Name: ____________________

Class: ____________________

2017

Term 4 Assessment (HSC)

Mathematics Extension 2

General Instructions

- Working time – 90 minutes + 5minutes reading
- Board approved calculators may be used
- Write using black or blue pen
- All necessary working should be shown in all questions
- Write your student number at the top of every page of your answer sheets.

4 questions – 15 marks each

Total marks – 60

Attempt all questions

Start each question on a new sheet of paper.

QUESTION 1 (Start a new page) 15 Marks

a) Let $z = 1 + 2i$ and $w = 2 - i$. Find in the form $x + iy$

i) $z\overline{w}$ **2**

ii) $\frac{1}{w^2}$ **2**

b) Let θ be a real number and consider $(cos\theta + isin\theta)^3$.
Prove $cos3\theta = cos^3\theta - 3\cos\theta\sin^2\theta$ **2**

c) Sketch on the Argand diagram the region which satisfies these expressions
$2 \leq |z| \leq 3$ and $\frac{\pi}{4} < \arg(z - i) < \frac{3\pi}{4}$ **3**

d) $z = x + iy$ is such that $\frac{z-i}{z+1}$ is purely imaginary. Find the equation of the locus of the point P representing z. Describe this locus geometrically. **3**

e) Given that i is a zero of $P(x) = x^4 - x^3 + 3x^2 - x + 2$, find all complex solutions to $x^4 - x^3 + 3x^2 - x + 2 = 0$. **3**

QUESTION 2 (Start a new page) 15 Marks

a) i) Find the square roots of $6i$. **2**

ii) Hence or otherwise solve $z^2-(1-i)z-2i=0$ **2**

b) The polynomial equation $x^3-2x^2+4x-5=0$ has roots α, β and γ.

i. Find the value of $\frac{1}{\alpha^3\beta^3\gamma^3}$ **1**

ii. Which polynomial equation has roots α^2, β^2 and γ^2? **2**

iii. Find the value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$. **2**

c) Let $z_1=4+i$ and $z_2=1+i$.

i) Sketch the locus of z on the Argand plane such that $arg\left(\frac{z-z_2}{z-z_1}\right)=\frac{\pi}{2}$ **2**

ii) Explain why $\arg\left(z-\frac{5}{2}-i\right)=2\arg(z-1-i)$. **2**

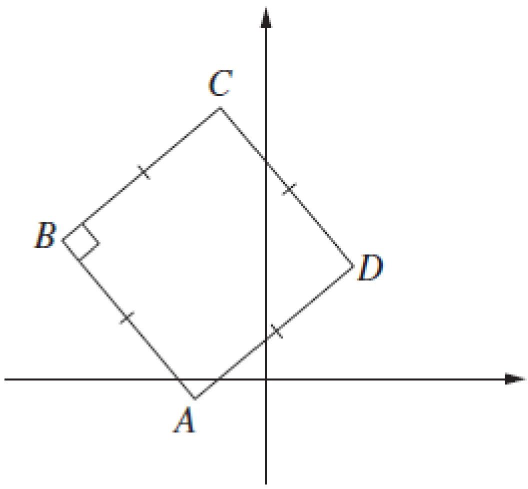
d) The polynomial $P(x)=x^4-3x^3-6x^2+28x-24$ has a root of multiplicity 3.
Find all the roots of $P(x)$ and hence factorise $P(x)$. **2**

QUESTION 3 (Start a new page) 15 Marks

a) If w is a complex cube root of unity, simplify $(1 + w - w^2)^7$ in terms of w^2. **1**

b) Find, in modulus-argument form, the three cube roots of -8 **2**

c) The points A, B, C and D on the Argand diagram below represent the complex numbers a, b, c and d respectively.



The points form a square as shown on the diagram above. By using vectors, or otherwise, show that $d = a(1 + i) - ib$. **2**

d) i) On an Argand diagram, shade in the region determined by the inequalities **3**

$1 \leq Im\, z \leq 2$ and $\frac{5\pi}{12} \leq \arg z \leq \frac{11\pi}{12}$

ii) Let z_0 be the complex number of maximum modulus satisfying the inequalities of d(i). Express z_0 in the form of $a + ib$ in exact form. **2**

e) Let $z = cos\theta + isin\theta$.

i) Show that $\sin n\theta = \frac{z^n - z^{-n}}{2i}$ and $\cos n\theta = \frac{z^n + z^{-n}}{2}$ **2**

ii) Hence, or otherwise, prove the identity
$32sin^4\theta cos^2\theta = cos6\theta - 2cos4\theta - cos2\theta + 2$ **3**

