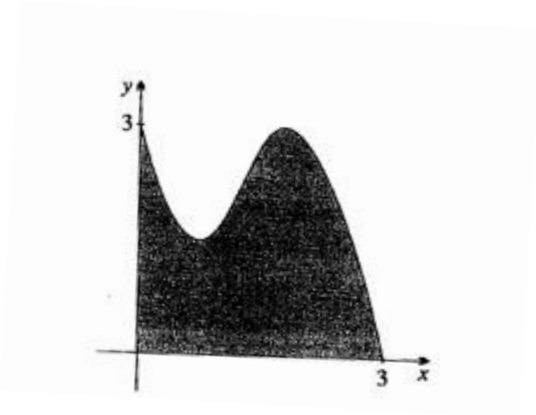
**QUESTION 1 (12 marks)** **Marks**

a) **2**




In the diagram above, the region under the curve $y = 3 - 4x + 4x^2 - x^3$ in the first quadrant is shaded. A solid of revolution is formed by rotating this region about the *y*-axis. Use the method of cylindrical shells to show the volume of this solid is given by

$$V = \pi \int_0^3 (6x - 8x^2 + 8x^3 - 2x^4)\, dx$$

b) The region in the number plane that contains all points satisfying simultaneously the inequalities **4**

$$x \leq 1, y \geq 1 \text{ and } y \leq e^x$$

is rotated through one complete revolution about the *x*-axis. Use the method of cylindrical shells to show that the volume of the resulting solid is

$$\frac{\pi}{2}(e^2 - 3).$$

c) How many arrangements of six A's, five B's, and four C's are there in which the first A precedes the first B which precedes the first C? Explain your answer. **2**

d) A particle of mass *m* is moving vertically in a resisting medium in which the resistance to motion has magnitude $\frac{1}{10}mv^2$, when the particle has velocity *v* ms$^{-1}$. The acceleration due to gravity is 10ms$^{-2}$.

(i) The particle is projected vertically upwards with speed *U* ms$^{-1}$. Show that during its upward motion, its acceleration *a* ms$^{-2}$ is given by **1**

$$a = -\frac{1}{10}(100 + v^2)$$

(ii) Hence show that its maximum height, *H* metres, is given by **3**

$$H = 5 \ln\left(\frac{U^2 + 100}{100}\right)$$

**QUESTION 2 (12 marks)** **START A NEW PAGE** **Marks**

$P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$ where $p$, $q > 0$ are two distinct points on the hyperbola $H$ with equation $xy = c^2$. You are given that the equation of the tangent to $H$ at $P\left(cp, \frac{c}{p}\right)$ is given by $x + p^2y = 2cp$ and that the normal to $H$ at $P\left(cp, \frac{c}{p}\right)$ is given by $y = p^2x + \frac{c}{p} - cp^3$.
DO NOT prove these results.

(i) The tangents to $H$ at $P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$ meet at $T$. Find the coordinates of $T$. **2**

(ii) Given that the chord $PQ$ passes through the point $(2c, 0)$ find a relationship between the parameters $p$ and $q$. **2**

(iii) Find the equation of the locus of $T$ as $P$ and $Q$ move about $H$ according to the restriction in part (ii). Give a complete description of this locus. **1**

(iv) The normal at $P$ intersects the second 'branch' of the hyperbola at $R$. Show that the $x$ coordinates of $P$ and $R$ are the roots of the equation **3**

$$x^2 - c\left(p - \frac{1}{p^3}\right)x - \frac{c^2}{p^2} = 0$$

and find the coordinates of $R$.

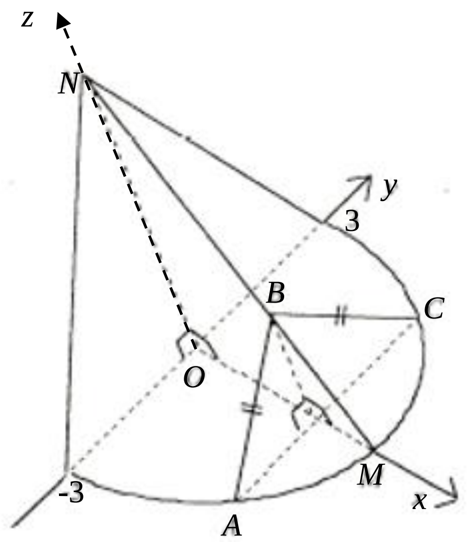
(v) The tangent at $P$ intersects the $x$ and $y$ axes at $A$ and $B$ respectively. Show that the area of triangle $ABR$ is given by **3**

$$c^2\left(p^2 + \frac{1}{p^2}\right)^2$$

(vi) Given the fact that the sum of any positive number and its reciprocal is greater than or equal to 2, find the minimum area of triangle $ABR$. **1**

**QUESTION 3 (12 marks)** **START A NEW PAGE** **Marks**

a)




A solid figure has a semi-circular base of radius 3cm. Cross sections taken perpendicular to the $x$-axis are isosceles triangles. The vertical cross section containing the radius $OM$ of the base of the solid is a right isosceles triangle $OMN$, where $OM = ON$.

