

Name: _____



THE SCOTS COLLEGE

August 2004

Year 12 – General Mathematics Assessment

Trial-HSC Examination

General Instructions

- ☐ Working time – $2\frac{1}{2}$ hours. (Plus 5 minutes reading time)
- ☐ Write using a black or blue pen.
- ☐ Use Multiple Choice Answer Sheet provided.
- ☐ Use Scatter plot sheet provided for question 25 (a)
- ☐ Board approved calculators may be used.
- ☐ All necessary working should be shown for every question.
- ☐ A separate Formula Sheet is provided.

Total marks – 100 marks

Section I

Total marks (22)

Attempt Questions 1-22

Allow about 30 minutes for this section

Section II

Total marks (78)

Attempt questions 23-28

Allow about 2 hours for this section

**STUDENTS ARE ADVISED THAT THIS IS A TRIAL EXAMINATION ONLY
AND CANNOT IN ANY WAY GUARANTEE THE CONTENT OR THE
FORMAT OF THE HIGHER SCHOOL CERTIFICATE EXAMINATION**

Section I
Total Marks (22)
Attempt questions 1-22

Allow about 30 minutes for this section

Select the alternative A, B, C, or D that best answers the question and indicate your choice with a cross (X) in the appropriate space on the **Section I answer sheet attached**.

1. If a car travels a distance of 70 km using 8 litres of petrol. What is its fuel consumption in litres per 100 km correct to two decimal places?

A) 11.43 litres B) 0.11 litres C) 9.54 litres D) 10.43 litres

2. Calculate is the sample standard deviation of the following numbers, leaving your answer correct to two decimal places.

107 118 132 169 146 126 181

A) 28.44 B) 29.06 C) 27.05 D) 27.65

3. The volume of a rectangular pyramid with base dimensions 14 cm by 9 cm and height 6 cm is:

A) 765 cm³ B) 567 cm³ C) ~~756 cm³~~
 252 D) 576 cm³

4. Given that $v = ut + \frac{1}{2}at^2$, then $u =$

A) $\frac{2v(t + t^2)}{a}$ B) $\frac{v - \frac{1}{2}at^2}{t}$ C) $\frac{v - at^2}{2t}$ D) $\frac{v - t}{2at^2}$

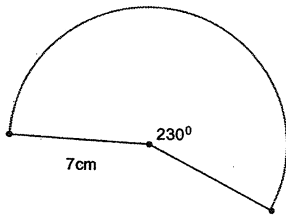
5. When completely simplified the expression, $5p(4q - 7) - 3q(5p + 2)$ is

- A) $5pq - 35q - 6p$
- B) $35pq - 35p + 6q$
- C) $5pq - 41p$
- D) $5pq - 35p - 6q$

6. For the straight line equation $2y = -8 + 6x$, the gradient and y -intercept are:

- A) $m = 6$, $b = -8$
- B) $m = 2$, $b = 6$
- C) $m = 3$, $b = -4$
- D) $m = -6$, $b = +4$

7. This is a sketch of a sector of a circle. Calculate the area of the sector correct to two decimal places.



- A) 89.35 cm^2
- B) 98.35 cm^2
- C) 102.46 cm^2
- D) 120.64 cm^2

8. The tax payable on taxable incomes in the range \$20001 to \$50001 is given by :

\$20001--\$50000	\$2380 plus 30 cents for each dollar over \$20000.
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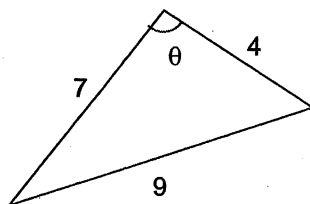
How much tax does John pay if his taxable income is \$34408?

- A) \$2410.00 B) \$4332.40 C) \$6702.40 D) \$10372.40

9. A cone has a volume of 260 cm^3 and a radius of 15 cm. Calculate its height correct to one decimal place.

A) 1.1 cm B) 1.3 cm C) 1.2 cm^3 D) 3.2 cm

10. Which of the following expressions represent the value of $\cos \theta$ in the triangle? All measurements in cm.



A) $\frac{9^2 + 7^2 - 4^2}{2 \times 9 \times 7}$

B) $\frac{9^2 + 4^2 - 7^2}{2 \times 9 \times 4}$

C) $\frac{4^2 + 7^2 - 9^2}{2 \times 7 \times 9}$

D) $\frac{4^2 + 7^2 - 9^2}{2 \times 4 \times 7}$

11. Calculate the area of the triangle above (in Q10) given that the value of θ is $106^\circ 36'$. Leave your answer to the nearest square centimetre.

