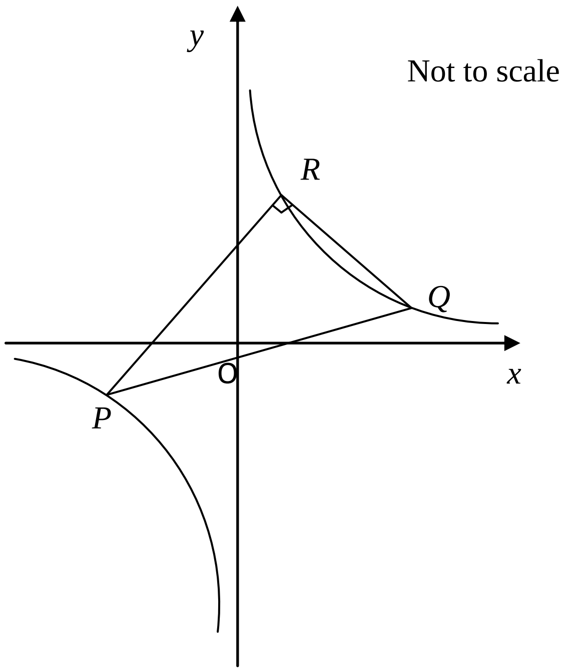
QUESTION 1 **12 Marks**

(a) At a particular school there are 10 HSC subjects. Of these three are Mathematics courses. In how many different orders can the 10 examination papers be set so that no two of the three Mathematics papers are consecutive. Justify your answer. 2

(b) Six students go to the canteen and buy one ice cream each. There are 4 flavours to choose from, but there are only 4 ice creams of each flavour. How many different ways are there for the students to buy the ice creams? Give a full explanation. 3

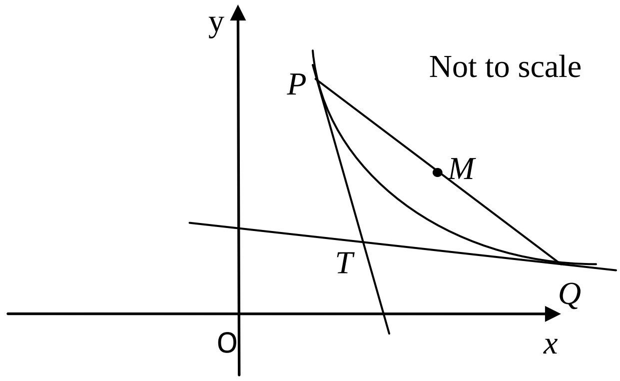
(c) How many ways can three integers be chosen which are in arithmetic progression from the numbers 1 to 101? Explain your answer. 3

(d) P, Q and R are three points on the hyperbola $xy = 16$ such that PQ subtends a right angle at R. Show that the tangent at R is perpendicular to PQ. 4



QUESTION 2 START A NEW PAGE **12 Marks**

(a) P and Q are two points on the hyperbola $xy = c^2$, with tangents TP and TQ . M is the midpoint of PQ. $AMBT$ is a rectangle with sides parallel to the coordinate axes. 4



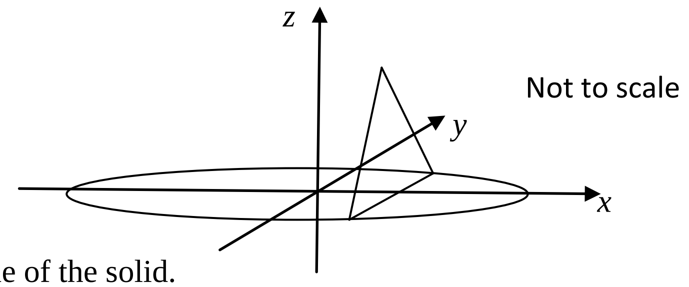
Copy the diagram and mark points A and B on it. Prove that A and B both lie on the hyperbola. You may assume that the tangent at the point $P\left[cp, \frac{c}{p}\right]$ on the hyperbola is $x + p^2y = 2cp$.

(b) A 1 kg object is moving in a straight line against a resistance force of kv^n Newton where $0 < n < 1$, v is the speed in m/s and k is a constant.

(i) Initially its speed is 30 m/s and after one second its speed is 15 m/s. Show that $15^{1-n}[1-2^{1-n}] = \frac{k(1-n)}{m}$ 2

(ii) The object stops after a further two seconds. Calculate the value of n to 2 decimals places. 3

(c) The base of a solid is in the region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Each cross-section perpendicular to the x-axis is an isosceles triangle with its base in the region and altitude equal to twice the base.



Find the volume of the solid. 3

QUESTION 3 START A NEW PAGE 12 Marks

(a) A 4 kg food package is dropped from a helicopter hovering above a campsite. As the package falls it experiences air resistance of $2v$ N.

(i) Draw a diagram showing the forces acting on the package as it falls. 1

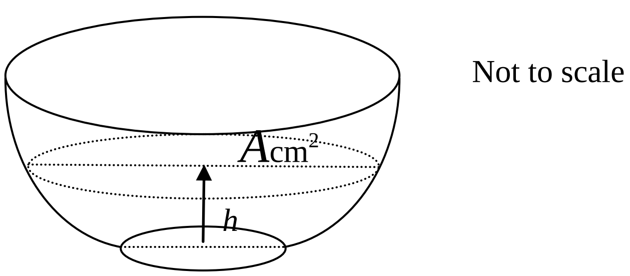
(ii) Write down the equation of motion for the package and find the value of the terminal velocity. Use $g = 10 \text{ ms}^{-2}$. 2

(iii) How far has the package fallen when its speed is 18 m/s? 3

(b) A 50g bullet is fired into a new experimental material and experiences a resistive force of R Newton such that $R = \sqrt{v} + 10$ where v is the speed of the bullet in m/s.

