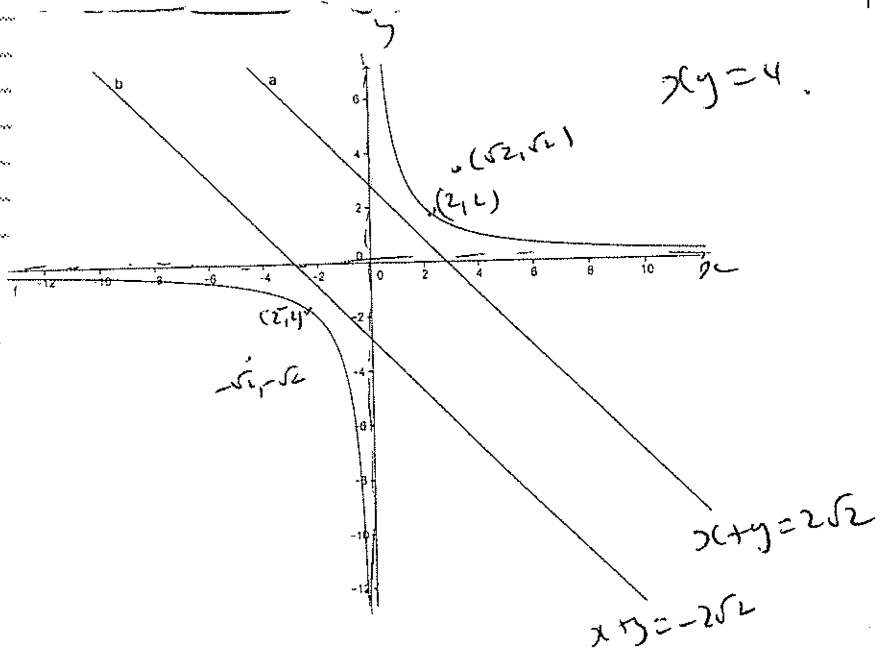
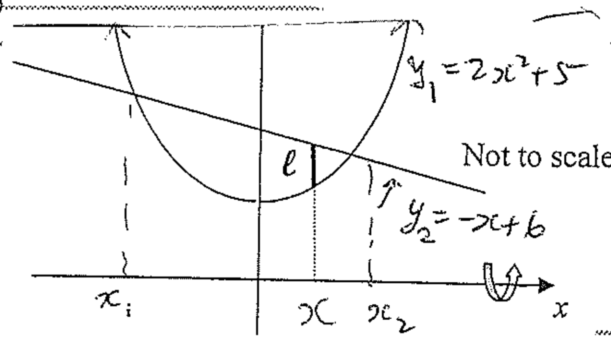
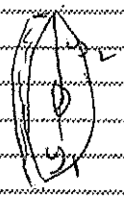
MATHEMATICS Extension 2 : Question 1

Suggested Solutions	Marks	Marker's Comments
a)	1	1 for vertex 1 for directrices and asymptote 1 for foci 1 for graph well done.



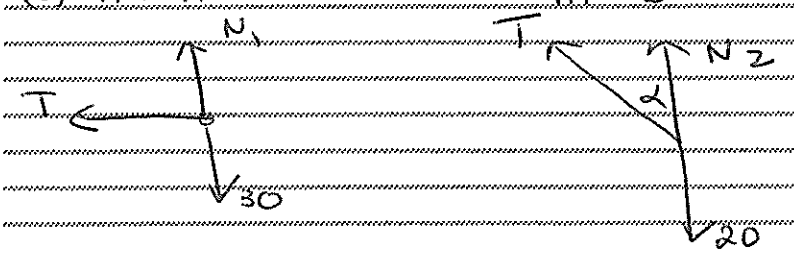
(b)





Suggested Solutions	Marks	Marker's Comments
(i) Area $= \pi\left(y_2^{\,2} - y_1^{\,2}\right)$ $= \pi\left((6-x)^2 - (2x^2+5)^2\right)$	1	Students not drawing slice more likely to get this incorrect
(ii) $6 - x = 2x^2 + 5$ $2x^2 + x - 1 = 0$ $(2x-1)(x+1) = 0$ $x = \frac{1}{2}$ or -1	1	poor algebra

MATHEMATICS Extension 1 : Question……..

Suggested Solutions	Marks	Marker's Comments
$\Delta V \doteq \pi(11-12x-19x^2-4x^4)\Delta x$		Students obtain -1 and $\frac{1}{2}$ but then only went from 0.
$V = \lim\limits_{\Delta x \to 0} \sum\limits_{x=-1}^{\frac{1}{2}} \pi(11-12x-19x^2-4x^4)\Delta x$	1	poor expansion
$V = \int_{-1}^{\frac{1}{2}} \pi(11-12x-19x^2-4x^4)\,dx$		
$= \pi\left[11x-\frac{12x^2}{2}-\frac{19x^3}{3}-\frac{4x^5}{5}\right]_{-1}^{\frac{1}{2}}$		
$= \pi\left[\frac{191}{60}+\frac{148}{15}\right]$	1	
$= \frac{261\pi}{20}\ u^3$		
c) Resolving forces		
(i) At A At B		1 for each
		Some students missed N_1 on A.
(ii) Resolving forces at A		
$T = m\omega^2 r \quad \therefore T = 3\times 2^2 \times r = 12r$	1	
Resolving forces at B		
horizontally: $T\sin\alpha = M\omega^2 R$		
vertically: $T\cos\alpha + N_2 = Mg$		

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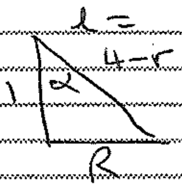
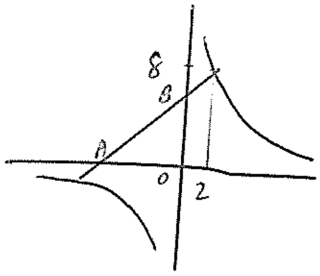
MATHEMATICS Extension 1 : Question……..

