + 2 - 6i + i$
$= 5 - 5i$

1 - correct answer

(ii) $z^2 - w^2$ **2**

$= (1 - 2i - 3 - i)(1 - 2i + 3 + i)$
$= (-2 - 3i)(4 - i)$
$= -8 + 2i - 12i - 3$
$= -11 + 10i$

1 - partially correct working
2 - correct answer

(iii) $\overline{\left(\frac{10}{z}\right)}$ **2**

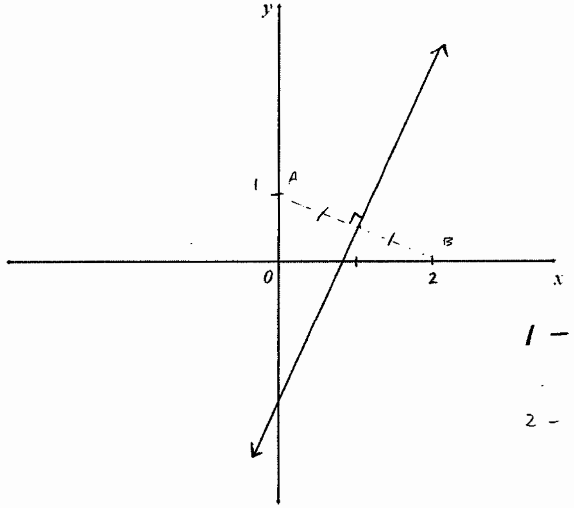
$= \frac{10}{\bar{z}} = \frac{10}{1+2i} \times \frac{1-2i}{1-2i}$
$= \frac{10(1-2i)}{5}$
$= 2(1-2i)$ or $2 - 4i$

1 - attempt to realise a denominator or express as $\frac{10}{1+2i}$
2 - correct solution

Question 7 (3 marks)

If $|z-2| = |z-i|$:

(i) Sketch the locus of z on an Argand diagram. **2**



1 – a straight line

2 – correct solution

(ii) Find the equation of the locus of z in Cartesian form. **1**

Perpendicular bisector of $\overset{A}{(0,1)}$ and $\overset{B}{(2,0)}$:

$m = 2$, midpt $= \left(\frac{1}{2}, \frac{1}{2}\right)$

$y - \frac{1}{2} = 2\left(x - \frac{1}{2}\right)$

$y - \frac{1}{2} = 2x - 1$

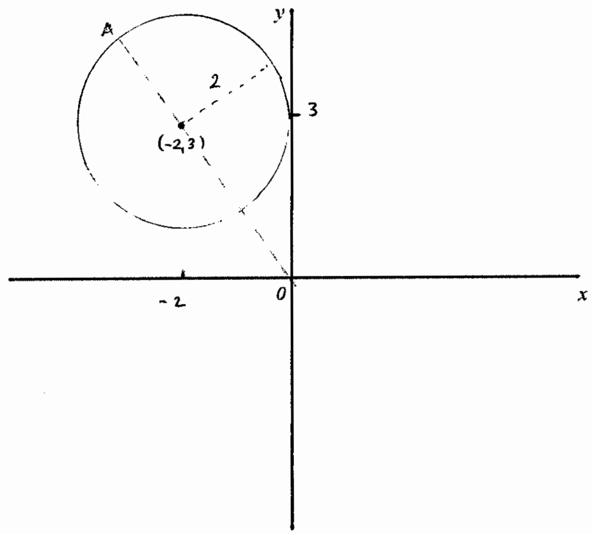
$4x - 2y - 3 = 0$ or $y = 2x - \frac{3}{2}$

1 – correct eqn

Question 8 (3 marks)

Given that $|z + 2 - 3i| = 2$:

(i) Draw the locus of z on the Argand diagram. **2**

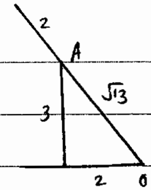


1 – circle with correct centre or radius

2 – correct circle

(ii) Find the maximum value of $|z|$. **1**

Maximum value of $|z|$ occurs at A

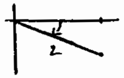


Max. value of $|z| = \sqrt{13} + 2$

1 – correct solution

Question 9 (5 marks)