QUESTION 4 (Start a new page) 15 Marks

a) If $|z| = |w|$, prove that $\frac{z+w}{z-w}$ is purely imaginary. By drawing a suitable diagram, give a geometrical interpretation of the result. **3**

b) i) Write $\sqrt{6} - \sqrt{2}i$ in modulus-argument form. **2**

ii) Hence, expand and simplify $\left(\frac{\sqrt{6}-\sqrt{2}i}{2-2i}\right)^{2000}$. Give your answer in the form of $x + iy$ **2**

c) Let α be the complex root of the polynomial $z^7 = 1$ with the smallest possible argument. Let $\theta = \alpha + \alpha^2 + \alpha^4$ and $\delta = \alpha^3 + \alpha^5 + \alpha^6$

i) Explain why $\alpha^7 = 1$ and $1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6 = 0$ **1**

ii) Show that $\theta + \delta = -1$ and $\theta\delta = 2$ and hence write a quadratic equation whose roots are θ and δ **3**

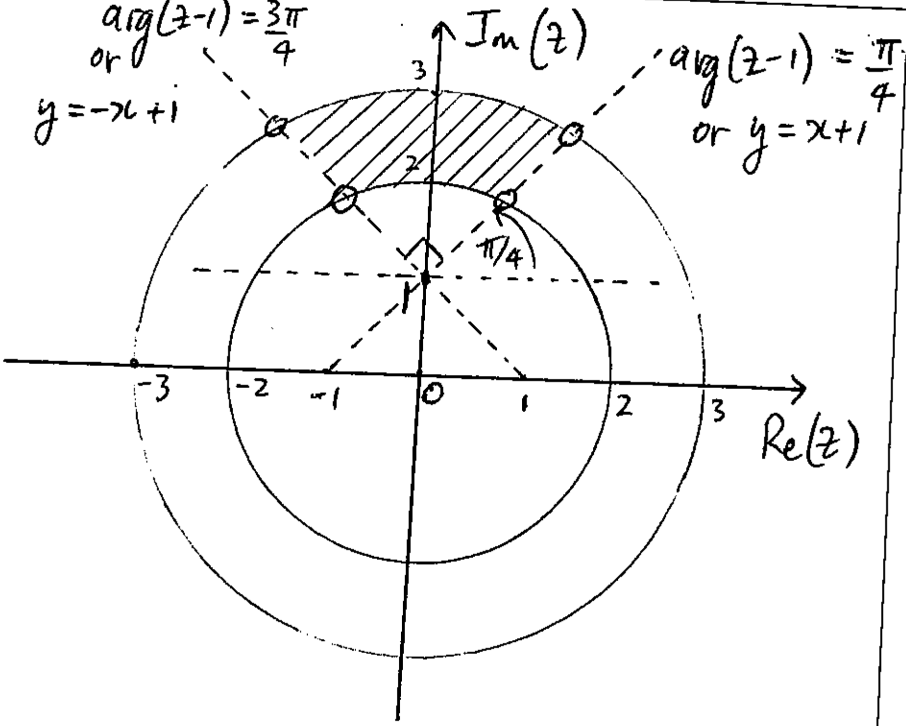
iii) Write down α in mod-arg form, and show that

$cos\frac{4\pi}{7} + \cos\frac{2\pi}{7} - cos\frac{\pi}{7} = -\frac{1}{2}$ and $sin\frac{4\pi}{7} + \sin\frac{2\pi}{7} - sin\frac{\pi}{7} = \frac{\sqrt{7}}{2}$ **4**