(i) Show that the area, $A$, of triangle $ABC$ (where $AB = BC$) is given by **2**

$$A = (3 - x)(9 - x^2)^{\frac{1}{2}}$$

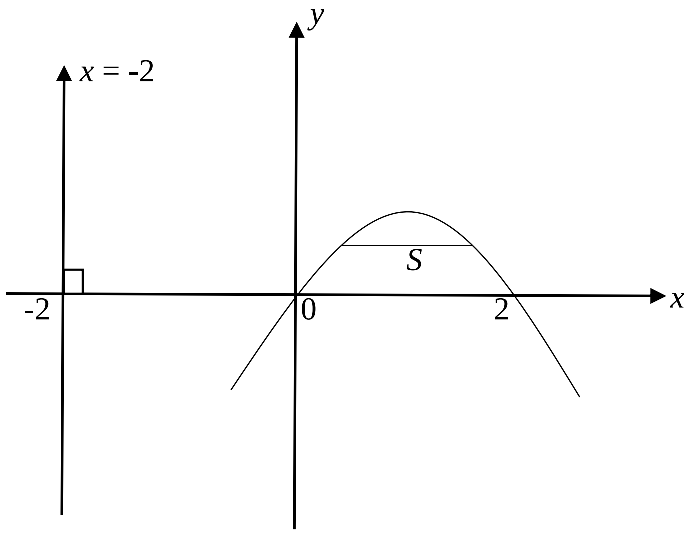
(ii) Find the volume of the solid. **4**

b) 21 books are to be distributed among 3 girls, Ava, Beth and Cate, so that Ava and Beth together receive twice as many books as Cate. Find the probability that each girl receives the same number of books. **3**

c) How many ways are there, on 6 successive nights, to enlist one of three siblings to help with dinner so that no one child is helping more than three times? Give a full explanation. **3**

**QUESTION 4 (12 marks)** **START A NEW PAGE** **Marks**

a)




The diagram shows the parabola $y = x(2 - x)$. When the region bounded by the parabola and the $x$-axis, $0 \le x \le 2$, is revolved about the line $x = -2$ to form a solid, the line segment $S$ at height $y$ sweeps out an annulus.

(i) Show that the area of the annulus is given by $A = 12\pi\sqrt{1-y}$. **3**

(ii) Hence find the volume of the solid formed. **2**

b) A particle of mass 2kg is propelled from the origin along the $x$-axis with an initial velocity of $Q$ m/s. The only forces acting on the body in the direction of the $x$-axis are friction, which is a constant 16 Newtons, and air resistance which equals $v^2$ Newtons where $v$ is the velocity of the particle $t$ seconds after leaving the origin. **5**

Show that

$$t = \frac{1}{2}\tan^{-1}\left(\frac{4Q - 4v}{16 + Qv}\right)$$

c) A particle of mass $m$ kg falls from rest in a medium where the resistance to motion is $mkv$ when the particle has a velocity of $v$ $\text{ms}^{-1}$ and $k$ is a constant. Take gravity to be $g$ $\text{ms}^{-2}$.

(i) Find the terminal velocity, $V$, of the particle in this medium. **1**

(ii) Hence, show that the equation of motion of the particle is $\ddot{x} = k(V - v)$ where $\ddot{x}$ is acceleration. **1**

**QUESTION 5 (12 marks)** **START A NEW PAGE** **Marks**

A camera of mass m kg has a parachute attached. A cameraman releases the camera from rest at the top of a building 50 metres high. The forces acting during its fall are gravity and, owing to the parachute, a resistance force of $\frac{1}{10}mv^2$ when the speed of the camera is $v$ ms$^{-1}$. Unfortunately, after 2ln2 seconds, the parachute detaches itself from the camera, and then the only force acting on the camera is gravity. The acceleration due to gravity is taken as $g$ = 10 ms$^{-2}$. At time $t$ seconds, the camera has fallen a distance $x$ metres from the top of the building, and its speed is $v$ ms$^{-1}$.

(i) Show that while the parachute is attached, $10\ddot{x} = 100 - v^2$ where $\ddot{x}$ is acceleration. **1**

(ii) Hence show that **6**

$$v = 10\left(\frac{e^{2t} - 1}{e^{2t} + 1}\right)$$

and

$$x = -5ln\left[1 - \left(\frac{v}{10}\right)^2\right]$$

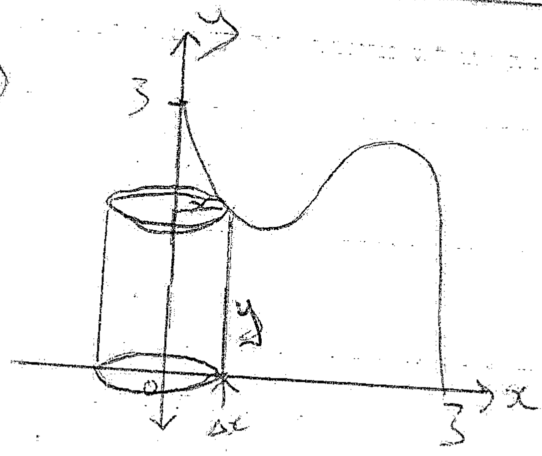
(iii) Find the exact speed of the camera and the exact distance fallen just before the parachute detaches. **2**

(iv) Find an expression for $\ddot{x}$ after the parachute detaches and find the speed of the camera just before it reaches the base of the building. Give your answer correct to 2 significant figures. **3**

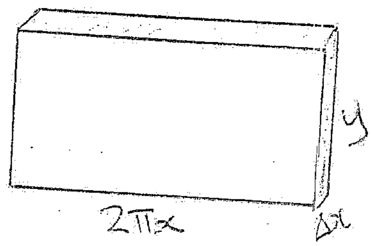
# Ext 2 – Q1

1/5

(a)




open the cylinder




$\Delta V \doteqdot 2\pi x y \Delta x$

but $y = 3 - 4x + 4x^2 - x^3$

$\therefore \Delta V = 2\pi x(3 - 4x + 4x^2 - x^3)\Delta x$

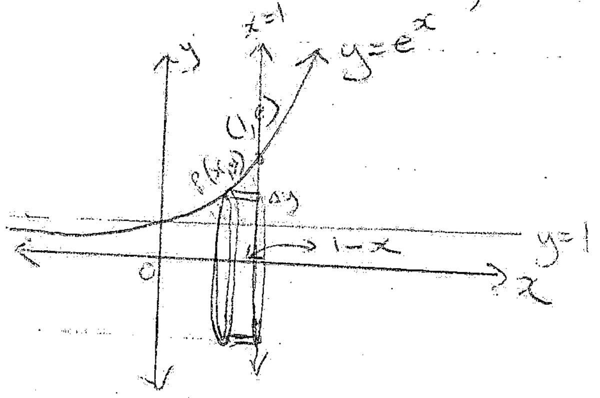
$= 2\pi(3x - 4x^2 + 4x^3 - x^4)\Delta x$ (1)