Given $z = \sqrt{3} - i$



(i) Express z in modulus-argument form. **2**

$2\left(\cos -\frac{\pi}{6} + i \sin \frac{-\pi}{6}\right)$

1 – correct mod or arg

or $2 \operatorname{cis} \frac{-\pi}{6}$

2 – both correct

(must be principal arg)

(ii) Hence or otherwise, evaluate z^{10}, giving your answer in Cartesian form. **2**

$z^{10} = 2^{10} \operatorname{cis}\left(10 \times \frac{-\pi}{6}\right)$

$= 1024 \operatorname{cis} \frac{-5\pi}{3}$

1 – attempts to apply De Moivre's theorem

$= 1024 \operatorname{cis} \frac{\pi}{3}$

2 – correct soln

$= 1024\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$

$= 512 + 512 i\sqrt{3}$

(iii) What is the value of the least positive integer for which z^n is purely imaginary? **1**

If purely imaginary, $n \times \frac{\pi}{6}$ is an odd multiple of $\frac{\pi}{2}$.

$n = 3$ is the first such value.

1 – correct answer.

Question 10 (4 marks)

(i) Find the complex numbers z such that $z^2 = 5 + 12i$. Answer in Cartesian form. **2**

Let $z = x + iy$, $(x+iy)^2 = 5 + 12i$

$x^2 - y^2 = 5$ —— (1)

$2xy = 12$

$xy = 6$ —— (2)

$x^2 - \left(\frac{6}{x}\right)^2 = 5$

$x^4 - 5x^2 - 36 = 0$

$(x^2 - 9)(x^2 + 4) = 0$

$x = \pm 3$, $y = \pm 2$

$z = \pm(3 + 2i)$

1 – obtains a correct pair of simult. equations and attempts to solve

2 – correct soln

(ii) Hence or otherwise, solve the equation $z^2 + 3z + 1 - 3i = 0$. **2**

$\Delta = b^2 - 4ac = 3^2 - 4(1 - 3i) = 5 + 12i$

$z = \dfrac{-3 \pm \sqrt{5 + 12i}}{2}$

$= \dfrac{-3 \pm (3 + 2i)}{2}$

1 – applies quadratic formula with correct Δ.

2 – correct solutions

$= i, \ -3 - i$

Question 11 (3 marks)

(i) Expand $(\cos\theta + i\sin\theta)^3$. **1**

$\cos^3\theta + 3\cos^2\theta . i\sin\theta + 3\cos\theta . i^2 \sin^2\theta + i^3 \sin^3\theta$

$= \cos^3\theta + 3i\cos^2\theta \sin\theta - 3\cos\theta \sin^2\theta - i \sin^3\theta$

1 – either of above lines.

(ii) Hence express $\sin 3\theta$ in terms of $\sin\theta$. **2**

$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i \sin 3\theta$

$\sin 3\theta = 3\cos^2\theta \sin\theta - \sin^3\theta$

$= 3(1 - \sin^2\theta)\sin\theta - \sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$.

1 – equates $\sin 3\theta$ with imaginary part of cis.

2 – correct solution

Question 12 (5 marks)

The polynomial $P(z) = 3z^4 - 4z^3 - 6z^2 + 20z - 16$ has a zero at $z = 1 + i$.

(i) Find a quadratic factor of $P(z)$. **2**

Since coefficients are real, $1-i$ is also a root.

$\alpha = \cancel{1+i}\ 1+i, \quad \beta = 1-i$ — 1 – identifies $1-i$ is a root

$\left.\begin{aligned} \alpha+\beta &= 2 \\ \alpha\beta &= 2 \end{aligned}\right\} \therefore \underline{z^2 - 2z + 2.}$ — 2 – correct factor

(ii) Express $P(z)$ as a product of linear factors. **3**

$P(z) = (z^2 - 2z + 2)(3z^2 + kz - 8)$

Comparing terms in z:

$16z + 2kz = 20z$

$k = 2$

$\therefore P(z) = (z^2 - 2z + 2)(3z^2 + 2z - 8)$

$= (z - 1 - i)(z - 1 + i)(3z - 4)(z + 2).$

– ~~obtains $(z-1-i)(z-1+i)$~~

1 – obtains $(3z^2 + 2z - 8)(z^2 - 2z + 2)$

2 – $(z-1-i)(z-1+i)(z-\frac{4}{3})(z+2)$, omits lead coefficient

– $(z-1-i)(z-1+i)$ and incorrect factors for the quadratic

3 – correct soln.