A) 13 cm^2 B) 21 cm^2 C) 33 cm^2 D) 31 cm^2

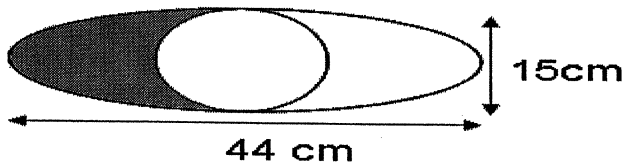
12. Five months after joining an aerobics class, Andrew has reduced his weight by 18% to 55.4 kg. What was his original weight?

A) 70.02 kg B) 69.56 kg C) 71.02 kg D) 67.56 kg

13. For a loan of \$40,000 the repayments over 5 years total \$70,865. Calculate the annual flat interest rate for this loan.

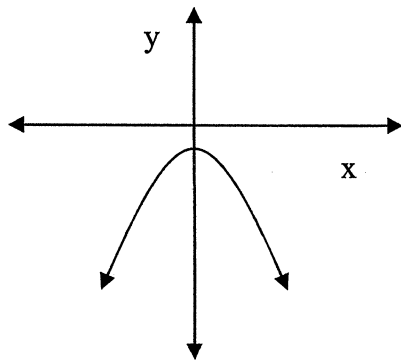
A) 15% B) 77% C) 65% D) 33%

14. Below is a figure of an ellipse with a circle inside. If both have the same centre, then the area of the shaded region is;



- A) 259.18 cm^2 B) 341.65 cm^2 C) 170.82 cm^2 D) 176.71 cm^2
15. If the length of a basketball court is 25 m to the nearest metre. Find the percentage error of this measurement?
- A) 2% B) 0.002% C) 20% D) 0.2%
16. Evaluate $\sqrt[3]{2.9 \times 10^{-7}}$ and express your answer in scientific notation correct to 3 significant figures.
- A) 3.07×10^2 B) 2.90×10^{-2} C) 1.426×10^2 D) 6.62×10^{-03}
17. In a certain test six students scored :
- 63 38 94 73 59 64**
- After further revision the test was repeated and the students' marks all increased by 5. When comparing the second set of results with the first :
- A) The mean had increased by 5 and the standard deviation remained the same.
 B) The mean had decreased by 5 and the standard deviation remained the same.
 C) The mean had increased by 5 and the standard deviation increased by 5.
 D) The mean had decreased by 5 and the standard deviation increased by 5.

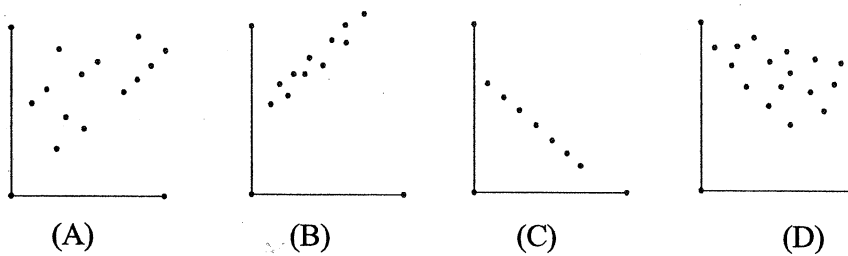
18. What could be the equation of this graph in the figure below?



- A) $y = -2x^2 + 2$
 B) $y = -x^2 - 2$
 C) $y = -2x^2$
 D) $y = x^2 - 2$
19. Which variation equation shows y being proportional to the square of x ?

A) $y = \frac{k}{x^2}$ B) $y = k\sqrt{x}$ C) $y = \frac{k^2}{x}$ D) $y = kx^2$

20. Which of the following scatter plots best represents a correlation of -1 ?



21. Calculate the angular distance between the points A(31° N, 108° E) and B(20° S, 108° E).
- A) 20° B) 11° C) 51° D) 108°

22. The marks on a Math test are normally distributed with a mean of 88% and a standard deviation of 1.5. What is the minimum percentage required to be in the top 2.5% of the test.