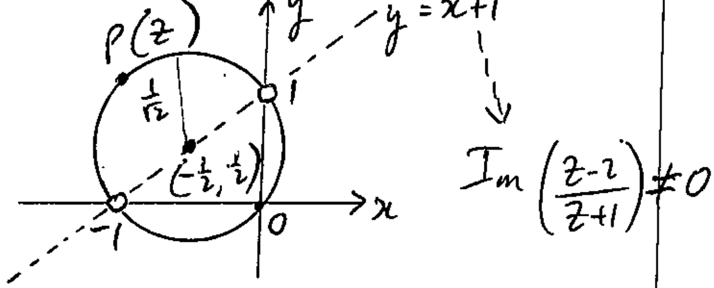
****END OF EXAMINATION****

Y12 Task 1: 2017 **MATHEMATICS Extension 2: Question** 1 JRAHS

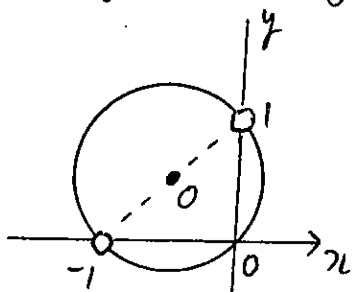
Suggested Solutions	Marks	Marker's Comments
a) (i) $z\bar{w} = (1+2i)(2+i)$	1	correct conjugate for w
$= 2+5i-2$ $= 0+5i$	1	correct product
(ii) $\frac{1}{w^2} = \frac{1}{(2-i)^2} = \frac{1}{4-4i-1}$		
$= \frac{1}{3-4i} \times \frac{3+4i}{3+4i}$ $= \frac{3+4i}{9+16}$	1	For squaring and realising the denominator
$= \frac{3}{25} + \frac{4}{25}i$	1	For correct simplified expression
b) $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$ ... ① by De Moivre's Theorem	1	For correct reason
But $(\cos\theta + i\sin\theta)^3$ $= \cos^3\theta + 2\cos^2\theta\, i\sin\theta + 2\cos\theta\, i^2\sin^2\theta + i^3\sin^3\theta$ $= \cos^3\theta - 2\cos\theta\sin^2\theta + i(2\cos^2\theta\sin\theta - \sin^3\theta)$ --- ②		Using Binomial Theorem or just cubing of a binomial
Equating real parts of ① and ②, it can be seen that $\cos 3\theta = \cos^3\theta - 2\cos\theta\sin^2\theta$	1	Correctly stating "Equate Reals"

Suggested Solutions	Marks	Marker's Comments
c) 	1 1 1	For correct shaded region for correct arguments for correct boundaries If 2 or more errors, then max 1 If no arg marked on diagram, max 2

Y12 : Task 1 : 2017 **MATHEMATICS Extension 2: Question** 1 JRAHS

Suggested Solutions	Marks	Marker's Comments
d) **Method 1:**		
$\dfrac{z-i}{z+1} = \dfrac{x+i(y-1)}{x+1+iy} \times \dfrac{x+1-iy}{x+1-iy}$	1	For realising denominator
$= \dfrac{x(x+1)+y(y-1)+i(x+1)(y-1)-ixy}{(x+1)^2+y^2}$		
For $\dfrac{z-i}{z+1}$ to be purely imaginary, $Re\left(\dfrac{z-i}{z+1}\right)=0$		
$\Rightarrow \dfrac{x(x+1)+y(y-1)}{(x+1)^2+y^2}=0$		
$\Rightarrow x^2+x+y^2-y=0$		
i.e $\left(x+\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{2}$	1	Equation of locus
$\therefore$ locus is a circle with centre $\left(-\frac{1}{2},\frac{1}{2}\right)$ and radius $\frac{1}{\sqrt{2}}$, with $(-1,0)$ and $(1,0)$ excluded.	1	Geometrical description
		If $\dfrac{z-i}{z+i}$, then max 1

Y12 : Task 1 : 2017 **MATHEMATICS Extension 2: Question** 1 JRAHS

Suggested Solutions	Marks	Marker's Comments
d) Method 2		
For $\frac{z-i}{z+1}$ to be purely imaginary, then		
$\arg\left(\frac{z-i}{z+1}\right) = \arg(z-i) - \arg(z+1) = \pm\frac{\pi}{2}$	1	For correct argument
		
$\therefore$ Coordinate of centre $\left(-\frac{1}{2}, \frac{1}{2}\right)$, with radius $\frac{1}{\sqrt{2}}$		
and hence a circle, excluding $(-1,0)$ and $(0,1)$, with	1	Equation of locus
equation $\left(x+\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{2}$	1	Geometrical description
Method 3 :		
Let $\frac{z-i}{z+1} = ki$ ... purely imaginary		Note:
$\Rightarrow k = -\frac{x}{y}$ and $k = \frac{y-1}{x+1}$	1	No marks were awarded for the restrictions
$\hookrightarrow x^2+x+y^2-1=0$		
$\hookrightarrow \left(x+\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{2}$ etc, etc.	1	
	1	description

Y12 : Task 1 : 2017 **MATHEMATICS Extension 2: Question** 1 JRAHS

Suggested Solutions	Marks	Marker's Comments
e) Since $P(x)$ has real coefficients, by the conjugate root theorem, complex roots appear in conjugate pairs i.e Since $P(i)=0$, then $P(-i)=0$ $\Rightarrow \pm i$ are roots of $P(x)$ i.e $(x+i)(x-i) = x^2+1$ are factors of $P(x)$.	1	For $-i$ is a root and that x^2+1 is a factor
Hence $P(x) = (x^2+1)\,Q(x)$ $Q(x)$ obtained by long division or by inspection		
$\therefore P(x) = (x^2+1)(x^2-x+2)$ $= (x+i)(x-i)\left(\left(x-\frac{1}{2}\right)^2+\frac{7}{4}\right)$ $= (x+i)(x-i)\left(x-\frac{1}{2}+\frac{\sqrt{7}}{2}i\right)\left(x-\frac{1}{2}-\frac{\sqrt{7}}{2}i\right)$	1	For the other factor of $P(x)$
$\therefore$ The roots are $\pm i, \quad \frac{1}{2} \pm \frac{\sqrt{7}}{2}i$	1	For stating all the roots