Question 13 (2 marks)

Simplify $\left(e^{i\theta} - e^{-i\theta}\right)^4$, expressing your answer as a trigonometric function. **2**

$= \left(\cos\theta + i\sin\theta - (\cos(-\theta) + i\sin(-\theta))\right)^4$

$= \left(\cos\theta + i\sin\theta - (\cos\theta - i\sin\theta)\right)^4$

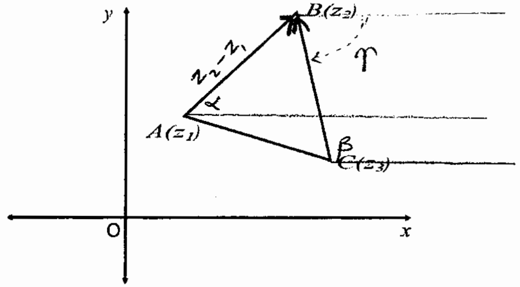
$= (2i\sin\theta)^4$ — 1 - significant progress

2 – correct solution

$= 16\sin^4\theta$

Question 14 (4 marks)

In the diagram below, ΔABC is an equilateral triangle with $\overrightarrow{OA} = z_1$, $\overrightarrow{OB} = z_2$, $\overrightarrow{OC} = z_3$ and $\arg(z_2 - z_1) = \alpha$:



(i) Find $\arg(z_2 - z_3)$ in terms of α. **1**

Let $\beta = \arg(z_2 - z_3)$.

$\left(\beta + \frac{\pi}{3}\right) + \left(\frac{\pi}{3} - \alpha\right) = \pi$ (cointerior $\angle$s, // lines)

$\beta + \frac{2\pi}{3} - \alpha = \pi$

$\therefore \beta = \arg(z_2 - z_3) = \alpha + \frac{\pi}{3}.$

1 – correct solution

(ii) Find the modulus and argument of $\frac{z_2 - z_1}{z_3 - z_2}$. **3**

Modulus $= 1$ (since Δ is equilateral)

$\operatorname{Arg}\left(\frac{z_2 - z_1}{z_3 - z_2}\right) = \arg(z_2 - z_1) - \arg(z_3 - z_2)$

$= \alpha - (\pi - \beta)$

$= \alpha - \pi + \alpha + \frac{\pi}{3}.$

$= 2\alpha - \frac{2\pi}{3}$

(1) – correct modulus

Then

(1) significant progress } for

(2) correct soln } arg.

Question 12 (5 marks)

The polynomial $P(z) = 3z^4 - 4z^3 - 6z^2 + 20z - 16$ has a zero at $z = 1 + i$.

(i) Find a quadratic factor of $P(z)$. **2**

Since coefficients are real, $1-i$ is also a root.

$\alpha = \cancel{1+1}\ 1+i, \quad \beta = 1-i$ — 1 - identifies $1-i$ is a root

$\left.\begin{array}{l}\alpha+\beta = 2\\ \alpha\beta = 2\end{array}\right\} \therefore \underline{z^2 - 2z + 2.}$ — 2 - correct factor

(ii) Express $P(z)$ as a product of linear factors. **3**

$P(z) = (z^2 - 2z + 2)(3z^2 + kz - 8)$

Comparing terms in z:

$16z + 2kz = 20z$

$k = 2$

$\therefore P(z) = (z^2 - 2z + 2)(3z^2 + 2z - 8)$

$= (z-1-i)(z-1+i)(3z-4)(z+2).$

~~- obtains $(z-1-i)(z-1+i)$~~

1 - obtains $(3z^2+2z-8)(z^2-2z+2)$

2 - $(z-1-i)(z-1+i)(z-\frac{4}{3})(z+2)$, omits lead coefficient

- $(z-1-i)(z-1+i)$ and incorrect factors e.g. for the quadratic

3 - correct soln.