$V = \lim_{\Delta x \to 0} \sum_{x=0}^{3} 2\pi(3x - 4x^2 + 4x^3 - x^4)\Delta x$ (1)

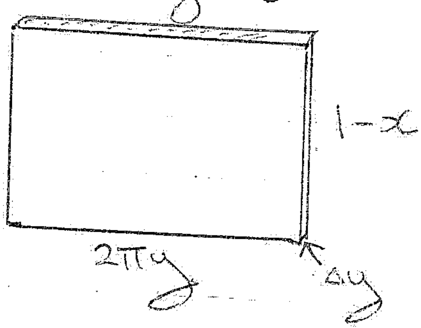
$= \pi \int_0^3 (6x - 8x^2 + 8x^3 - 2x^4)\,dx$

* Some diagrams were very disappointing, you needed to clearly show where your slice came from and its measurements, otherwise you lost the 1st mark.

(b)




unfold the cylinder




$\Delta V \doteqdot 2\pi y(1-x)\Delta y$

$\Delta V = \lim_{\Delta y \to 0} \sum_{y=1}^{e} 2\pi y(1-x)\Delta y$

$\therefore V = 2\pi \int_1^e y(1-x)\,dy$ (1)

$= 2\pi \int_1^e y(1 - \ln y)\,dy = 2\pi \int_1^e y\,dy - 2\pi \int_1^e y \ln y\,dy$

let $u = \ln y$ $\quad v' = y$
$u' = \frac{1}{y}$ $\quad v = \frac{1}{2}y^2$

$$V = 2\pi \left[\frac{y^2}{2}\right]_1^e - 2\pi\left\{\left[\tfrac{1}{2}y^2 \ln y\right]_1^e - \int_1^e \tfrac{1}{2} y \, dy\right\}$$

$$= 2\pi\left(\frac{e^2}{2} - \frac{1}{2}\right) - 2\pi\left\{\left(\tfrac{1}{2}e^2 - 0\right) - \tfrac{1}{2}\left[\frac{y^2}{2}\right]_1^e\right\}$$

$$= 2\pi\left(\frac{e^2}{2} - \frac{1}{2}\right) - 2\pi\left(\tfrac{1}{2}e^2 - \tfrac{1}{2}\left(\frac{y^2}{2} - \frac{1}{2}\right)\right)$$

$$= 2\pi\left(\frac{e^2}{2} - \frac{1}{2} - \frac{e^2}{2} + \frac{e^2}{4} - \frac{1}{4}\right)$$

$$= 2\pi\left(\frac{e^2}{4} - \frac{3}{4}\right)$$

$$= \frac{\pi}{2}\left(e^2 - 3\right)$$

2nd mark for correctly doing the Integration by parts
3rd mark for putting it altogether <u>and</u> factoring $2\pi$.
4th mark for simplifying correctly <u>without</u> fudging.
* If you did it by slices then it was a maximum of 2 marks.

(c)
no. of arrangements $= 1 \times {}^{14}C_5 \times 1 \times \frac{8!}{4!4!}$ $\rightarrow$ place the rest of B's and <u>C's</u>

must start with an A

place the other A's

B must go first in the remaining places.

$= 1 \times$

$= 140140$

alternative 1 $\quad 3/5$

$A_1 B_1 \quad \dfrac{13!}{\underset{(C's)}{4!}\,\underset{(A's)}{5!}\,\underset{(B's)}{4!}} \quad 90\,090$

$A_1 AB_1 \quad + \dfrac{12!}{4!\,4!\,4!} \quad + 34650$

$A_1 AAB_1 \quad + \dfrac{11!}{4!\,3!\,4!} \quad + 11550$

$A_1 AAAB_1 \quad + \dfrac{10!}{4!\,2!\,4!} \quad + 3150$

$A_1 AAAAB_1 \quad + \dfrac{9!}{4!\,1!\,4!} \quad + 630$

$A_1 AAAAAB_1 \quad + \dfrac{8!}{4!\,4!} \quad + 70$

$= 140140$

Since only matters when the first B occurs.

Alternative 2

A ① A ② A ③ A ④ A ⑤ A ⑥ $\quad$ ie 6 Gaps

case 1: 1st B in 1st gap
- ways to replace remaining B's $= \dfrac{6\times7\times8\times9}{4!} = 126$
- ways to place the 4 C's $= \dfrac{10\times11\times12\times13}{4!} = 715$

Total for case $1 = 126 \times 715 = 90090$ ways

case 2: 1st B in second gap
- ways to replace remaining B's $= \dfrac{5\times6\times7\times8}{4!} = 70$
- ways to place the 4 C's $= \dfrac{9\times10\times11\times12}{4!} = 495$

Total for case $2 = 70 \times 495 = 34650$

case 3: 1st B in 3rd gap
- ways to replace remaining B's $= \dfrac{4\times5\times6\times7}{4!} = 35$
- ways to replace the 4 C's $= \dfrac{8\times9\times10\times11}{4!} = 330$

Total for case $3 = 35 \times 330 = 11550$ ways

Case 4: 1st B in 4th gap

- ways to place the remaining B's = $\frac{3\times4\times5\times6}{4!} = 15$
- ways to place the 4 C's = $\frac{7\times8\times9\times10}{4!} = 210$

Total for case 4 = $15 \times 210 = 3150$

Case 5: 1st B in 5th gap

- ways to place the remaining B's = $\frac{2\times3\times4\times5}{4!} = 5$
- ways to place the 4 C's = $\frac{6\times7\times8\times9}{4!} = 126$

Total for case 5 = $5 \times 126 = 630$ ways

Case 6: 1st B in 6th gap

- ways to place the remaining B's = 1
- ways to place the 4 C's = $\frac{5\times6\times7\times8}{4!} = 70$

Total = 70

∴ Total number of ways = 90090 + 34650 + 11550 + 3150 + 630 + 70
= 140 140

* If you attempted alternative ① or ② and showed working with an explanation, then you scored 1 mark if you had a correct case.