Suggested Solutions	Marks	Marker's Comments
by equating forces		
~~q~~ Now $R = (4-r)\sin\alpha$		
$T\sin\alpha = 2\times(4-r)\sin\alpha\times 4$	1	poorly done those who tried to use pythagoras rather than trig were less successful.
$\therefore T = 8(4-r)$ 		
but $T = 12r$		
$\therefore 12r = 32 - 8r$		
$20r = 32$		
$r = \frac{8}{5}$		
So $T = 12\times\frac{8}{5}$	1	
$= \underline{19.2\text{ N}}$		
(iii)		
$T\cos\alpha + N = mg$		poorly done.
$N = 20 - T\cos\alpha$		students could have used part above.
$= 20 - 19.2\cos\alpha$	1	
$= 20 - 19.2\times\frac{5}{12}$		
$= 12\text{ N}$	1	
since $l = 4 - 1.6$		
$= \underline{2.4}$		

2a i) $y' = \frac{-16}{x^2}$

At $x = 2$ $y' = -4$ — 1m

$m^{\perp} = \frac{1}{4}$



well done

EQ Normal at $(2, 8)$

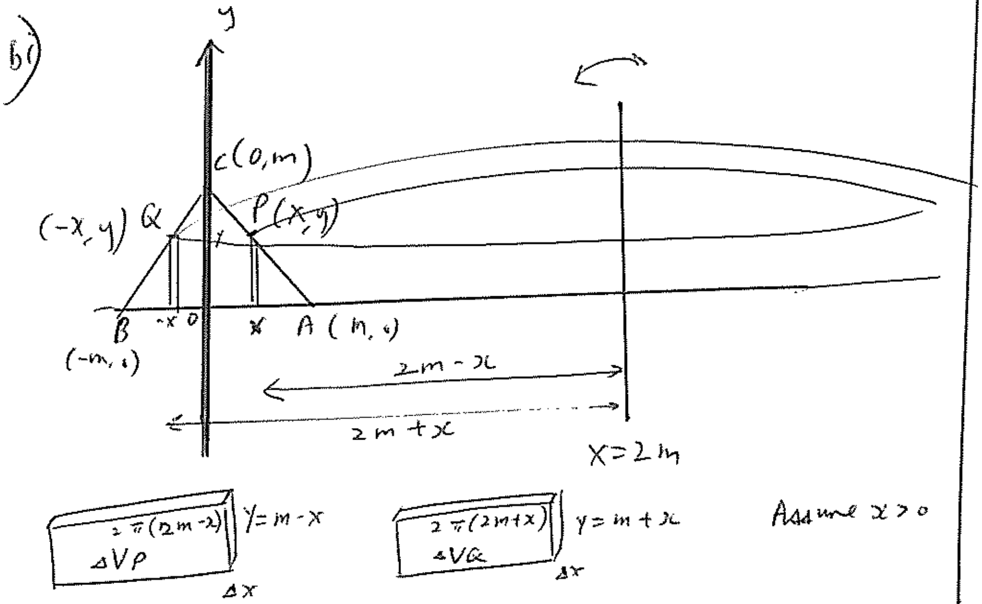
$y - 8 = \frac{1}{4}(x - 2)$

$y = \frac{x}{4} + 15/2$ or $x - 4y + 30 = 0$ — 1m

At A $y = 0$, $x = -30$
At B $x = 0$, $y = 15/2$ — 1m

Area $\Delta AOB = \frac{1}{2} \times 30 \times 15/2 = 112.5$ unit^2 — 1m

b i)



✓ few students use this method

Assume $x > 0$

1m for diagram

$$\Delta V = \left[2\pi(2m - x)y + 2\pi(2m + x)y\right]\Delta x$$ — 1m

$$= 2\pi \, 4my \, \Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=0}^{m} 4\pi m \, 2y \, \Delta x$$

$$V = 4\pi m \, 2\int_0^m y \, dx$$

$$A = \text{Area } \Delta ABC = 2\int_0^m y \, dx$$ (ΔABC is symmetrical about y-axis) — 1m