Question 13 (2 marks)

Simplify $(e^{i\theta} - e^{-i\theta})^4$, expressing your answer as a trigonometric function. **2**

$= \left(\cos\theta + i\sin\theta - (\cos(-\theta) + i\sin(-\theta))\right)^4$

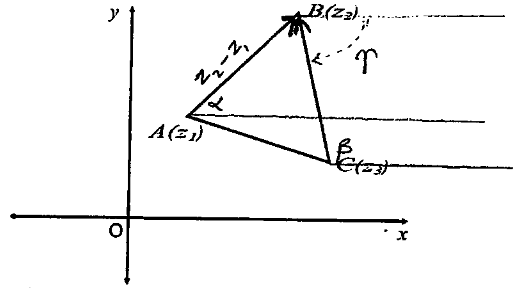
$= \left(\cos\theta + i\sin\theta - (\cos\theta - i\sin\theta)\right)^4$

$= (2i\sin\theta)^4$ — 1 - significant progress

$= 16\sin^4\theta$ — 2 - correct solution

Question 14 (4 marks)

In the diagram below, ΔABC is an equilateral triangle with $\overrightarrow{OA} = z_1$, $\overrightarrow{OB} = z_2$, $\overrightarrow{OC} = z_3$ and $\arg(z_2 - z_1) = \alpha$:



(i) Find $\arg(z_2 - z_3)$ in terms of α. **1**

Let $\beta = \arg(z_2 - z_3)$.

$\left(\beta + \frac{\pi}{3}\right) + \left(\frac{\pi}{3} - \alpha\right) = \pi$ (cointerior ∠s, // lines)

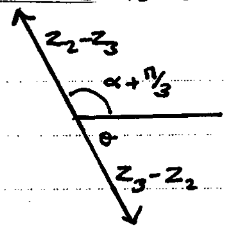
$\beta + \frac{2\pi}{3} - \alpha = \pi$

$\therefore \beta = \arg(z_2 - z_3) = \alpha + \frac{\pi}{3}.$

1 - correct solution

(ii) Find the modulus and argument of $\frac{z_2 - z_1}{z_3 - z_2}$. **3**

Modulus = 1 (since Δ is equilateral)



In the diagram,

$\theta + \alpha + \frac{\pi}{3} = \pi$ (∠ sum of straight ∠)

$\theta = \frac{2\pi}{3} - \alpha$

$\arg(z_3 - z_2) = -\theta = \alpha - \frac{2\pi}{3}$

$\therefore \arg\left(\frac{z_2 - z_1}{z_3 - z_2}\right) = \arg(z_2 - z_1) - \arg(z_3 - z_2)$

$= \alpha - \left(\alpha - \frac{2\pi}{3}\right)$

$= \frac{2\pi}{3}$

(1) - correct modulus

Then

(1) significant progress
(2) correct soln
for arg.

Question 15 (6 marks)

Given $P(z) = z^5 - 1$ and $z \in C$:

(i) Find the roots of $P(z) = 0$. **2**

$1, \ cis \pm\frac{2\pi}{5}, \ cis \pm\frac{4\pi}{5}$

1 - obtain 1 and one other correct root

2 - correct roots (all)

(ii) Express $P(z)$ as a product of real linear and real irreducible quadratic factors. **2**

$P(z) = (z-1)\left(z^2 - 2z\cos\frac{2\pi}{5} + 1\right)\left(z^2 - 2z\cos\frac{4\pi}{5} + 1\right)$

1 - obtain $(z-1)$ and two partially correct quad. factors

2 - correct factors

(iii) Given the identity:

$$z^5 - 1 = (z-1)(z^4 + z^3 + z^2 + z + 1)$$

Find the value of : $\left(1 + \cos\frac{2\pi}{5}\right)\left(1 + \cos\frac{4\pi}{5}\right)$ **2**

$P(z) = z^5 - 1$. Let $Q(z) = \left(z^2 - 2z\cos\frac{2\pi}{5} + 1\right)\left(z^2 - 2z\cos\frac{4\pi}{5} + 1\right)$

Method 1 $\qquad = z^4 + z^3 + z^2 + z + 1$

Now $Q(-1) = \left(1 + 2\cos\frac{2\pi}{5} + 1\right)\left(1 + 2\cos\frac{4\pi}{5} + 1\right) = 1 - 1 + 1 - 1 + 1$

$\therefore 2\left(1 + \cos\frac{2\pi}{5}\right) \cdot 2\left(1 + \cos\frac{4\pi}{5}\right) = 1$

(1) substitute $z = -1$

(2) correct soln.