Suggested Solutions	Marks	Marker's Comments
e) Method 2:		
$P(i) = 0 \cdots$ given		
$P(-i) = 0 \cdots P(x)$ has real coefficients and hence roots come in conjugate pairs.	1	Identifying $-i$ is also a root
Let $\alpha, \bar{\alpha}$ be the other roots		
$\sum \alpha = 1 \Rightarrow \alpha + \bar{\alpha} = 1$		
$\Rightarrow 2x = 1$ if $\alpha = x + iy$		
$x = \frac{1}{2}$	1	For obtaining 2 of the roots
$\sum \alpha\beta\gamma\delta = 2 \Rightarrow \alpha\bar{\alpha} = 2$		
$\Rightarrow x^2 + y^2 = 2$		
$\therefore y = \pm \frac{\sqrt{7}}{2}$		
$\therefore$ The roots are $\pm i, \ \frac{1}{2} \pm \frac{\sqrt{7}}{2} i$	1	For stating all the roots

Mathematics Ext 2 Question 2.

Suggested Solutions	Marks	Marker's Comments

a) i Let $z = x+iy$

$z^2 = (x^2-y^2) + 2xyi$

$z^2 = 6i \Rightarrow x^2 - y^2 = 0$ Eqn 1

$2xy = 6$

Equating Re(z) and Im(z) ——— 1

$xy = 3$

$y = \frac{3}{x}$ Eqn 2

Sub Eqn 2 into Eqn 1

$x^2 - \frac{9}{x^2} = 0$

$x^4 - 9 = 0$

$(x^2+3)(x^2-3) = 0$

$x = \pm\sqrt{3}$ $(x \in \mathbb{R})$

when $x = \sqrt{3}$, $y = \sqrt{3}$,

$x = -\sqrt{3}$, $y = -\sqrt{3}$

Final Answer

$\therefore \sqrt{6i} = \sqrt{3} + \sqrt{3}i$ or $-\sqrt{3} - \sqrt{3}i$ ——— 1

ii $\therefore z = \dfrac{(1-i) \pm \sqrt{(1-i)^2 - 4(1)(-2i)}}{2}$

$= \dfrac{(1-i) \pm \sqrt{-2i + 8i}}{2}$

$= \dfrac{(1-i) \pm \sqrt{6i}}{2}$ ——— Getting to this step ——— 1

$= \dfrac{(1-i) \pm (\sqrt{3}+\sqrt{3}i)}{2} = \dfrac{(1+\sqrt{3})}{2} + \dfrac{(\sqrt{3}-1)}{2}i$ or $\dfrac{1-\sqrt{3}}{2} - \dfrac{(1+\sqrt{3})}{2}i$ ——— 1

Mathematics ______ Question _____

Suggested Solutions	Marks	Marker's Comments
b) i $\dfrac{1}{\alpha^3\beta^3\gamma^3} = \dfrac{1}{(\alpha\beta\gamma)^3} = \dfrac{1}{5^3} = \dfrac{1}{125}$	1	
ii <u>Method 1</u>		
Polynomial will have equation		
$(\sqrt{x})^3 - 2(\sqrt{x})^2 + 4\sqrt{x} - 5 = 0$ Sub $\sqrt{x}$ into eqn	1	
$x\sqrt{x} + 4\sqrt{x} = 5 + 2x$		
$x^3 + 8x^2 + 16x = 25 + 20x + 4x^2$		
$x^3 + 4x^2 - 4x - 25 = 0$ answer	1	
<u>Method 2</u>		
$\alpha^2\beta^2\gamma^2 = (\alpha\beta\gamma)^2 = 5^2 = 25$		
$(\alpha^2+\beta^2+\gamma^2) = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$	1	Need Both values.
$= (2)^2 - 2(4)$		
$= -4$		
$\alpha^2\beta^2 + \alpha^2\gamma^2 + \beta^2\gamma^2 = (\alpha\beta+\beta\gamma+\alpha\gamma)^2 - 2(\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2)$		
$= (\alpha\beta+\beta\gamma+\alpha\gamma)^2 - 2\alpha\beta\gamma(\alpha+\beta+\gamma)$		
$= 4^2 - 2(5)(2)$		
$= 16 - 20$		
$= -4$		
$\therefore$ Polynomial is given by $x^3 + 4x^2 - 4x - 25 = 0$	1	