$$\therefore V = 4\pi m A$$

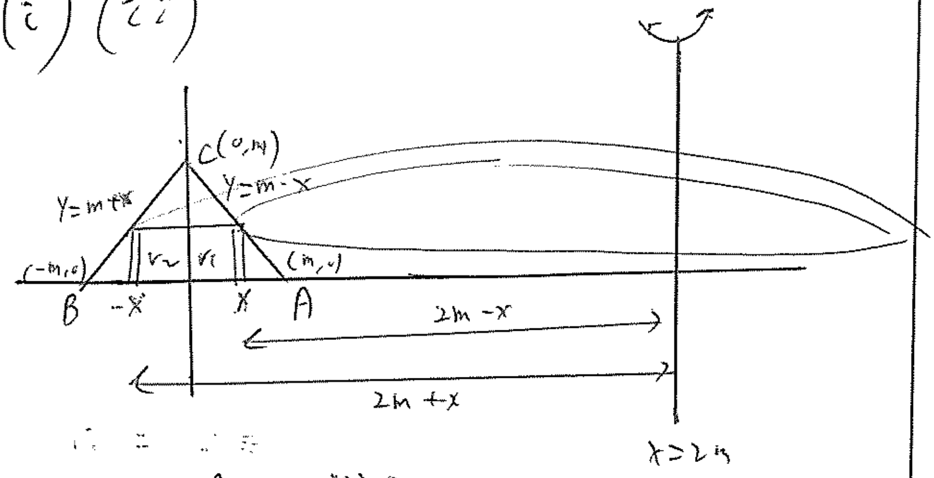
ii) Area $\Delta ABC = \frac{1}{2} 2m \cdot m = m^2$

$\therefore$ Vol of solid formed $= 4\pi m \, m^2$
$= 4\pi m^3$ unit^3 — 1m

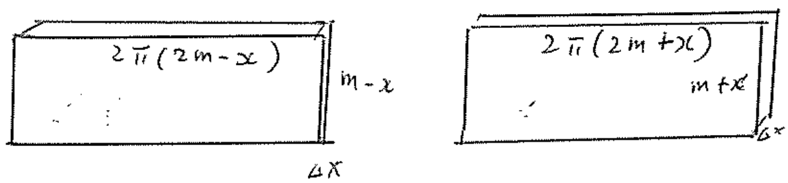
Ext 2

P2

b (i) (ii)



Assume $x > 0$



$\Delta V_1 = 2\pi(2m-x)(m-x)$ $\qquad$ $\Delta V_2 = 2\pi(2m+x)(m+x)$

1 m

$$V_1 = \lim_{\Delta x \to 0} \sum_0^m 2\pi(2m-x)(m-x)\,dx$$

$$V_1 = 2\pi\int_0^m (2m^2 - 3mx + x^2)\,dx$$

$$V_1 = 2\pi\left(2m^2x - \frac{3mx^2}{2} + \frac{x^3}{3}\right)\Big]_0^m$$

$$V_1 = 2\pi\left(2m^3 - \frac{3}{2}m^3 + \frac{m^3}{3}\right) = \frac{5\pi}{3}m^3$$

Similarly

$$V_2 = 2\pi\int_{-m}^0 (2m-x)(m+x)\,dx$$

$$= 2\pi\int_{-m}^0 (2m^2 + mx - x^2)\,dx$$

$$= 2\pi\left(2m^2x + \frac{mx^2}{2} - \frac{x^3}{3}\right)\Big]_{-m}^0$$

$$= \frac{7\pi}{3}m^3$$

$$\therefore V_1 + V_2 = \frac{5\pi + 7\pi}{3}m^3 = 4\pi m^3$$

$\therefore$ Vol. of the solid formed $= 4\pi m^3$

A = Area of $\triangle ABC = \frac{1}{2}(2m)m = m^2$

$\therefore$ vol of solid formed $= \underline{\underline{4\pi m A}}$

A lot of students have tried to use this method but failed to see the limits of V_1, V_2 are different.

$\therefore$ failed to arrive at the correct answer.

1 m for

$$V_1 + V_2 = \lim_{\Delta x \to 0}\sum_0^m 2\pi(2m-x)(m-x)\,dx + \lim_{\Delta x \to 0}\sum_{-m}^0 2\pi(2m-x)(m+x)\,dx$$

or 1 m for $\frac{5\pi}{3}m^3$ or $\frac{7\pi}{3}m^3$ together with $|ABC| = m^2$

1 m for reaching $4\pi m A$

1 m for $4\pi m^3$

c i) $t=0,\ v=0,\ x=0$

$\downarrow 2g \quad \uparrow kv$

+ve $\downarrow$

1m for diagram with +ve direction

ii) $m\ddot{x} = mg - kv$ (Newton's Law of Motion)

$2\ddot{x} = 2g - kv \quad (m=2)$

must show $m=2$

$\ddot{x} = g - \frac{kv}{2}$

Terminal velocity when $\ddot{x} = 0$

$0 = g - \frac{kv}{2}$

1m

$V_T = \frac{2g}{k}$ m/s #

iii) $\frac{dv}{dt} = \frac{2g-kv}{2}$

1m

$\int_0^v \frac{dv}{2g-kv} = \int_0^t \frac{dt}{2}$

$-\frac{1}{k}\ln(2g-kv)\Big]_0^v = \frac{t}{2}$

$-\frac{1}{k}\ln\left(\frac{2g-kv}{2g}\right) = \frac{t}{2}$

$t_d = \frac{-2}{k}\ln\left(\frac{2g-kv}{2g}\right) \quad \therefore e^{-\frac{kt}{2}} = 1 - \frac{kv}{g}$