A) 92.5% B) 89.5% C) 91% D) 96%

End of Section I

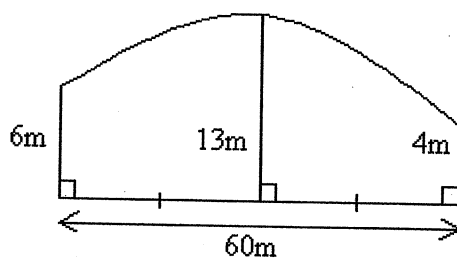
Section II
Total Marks (78)

Attempt questions 23 – 28
Allow about 2 hours for this section.

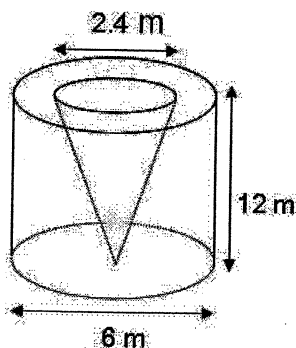
Answer the questions on the booklets provided, beginning each new question on a new booklet.

Question 23 (13 marks) Start a new booklet

- a) The diagram below represents a top view of a local dam.



- i) Using Simpson's Rule calculate the surface area of the dam, leaving your answer to the nearest square metre. [2]
 - ii) If the average depth of the dam is 3.8 cm, calculate its approximate volume using your answer from (i), to the nearest cubic metre. [2]
- b) i) The solid below has been formed from a cylinder with a cone section removed. Find the volume of the solid leaving your answer to the nearest cubic metre. [5]



Question 23 continued

- c) This table shows part of Eric's calculations using his Home Loan Repayment Table.

Home Loan Repayment Table			
Amount borrowed c(i) Interest rate = 10%p.a. Monthly repayment = \$1300			
Month	Principal (P)	Interest(I)	Amount Owing
1	\$120000	\$1000	\$119700
2	A	B	C
3			

- i) How much did Eric borrow? [1]
- ii) What value should Eric write in the section of the table labelled A? [1]
- iii) Calculate the amount of interest that Eric should insert in the section of the table labelled B. [1]
- iv) How much should Eric insert in the Amount Owing section of the table labelled C? [1]

Question 24 (13 marks) Start a new booklet

- a) George wants to have \$28,000 in 5 years time for a home loan deposit. How much would he need to deposit monthly into an annuity that pays 4.8% pa interest? (Leave your answer in whole dollars.) [3]
- b) Joseph requires \$ 520 per month for 3 years while he studies at the University. His parents wish to provide the finance. What amount of money do they need to invest **now** at 5.5 % pa compounding monthly to provide Joseph the money he needs? [3]

Question 24 continued

- c) James purchased a car for \$38,000 and it depreciates by 2,100 per year.
- i) Using the straight-line method of depreciation, calculate the value of the car after 6 years. [2]
 - ii) After how many years will the car be worth half of its original purchase price? [2]
 - iii) The same car was valued at \$30,000 after 3 years. Using the declining-balance method calculate the rate of depreciation, as a percentage, to one decimal place. [2]
 - iv) What is the difference between the straight-line method and declining balance method of depreciation? [1]

Question 25 (13 marks) Start a new booklet

- a) This question requires you to use the data plotted on the **scatter plot** attached to this exam.
- i) Calculate the three medians used for finding the regression line and draw the median regression line, showing your working in the answer booklet. [3]
 - ii) Write down the value of the vertical intercept of your regression line. [1]

Question 25 (continued)

- b) Results (marks) from two different college Mathematics departments have been collected and recorded in a back-to-back stem and leaf plot shown below.

Maths marks										
College A						College B				
				3	1	6				
			8	6	4	2	1	2	5	
		6	5	4	2	3	3	4	5	6
	9	9	7	7	6	4	2	3	3	6
8	8	8	6	6	4	5	3	7	8	8
			4	3	2	6	4	6	6	
						7	5	5		
						8	2			
						9				

The five number summary for the sample from **College A** is shown in the table below:

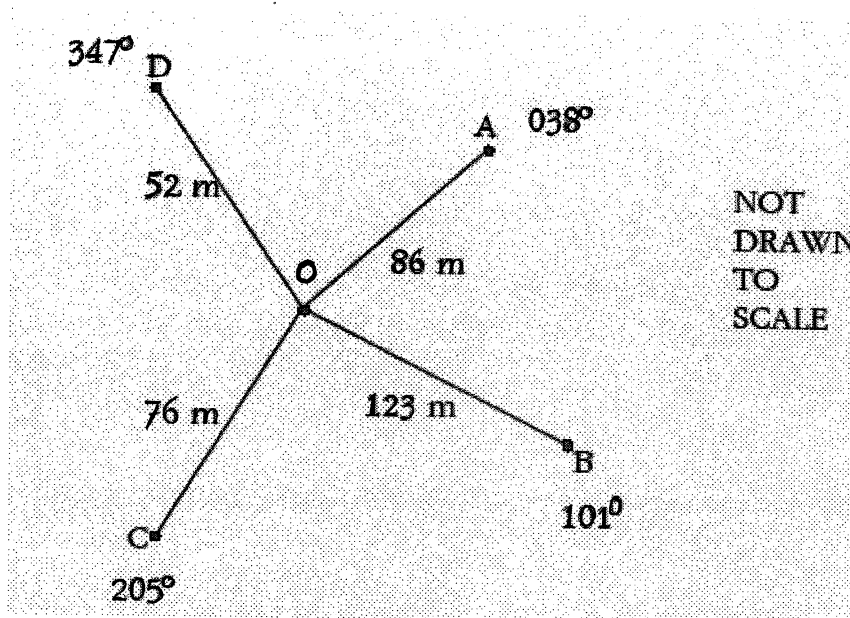
Sample	College A	College B
Minimum mark	23	
Lower Quartile	44	
Median	58	
Upper Quartile	68	
Maximum mark	74	

- i) Using the data in the stem-and-leaf plot, write down, **in your booklet**, the five number summaries for **College B**. (Complete the table above). [5]
- ii) Calculate the mean and sample standard deviation for **College B**. [2]
- iii) Compare and contrast the spread of scores for each college. [2]

Question 26 (13 marks) Start a new booklet

- a) Answer the following questions using the radial survey diagram below.

Diagram:



- i) Write down the size of $\angle DOA$. [1]
 - ii) Find the area of triangle AOB. [2]
 - iii) Calculate the distance AB correct to two decimal places. [2]
- b) The city of Beijing is located at $(40^\circ\text{N}, 116^\circ\text{E})$. Perth is located at $(32^\circ\text{S}, 116^\circ\text{E})$.
- i) What is the difference in Latitude between these two cities? [1]
 - ii) Calculate the distance between these two cities in nautical miles. [2]
 - iii) If a plane left Beijing for Perth at 10:30 pm on Monday (Beijing Time) and the flying time from Beijing to Perth is eleven hours. When the plane lands in Perth, what is the local time and day? [2]

Question 26 continued

- c) In the diagram below, find the value of d , leaving your answer correct to two decimal places.

[3]

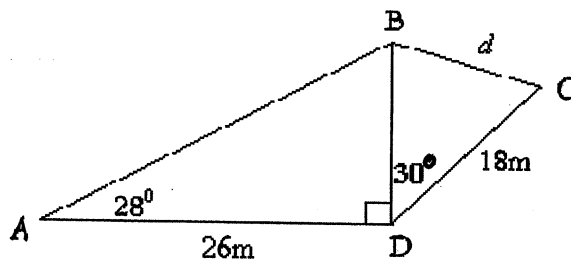


Diagram NOT to scale

Question 27 (13 marks) Start a new booklet

- a) In a bag there are 12 coloured discs, 8 red and 4 blue. At random Chris selected a disc from the bag. He then selected a second disc at random from the bag, without putting the first disc back in the bag.

i) What is the probability that the first disc he selected was red?

[1]

ii) Complete a tree diagram by writing the appropriate probabilities.

[3]

- b) A student answers question Q7 and question Q9 in a class test. His chance of getting Q7 correct is 0.7 and chance of getting Q9 correct is 0.65. Find the probability that he gets only one of them correct.

[2]

- c) i) Solve the equation $\frac{4x}{2x-8} = \frac{4}{6}$

[2]

ii) Solve the equation $1.05^x = 2$, using the estimation and refinement method to find the value of x .

[2]

Question 27 continued

- d) The number of men (m) it takes to dig a ditch with a diameter 10m and height 6m varies inversely with the time taken (t). One of the ditches was completed by 5 men in 8 hours.

ii) Write an equation to represent this relationship.

[1]

ii) A similar ditch has 2 men working on it, how long will it take to completely dig it out?