* If you misunderstood the question, and therefore made it easier, your answer was 27 720, then you scored a maximum of one mark, as long as you had an explanation.
ie 27 720 with no words = 0 marks

* 105 105 = 0 marks.

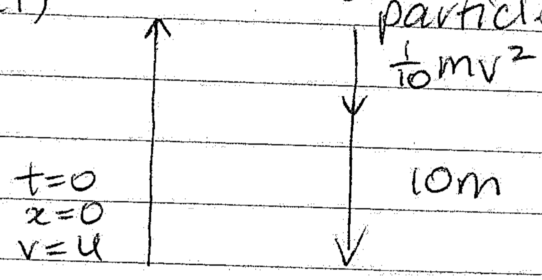
Altenate ③ sit A, B and C

- There are 14 places that A can choose,

$^{14}C_5 \times {^8C_4} \times {^4C_4} = 140140$

d) (i)

Forces on particle




If "m" is missing then they lose one mark!

$$m\ddot{x} = -\left(10m + \tfrac{1}{10}mv^2\right)$$

$$= -\tfrac{1}{10}\left(100m + mv^2\right) \qquad 1$$

$$\therefore \quad \ddot{x} = -\tfrac{1}{10}\left(100 + v^2\right)$$

ie $\quad a = -\frac{1}{10}\left(100 + v^2\right)$

(ii) $\quad v\dfrac{dv}{dx} = \dfrac{-1}{10}\left(100 + v^2\right)$

$$\frac{dx}{dv} = \frac{-10v}{100 + v^2}$$

$$= -5\left(\frac{2v}{100 + v^2}\right)$$

1 $\quad x = -5\ln\left(100 + v^2\right) + c$

$x = 0,\ v = u$

$$0 = -5\ln\left(100 + u^2\right) + c$$

$$c = +5\ln\left(100 + u^2\right)$$

1 $\quad \therefore x = -5\ln\left(100 + v^2\right) + 5\ln\left(100 + u^2\right)$

$$= 5\ln\left(\frac{100 + u^2}{100 + v^2}\right)$$

At max height, $H$, $v = 0$

1 $\quad \therefore H = 5\ln\left(\dfrac{u^2 + 100}{100 + 0}\right)$

$$= 5\ln\left(\frac{u^2 + 100}{100}\right)$$

or $\quad v\dfrac{dv}{dx} = \dfrac{-1}{10}\left(100 + v^2\right)$

$$\int_u^0 \frac{v\,dv}{100 + v^2} = \frac{-1}{10}\int_0^H dx \qquad 1$$

$$\left[\tfrac{1}{2}\ln\left(100 + v^2\right)\right]_u^0 = \frac{-1}{10}\left[x\right]_0^H$$

$$1 \quad \tfrac{1}{2}\left[\ln(100) - \ln\left(100 + u^2\right)\right] = \frac{-1}{10}(H)$$

$$-\tfrac{10}{2}\left(\ln 100 - \ln\left(100 + u^2\right)\right) = H$$

$$\therefore H = 5\left(\ln\left(100 + u^2\right) - \ln 100\right)$$

$$\therefore H = 5\ln\left(\frac{u^2 + 100}{100}\right)$$

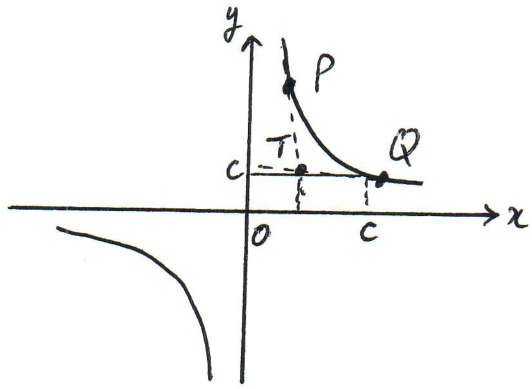
for this method students had to show the substitution

JRAHS Y12 T3 **MATHEMATICS Extension 2: Question** 2 Task 3 : 2017

**Suggested Solutions**

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (i) Tangent at $P$: $x + p^2 y = 2cp \quad \cdots (1)$ <br> " " $Q$: $x + q^2 y = 2cq \quad \cdots (2)$ <br> (1) − (2): $y(p^2 - q^2) = 2c(p-q)$ <br> $y = \dfrac{2c(p-q)}{(p-q)(p+q)}$ <br> $= \dfrac{2c}{p+q} \quad \cdots (3)$ | 1 | For correct x-coordinate |
| Sub (3) into (1) <br> $x = -p^2\left(\dfrac{2c}{p+q}\right) + 2cp$ <br> $= \dfrac{-2cp^2 + 2cp^2 + 2cpq}{p+q}$ <br> $= \dfrac{2cpq}{p+q}$ <br> $\therefore T\left(\dfrac{2cpq}{p+q}, \dfrac{2c}{p+q}\right)$ | 1 | For correct y-coordinate |
| (ii) Equation of $PQ$: $y - y_1 = \left(\dfrac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$ <br> $\Rightarrow \quad y - \dfrac{c}{p} = -\dfrac{1}{pq}(x - cp)$ | 1 | For equation of PQ |
| Sub $(2c, 0)$: <br> $-\dfrac{c}{p} = -\dfrac{1}{pq}(2c - cp)$ <br> $\Rightarrow p + q = 2$ | 1 | For correct relationship |

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (iii) From (i) $y = \frac{2c}{p+q}$ and from (ii), $p+q=2$ <br> $\therefore y = \frac{2c}{2} = c$ <br> But $p, q > 0$ and T cannot possibly be on $xy=c^2$ as $p$ and $q$ are distinct. Hence the locus is $\underline{y = c \text{ with } 0 < x < c}$ | 1 | For both the locus equation <u>and</u> its restriction |