How long does it take the bullet to come to rest if it enters the material at 400 m/s? 3

(c) A sculpture is designed such that its cross-sectional area A cm^2 at height h cm above the base is given by the formula $A = 5 + \sqrt{h}$. 3



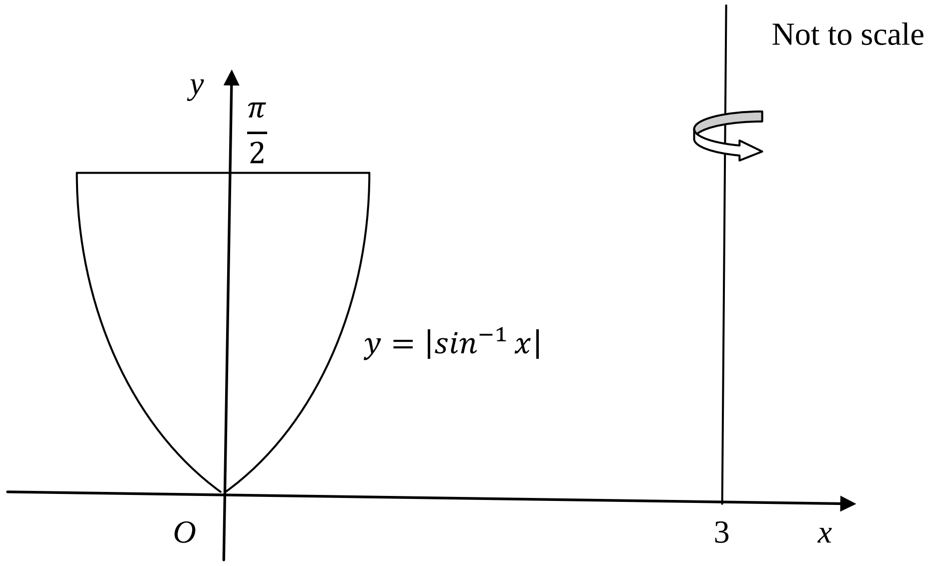
Calculate the volume of the sculpture if the area of the top surface is 20 cm^2.

QUESTION 4 START A NEW PAGE 12 Marks

(a) How many different sums of money can be made from five \$20 notes, one \$10 note, four \$2 coins, three 50c coins and one 10 c coin? 1

(b) Four sets of twins go to a party. In how many ways can these children be arranged in pairs so that no child is with his twin? 2

(c) The region bounded by the curve $y = |sin^{-1} x|$ and $y = \frac{\pi}{2}$ is revolved about the line $x = 3$.



Use the method of cylindrical shells or otherwise to show that the volume can be expressed as $12\int_0^{\frac{\pi}{2}} \sin x dx$. 4

(d) A toy rocket with a mass of 200 g is fired vertically upwards with a speed of 20 m/s.

(i) Write the equation of motion if the air resistance is $0.5v^2$ N, where *v* is the speed of the rocket. 1

(ii) Calculate the time for the rocket to reach its maximum height. Use $g = 10$ ms^{-2} 2

(iii) Find the maximum height reached by the rocket. 2

QUESTION 5 START A NEW PAGE 12 Marks

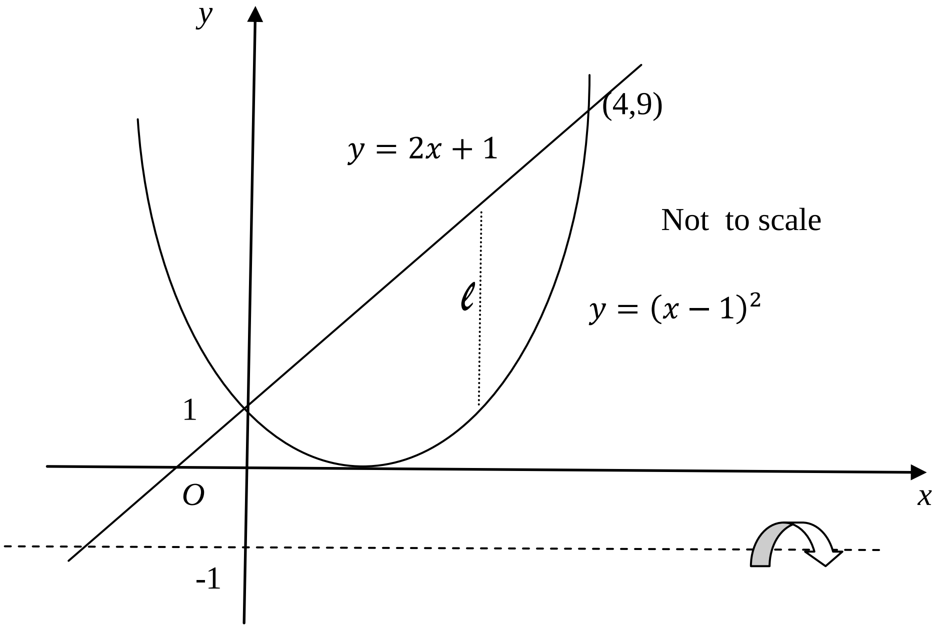
(a) $P(x_1, y_1)$ is a point on the hyperbola $xy = c^2$.

(i) Show that the equation of the normal at *P* is $xx_1 - yy_1 = {x_1}^2 - {y_1}^2$. 2

(ii) The normal at *P* meets the hyperbola again at $Q(x_2, y_2)$. 3
Express x_2 in terms of only x_1 and y_1.

(b) In addition to point *A*, there are *m* points marked on the straight line *AB* and *n* points on *AC*. 2
How many triangles can be formed with these points as vertices.
Give your answer in simplified algebraic form.

(c) The region bounded by the curves $y = (x-1)^2$ and $y = 2x+1$ is rotated around the line $y = -1$.



(i) When the region is rotated, the vertical line segment ℓ sweeps out an annulus. Show that the area of the annulus A is given by **3**

$$A = \pi[16x - 4x^2 + 4x^3 - x^4] \text{ for } 0 \le x \le 4$$

(ii) Find the volume of the solid generated. **2**

End of Examination

YEAR 12 ASS.
TEST 3 2016

MATHEMATICS Extension 2: Question 1

Suggested Solutions	Marks	Marker's Comments
QUESTION 1		
a) _ _ _ _ _ _ _ ^ ^ ^ ^ ^ ^ ^ ^		
Number of ways for 7 non maths papers = $7!$	1	For $7!$ or $^7P_7$
8 places for 3 maths papers = $^8C_3$		or
Number of ways for 3 maths papers = $3!$		$^8C_3 \times 3!$
$\therefore$ Total number of ways $= 7! \times {}^8C_3 \times 3!$		
OR $= 7! \times {}^8P_3$		
$= 1\ 693\ 440$	1	For correct answer
OR		
Number of ways = Number of Restrictions − 3 together − 2 together	1	For $3!8!$
$= 10! - 3!8! - ({}^3C_2 \times 7! \times 8 \times 7) \times 2$		OR $({}^3C_2 \times 7! \times 8 \times 7) \times 2$
$= 3628800 - 241920 - 1693440$		
$= 1\ 693\ 440$	1	For correct answer