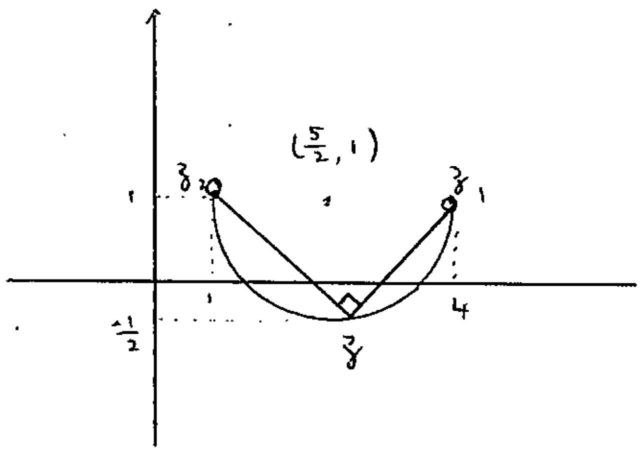
Mathematics ______ Question _____		
Suggested Solutions	Marks	Marker's Comments

iii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$

$= \frac{\beta^2\gamma^2 + \alpha^2\gamma^2 + \alpha^2\beta^2}{\alpha^2\beta^2\gamma^2}$ —— (1)

$= \frac{-4}{25}$ From equation found in part ii) —— (1)

c) i



(1) —— Clearly demonstrating a semi-circle By:

- Marking a centre <u>or</u>
- Marking the angle subtended at z

(1) —— Correctly getting the bottom half

\+

open dots at z_1 and z_2

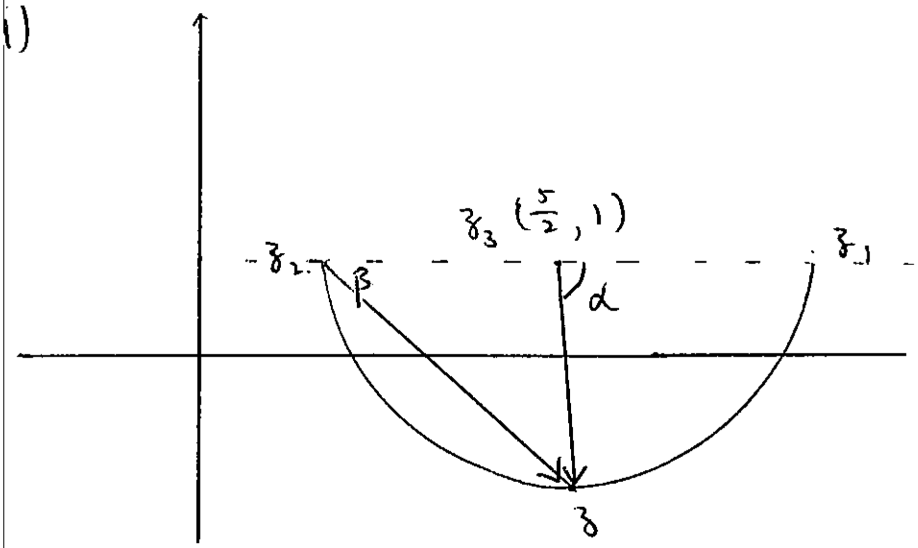
\+

correct scale

Mathematics ______ Question ______

Suggested Solutions	Marks	Marker's Comments

ii)



$$\text{Arg}(z - \tfrac{5}{2} - i) = \text{Arg}(z - (\tfrac{5}{2} + i))$$
$$= \alpha \text{ (see diagram)}$$

$$\text{Arg}(z - 1 - i) = \text{Arg}(z - (1 + i))$$
$$= \beta \text{ (see diagram)}$$

(1) → Correctly demonstrating what $\text{Arg}(z - \frac{5}{2} - i)$ and $\text{Arg}(z-1-i)$ is.

<u>Method 1</u>

$\alpha = 2\beta$ (angle at the centre is twice the angle at the circumference standing on the same arc).

$\therefore \text{Arg}(z - \frac{5}{2} - i) = 2\text{Arg}(z - 1 - i)$ Both are needed (1)

<u>Method 2</u>

$z_3 z_2 = z_3 z$ (radii of a circle)

$\therefore \angle z_3 z z_2 = \beta$ (equal angles are opposite to equal sides in $\Delta z_2 z_3 z$

$\therefore \alpha = 2\beta$ (exterior angle of $\Delta z_2 z_3 z$) (1)

Mathematics ____ Question ____		
Suggested Solutions	Marks	Marker's Comments

d) $P(x) = x^4 - 3x^3 - 6x^2 + 28x - 24$

$P'(x) = 4x^3 - 9x^2 - 12x + 28$

$P''(x) = 12x^2 - 18x - 12$

$= 6(2x^2 - 3x - 2)$

$= 6(x-2)(2x+1)$

$\therefore P''(x) = 0 \Rightarrow x = 2$ or $\frac{-1}{2}$ —— 1

$P(2) = (2)^4 - 3(2)^3 - 6(2)^2 + 28(2) - 24$

$= 16 - 24 - 24 + 56 - 24$

$= 0$

$\therefore x = 2$ is a root with multiplicity 3.