1m (many forgot V expression)

$V = \frac{2g}{k}\left(1 - e^{-\frac{kt}{2}}\right)$ #

2nd particle

$\uparrow +$ $\quad t=0,\ v=u,\ x=0$ $\quad \downarrow kv \ \downarrow mg$

$2\ddot{x} = -(2g+kv)$ Newton's Law

$\ddot{x} = -\left(\frac{2g+kv}{2}\right)$

(At rest $v=0$)

1m

$\int_u^0 \frac{2\,dv}{-(2g+kv)} = \int_0^t dt$

$$t = -\frac{2}{k}\ln(kv+2g)\Big]_u^0$$

$$t_u = \frac{-2}{k}\ln\left(\frac{2g}{ku+2g}\right)$$

$t_d = t_u$

$$\cancel{\frac{-2}{k}}\ln\left(\frac{2g-kv}{2g}\right) = \cancel{\frac{-2}{k}}\ln\left(\frac{2g}{ku+2g}\right)$$ 1m

$t_d = t_u$

$$\frac{2g-kv}{2g} = \frac{2g}{ku+2g}$$

$$4\cancel{g^2} = 2kug - k^2uv + 4\cancel{g^2} - 2kvg$$

$$uvk^2 + 2kVg = 2kug$$

$$uv + \frac{2Vg}{k} = \frac{2ug}{k}$$

let $V_T = \frac{2g}{k}$

$$\therefore uV + VV_T = V_T u$$

$$\therefore V(u+V_T) = uV_T$$

$$v = \frac{uV_T}{u+V_T}$$ 1m

Let $V = V_T$

$$\therefore v = \frac{uV}{u+V}$$

Note Some students wrote $t_d = -\frac{1}{k}\ln\left(\frac{2g-kv}{2g}\right)$

$t_u = \frac{-1}{k}\ln\left(\frac{2g}{ku+2g}\right)$

hence double mistake will correct itself in prove

$v = \frac{uV}{u+V}$

max get $3/5^m$

1/3

MATHEMATICS Extension 2: Question 3		
Suggested Solutions	**Marks**	**Marker's Comments**

(i) $y = \frac{c^2}{x}$

$\frac{dy}{dx} = \frac{-c^2}{x^2}$

when $x = cp$, $m_T = \frac{-c^2}{c^2p^2} = \frac{-1}{p^2}$

eqn of tangent: $y - y_1 = m(x - x_1)$

had to show working, ie deive the formula

$y - \frac{c}{p} = \frac{-1}{p^2}(x - cp)$

$y = \frac{-1}{p^2}x + \frac{c}{p} + \frac{c}{p}$

$p^2y = -x + 2cp$ — 1

$x + p^2y = 2cp$ ...(1)

similarly tangent at Q is $x + q^2y = 2cq$ ...(2)

Solve (1) & (2)

$x\left(\frac{1}{p^2} - \frac{1}{q^2}\right) = 2c\left(\frac{1}{p} - \frac{1}{q}\right)$

$x\left(\frac{1}{p} + \frac{1}{q}\right) = 2c$

$x\left(\frac{q+p}{pq}\right) = 2c$

$\therefore x = \frac{2cpq}{p+q}$ — 1

$\therefore y = \frac{-1}{p^2}\left(\frac{2cpq}{p+q}\right) + \frac{2c}{p}$

$= \frac{-2cq + 2cp + 2cq}{p(p+q)}$

$= \frac{2c}{p+q}$ — 1

(ii) eqn of chord PQ is:

$y - y_1 = m(x - x_1)$

$y - \frac{c}{p} = \frac{c/p - c/q}{cp - cq}(x - cp)$

$y - \frac{c}{p} = \frac{-1}{pq}(x - cp)$

had to show working, if you just quoted the formula you got 0mks.

$pqy - cq = -x + cp$ — 1

$pqy + x = cp + cq$

$pqy + x = c(p+q)$

MATHEMATICS Extension 2: Question 3 continued.

Suggested Solutions	Marks	Marker's Comments
(iii) Eqn of OT is: $y - 0 = \frac{2c}{2cpq}(x - 0)$		* a number of students didn't use $O(0,0)$ as the (x_1, y_1)
$y = \frac{x}{pq}$		
$pqy = x$		1
R is intersection of OT and PQ:		
$pqy = -pqy + c(p+q)$		
$2pqy = c(p+q)$		
$y = \frac{c(p+q)}{2pq}$		1
$\therefore x = p \times q \times \frac{c(p+q)}{2pq}$		
$x = \frac{c(p+q)}{2}$		
$\therefore R\left(\frac{c(p+q)}{2}, \frac{c(p+q)}{2pq}\right)$		1
(iv) Midpoint of PQ is: $\left(\frac{cp+cq}{2}, \frac{\frac{c}{p}+\frac{c}{q}}{2}\right)$		
$= \left(\frac{c(p+q)}{2}, \frac{cq+cp}{2pq}\right)$		
$= \left(\frac{c(p+q)}{2}, \frac{c(p+q)}{2pq}\right)$		
$= R$		1
$\therefore R$ bisects PQ		
(b)(i) $a_T = r\ddot{\theta}$	1	
$a_N = r(\ddot{\theta})^2$	1	

MATHEMATICS Extension 2: Question 3 continued.