YEAR 12 ASS.
TEST 3 2016

MATHEMATICS Extension 2: Question 1

Suggested Solutions	Marks	Marker's Comments
QUESTION 1		
b Number of ways for each person to have 4 icecreams if there are no restrictions $= 4^6$	1	For 4^6
Number of ways 6 people choose the same one $= 4$		
Number of ways 5 people choose the same one and one person different $= {}^6C_5 \times {}^4C_1 \times {}^3C_1$	1	For ${}^6C_5 \times {}^4C_1 \times {}^3C_1$
$\underbrace{{}^6C_5}_{\text{5 people}} \times \underbrace{{}^4C_1}_{\text{choose 1 icecream}} \times \underbrace{{}^3C_1}_{\text{other perso chooses ice cream}}$		
$\therefore$ Total $= 4^6 - 4 - {}^6C_5 \times {}^4C_1 \times {}^3C_1$		
$= 4096 - 4 - 72$		
$= 4020$	1	For correct answer

YEAR 12 ASS
TEST 3 2016

MATHEMATICS Extension 2: Question 1

Suggested Solutions	Marks	Marker's Comments
QUESTION 1		
b) Alternate Answer		
All 4 flavours		
1113 $\quad {}^4C_1 \times \frac{6!}{3!} = 480$ (A) eg ABCDDD	1	For any 2 of ABCDEFG
1122 $\quad {}^4C_2 \times \frac{6!}{2!2!} = 1080$ (B) eg ABCCDD		
3 flavours		
114 $\quad {}^4C_1 \times {}^3C_1 \times \frac{6!}{4!} = 360$ (C)	1	For any 4 of ABCDEFG
123 $\quad {}^4C_1 \times 3! \times \frac{6!}{3!2!} = 1440$ (D)		
222 $\quad {}^4C_1 \times \frac{6!}{2!2!2!} = 360$ (E)		
2 flavours		
3, 3 $\quad {}^4C_2 \times \frac{6!}{3!3!} = 120$ (F)		
2, 4 $\quad {}^4C_2 \times 2 \times \frac{6!}{2!4!} = 180$ (G)		
$\therefore$ Total $= 4020$ ways.	1	For correct answer

YEAR 12 ASS
TEST 3 2016

MATHEMATICS Extension 2: Question 1

Suggested Solutions	Marks	Marker's Comments
QUESTION 1		
c) If $d = 1$ 99 AP's; $d = 2$ 97 AP's; $d = 3$ 95 AP's; ...; $d = 50$ 1 AP — 50 terms.	1	For recognizing the pattern
	1	For attempting to sum a pattern with minor error
$S_n = \frac{50}{2}[1+99] = 2500$ possible AP's	1	For correct answer
ALTERNATE SOLUTION		
Choose the first number and the last number. Their average will determine the middle number ie $a, \frac{a+c}{2}, c$. Since $\frac{a+c}{2}$ is an integer, a and c must both be even or both odd.	1	For setting up the explanation noting that a and c must both be even or both odd.
$\therefore$ From 1 to 101 there are 51 odd numbers and 50 even numbers	1	For recognising there are 51 odd numbers and 50 even.
$\therefore$ No of ways $= \binom{50}{2} + \binom{51}{2}$		
$= 2500$	1	For correct answer.

YEAR 12 ASS
TEST 3 2016

MATHEMATICS Extension 2: Question 1

Suggested Solutions	Marks	Marker's Comments
QUESTION 1		
d)		
Let P be $\left[4p, \frac{4}{p}\right]$, $R\left[4r, \frac{4}{r}\right]$ and $Q\left[4q, \frac{4}{q}\right]$		
$M_{PR} = \dfrac{\frac{4}{r} - \frac{4}{p}}{4r - 4p} = \dfrac{p-r}{rp(r-p)} = \dfrac{-1}{rp}$		
Similarly $M_{RQ} = \dfrac{-1}{rq}$		
As $\angle PRQ = 90^\circ$		
$\dfrac{-1}{rp} \cdot \dfrac{-1}{rq} = -1$		
$\therefore r^2pq = -1$	1	For proving that $r^2pq = -1$
For the gradient of the tangent at R		
$xy = 16$		
$y = \dfrac{16}{x}$		
$\dfrac{dy}{dx} = \dfrac{-16}{x^2}$		

YEAR 12 ASS
TEST 3 2016

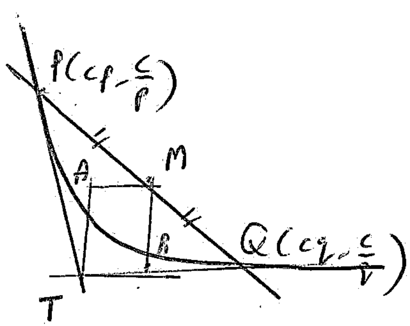
MATHEMATICS Extension 2: Question 1

Suggested Solutions	Marks	Marker's Comments
QUESTION 1		
d) Continued		
When $x = 4r$, $m_T = \frac{-16}{16r^2}$		
$= -\frac{1}{r^2}$	1	For finding that $m_T = -\frac{1}{r^2}$
$m_{PQ} = \frac{\frac{4}{q} - \frac{4}{p}}{4q - 4p} = \frac{p-q}{pq(q-p)}$		
$= -\frac{1}{pq}$	1	For finding that $m_{PQ} = -\frac{1}{pq}$.
but $r^2 pq = -1$		
$r^2 = -\frac{1}{pq}$		
$\therefore m_{T \text{ at } R} = pq$		
$\therefore m_{PQ} \times m_{T \text{ at } R} = -\frac{1}{pq} \times pq$	1	For conclusion with reasoning.
$= -1$		
$\therefore$ tangent at R is $\perp$ PQ		

Q2 Ext 2 Yr 12 T2 (CHAN)

a) TANGENT at P $x+p^2y=2cp$ ①

Q $x+q^2y=2cq$ ②



①-② $y(p^2-q^2)=2c(p-q)$

$$y=\frac{2c(p-q)}{(p-q)(p+q)} \quad p\neq q$$

$$y=\frac{2c}{p+q}$$

$$x=2cp-p^2\left[\frac{2c}{p+q}\right]=\frac{\cancel{2cp^2}+2cpq-\cancel{2cp^2}}{p+q}$$

$$x=\frac{2cpq}{p+q} \qquad \therefore T=\left[\frac{2cpq}{p+q},\ \frac{2c}{p+q}\right]$$