$\therefore$ Let roots of $P(x)$ be $2, 2, 2$ and α

$2 + 2 + 2 + \alpha = 3$ (Sum of roots)

$6 + \alpha = 3$

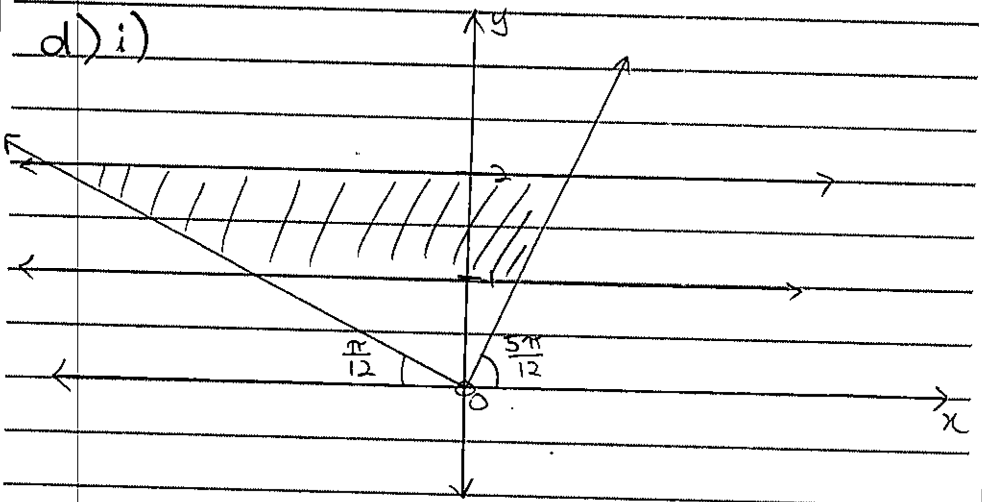
$\alpha = -3$.

$\therefore P(x) = (x-2)^3(x+3)$ —— 1

MATHEMATICS Extension 2: Question...3..

Suggested Solutions	Marks	Marker's Comments
a) $\omega^3 - 1 = 0$		
$(\omega - 1)(\omega^2 + \omega + 1) = 0$		
ω is complex $\therefore \omega \neq 1$		
$\therefore \omega^2 + \omega + 1 = 0$		
$1 + \omega = -\omega^2$		
$(1 + \omega - \omega^2)^7 = (-\omega^2 - \omega^2)^7$		
$= (-2\omega^2)^7$		
$= -128(\omega^2)^7$		
$= -128\,\omega^{14}$		
$= -128\,\omega^2 \times (\omega^3)^4$		
$= -128\,\omega^2$	1	
b) $z^3 = -8$		
let $z = r(\cos\theta + i\sin\theta)$		
$\therefore z^3 = r^3(\cos\theta + i\sin\theta)^3$		
$= r^3(\cos 3\theta + i\sin 3\theta)$ (De Moivre's)		
$-8 = 8(\cos\pi + i\sin\pi)$		
$\therefore r^3(\cos 3\theta + i\sin 3\theta) = 8(\cos\pi + i\sin\pi)$		
$\therefore r^3 = 8$		
$r = 2$		
$3\theta = \pi + 2k\pi$ where $k \in \mathbb{Z}$	1	
$\theta = \dfrac{\pi + 2k\pi}{3}$		
when $k=0$, $z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$		
$k=1$, $z_2 = 2(\cos\pi + i\sin\pi)$		
$k=-1$, $z_3 = 2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$		
$\therefore$ roots are $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$,		
$2(\cos\pi + i\sin\pi)$, $2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$	1	

MATHEMATICS Extension 2: Question 3

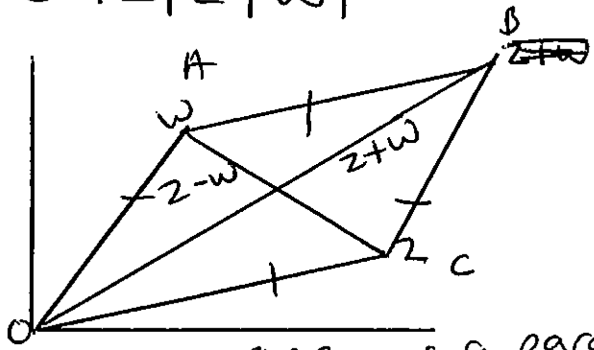
Suggested Solutions	Marks	Marker's Comments
c) $\vec{AD} = -i\vec{AB}$ ($\vec{AD}$ is $\vec{AB}$ rotated clockwise by $\frac{\pi}{2}$)	1	
$d-a = -i(b-a)$ $d = a - ib + ia$	1	
$= a(1+i) - ib$		
d) i) 		1 showing angles 1 correct region 1 correct lines
ii) $\lvert z_0 \rvert = \dfrac{2}{\sin\frac{\pi}{12}}$ $\arg(z_0) = \frac{11\pi}{12}$	1	
$\therefore z_0 = \dfrac{2}{\sin\frac{\pi}{12}}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$ $= \dfrac{2}{\sin\frac{11\pi}{12}}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$ since $\sin\frac{\pi}{12} = \sin\frac{11\pi}{12}$		
$= 2\cot\frac{11\pi}{12} + 2i$	1	
$\dfrac{2}{\tan\frac{11\pi}{12}} = \dfrac{2}{\tan\left(\frac{\pi}{4}+\frac{2\pi}{3}\right)}$ $= \dfrac{2\left(1-\tan\frac{\pi}{4}\tan\frac{2\pi}{3}\right)}{\tan\frac{\pi}{4}+\tan\frac{2\pi}{3}}$ $= \dfrac{2(1+\sqrt{3})}{(1-\sqrt{3})}$ $= -4-2\sqrt{3}$		exact form is: $z_0 = -4-2\sqrt{3} + 2i$