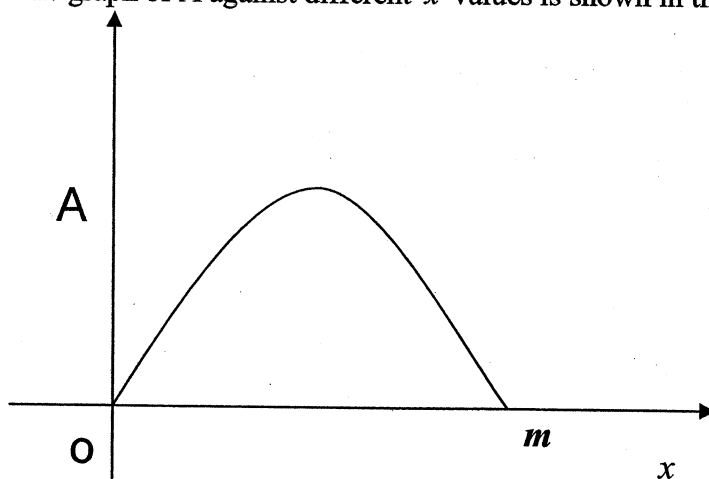
[2]

Question 28 (13 marks) Start in a new booklet

- a) The area (A) of rectangular farmyard of length x metres is given by the formula below :

$$A = 50x(40 - x) \text{ where } A \text{ is in square metres}$$

The graph of A against different x values is shown in the diagram below:



- (i) Write down the value of m on the graph. [1]
- (ii) What should the farmyard length be in order to have maximum area? [1]
- (iii) Determine the maximum area of the farmyard. [1]
- (iv) Explain, without any calculation, why $x = 25$ will give the same answer as $x = 15$. [1]