Suggested Solutions	Marks	Marker's Comments
(ii) $\ddot{s} = r\ddot{\theta}$		
$= r\dfrac{d\dot{\theta}}{dt}$		
$= r\dfrac{d\dot{\theta}}{d\theta} \times \dfrac{d\theta}{dt}$ →		necessary line!! 1
$= r\dot{\theta}\dfrac{d\dot{\theta}}{d\theta}$		
$= r\left(\dfrac{d(\frac{1}{2}\dot{\theta}^2)}{d\dot{\theta}}\right)\cdot\dfrac{d\dot{\theta}}{d\theta}$		
$= r\dfrac{d(\frac{1}{2}\dot{\theta}^2)}{d\theta}$ $\quad v = r\dot{\theta}$ →		need to see this (if its missing = 0 mks) 1
$= \dfrac{r}{r^2}\dfrac{d(\frac{1}{2}v^2)}{d\theta}$ $\quad \dfrac{v}{r} = \dot{\theta}$		
$= \dfrac{d(\frac{1}{2}v^2)}{r\,d\theta}$		* Don't use ω because it is not a VARIABLE!!
(iii)(α) $F_T = m\ddot{s}$		
$= mr\ddot{\theta}$		
$-0.1 = 0.04 \times 3 \times \ddot{\theta}$		
$\ddot{\theta} = -\frac{5}{6}$	1	
(β) $\ddot{s} = \dfrac{d(\frac{1}{2}v^2)}{r\,d\theta}$		
now, $r\ddot{\theta} = 3 \times -\frac{5}{6} = -\frac{5}{2}$		
$r\ddot{\theta} = \ddot{s}$		
$r\ddot{\theta} = \dfrac{d(\frac{1}{2}v^2)}{ds}$		
$\displaystyle\int_5^0 \frac{d(\frac{1}{2}v^2)}{ds}\,ds = \int_0^S -\frac{5}{2}\,ds$	1	
$\left[\frac{1}{2}v^2\right]_5^0 = -\frac{5}{2}S$		
$-\frac{25}{2} = -\frac{5}{2}S$		
$S = 5$		
distance $= 5$m	1	

MATHEMATICS Extension 2: Question 4

Suggested Solutions	Marks	Marker's Comments
(i) $m\ddot{x} = -mk(c+v^2)$ $\ddot{x} = -k(c+v^2)$	1	Must show "$-$" sign as it is a resistive force
(ii) $v\dfrac{dv}{dx} = -k(c+v^2)$ $\displaystyle\int_u^0 \frac{v}{c+v^2} = -k\int_0^D dx$ $\frac{1}{2}\Big[\ln[c+v^2]\Big]_u^0 = -kD \quad (i)$ $-\frac{1}{2k}\ln\left[\dfrac{c}{c+u^2}\right] = D$	1	Integral - integral should be shown.
$D = \dfrac{1}{2k}\ln\left[\dfrac{c+u^2}{c}\right] \quad (ii)$	1	correct answer with working as answer was given.
(iii) From (i) $-\dfrac{kD}{2} = \frac{1}{2}\Big(\ln\left[c+v^2\right]\Big)_u^{u/2}$		
OR. $\left[-\dfrac{kD}{2} = \frac{1}{2}\Big[\ln(c+v^2)\Big]_{u/2}^{0}\right]$		$\leftarrow$ ALTERNATIVE
$\dfrac{D}{2} = \dfrac{-1}{2k}\left[\ln\left(\dfrac{c+u^2/4}{c+u^2}\right)\right]$ $= \dfrac{2}{2k}\left[\ln\left(\dfrac{4(c+u^2)}{4c+u^2}\right)\right]$ $= \dfrac{1}{2k}\left[\ln\left(\dfrac{4(c+u^2)}{4c+u^2}\right)^2\right] \quad (ii)$		
Equate logs. $\dfrac{c+u^2}{c} = \dfrac{16(c+u^2)^2}{(4c+u^2)^2}$ $\dfrac{1}{c} = \dfrac{16(c+u^2)}{16c^2+8cu^2+u^4}$ $16c^2+16u^2c = 16c^2+8cu^2+u^4$ $8cu^2 = u^4 \quad u\neq 0$ $8c = u^2$	1	Several methods possible but must relate D and $D/2$ and remove logs
$D = \dfrac{1}{2(0{\cdot}01)}\ln\left[\dfrac{c+8c}{c}\right] = 5\ln 9$ $k=0{\cdot}01$ $= 10\ln 3$ $\approx 10{\cdot}99$	1	correct answer.