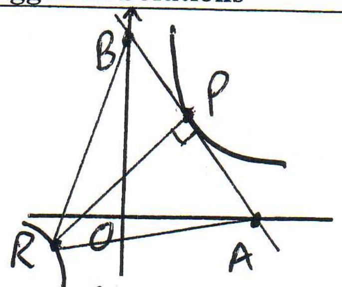



| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (iv) ($\alpha$): Normal at P: $y = p^2x + \frac{c}{p} - cp^3$ <br> Sub $y = c^2/x$ into above <br> $\Rightarrow c^2 = p^2x^2 + \frac{c}{p}x - cp^3x$ <br> $p$: $c^2/p^2 = x^2 + \frac{cx}{p^3} - cp\,x$ <br> $\therefore x^2 - c(p - \frac{1}{p^3})x - c^2/p^2 = 0$ | 1 | For substitution and simplification and obtaining the desired result. |

JRAHS Y12 T3 **MATHEMATICS Extension 2: Question** 2 Task 3 : 2017

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (iv) ($\beta$): continued: | | |
| from $x^2 - c\left(p - \frac{1}{p^3}\right)x^2 - \frac{c^2}{p^2} = 0$ | | |
| Sum of roots, $x_1 + x_2 = \frac{-b}{a}$ | | |
| $= c\left(p - \frac{1}{p^3}\right)$ | | |
| But $x_1 = cp = P_x$ | | |
| $\therefore x_2 = cp - c/p^3 - cp$ | | |
| $= -c/p^3 = R_x$ | | |
| Sub $x = -c/p^3$ into $y = c^2/x^2$ | | |
| $\therefore y = -cp^3$ | | |
| $\therefore R\left(-c/p^3, -cp^3\right)$ | 1+1 | For each coordinate of R |
| **OR**: $x^2 - c\left(p - \frac{1}{p^3}\right)x - c^2/p^2 = 0$ | | |
| $\Rightarrow (x - cp)(x + c/p^3) = 0$ | | Some alternate ways of $R_x$ |
| **OR**: $x = \dfrac{c\left(p - \frac{1}{p^3}\right) \pm \sqrt{c^2\left(p - \frac{1}{p^3}\right)^2 + 4c^2/p^2}}{2}$ | | |

JRAHS Y12 T3 **MATHEMATICS Extension 2: Question** 2 Task 3 : 2017

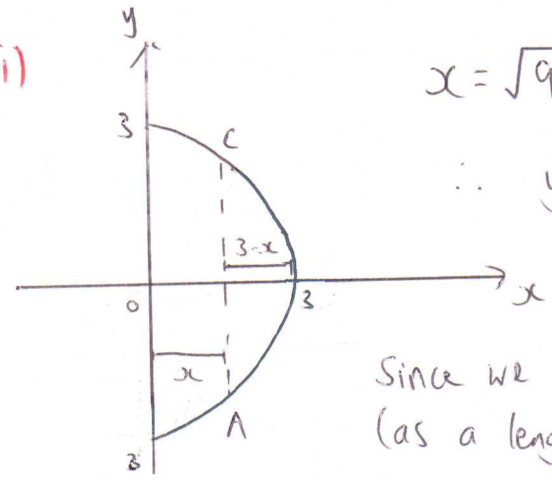
| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (v) Tangent at P: $x+p^2y=2cp$   | | $d=\dfrac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $=\dfrac{\lvert -c/p^3-cp^5-2c}{\sqrt{1+p^4}}$ |
| Coordinates of A : $(2cp,0)$ <br> " " B : $\left(0,\frac{2c}{p}\right)$ | | |
| $AB=\sqrt{(2cp-0)^2+(0-2c/p)^2}$ <br> $=2c\sqrt{p^2+\frac{1}{p^2}}$ or $2c/p\sqrt{1+p^4}$ | 1 | For length of AB |
| $PR=\sqrt{(cp+c/p^3)^2+(c/p+cp^3)^2}$ <br> $=\sqrt{c^2(p^6+3p^2+3/p^2+\frac{1}{p^6})}$ <br> $=c\sqrt{(p^2+1/p^2)^3}$ or $c/p^4\sqrt{(p^4+1)^3}$ | 1 | for length of PR |
| $\therefore \lvert\Delta ABR\rvert=\frac{1}{2}AB.PR$ <br> $=\frac{1}{2}.2c\sqrt{p^2+\frac{1}{p^2}}\,.\,c\sqrt{(p^2+1/p^2)^3}$ <br> $=c^2\left(p^2+\frac{1}{p^2}\right)^2$ | 1 | For derivation for Area |
| (vi) $p+\frac{1}{p}\geqslant 2$ <br> $\Rightarrow p^2+\frac{1}{p^2}\geqslant 2$ (How?) | | Incorrect R, then max 2 |
| $\therefore$ Minimum Area $\Delta ABR=4c^2$ | 1 | For correct minimum area |

J:\Maths\marking templates\Suggested Mk solns template_V2_no Ls.doc

Mathematics Ext.2 Question 3.

| Suggested Solutions | Marks | Marker's Comments |
| --- | --- | --- |

a) i)



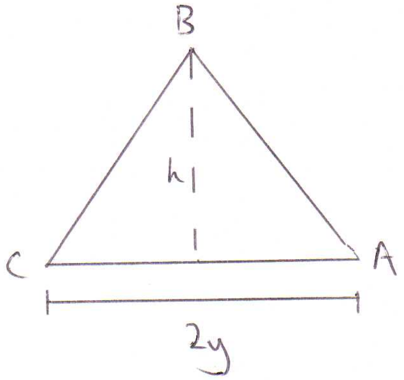

$x = \sqrt{9-y^2}$

$\therefore \quad y^2 = 9 - x^2$

$y = \pm\sqrt{9-x}$

Since we require $y \geq 0$ (as a length)

$y = \sqrt{9-x^2}$ — 1 — Some explanation




Since $h = 3 - x$ (vertical cross section is an Isosceles triangle) — 1 — Some explanation/diagram

then $A = \frac{1}{2} \times h \times 2y$

$= \frac{1}{2}(3-x) \times 2\sqrt{9-x^2}$

$A = (3-x)\sqrt{9-x^2}$

<u>or</u>

$h = 3 - x$ could be proven by

- Similar triangles
- finding the linear equation from M to N getting $z = 3 - x$.