2 m for T

$$M=\left[\frac{c}{2}(p+q)\ \ \frac{c}{2}\left(\frac{1}{p}+\frac{1}{q}\right)\right]=\left[\frac{c}{2}(p+q),\ \frac{c}{2}\left(\frac{p+q}{pq}\right)\right]$$

$$A=\left[\frac{2cpq}{p+q},\ \frac{c}{2}\left(\frac{p+q}{pq}\right)\right] \qquad X_A\cdot Y_A=\frac{\cancel{2}c\cancel{pq}}{\cancel{p+q}}\cdot\frac{c}{\cancel{2}}\frac{\cancel{p+q}}{\cancel{pq}}$$

1 m Prove A is on $xy=c$

$$B=\left[\frac{c}{2}(p+q),\ \frac{2c}{p+q}\right] \qquad X_B\cdot Y_B=\frac{c}{\cancel{2}}\cancel{(p+q)}\cdot\frac{\cancel{2}c}{\cancel{(p+q)}}$$

1 m Prove B is on $xy=c$

$\therefore$ both A & B lie on the hyperbola $xy=c^2$

b) $\xleftarrow{kv^n}$ $\xrightarrow{\quad} +$ $\quad t=0,\ v=30$ $\quad 0<n<1$

$t=1,\ v=15$

$m\ddot{x}=-kv^n$ (Newton's 2nd Law of Motion)

$\ddot{x}=-kv^n$ $(m=1)$

$$\frac{dv}{dt}=-kv^n \qquad \int_{30}^{15}\frac{dv}{-kv^n}=\int_0^1 dt$$

1 m for correct integral with limit

$$\frac{1}{k}\left[\frac{v^{-n+1}}{-n+1}\right]_{15}^{30}=1$$

★ $$\frac{1}{k}\frac{1}{(1-n)}\left[30^{-n+1}-15^{-n+1}\right]=1$$

$$15^{-n+1}\left[1-2^{-n+1}\right]=k(n-1) \qquad \text{Since } 30^{-n+1}=15^{-n+1}\cdot 2^{-n+1}$$

1 m for final proof

3/2

ii) At $t=3$, $V=0$ (stop) $\therefore \int_{30}^{0} -\frac{dv}{kv^n} = \int_0^3 dt$

using ★ $\frac{1}{k(1-n)}\left[30^{-n+1} - 0\right] = 3$

$\therefore \frac{30^{1-n}}{3} = k(n-1)$

1m for $\frac{-30^{1-n}}{3} = k(n-1)$

FROM (a) $k(n-1) = 15^{1-n}(1-2^{-n+1})$

$\therefore \frac{-30^{1-n}}{3} = 15^{1-n}(1-2^{-n+1})$

$-2^{1-n}\cdot \cancel{15^{1-n}} = 3\times \cancel{15^{1-n}}(1-2^{1-n})$

$-2^{1-n} = 3 - 3(2^{1-n})$

$2\cdot 2^{1-n} = 3$

$2^{2-n} = 3$

1m for $2^{2-n} = 3$

$\therefore (2-n)\log 2 = \log 3$

$2-n = \frac{\log 3}{\log 2}$

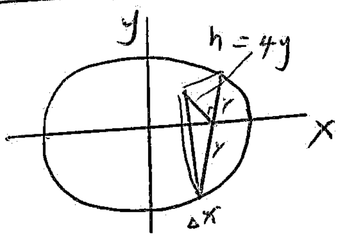
$n = 2 - \frac{\log 3}{\log 2} = 0.42$ (2dp)

1m final answer

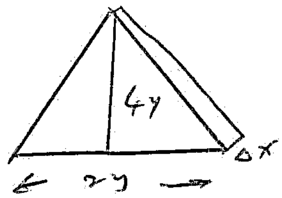
c) Area of triangle

$= \frac{1}{2}\times 2y \times 4y = 4y^2$

1m for area of $\Delta = 4y^2$



$V = \lim_{\Delta x\to 0} \sum_{-3}^{3} 4y^2 \Delta x$



$= \lim_{\Delta x\to 0} \sum_{-3}^{3} 4\times\frac{4}{9}\left[9-x^2\right]\Delta x$

$= \frac{4\times 4}{9}\int_{-3}^{3} 9-x^2\,dx$

1m for setting up integration in terms of x with correct limits

$\frac{x^2}{9}+\frac{y^2}{4}=1$

$y^2 = 4\left[\frac{9-x^2}{9}\right]$

$= \frac{16\times 2}{9}\int_0^3 9-x^2\,dx$ (even function)

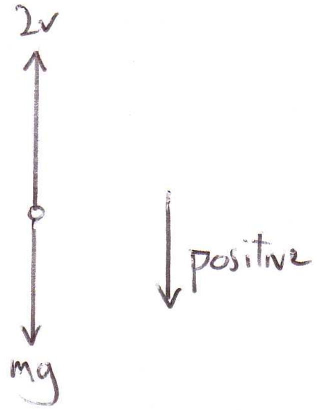
must mention even function or symmetrical

$= \frac{32}{9}\left(9x - \frac{x^3}{3}\right)\Big]_0^3 = \frac{32}{9}(27-9)$

$= 64$ unit^3 #

1m final answer

Q3. a) i



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ii) $m\ddot{x} = 40 - 2v$ 1

$4\ddot{x} = 40 - 2v$

$\ddot{x} = 10 - \frac{1}{2}v$

$\ddot{x} = 0 \Rightarrow 10 - \frac{1}{2}v = 0$

$\therefore \frac{1}{2}v = 10$

$v = 20\,m/s$ 1

iii) $v\frac{dv}{dx} = 10 - \frac{1}{2}v$

$\int \frac{v}{10 - \frac{v}{2}}\,dv = \int dx$ 1

$\int_0^{18} \frac{2v}{20 - v}\,dv = \int_0^x dx$

$\int_0^{18} -2 + \frac{40}{20 - v}\,dv = x$

$\therefore x = \left[-2v - 40\ln|20 - v|\right]_0^{18}$ 1

$= -36 - 40\ln|2| + 40\ln|20|$

$= 40\ln 10 - 36$ 1

Note: Those who had $x = -36 - 40\ln(-2) + 40\ln(-20)$

$= -36 + 40\ln\left(\frac{-20}{-2}\right)$

$= 40\ln 10 - 36 \longrightarrow -1$ mark

b) $ma = -\sqrt{v} - 10$

$ma = -(\sqrt{v} + 10)$

$\frac{dv}{dt} = \frac{-(\sqrt{v}+10)}{m}$ —— 1

$\frac{dt}{dv} = \frac{0.05}{-(\sqrt{v}+10)}$

$20\int_0^T t = -\int_{400}^{0} \frac{1}{\sqrt{v}+10}\, dv$

$20T = \int_0^{400} \frac{1}{\sqrt{v}+10}\, dv$ (where T is the time taken for $v=0$)