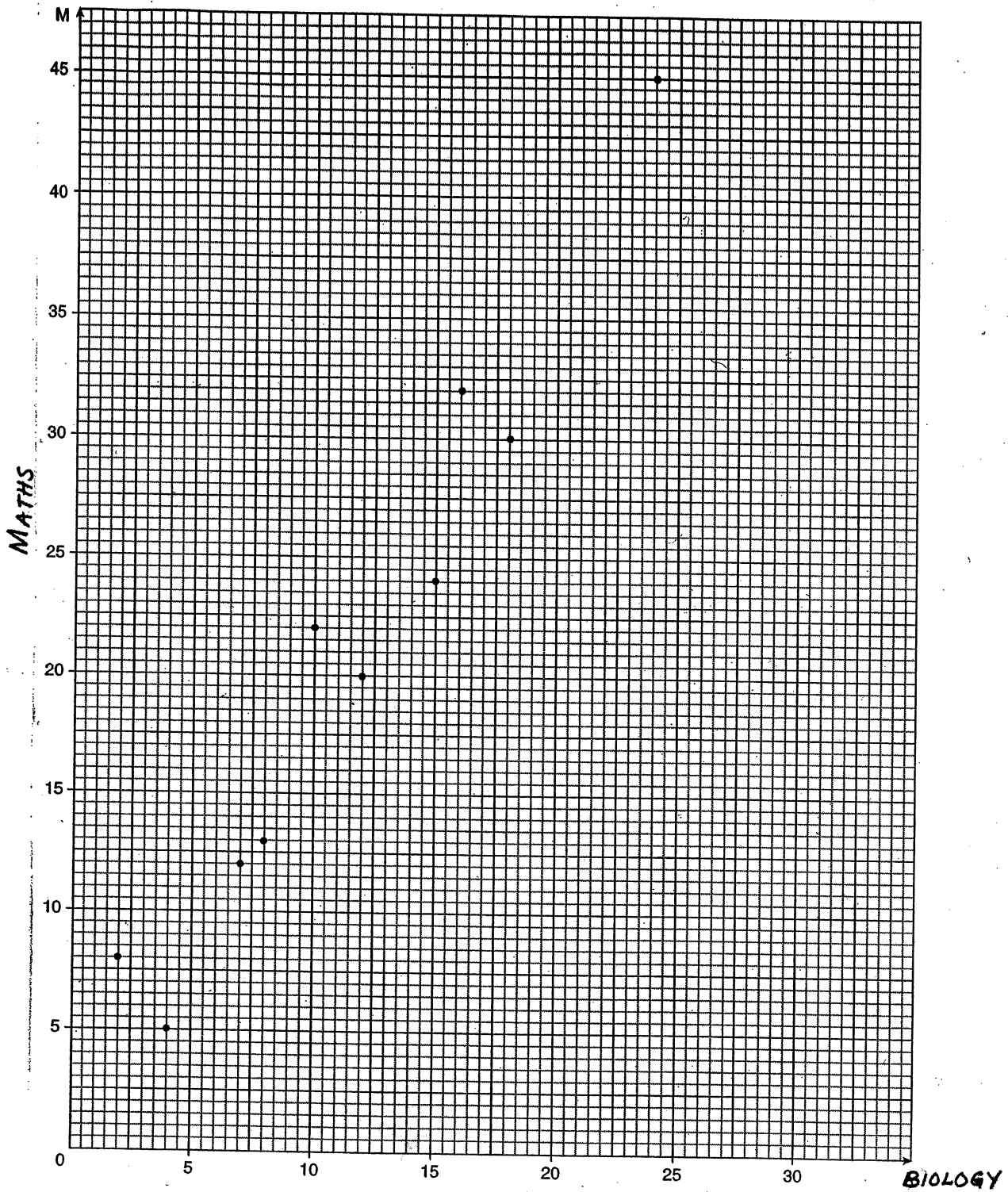
Question 28 continued

- b) An overseas production plant manufactures microprocessors for desktop computers the plant claims that any large shipment of microprocessors will have a life that normally distributed with the microprocessors having a mean life of 20500 hours and a standard deviation of 425 hours. The Australian defence ascendency orders 100,000 microprocessors.
- (i) Explain what a Z-score of -1 would represent in relation to the life expectancy of the microprocessors in the shipment. [2]
- (ii) Calculate the approximate number of microprocessors in the shipment that should have a life expectancy greater than 21775 hours? [2]
- c) Solve these equations for n :
- (i) $\sqrt{4n-3} = 9$ [3]
- (ii) $3n^3 = 192$ [2]

END OF SECTION II

Scatter plot for question 25(a)

STUDENT NO.:



Solution

SECTION 1 - MULTIPLE CHOICE ANSWER SHEET

Total Marks (22)

Attempt Questions 1 - 22

Allow about 30 minutes for this part

original. Sol.

			Student Number	
1	A ☒	B ☐	C ☐	D ☐
2	A ☐	B ☐	C ☒	D ☐
3	A ☒	B ☐	C ☐	D ☐
4	A ☐	B ☒	C ☐	D ☐
5	A ☐	B ☐	C ☐	D ☒
6	A ☐	B ☐	C ☒	D ☐
7	A ☐	B ☒	C ☐	D ☐
8	A ☐	B ☐	C ☒	D ☐
9	A ☒	B ☐	C ☐	D ☐
10	A ☐	B ☐	C ☐	D ☒
11	A ☒	B ☐	C ☐	D ☐
12	A ☐	B ☐	C ☐	D ☒
13	A ☒	B ☐	C ☐	D ☐
14	A ☐	B ☐	C ☒	D ☐
15	A ☒	B ☐	C ☐	D ☐
16	A ☐	B ☐	C ☐	D ☒
17	A ☒	B ☐	C ☐	D ☐
18	A ☐	B ☒	C ☐	D ☐
19	A ☐	B ☐	C ☐	D ☒
20	A ☐	B ☐	C ☒	D ☐
21	A ☐	B ☐	C ☒	D ☐
22	A ☒	B ☐	C ☐	D ☐

+1 for all

Q. 23.

(a) (i) $A \approx \frac{h}{3} (d_f + 4d_m + d_1)$

$h = \frac{60}{2}$
 $= 30$

$A \approx \frac{30}{3} (6 + 4 \times 13 + 4) \quad \frac{1}{2}$

$\approx 10 (6 + 52 + 4) \quad \frac{1}{2}$

$\approx 10 (62)$

$\approx \underline{620} \text{ m}^2 \quad \frac{1}{2} \frac{1}{2}$

(2)

(ii)

$V = 620 \text{ m}^2 \times 0.038 \text{ m} \quad \leftarrow 620 \times 0.038$

$= 2356 \text{ m}^3 \quad \leftarrow \frac{1}{2} \frac{1}{2}$

$\approx \underline{24} \text{ m}^3$

(2)

(b)

$V_{\text{cyl}} = \pi r^2 h$

$= \pi \underline{3}^2 \times 12 \quad \checkmark$

$= 339.29 \text{ m}^3 \quad \checkmark$

$V_{\text{cone}} = \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \pi \underline{1.2}^2 \times 12 \quad \checkmark$

$= \frac{54.2867 \text{ m}^3}{3} \quad \checkmark = 18.095 \text{ m}^3$

$V_{\text{cyl}} - V_{\text{cone}} = \underline{\underline{321.195}} \text{ m}^3$
 $\approx \underline{\underline{321.20}} \text{ m}^3 \quad \checkmark \checkmark$