Mathematics ________ Question ______

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| | | |

a)ii) $V = \lim_{\Delta x \to 0} \sum_{0}^{3} (3-x)(9-x^2)^{\frac{1}{2}} \Delta x$

$= \int_0^3 (3-x)(9-x^2)^{\frac{1}{2}} dx$ — 1

$= \int_0^3 3(9-x^2)^{\frac{1}{2}} dx - \int_0^3 x(9-x^2)^{\frac{1}{2}} dx$

$= 3 \times \text{Area (quadrant radius 3)} + \frac{1}{2}\int_0^3 (-2x)(9-x^2)^{\frac{1}{2}} dx$

1 value — $= \frac{27\pi}{4} + \frac{1}{2}\left[\frac{(9-x^2)^{3/2}}{3/2}\right]_0^3$ — 1 Integration

$= \frac{27\pi}{4} + \left(0 - \frac{9(3)}{3}\right)$

$= \frac{27\pi}{4} - 9$ — 1

Students who used Trig substitution:

1 Writing the integral

1 Writing the integral in terms of $\theta$ and no square roots

i.e $\int_0^{\frac{\pi}{2}} (3 - 3\sin\theta) \times 9\cos^2\theta \, d\theta$

1 Integrate

1 Evaluate

Mathematics Ext.2 Question 3

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| b) Ava and Beth receive 14<br>Cate receives 7 | 1 | |
| Remaining 2 marks :<br>Method 1<br>Number of ways to distribute into (7,7,7)<br>$= \underbrace{{}^{21}C_{14}}_{\text{Ways to choose 14 for Ava and Beth}} \times \underbrace{{}^{14}C_{7}}_{\text{Ways to choose 7 to belong to Ava}} = 39907296 0$ | 1 | Expression will suffice |
| Number of ways to distribute into (7,14)<br>$= \underbrace{{}^{21}C_{14}}_{\text{Ways to choose 14 for Ava and Beth}} \times \underbrace{(2^{14}-2)}_{\text{ways to divide 14 into 2 group each person gets something!!!}}$ | | |
| $\therefore P = \dfrac{{}^{21}C_{14} \times {}^{14}C_{7}}{{}^{21}C_{14} \times (2^{14}-2)} = \dfrac{{}^{14}C_{7}}{2^{14}-2} = \dfrac{1716}{8191}$<br>or<br>$0.209498$ | 1 | |
| If you forget to take away 2 from denominator you will get $\dfrac{429}{2048}$ or $0.209472$ | | Wrong!!! |

| Mathematics ______ Question _____ | | |
|---|---|---|
| Suggested Solutions | Marks | Marker's Comments |
| **Method 2**<br><br>$P(7,7,7) = \frac{21!}{7!7!7!}\left(\frac{1}{3}\right)^7\left(\frac{1}{3}\right)^7\left(\frac{1}{3}\right)^7$<br><br>$P(7,14) = \frac{21!}{7!\,14!}\left(\frac{1}{3}\right)^7\left(\frac{2}{3}\right)^{14}$ | 1 | |
| $\therefore P = \dfrac{\frac{\cancel{21!}}{7!7!7!}\left(\frac{1}{3}\right)^{21}}{\frac{\cancel{21!}}{7!\,14!}\left(\frac{1}{3}\right)^7\left(\frac{2}{3}\right)^{14}}$<br><br>$= \frac{14!}{7!7!} \times \frac{\left(\frac{1}{3}\right)^{14}}{\left(\frac{2}{3}\right)^{14}}$<br><br>$= \frac{{}^{14}C_7\left(\frac{1}{3}\right)^{14}}{2^{14}\left(\frac{1}{3}\right)^{14}}$<br><br>$= \frac{{}^{14}C_7}{2^{14}}$<br><br>Since sample space (denominator) must subtract 2 as explained above. | | |
| $\therefore P = \frac{{}^{14}C_7}{2^{14}-2} = \frac{1716}{8191}$ | 1 | |

Mathematics Ext2 Question 3.

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| | | |

c) Method 1

- Each child helps out 2 nights (2, 2, 2)

$= \dfrac{6!}{2!2!2!}$ — Ways to line 6 up in order

— 3 repeats of 2

$= \underline{\underline{90}}$

- One child one night, one child two nights, one child 3 nights (1, 2, 3)

Ways to line 6 up — $= \dfrac{6!}{3!2!} \times 3!$ — ways to arrange who does 1, 2, 3 nights.

a repeat of 3 and a repeat of 2

$= \underline{\underline{360}}$

- 2 Children help 3 nights each

line 6 up — $= \dfrac{6!}{3!3!} \times 3$ — Ways to choose who misses out.

2 repeats of 3.