let $u = \sqrt{v}$ when $v = 400$, $u = 20$

$u^2 = v$ $v = 0$, $u = 0$

$\therefore \frac{dv}{du} = 2u$

$dv = 2u\, du$

$\therefore 20T = \int_0^{20} \frac{2u}{u+10}\, du$ —— 1

$= 2\int_0^{20} 1 - \frac{10}{u+10}\, du$

$10T = \left[u - 10\ln(u+10)\right]_0^{20}$

$= \left[20 - 10\ln 30 + 10\ln 10\right]$

$10T = 20 - 10\ln 3$

$\therefore T = 2 - \ln 3\, s$

or

$2 - \ln 30 + \ln 10$

or

$2 + \ln\left(\frac{1}{3}\right)$ —— 1

or

$$20T = \int_0^{400} \frac{1}{\sqrt{v}+10}\,dv$$

let $u = \sqrt{v} + 10$ When $v = 400$, $u = 30$

$v = 0$, $u = 10$

$$\frac{du}{dv} = \frac{1}{2}v^{-\frac{1}{2}}$$

$$\frac{dv}{du} = 2\sqrt{v}$$

$$dv = 2(u-10)\,du$$

$$\therefore 20T = \int_{10}^{30} \frac{2(u-10)}{u}\,du$$ (1)

$$10T = \int_{10}^{30} 1 - \frac{10}{u}\,du$$

$$10T = \left[u - 10\ln u\right]_{10}^{30}$$

$$= 30 - 10\ln 30 - 10 + 10\ln 10$$

$$10T = 20 - 10\ln 3$$

$$\therefore T = 2 - \ln 3 \text{ s.}$$ (1)

$$50a = -(\sqrt{v} + 10)$$

$$a = \frac{-(\sqrt{v}+10)}{50}$$

$$\frac{dv}{dt} = \frac{-(\sqrt{v}+10)}{50}$$

$$\frac{dt}{dv} = \frac{-50}{(\sqrt{v}+10)}$$

$$\int_0^T dt = -50\int_{400}^{0} \frac{1}{\sqrt{v}+10}\,dv$$

$$T = 50\int_0^{400} \frac{1}{\sqrt{v}+10}\,dv$$

let $u = \sqrt{v}$

$u^2 = v$

$\frac{dv}{du} = 2u$

$dv = 2u\,du$

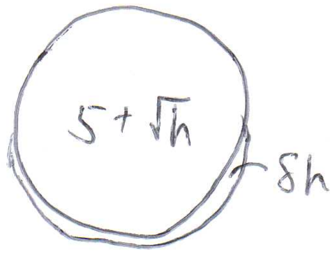
$$T = 100\int_0^{20} \frac{2u}{u+10}\,du$$

$$= 100\left[u - 10\ln(u+10)\right]_0^{20}$$

$$= 100\left[20 - 10\ln(30) + \ln(10)\right]$$

$$= 2000 - 1000\ln(3) \quad u$$

c) $A = 5 + \sqrt{h}$



when $A = 20$, $5 + \sqrt{h} = 20$

$\therefore \sqrt{h} = 15$

$h = 225$ 1

$\therefore V_{slice} = (5+\sqrt{h})\,\delta h$

$\therefore V_{solid} = \lim_{\delta h \to 0} \sum_{0}^{225} (5+\sqrt{h})\,\delta h$

$= \int_0^{225} (5+\sqrt{h})\,dh$ 1

$= \left[5h + \frac{2h^{3/2}}{3}\right]_0^{225}$

$= 1125 + 2250$

$= 3375\ u^3$ 1

1/4

MATHEMATICS Extension 2: Question 4

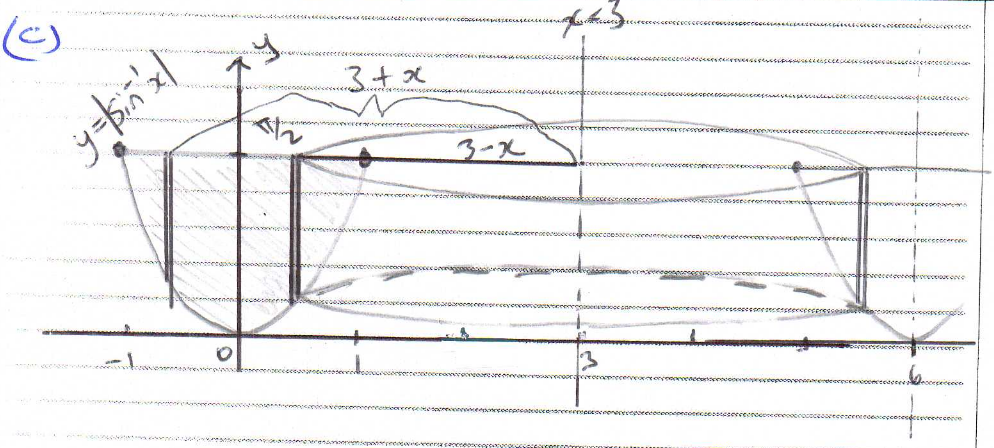
Suggested Solutions	Marks	Marker's Comments
(a) * can choose to spend or not 5 × \$20 notes = 6 choices (0,1,2,3,4,5) 1 × \$10 notes = 2 " (0,1) 4 × \$2 coins = 5 " (0,1,2,3,4) 3 × 50c coins = 4 " (0,1,2,3) 1 × 10c coin = 2 " (0,1) $\text{Total} = 6 \times 2 \times 5 \times 4 \times 2 - 1$ $= 480 - 1$ $= 479$	1	right or wrong
(b) A1 A2 B1 B2 C1 C2 D1 D2 A_1 has a choice of 6 A_2 has a choice of 5 3rd has 2 choices of 2 $\text{Total} = 6 \times 5 \times 2 = 60$	1 2	
or Total of choosing 4 groups of 2 $= \dfrac{{}^8C_2 \times {}^6C_2 \times {}^4C_2 \times {}^2C_2}{2!\,2!\,2!\,2!\,4!}$ → were not interested in their arrangements $= 105$	or 1	but only if there's more calculations
"105" includes all twins being together or 1 pair of twins together or 2 twins together. one twin → ${}^4C_1 \times 1 \times 4 \times 2 \times 1 = 32$ two twins → ${}^4C_2 \times 2 \times 1 = 12$ 3/4 sets of twins → $1 \times {}^4C_4 = 1$ $\therefore \text{Total} = 105 - 32 - 12 - 1$ $= 60$	1	
or 2 pairs swap $A_1A_2 \to B_1B_2$ $C_1C_2 \to D_1D_2$ $\text{Total} = 2 \times 2 \times 3 = 12$ 4 pairs all swap A_1 goes with 6 A_2 goes with 4 3rd twin goes with 2 → $6 \times 4 \times 2$ $\text{Total} = 12 + 48 = 60$		