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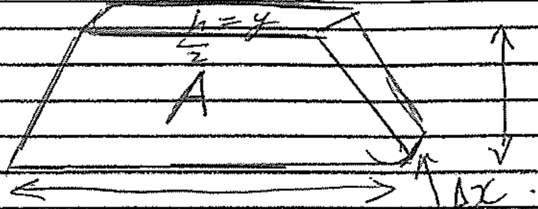
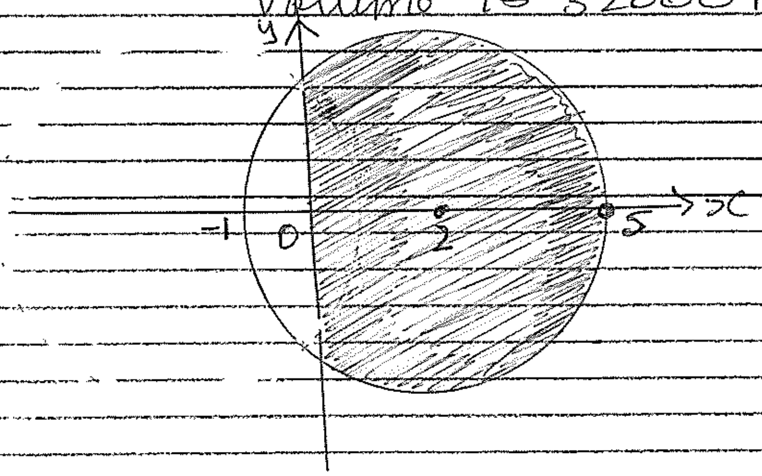
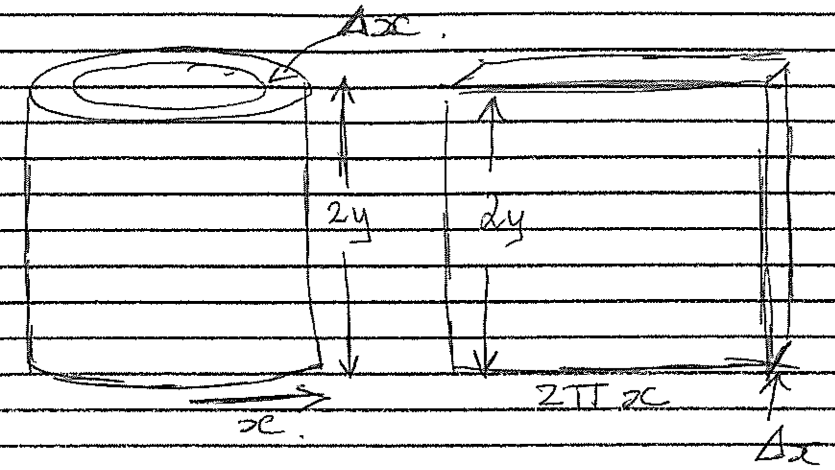
MATHEMATICS Extension 2: Question 4

Suggested Solutions	Marks	Marker's Comments
b) 		
$h = 2y$		
$x^2 + y^2 = 400$ $y = \sqrt{400 - x^2}$		
$\Delta V = \frac{2y}{2}(2y + y)\Delta x$ $= 3y^2 \Delta x$	1	$\Delta V = 3y^2 \Delta x$
$V = \lim_{\Delta x \to 0} \sum_{-20}^{20} 3y^2 \Delta x$		
$V = \int_{-20}^{20} 3(400 - x^2)\,dx$	1	sub $y = \sqrt{400 - x^2}$
Even Function $V = 2 \times 3 \int_0^{20} (400 - x^2)\,dx$		
$= 6\left[400x - \frac{x^3}{3}\right]_0^{20}$	1	integration
$= 6\left[8000 - \frac{8000}{3}\right]$		
$= 32\,000$ Volume is $32000\ m^2$	1	answer
	1	correct graph and shaded area

MATHEMATICS Extension 2: Question……..

Suggested Solutions	Marks	Marker's Comments

(ii)



$\Delta V \doteqdot 2\pi x(2y)\Delta x$ — ① — Must show a diagram and working

OR $\Delta V = 2\pi\left((x+\Delta x)^2 - x^2\right)2y$

$= 2\pi(x)\Delta x \cdot 2y$

$y^2 = 9-(x-2)^2$

$y = \sqrt{9-(x-2)^2}$

$\Delta V \doteqdot 2\pi x\, 2\sqrt{9-(x-2)^2}\,\Delta x$ — ① — substitute y expression

$= 4\pi x\sqrt{9-(x-2)^2}\,\Delta x$

(iii) $V = \lim_{\Delta x \to 0} \sum_{x=0}^{5} 4\pi x\sqrt{9-(x-2)^2}\,\Delta x$

$V = 4\pi \int_0^5 x\sqrt{9-(x-2)^2}\,dx$

Let $(x-2) = 3\sin\theta \qquad 3\cos\theta = \sqrt{9-(x-2)^2}$

$\frac{du}{dx} = 3\cos\theta$

$x = 5 \quad \sin\theta = 1 \quad \theta_1 = \pi/2$

$x = 0 \quad \sin\theta = -2/3 \quad \theta_2 = \sin^{-1}(-2/3)$

$V = 4\pi \int_{\theta_1}^{\theta_2} (3\sin\theta + 2)\times 3\cos\theta \times 3\cos\theta\, d\theta$ — ① — substitution

$= 4\pi \int_{\theta_2}^{\theta_1} 27(\sin\theta\cos^2\theta) + 18\cos^2\theta\, d\theta$ — simplify and limit values

$= 4\pi \int_{\theta_2}^{\theta_1} -27(-\sin\theta\cos^2\theta) + \frac{18}{2}(1+\cos 2\theta)\,d\theta$ — ①