MATHEMATICS Extension 2: Question 3

Suggested Solutions	Marks	Marker's Comments
e) i) $z^n = (\cos\theta + i\sin\theta)^n$		
$= \cos n\theta + i\sin n\theta$ (De Moivre's) ①		
$z^{-n} = (\cos\theta + i\sin\theta)^{-n}$		
$= (\cos(-n\theta) + i\sin(-n\theta))$ (De Moivre's)		
$= \cos n\theta - i\sin n\theta$ ②	1	
since $\cos(-n\theta) = \cos n\theta$.		
$\sin(-n\theta) = -\sin n\theta$.		
$\therefore$ ① + ②:		
$z^n + z^{-n} = \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$		
$= 2\cos n\theta$.		
$\therefore \cos n\theta = \dfrac{z^n + z^{-n}}{2}$		
① − ②	1	
$z^n - z^{-n} = \cos n\theta + i\sin n\theta - \cos n\theta + i\sin n\theta$		
$= 2i\sin n\theta$		
$\therefore \sin n\theta = \dfrac{z^n - z^{-n}}{2i}$		
ii) $32\sin^4\theta\cos^2\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$		
LHS $= 32\left(\dfrac{z - z^{-1}}{2i}\right)^4\left(\dfrac{z + z^{-1}}{2}\right)^2$	1	using part (i)
$= 32\,\dfrac{(z - z^{-1})^4}{16}\,\dfrac{(z + z^{-1})^2}{4}$		
$= \frac{1}{2}(z - z^{-1})^2(z + z^{-1})^2(z - z^{-1})^2$		
$= \frac{1}{2}(z^2 - z^{-2})^2(z^2 - 2 + z^{-2})$		
$= \frac{1}{2}(z^4 - 2 + z^{-4})(z^2 - 2 + z^{-2})$		
$= \frac{1}{2}(z^6 - 2z^4 + z^2 - 2z^2 + 4 - 2z^{-2} + z^{-2} - 2z^{-4} + z^{-6})$	1	expanding
$= \frac{1}{2}(z^6 + z^{-6} - 2(z^4 + z^{-4}) - (z^2 + z^{-2}) + 4)$		
$= \dfrac{z^6 + z^{-6}}{2} - \dfrac{2(z^4 + z^{-4})}{2} - \dfrac{z^2 + z^{-2}}{2} + 2$	1 $\Rightarrow$	changing back to correct form

MATHEMATICS Extension 2: Question 3

Suggested Solutions	Marks	Marker's Comments
$= \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$		
$=$ RHS.		
OR $= \frac{1}{2}(z - z^{-1})^4 (z + z^{-1})^2$		
$= \frac{1}{2}(z^4 - 4z^2 + 6 - 4z^{-2} + z^{-4})(z^2 + 2 + z^{-2})$		
$= \frac{1}{2}(z^6 + 2z^4 + z^2 - 4z^4 - 8z^2 - 4 + 6z^2 + 12 + 6z^{-2} - 4 - 8z^{-2} - 4z^{-4} + z^{-2} + 2z^{-4} + z^{-6})$		⇐ different expansion all else is the same.
$= \frac{1}{2}(z^6 + z^{-6} - 2(z^4 + z^{-4}) - (z^2 + z^{-2}) + 4)$		