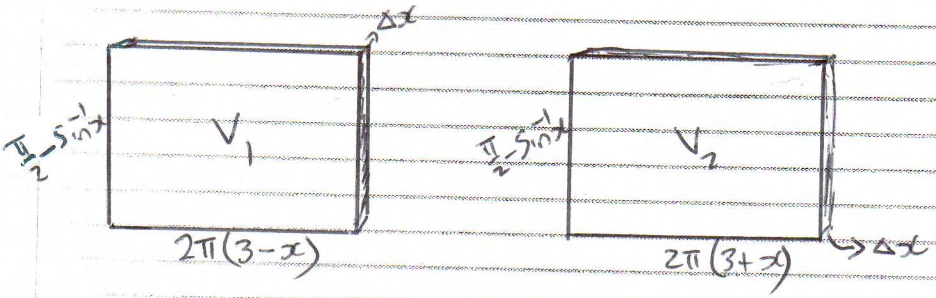
2/4

MATHEMATICS Extension 2: Question ....4 cont.

Suggested Solutions | **Marks** | **Marker's Comments**

(c)





$A_1 = 2\pi(3-x)(\pi/2-\sin^{-1}x)$ $\qquad$ $A_2 = 2\pi(3+x)(\pi/2-\sin^{-1}x)$

$\Delta V_1 = 2\pi(3-x)(\pi/2-\sin^{-1}x)\Delta x$ $\qquad$ $\Delta V_2 = 2\pi(3+x)(\pi/2-\sin^{-1}x)\Delta x$

$V_1 = \lim_{\Delta x\to 0}\sum_{x=0}^{1} 2\pi(3-x)(\pi/2-\sin^{-1}x)\Delta x$

$V_2 = \lim_{\Delta x\to 0}\sum_{x=0}^{1} 2\pi(3+x)(\pi/2-\sin^{-1}x)\Delta x$ $\qquad$ 1

$V = V_1 + V_2$

$= \lim_{\Delta x\to 0}\sum_{x=0}^{1}\left[2\pi(3-x)(\pi/2-\sin^{-1}x)\Delta x + 2\pi(3+x)(\pi/2-\sin^{-1}x)\right]$ $\qquad$ 1

$= \lim_{\Delta x\to 0}\sum_{x=0}^{1} 2\pi(\pi/2-\sin^{-1}x)(3-x+3+x)\Delta x$

$= \lim_{\Delta x\to 0}\sum_{x=0}^{1} 2\pi(\pi/2-\sin^{-1}x)6\,\Delta x$

$= \int_0^1 12\pi(\pi/2-\sin^{-1}x)\,dx$ $\leftrightarrow$ this represents area, which is the same as $\int_0^{\pi/2}\sin y\,dy$ $\qquad$ 1

$= 12\pi\int_0^{\pi/2}\sin y\,dy$

$= 12\pi\int_0 \sin x\,dx$ (dummy variable)

Marker's Comments:

* very disappointing to see the lack of effort. It was 4 marks for setting up the integral, needed to see every step and diagrams to get the full 4 marks.

* Some students didn't draw the graph or show their slices/shells.

(1) first mark was for drawing the correct diagram and showing the slice/shell

(1) showing both volumes and simplifying them, show limiting volume

(1) for simplifying addition of both volumes

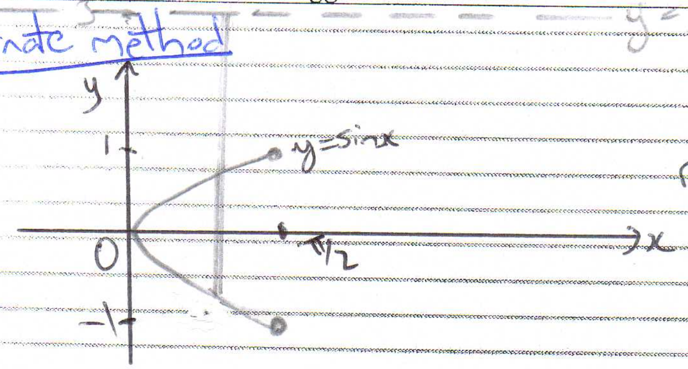
(1) discussing area and using dummy variable.

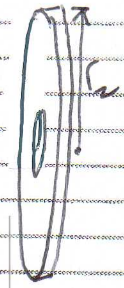
3/4

MATHEMATICS Extension 2: Question 4 cont.

Suggested Solutions	Marks	Marker's Comments

alternate method





(1)

clear, neat diagrams showing the slice with dimensions

$\Delta V = [\pi(3+x)^2 - \pi(3-x)^2]\,\Delta y$

$= [\pi(9+6x+x^2) - \pi(9-6x+x^2)]\,\Delta y$

$= 12\pi x\,\Delta y$ (1)

$V = \lim_{\Delta y \to 0} \sum_{y=0}^{\pi/2} 12\pi x \Delta y$ (1)

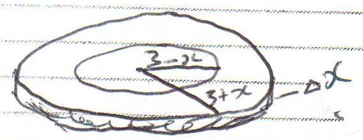
or could use the annulus and its very similar

$= 12\pi \int_0^{\pi/2} \sin y\, dy$

$= 12\pi \int_0^{\pi/2} \sin x\, dx$ (dummy varible) (1)

alternate method

~~taking the inverse~~ the volume of the area bounded by $y=|\sin x|$ and $y=\pi/2$ rotated about $x=3$ is equal to $y=\pm\sin x$ from $x=0$ to $x=\pi/2$, rotated about $y=3$ (1)