$\therefore \left(1 + \cos\frac{2\pi}{5}\right)\left(1 + \cos\frac{4\pi}{5}\right) = \frac{1}{4}$

Method 2.

$\left(z^2 - 2z\cos\frac{2\pi}{5} + 1\right)\left(z^2 - 2z\cos\frac{4\pi}{5} + 1\right) = z^4 + z^3 + z^2 + z + 1 = Q(z)$

The roots of $Q(z)$ are $cis\left(\pm\frac{2\pi}{5}\right)$, $cis\left(\pm\frac{4\pi}{5}\right)$

$\alpha + \beta + \gamma + \delta = \frac{-b}{a} = cis\frac{2\pi}{5} + cis\frac{-2\pi}{5} + cis\frac{4\pi}{5} + cis\frac{-4\pi}{5}$

$-1 = 2\cos\frac{2\pi}{5} + 2\cos\frac{4\pi}{5}$

$\therefore \cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}$ (1)

$\Sigma\alpha\beta = +\frac{c}{a} = cis\frac{2\pi}{5} \cdot cis\frac{-2\pi}{5} + cis\frac{2\pi}{5} \cdot cis\frac{4\pi}{5} + cis\frac{2\pi}{5} \cdot cis\frac{-4\pi}{5}$

$+ cis\frac{-2\pi}{5} \cdot cis\frac{4\pi}{5} + cis\frac{-2\pi}{5} \cdot cis\frac{-4\pi}{5}$

$+ cis\frac{4\pi}{5} \cdot cis\frac{-4\pi}{5}$

$1 = 1 + cis\frac{2\pi}{5} \cdot cis\frac{4\pi}{5} + cis\frac{2\pi}{5} \cdot cis\frac{-4\pi}{5}$

$+ cis\frac{-2\pi}{5} \cdot cis\frac{4\pi}{5} + cis\frac{-2\pi}{5} \cdot cis\frac{-4\pi}{5} + 1$

$-1 = cis\frac{2\pi}{5}\left(cis\frac{4\pi}{5} + cis\frac{-4\pi}{5}\right) +$

$cis\frac{-2\pi}{5}\left(cis\frac{4\pi}{5} + cis\frac{-4\pi}{5}\right)$

(1) correct $\Sigma\alpha$.

$= \left(cis\frac{4\pi}{5} + cis\frac{-4\pi}{5}\right)\left(cis\frac{2\pi}{5} + cis\frac{-2\pi}{5}\right)$

(2) correct soln

$= 2\cos\frac{4\pi}{5} \cdot 2\cos\frac{2\pi}{5}$

$\therefore \cos\frac{2\pi}{5} \cdot \cos\frac{4\pi}{5} = -\frac{1}{4}$ (2)

Now $\left(1 + \cos\frac{2\pi}{5}\right)\left(1 + \cos\frac{4\pi}{5}\right) = 1 + \left(\cos\frac{4\pi}{5} + \cos\frac{2\pi}{5}\right) + \cos\frac{2\pi}{5}\cos\frac{4\pi}{5}$

$= 1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{4}\right)$

$= \frac{1}{4}$

Method 3.

$$\left(z^2 - 2z\cos\frac{2\pi}{5} + 1\right)\left(z^2 - 2z\cos\frac{4\pi}{5} + 1\right) = z^4 + z^3 + z^2 + z + 1$$

Equating coefficients of z :

$$-2\cos\frac{2\pi}{5} - 2\cos\frac{4\pi}{5} = 1$$

$$\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}.$$

Equating coefficients of z^2

$$1 + 1 + 4\cos\frac{2\pi}{5}\cos\frac{4\pi}{5} = 1$$

$$4\cos\frac{2\pi}{5}\cos\frac{4\pi}{5} = -1$$

$$\cos\frac{2\pi}{5}\cos\frac{4\pi}{5} = -\frac{1}{4}$$

Now $\left(1 + \cos\frac{2\pi}{5}\right)\left(1 + \cos\frac{4\pi}{5}\right)$

$$= 1 + \left(\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5}\right) + \cos\frac{2\pi}{5}\cos\frac{4\pi}{5}$$

$$= 1 + -\frac{1}{2} + -\frac{1}{4}$$

$$= \frac{1}{4}$$

(1) Correctly equate coefficients

(2) correct solution