Suggested Solutions	Marks	Marker's Comments
Since $\lvert z\rvert=\lvert w\rvert$		1 for explaining why a rhombus
		1 for rhombus diagonals bisect at right angles (poorly done)
OABC is a parallelogram		
since $\lvert z\rvert=\lvert w\rvert$ OABC is a rhombus (4 equal sides)		Students needed to read whole question to help approach question.
Diagonals of a rhombus bisect at right angles		
$\therefore \arg(z+w)-\arg(z-w)=\pm\frac{\pi}{2}$		1 for ~~diagram~~ arg statement proving purely imaginary.
so $\frac{z+w}{z-w}$ is purely imaginary		
Method 2		
$\frac{(x+a)+i(y+b)}{(x-a)+i(y-b)}\times\frac{(x-a)-i(y-b)}{(x-a)-i(y-b)}$	1	1 for correct start.
$\frac{x^2-a^2+y^2-b^2+i\left((y+b)(x-a)-(x+a)(y+b)\right)}{(x-a)^2+(y-b)^2}$		
Since $\lvert z\rvert=\lvert w\rvert$ $x^2-a^2+y^2-b^2=0$	1	for using $\lvert z\rvert=\lvert w\rvert$
$\therefore \operatorname{Re}(z)=0$ so purely imaginary		1 for geometry

Suggested Solutions	Marks	Marker's Comments
~~$\sqrt{6}-\sqrt{2}i$~~		NB: A 2 mark question requires working
$z=\sqrt{6}-\sqrt{2}i$ $\quad \arg(z)=\tan^{-1}-\frac{1}{\sqrt{3}}$		
$\lvert z\rvert=\sqrt{6+2}$ $\quad =-\frac{\pi}{6}$		1 for 1 correct
$=2\sqrt{2}$		
$\therefore \sqrt{6}-\sqrt{2}i=2\sqrt{2}\left(\cos\left(\frac{-\pi}{6}\right)+i\sin\left(\frac{-\pi}{6}\right)\right)$		2 for correct answer
(ii) $\left(\frac{\sqrt{6}-\sqrt{2}i}{2-2i}\right)^{2000}$		$2\sqrt{2}\,\text{cis}\left(\frac{-\pi}{6}\right)$ is not accepted in HSC. lost a mark.
$\left(\frac{2\sqrt{2}\cos\left(\frac{-\pi}{6}\right)+i\sin\left(\frac{-\pi}{6}\right)}{2\sqrt{2}\cos\frac{-\pi}{6}\ i\sin\frac{-\pi}{4}}\right)^{2000}$		1
$=\left(\cos\left(\frac{-\pi}{6}+\frac{\pi}{4}\right)+i\sin\left(\frac{-\pi}{6}+\frac{\pi}{4}\right)\right)^{2000}$		
$=\cos\frac{500\pi}{3}+i\sin\frac{500\pi}{3}$ (De Moivre)		
$=\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)$		
$=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$		1

Suggested Solutions	Marks	Marker's Comments
c (i) $z^7 = 1$ since α a root $\therefore \alpha^7 = 1$ $\alpha^7 - 1 = 0$ $(\alpha - 1)(1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6) = 0$ $1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6 = 0$ (α complex)	1	
(ii) $\theta + \delta = \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6$ $= -1$ (from part i)	1	
$(\theta\delta) = (\alpha + \alpha^2 + \alpha^4)(\alpha^3 + \alpha^5 + \alpha^6)$ $= \alpha^4 + \alpha^6 + \alpha^7 + \alpha^5 + \alpha^7 + \alpha^8 + \alpha^7 + \alpha^9 + \alpha^{10}$ since $\alpha^7 = 1$ $= 3 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 + \alpha^6$ $= 3 - 1$ $= 2$	1	well done.
quadratic equation $x^2 + x + 2 = 0$	1	

MATHEMATICS Extension 2: Question page 4

Suggested Solutions	Marks	Marker's Comments
$z^7 = 1$		
$z^7 = \text{cis}\, 2k\pi$.		
$\therefore z = \text{cis}\,\frac{2k\pi}{7}$ (de moivre)	1	
Solving $x^2+x+2=0$		
$x = \frac{-1 \pm \sqrt{7}}{2}$	1	Students who didn't find this struggled with second part of question
$\theta = \alpha + \alpha^2 + \alpha^4$		
$= \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7} + \cos\frac{4\pi}{7} + i\sin\frac{4\pi}{7} + \cos\frac{8\pi}{7} + i\sin\frac{8\pi}{7}$		
Equating real		
$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7} = -\frac{1}{2}$		
but $\cos\frac{8\pi}{7} = \cos\left(\pi + \frac{\pi}{7}\right)$ ~~also~~ $= -\cos\frac{\pi}{7}$	1	
$\therefore \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} - \cos\frac{\pi}{7} = -\frac{1}{2}$		
Equating imaginary		
$\sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} - \sin\frac{\pi}{7} = \frac{\sqrt{7}}{2}$		
Since $\sin\frac{2\pi}{7} > \sin\frac{\pi}{7}$ and $\sin\frac{2\pi}{7}, \sin\frac{4\pi}{7} > 0$	1	needed to explain why positive