(1)

Taking Annulus slices $A = \pi(r_2^2 - r_1^2)$ where $r_2 = 3+\sin x$ $r_1 = 3-\sin x$

$A = \pi[(3+\sin x)^2 - (3-\sin x)^2]$

$= \pi(9 + 6\sin x + x^2 - 9 + 6\sin x - \sin^2 x)$ (1)

$= 12\pi \sin x$

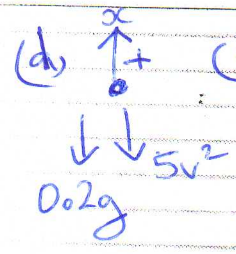
$\delta V = 12\pi \sin x\, \delta x$

$V = \lim_{\delta x \to 0} \sum_{x=0}^{\pi/2} 12\pi \sin x\, \delta x$ (1)

$= 12\pi \int_0^{\pi/2} \sin x\, dx$

4/4

MATHEMATICS Extension 2: Question 4 cont.

Suggested Solutions	Marks	Marker's Comments
(d) (i)  $0.2\ddot{x} = -(0.5v^2 + 0.2g)$		If you left it as m=200g you lost the mark.
$\ddot{x} = -(\frac{5}{2}v^2 + g)$	1	
or $\ddot{x} = -(\frac{5}{2}v^2 + 10)$		
(ii) $\frac{dv}{dt} = -(\frac{5}{2}v^2 + 10)$		
$\int_{20}^{0} \frac{dv}{\frac{5}{2}v^2+10} = -\int_0^T dt$		
$\frac{1}{5/2}\int_{20}^{0} \frac{dv}{v^2+4} = -T$	1	
$\frac{2}{5}\left[\frac{1}{2}\tan^{-1}\frac{v}{2}\right]_0^{20} = +T$		
$T = \frac{2}{5}\times\frac{1}{2}(\tan^{-1}10 - \tan^{-1}0)$		
$= \frac{1}{5}\tan^{-1}10$	1	a lot of students didn't have their calculator in radians and so got the wrong value for t :(
$= 0.294$ seconds		
(iii) $v\frac{dv}{dx} = -\frac{5}{2}v^2 + 10$		
$\frac{2}{5}\int_{20}^{0} \frac{v}{v^2+4}\,dv = -\int_0^H dx$	1	
$-\frac{1}{5}\left[\ln(v^2+4)\right]_{20}^{0} = H$		
max. height $= -\frac{1}{5}(\ln 4 - \ln 404)$		
$= \frac{1}{5}\ln\left(\frac{404}{4}\right)$		
$= \frac{1}{5}\ln 101$	1	
$= 0.92$m		
* now, if you took m=200g your answer to (ii) $T = 2\sqrt{10}\tan^{-1}\frac{1}{\sqrt{10}}$ (iii) $H = 200\ln\left(\frac{11}{10}\right)$		* max. marks would be 4 out of 5.

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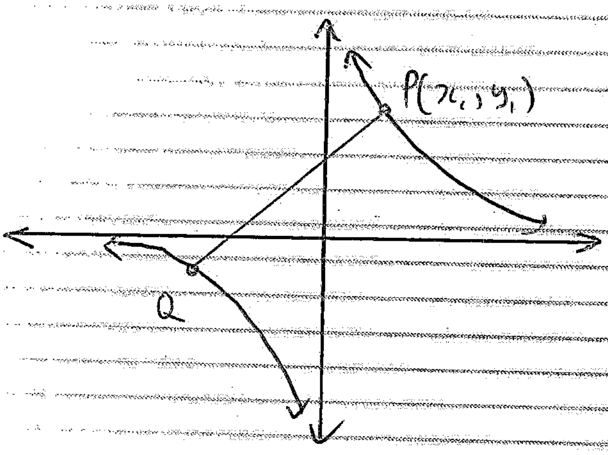
MATHEMATICS Extension 2: Question 5 (a)

Suggested Solutions	Marks	Marker's Comments
		
Gradient of normal:		
$xy = c^2$		
$y = \frac{c^2}{x}$		
$\frac{dy}{dx} = \frac{-c^2}{x^2}$		
at (x_1, y_1)		
$m_T = \frac{-c^2}{x_1^2}$		
$m_N = \frac{x_1^2}{c^2}$		
$= \frac{x_1^2}{x_1 y_1}$		
$= \frac{x_1}{y_1}$	1	
Alternative:		
$xy = c^2$		
$\frac{d}{dx}(xy) = \frac{d}{dx}(c^2)$		
$y + x\frac{dy}{dx} = 0$		
$\frac{dy}{dx} = \frac{-y}{x} \implies M_N = \frac{x}{y}$		

at (x_1, y_1) $\quad M_N = \frac{x_1}{y_1}$ 1

MATHEMATICS Extension 2: Question 5 (a)

Suggested Solutions	Marks	Marker's Comments
Point-Slope to find normal: $y - y_1 = \frac{x_1}{y_1}(x - x_1)$ $yy_1 - y_1^2 = xx_1 - x_1^2$ $x_1^2 - y_1^2 = xx_1 - yy_1$	1	
(ii) We're looking for intesection with $xy = c^2$ $y = \frac{c^2}{x}$ Solving simultaneously: $xx_1 - y_1\frac{c^2}{x} = x_1^2 - y_1^2$ $x^2 x_1 - y_1 c^2 = x_1^2 x - y_1^2 x$ $x^2 x_1 - x(x_1^2 - y_1^2) - y_1 c^2 = 0$	1	If students made correct substitution to form a quadratic in x then the mark was awarded
Solve the quadratic to find the intersection point. x_1 and x_2 are the roots of this quadratic. **Method 1:** Use product of roots: $x_1 x_2 = -\frac{y_1 c^2}{x_1}$ $c^2 = x_1 y_1 \Rightarrow x_2 = \frac{-y_1^2}{x_1}$	1 1	1 mark for identifying (clearly) how roots would be found 1 mark for answer.
	OR	
Method 2: Use sum of roots: $x_1 + x_2 = \frac{x_1^2 - y_1^2}{x_1}$ $x_1^2 + x_1 x_2 = x_1^2 - y_1^2$ $x_2 = \frac{-y_1^2}{x_1}$	1 1	1 mark for clear identification of how roots would be found 1 mark for answer.