(5)

(c)

(i) $\$ 120000 \quad \checkmark$

(ii) $\$ 119700 \quad \checkmark$

(iii) $\$ \frac{11970}{12} = 997.5 \quad \checkmark$

(iv) $\$ 119397.5 \quad \checkmark$

Solution

Q. 24.

(a)

$$A = \frac{M [(1+r)^n - 1]}{r}$$

$$28000 = \frac{M [(1 + \frac{0.048}{12})^{12 \times 5} - 1]}{\frac{0.048}{12}}$$

$$28000 = M \frac{[1.004^{60} - 1]}{0.004}$$

$$\Rightarrow M = \frac{28000 \times 0.004}{[1.004^{60} - 1]}$$

$$= 413.83$$

$$\approx \text{₹} 414$$

(b)

$$N = \frac{M [(1+r)^n - 1]}{r(1+r)^n}$$

$$= \frac{520 [(1 + \frac{0.055}{12})^{36} - 1]}{\frac{0.055}{12} (1 + \frac{0.055}{12})^{36}}$$

$$= \frac{520 [1.17895 - 1]}{0.00458 (1.17895)}$$

$$= \text{₹} 17220.858$$

Solution

Q.24 continued

(c) (i) $S = V_0 - Dn$

$$= 38,000 - 2,100 \times 6 \quad \checkmark$$

$$= \$25400 \quad \checkmark$$

2

(ii) $\frac{38000}{2} = 38000 - 2100 \times n \quad \frac{1}{2}$

$$19000 = 38000 - 2100n$$

$$2100n = 38000 - 19000 \quad \frac{1}{2}$$

$$n = \frac{19000}{2100} \quad \frac{1}{2}$$

$$= 9.0476$$

$$\approx 9 \text{ years} \quad \frac{1}{2}$$

2

(iii) $S = V_0 (1-r)^n$

$$30000 = 38000 (1-r)^3 \quad \frac{1}{2}$$

$$\Rightarrow (1-r)^3 = \frac{30000}{38000} \quad \frac{1}{2}$$

$$(1-r)^3 = 0.7895$$

$$1-r = \sqrt[3]{0.7895}$$

$$1-r = 0.92423$$

$$r = 1 - 0.92423$$

$$= 0.07577 \quad \frac{1}{2}$$

$$\approx 7.58 \% \quad \frac{1}{2}$$

2

(iv) In the straight line method the salvage value will be zero in about 18 years. but in the declining-balance method it will take much much longer.