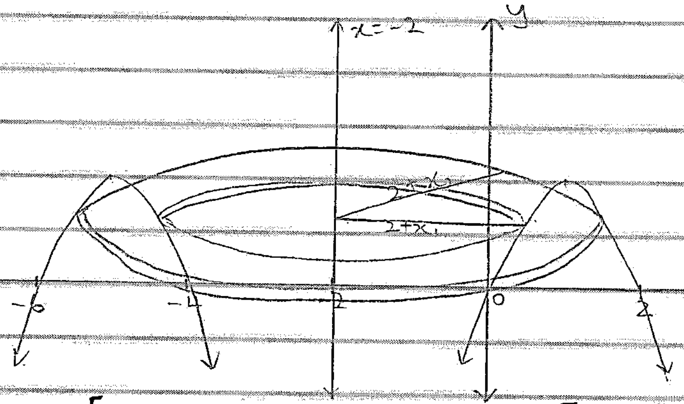
$= \underline{\underline{60}}$

$\therefore$ Total $= 90 + 360 + 60$

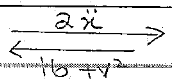
$= \underline{\underline{510}}$

| Mathematics ______ Question _____ | | |
|---|---|---|
| Suggested Solutions | Marks | Marker's Comments |
| c) **Method 2**<br><br>Total number of arrangements with no restrictions<br>$= 3^6$<br>$= \underline{\underline{729}}$<br><br>Number (1 person does 6 nights) $= \underline{\underline{3}}$ — 3 different childs<br><br>Number (1 does 5 nights) $= \underbrace{3}_{\text{3 different child to do 5 nights}} \times \underbrace{{}^6C_5}_{\text{choose 5 nights from 6}} \times \underbrace{2}_{\text{2 children to do the last night.}}$<br>$= \underline{\underline{36}}$<br><br>Number (1 does 4 nights) $= \underbrace{3}_{\text{3 different child to do 4 nights}} \times \underbrace{{}^6C_4}_{\text{choose 4 nights from 6}} \times \underbrace{2 \times 2}_{\text{2 children to choose to do 2 nights}}$<br>$= \underline{\underline{180}}$<br><br>$\therefore$ Answer $= 729 - 3 - 36 - 180$<br>$= \underline{\underline{510}}$<br><br>1st mark - Getting 729 AND one of 3, 36 or 180<br>2nd mark - Getting a 2nd restriction<br>3rd mark - Final answer. | | |

**MATHEMATICS Extension 2: Question ...4...**

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| 4. a) i) $y = 2x - x^2$ | | |
| $-y = x^2 - 2x$ | | |
| $1 - y = x^2 - 2x + 1$ | | |
| $1 - y = (x-1)^2$ | | |
| $x - 1 = \pm\sqrt{1-y}$ | | |
| $x = 1 \pm \sqrt{1-y}$  $x_2 = 1 + \sqrt{1-y}$, $x_1 = 1 - \sqrt{1-y}$ | 1 | |
|  | | |
| $A = \pi[(2+x_2)^2 - (2+x_1)^2]$ | | |
| $= \pi[(2+1+\sqrt{1-y})^2 - (2+1-\sqrt{1-y})^2]$ | 1 | show correct equation <u>and</u> substitution |
| $= \pi[(3+\sqrt{1-y})^2 - (3-\sqrt{1-y})^2]$ | | |
| $= \pi[(3+\sqrt{1-y}+3-\sqrt{1-y})(3+\sqrt{1-y} - (3-\sqrt{1-y}))]$ | 1 | correct expansion and work |
| $= \pi[(6)(2\sqrt{1-y})]$ | | |
| $= 12\pi\sqrt{1-y}$ | | |
| <u>OR</u> $1 - x = \sqrt{1-y}$ | 1 | |
| $A = \pi[(4-x)^2 - (2+x)^2]$ | | |
| $= \pi[16 - 8x + x^2 - 4 - 4x - x^2]$ | 1 | correct equation <u>and</u> expansion |
| $= \pi(12 - 12x)$ | | |
| $= 12\pi(1-x)$ | 1 | correct working <u>and</u> substitution. |
| $= 12\pi\sqrt{1-y}$ | | |
| ii) $V = \lim_{\Delta y \to 0} \sum_{y=0}^{1} 12\pi\sqrt{1-y}\,\Delta y$ | | ① limits |
| $= 12\pi\int_0^1 \sqrt{1-y}\,dy$ | | |
| $= 12\pi\left[-\frac{2}{3}\times(1-y)^{\frac{3}{2}}\right]_0^1$ | | ① correct integration and solution. |
| $= -8\pi(0-1) = 8\pi$ | | |

MATHEMATICS Extension 2: Question…4…

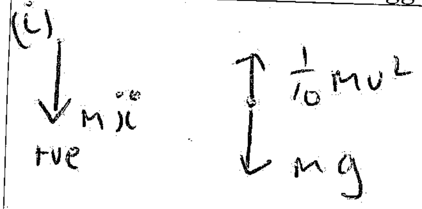
| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| b) $F = ma$   | | |
| $\therefore 2\ddot{x} = -16 - v^2$ | | |
| $\ddot{x} = \dfrac{-16 - v^2}{2}$ | 1 | |
| $\dfrac{dv}{dt} = -\dfrac{1}{2}(16 + v^2)$ | | |
| $\displaystyle\int_Q^v \frac{dv}{16+v^2} = -\frac{1}{2}\int_0^t dt$ | | ① correct limits. |
| $\dfrac{1}{4}\left[\tan^{-1}\dfrac{v}{4}\right]_Q^v = -\dfrac{1}{2}\left[t\right]_0^t$ | | |
| $t = -\dfrac{1}{2}\left[\tan^{-1}\dfrac{v}{4} - \tan^{-1}\dfrac{Q}{4}\right]$ | | |
| $t = \dfrac{1}{2}\left[\tan^{-1}\dfrac{Q}{4} - \tan^{-1}\dfrac{v}{4}\right]$ | | ① correct integration <u>and</u> formula for $t$. |
| $2t = \tan^{-1}\left(\dfrac{Q}{4}\right) - \tan^{-1}\left(\dfrac{v}{4}\right)$ | | |
| $\tan(2t) = \tan\left[\tan^{-1}\left(\dfrac{Q}{4}\right) - \tan^{-1}\left(\dfrac{v}{4}\right)\right]$ | | |
| $= \dfrac{\tan(\tan^{-1}\frac{Q}{4}) - \tan(\tan^{-1}\frac{v}{4})}{1 + \tan(\tan^{-1}\frac{Q}{4})\tan(\tan^{-1}\frac{v}{4})}$ | | ① taking tan of both sides |
| $= \dfrac{\frac{Q}{4} - \frac{v}{4}}{1 + \frac{Q}{4}\times\frac{v}{4}} \times \dfrac{16}{16}$ | | ① getting $\dfrac{\frac{Q}{4} - \frac{v}{4}}{1 + \frac{Q}{4}\times\frac{v}{4}}$ <u>and</u> getting final solution. |
| $= \dfrac{4Q - 4v}{16 + Qv}$ | | |
| $\therefore 2t = \tan^{-1}\left(\dfrac{4Q - 4v}{16 + Qv}\right)$ | | |
| $t = \dfrac{1}{2}\tan^{-1}\left(\dfrac{4Q - 4v}{16 + Qv}\right)$ | | |
| c) i)   $m\ddot{x} = mg - mkv$ | | |
| $\ddot{x} = g - kv$ | | |
| for terminal velocity $V$, $\ddot{x} = 0$ | 1 | |
| $\therefore 0 = g - kV$ | | |
| $\therefore V = \dfrac{g}{k}$ | | |