MATHEMATICS Extension 2: Question 5 (a)

Suggested Solutions	Marks	Marker's Comments
	OR	

Method 3

Use quadratic formula:

$$x = \frac{x_1^2 - y_1^2 \pm \sqrt{(x_1^2 - y_1^2)^2 + 4c^2 x_1 y_1}}{2x_1}$$

$c_2 = x_1 y_1$:

$$= \frac{x_1^2 - y_1^2 \pm \sqrt{(x_1^2 - y_1^2)^2 + 4x_1^2 y_1^2}}{2x_1}$$ **1**

$$= \frac{x_1^2 - y_1^2 \pm \sqrt{(x_1^2 + y_1^2)^2}}{2x_1}$$

$$= \frac{x_1^2 - y_1^2 \pm (x_1^2 + y_1^2)}{2x_1}$$

$$= \frac{2x_1^2}{2x_1} \quad \text{or} \quad = \frac{-2y_1^2}{2x_1}$$

$$= x_1 \qquad\qquad = \frac{-y_1^2}{x_1}$$

We know $x_p = x_1 \Rightarrow x_2 = \frac{-y_1^2}{x_1}$ **1**

Alternative: note only "−" sign

$$= \frac{x_1^2 - y_1^2 - \sqrt{(x_1^2 - y_1^2)^2 + 4x_1^2 y_1^2}}{2x_1}$$ **1**

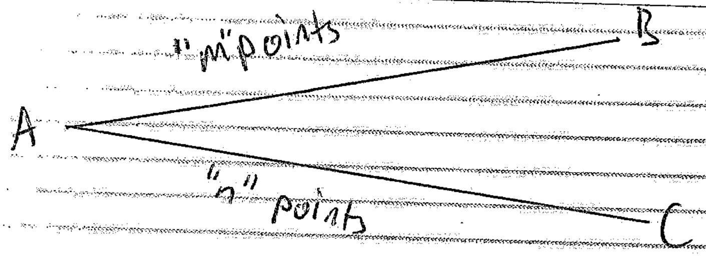
An adequate, clear explanation was needed as to why the other solution could be disregarded. Vague notions of "quadrants" or "other solution negative" was insufficient. (x_1, y_1) could be on either arc of the hyperbola.

Marker's Comments:

If student noted x_1 and x_2 were roots of equation and applied the formula in terms of only x_1 and y_1, then the mark was awarded.

Many students applied the formula and did not eliminate c. This earned no marks.

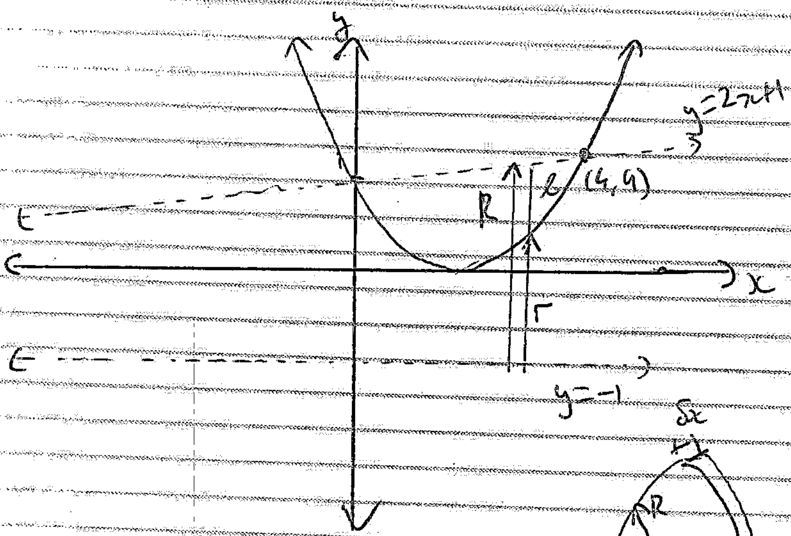
MATHEMATICS Extension 2: Question 5 (b)

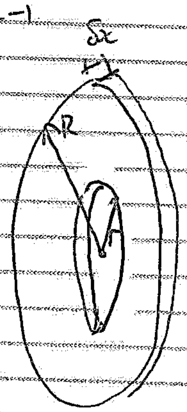
Suggested Solutions	Marks	Marker's Comments
		1 mark was awarded for considering each case and arriving at a correct number of ways for each.
Three points (non-collinear) form a triangle **Case 1: A is one vertex** Choose 1 point towards B = m ways Choose 1 point towards A = n ways Total $= 1 \times m \times n$ $= mn$ ways	1	Student did not need to simplify their algebraic expressions but ${}^nC_r$ notation had to be re-written as factorials.
Case 2: Excluding point A Choose 2 points towards A, 1 point towards B **or** 1 point toward A, 2 points towards B $= {}^mC_2 \times n + {}^nC_2 \times m$ $= \dfrac{m!\,n}{(m-2)! \times 2} + \dfrac{n!\,m}{(n-2)! \times 2}$	1	
Total $= \dfrac{m(m-1)n + n(n-1)m}{2} + nm$ $= \dfrac{m^2n + n^2m - 2mn + 2mn}{2}$ $= \dfrac{m^2n + n^2m}{2}$		

Alternative If student clearly showed the correct idea: fixing point A, then choosing two then one from each of the other intervals, without quantification, then 1 mark was awarded

MATHEMATICS Extension 1 Question 5 (c)

Suggested Solutions	Marks	Marker's Comments





Students needed the slice and the dimensions to get the mark.

$r = (x-1)^2 + 1$

$R = 2x+2$ — 1

$A = \pi\left[R^2 - r^2\right]$

$= \pi\left[(2x+2)^2 - \left((x-1)^2+1\right)^2\right]$ — 1

$= \pi\left[(2x+2-x^2+2x-2)(2x+2+x^2-2x+2)\right]$

$= \pi\left[(4x-x^2)(x^2+4)\right]$

$= \pi\left[4x^3+16x-x^4-4x^2\right]$

$= \pi\left[16x-4x^2+4x^3-x^4\right]$ — 1

$V \doteqdot A\delta x$

$V = \lim_{\delta x \to 0} \sum_{x=0}^{4} (16x-4x^2+4x^3-x^4)$

$= \pi\int_0^4 16x-4x^2+4x^3-x^4\,dx$ — 1

$= \pi\left[8x^2-\frac{4}{3}x^3+x^4-\frac{x^5}{5}\right]_0^4$

$= \frac{1408\pi}{15}$ — 1