$= 4\pi\left[-\frac{27}{3}\cos^3\theta + 9\theta + \frac{9}{2}\sin 2\theta\right]_{\sin^{-1}(-2/3)}^{\pi/2}$ — * note $\cos\theta_2 = \sqrt{5}/3$

$= 4\pi\left[\frac{9\pi}{2} + \frac{5\sqrt{5}}{3} + 9\sin^{-1}(2/3) + \frac{9}{2}\left(\frac{-2}{3}\right)\left(\frac{\sqrt{5}}{3}\right)\right]$

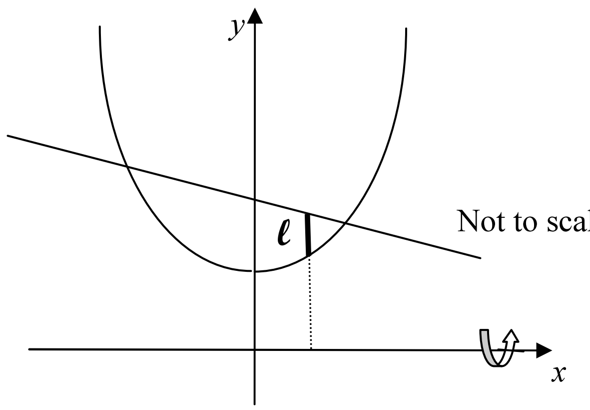
$= 18\pi^2 + \frac{44\sqrt{5}}{3}\pi + 36\sin^{-1}(2/3)$ — ① — correct answer

$\approx 363\cdot 21$

QUESTION 1 (15 Marks) **Marks**

(a) Neatly draw the graph of the curve $xy = 4$. 4
Show clearly the vertices, foci and the equations of the directrices and asymptotes.

(b) The area bounded by the curves $y = 2x^2 + 5$ and $x + y - 6 = 0$ is rotated about the x-axis as shown in the diagram below. When the region is rotated, the vertical line segment ℓ, at a distance x from the y-axis, sweeps out an annulus.

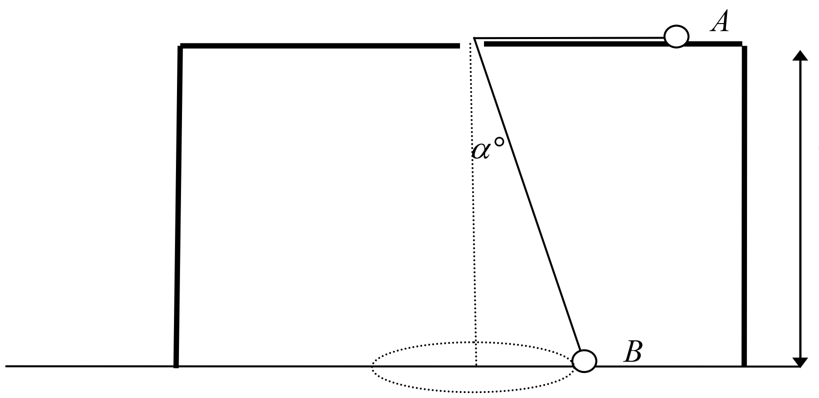


Not to scale

(i) Find an expression for the area of the annulus swept out by ℓ. 1

(ii) Hence calculate the volume of the solid of revolution. 3

(c) Two masses A and B, 3 kg and 2 kg respectively, are attached by a 4 m light inextensible string which passes through a small hole in a smooth horizontal table. Mass A moves in a circle on the table at a uniform angular speed of 2 radians/sec. Mass B moves as a conical pendulum with the same angular speed as mass A and is in contact with the smooth floor so that the string remains taut. (take $g = 10$ m/s^2.)



Not to scale

(i) Draw force diagrams for masses A and B. 2

(ii) Show that the tension in the string is $19{\cdot}2$ N. 3

(iii) Calculate the value of the normal reaction that the floor exerts on mass B, if the table is 1 m above the floor. 2

QUESTION 2 (15 Marks) START A NEW PAGE **Marks**

(a) (i) Find the equation of the normal to the curve $xy = 16$ at the point when $x = 2$. 2

(ii) This normal cuts the x-axis at A and the y-axis at B. Find the area of the ΔAOB, where O is the origin. 2

(b) ΔABC has vertices $A(m, 0)$, $B(-m, 0)$ and $C(0, m)$ as shown in the diagram below.

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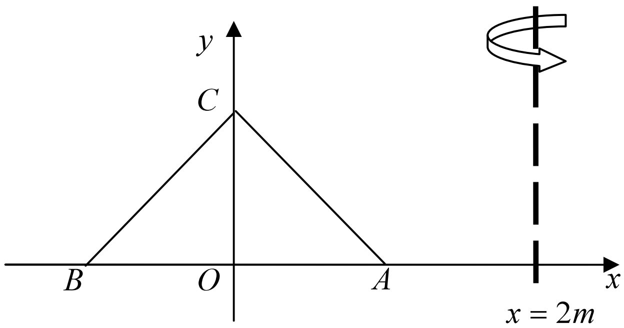


Not to scale

The triangle is rotated about the line $x = 2m$.