MATHEMATICS Extension 2: Question 4

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| ii) $\ddot{x} = g - kv$ | | |
| from (i) | | |
| $V = \frac{g}{k}$ | | |
| $\therefore g = kV$ | | |
| $\therefore \ddot{x} = kV - kv$ | 1 | |
| $= k(V - v)$ | | |

**MATHEMATICS Extension 2: Question...5...**

1/4

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (i)  | | |
| $m\ddot{x} = mg - \frac{1}{10}mv^2$<br>$\ddot{x} = 10 - \frac{1}{10}v^2$<br>$10\ddot{x} = 100 - v^2$ | 1 | Had to have a diagram clearly indicating positive direction. |
| (ii) $10\frac{dv}{dt} = 100 - v^2$<br>$\frac{dt}{dv} = \frac{10}{100-v^2}$<br>Now $\frac{10}{100-v^2} = \frac{a}{10-v} + \frac{b}{10+v}$.<br>$10 = a(10+v) + b(10-v)$<br>$v=10$ $\quad 20a = 10$<br>$a = \frac{1}{2}$.<br>$v = 0$ $\quad 10 = 10a + 10b$.<br>$10 = 5 + 10b$<br>$\therefore b = \frac{1}{2}$ | | |
| $\therefore \int_0^t dt = \frac{1}{2}\int_0^v \frac{1}{10+v} + \frac{1}{10-v}\,dv$. | | cannot quote this. |

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| OR $\dfrac{10}{100-v^2} = \dfrac{1}{2}\left(\dfrac{10+v+10-v}{(10-v)(10+v)}\right)$ | | If no indication of where $\frac{1}{2}$ came from lost this mark |
| $= \dfrac{1}{2}\left[\dfrac{1}{10-v} + \dfrac{1}{10+v}\right]$ | 1 | |
| $\therefore t = \dfrac{1}{2}\left[\ln(10+v) - \ln(10-v) - \dfrac{1}{2}(\ln 10 - \ln 10)\right]$ | | |
| $= \dfrac{1}{2}\ln\left(\dfrac{10+v}{10-v}\right)$ | | |
| $2t = \ln\left(\dfrac{10+v}{10-v}\right)$ | 1 | |
| $e^{2t} = \dfrac{10+v}{10-v}$ | | |
| $10e^{2t} - ve^{2t} = 10 + v$ | | |
| $v(e^{2t}+1) = 10(e^{2t}-1)$ | | |
| $v = 10\left(\dfrac{e^{2t}-1}{e^{2t}+1}\right)$ | 1 | |

| MATHEMATICS Extension 2: Question........ 3/4 | | |
|---|---|---|
| **Suggested Solutions** | **Marks** | **Marker's Comments** |
| (ii) $10v\frac{dv}{dx} = 100 - v^2$ | | |
| $\frac{dv}{dx} = \frac{100-v^2}{10v}$ | 1 | |
| $\frac{dx}{dv} = \frac{10}{100-v^2}$. | | |
| $\int_0^v \frac{10v}{100-v^2}\,dv = \int_0^x dx$. | | |
| $\therefore \left[-5\ln(100-v^2)\right]_0^v = x$. | 1 | |
| $-5\ln(100-v^2) - 5\ln 100 = x$. | | Should have put this step in but not penalised. |
| $x = -5\ln\left(\frac{100-v^2}{100}\right)$ | | |
| $= -5\ln\left(1 - \frac{v^2}{100}\right)$ | 1 | |
| (iii) when $t = 2\ln 2$ | | |
| $v = 10\left(\frac{e^{4\ln 2} - 1}{e^{4\ln 2} + 1}\right)$ | | students calculated $e^{2\ln 2}$ losing the 2. |
| $= 10\left(\frac{16-1}{16+1}\right)$ | | |
| $= 150/17$ m/s | 1 | |

**MATHEMATICS Extension 2: Question……..** 4/4

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| $x = -5\ln\left(1-\left(\frac{150}{17}\right)^2\right)$ | | |
| $= -5\ln\frac{64}{289}$ | | |
| or $10\ln\frac{17}{8}$ m. | | |
| (iv) | 1 | |
| $\downarrow x \quad \downarrow mg \qquad m\ddot{x} = mg$ | | |
| $\ddot{x} = 10$ | | |
| $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 10$ | 1 | |
| $\frac{1}{2}v^2 = 10x + C_3$ | | Students had incorrect limits |
| when $x = 10\ln\frac{17}{8}$ $v = \frac{150}{17}$ | | |
| $C_3 = -36.4498$ | | Those who used t instead of x were unsuccessful |
| $v^2 = 20x - 72.8996$ | 1 | |
| $\therefore$ when $x = 50$ | | |
| $v^2 = 20\times 50 - 72.8996$ | | |
| $v \doteqdot 30$ m/s | | |
| OR $10\int_{10\ln\frac{17}{8}}^{50} x\,dx = \int_{\frac{150}{17}}^{v} v.$ | 1 | |
| $\frac{v^2}{2} - \frac{\left(\frac{150}{17}\right)^2}{2} = 10\left(50 + 10\ln\frac{17}{8}\right)$ | | |
| $= 30$ m/s. | | |