(i) Use cylindrical shells to show that the volume of the solid of rotation is given by $V = 4\pi\, mA$, where A is the area of ΔABC. 3

(ii) Hence find the volume of the solid formed. 1

(c) A particle of mass 2 kg falls vertically from rest, from a point P, through a substance with resistance of kv Newton, where k is a positive constant and v m/s is the velocity of the mass at time t seconds.

(i) Draw a diagram showing all forces acting on the particle. 1

(ii) Show that the terminal velocity of the particle is $V = \dfrac{2g}{k}$ m/s, 1

where g is the acceleration due to gravity.

(iii) Find an expression for the velocity of the particle at time t. 2

(iv) At the same time that the first mass was released, an equal particle is projected vertically upwards from point P with an initial velocity U m/s, in the same substance. 3

Show that the velocity of the first mass when the second mass is momentarily at rest is given by $v = \dfrac{VU}{(V+U)}$ where V is the terminal velocity of the first particle.

QUESTION 3 (15 Marks) START A NEW PAGE **Marks**

(a) (i) Prove that the point of intersection T of the tangents at $P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$, 3

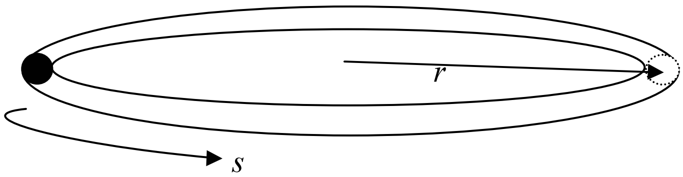
on the hyperbola $xy = c^2$, is $T\left(\frac{2cpq}{p+q}, \frac{2c}{p+q}\right)$.

(ii) Find the equation of the chord PQ. 1

(iii) The line though points O and T meets the chord PQ at R, where O is the origin. Find the coordinates of R. 3

(iv) Show that R bisects the interval PQ. 1

(b) The diagram shows a particle moving in a circular path of radius r metres with variable angular velocity $\dot{\theta}$ rad/sec. .



(i) Write down the tangential and normal components of the acceleration for the particle in terms of $\dot{\theta}$ and $\ddot{\theta}$. 2

(ii) When the particle has moved through an arc length of s metres and an angular distance of θ radians, show that 2

$$\ddot{s} = \frac{d\left(\frac{1}{2}v^2\right)}{r\,d\theta}$$ where v m/s is the speed of the particle.

(iii) A small ball of mass 40 g slides around in a narrow horizontal circular tube of radius 3 m. The initial speed of the particle is 5 m/s and there is a frictional resistance force in the tube of $0 \cdot 1$ N.

(α) Write down the equation of motion of the ball in terms of $\ddot{\theta}$. 1

(β) Calculate the distance travelled by the ball before coming to rest. 2

QUESTION 4 (15 Marks) **Marks**

(a) A resistance force of $mk\left(c+v^2\right)$ N acts directly against the motion of a particle moving in a horizontal straight line. The mass of the particle is m kg, v m/s is its velocity and k and c are positive constants. Initially the particle has a velocity U m/s (U>0) and comes to rest after it has travelled D metres.

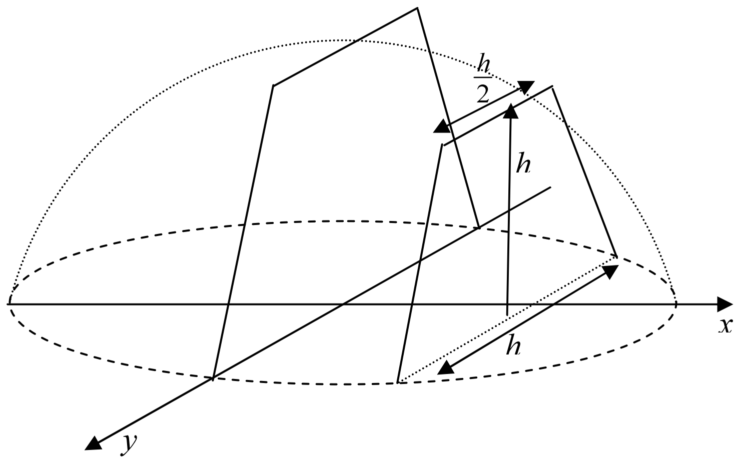
When it has travelled half this distance its speed is $\frac{U}{2}$ m/s.

(i) Write the equation of motion for the particle. 1

(ii) Show that $D=\frac{1}{2k}\log_e\left(\frac{c+U^2}{c}\right)$ 2

(iii) Find the total distance D covered when $k = 0{\cdot}1$. 2

(b) A storage shed is built with a horizontal circular base of radius 20 m. Vertical cross-sections are trapezia with heights equal to their base lengths and tops half the base length as shown in the diagram below.



Not to scale

Calculate the volume to the storage shed. 4

(c) (i) Shade the region represented by $(x-2)^2+y^2\le 9$ for $x\ge 0$ on a number plane 1

(ii) The shaded region described above is rotated about the y-axis, 2
Use the method of cylindrical shells to show that
$\Delta V \approx 4\pi x\sqrt{9-(x-2)^2}\ \Delta x$ for a cylinder of radius x units.

(iii) Hence find the volume of the solid formed. 3

END OF EXAMINATION