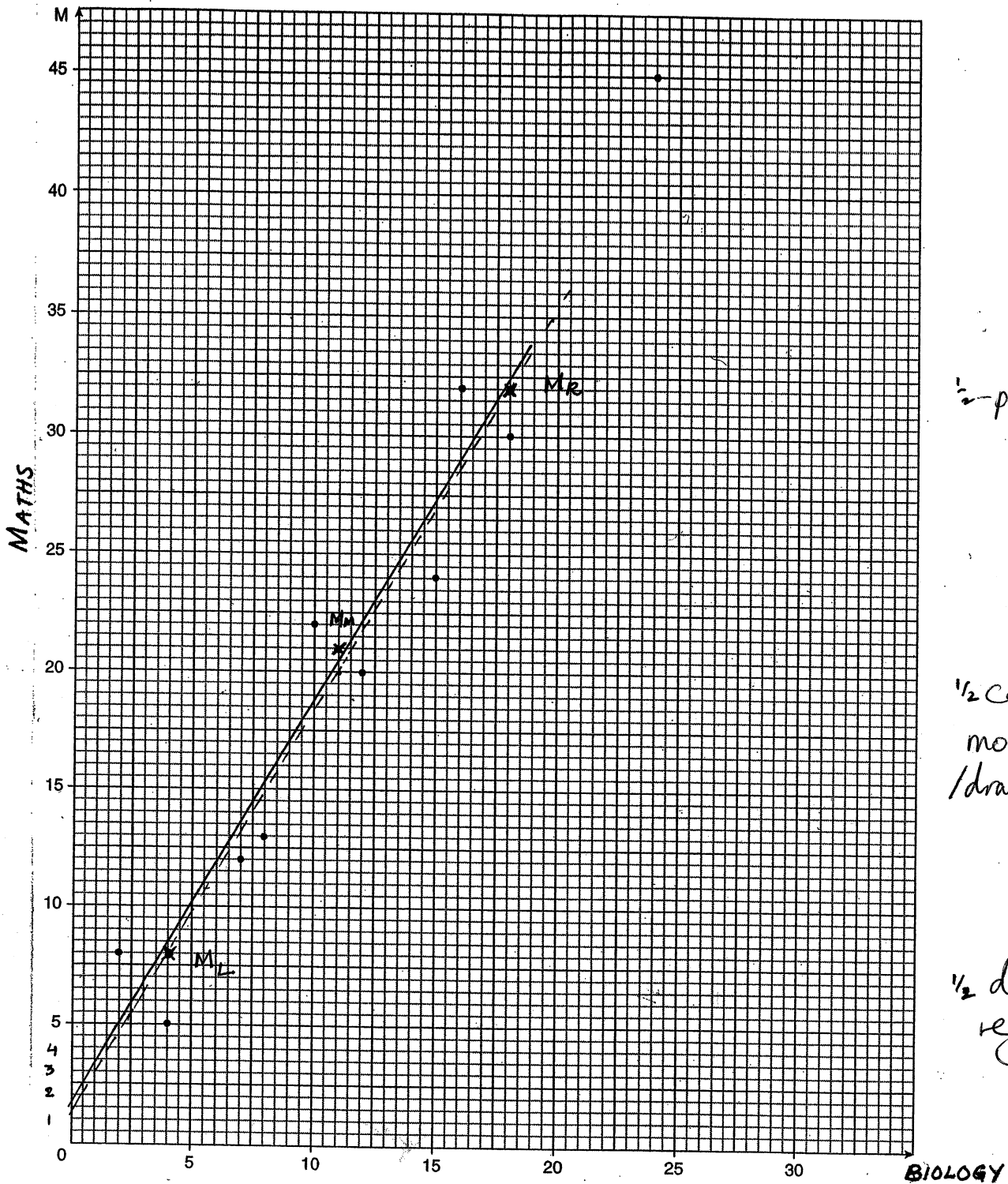
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25(a)

Scatter plot for question 25(a)

Solution

STUDENT NO:

 $\frac{1}{2}$ plotted $\frac{1}{2}$ calculated
movement
/ drawn dotted
line $\frac{1}{2}$ drawn
regression
line

$$\begin{array}{c|c|c}
 (i) \quad (2,8), (4,5), (7,12) & (8,13), (10,22), (12,20), (15,24) & (16,32), (18,30), (24,45) \\
 M_L (4,8) & M_M (11,21) & M_R (18,32) \\
 \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
 \end{array}$$

3

(ii) y-intercept $\rightarrow$ accept between $1.5 \leftrightarrow 2.5$ ✓

1

Solution

Q. 25 cont.

	College B.	
(b) (i) min mark	16	✓
Lower Q	34	✓
Median	43	✓
Upper Q	64	✓
Max mark	82	✓

5

(ii) $\bar{x}$ = mean = 46.8261 ✓
 S_x = std. dev. = 18.5807 ✓

$(\sigma_x = 18.1723) \frac{1}{2}$

2

(iii) compare IQR

College A IQR_A = ~~66~~ - 45 = 21 ✓
 College B IQR_B = ~~64~~ - 34 = 30 ✓

College B has higher spread ✓

or SD's, college B will be higher

2

$\frac{1}{2}$: Range + compared

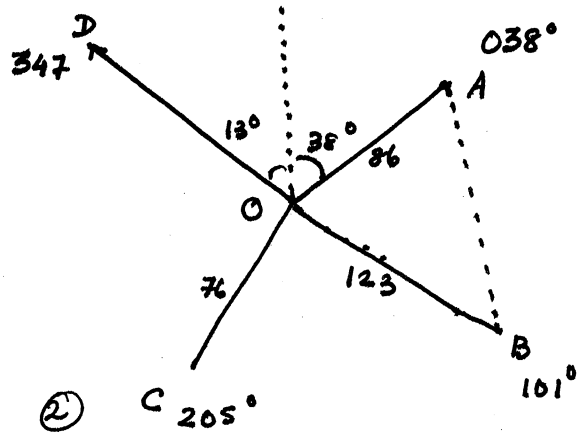
$\frac{1}{2}$: worked out range

Solution

Q.26

(a)

- (i) $\angle DOA = 51^\circ$ ✓
 $\frac{1}{2}$ no units: 51 ①
- (ii) $A = \frac{1}{2} ab \sin C$
 $= \frac{1}{2} \times 86 \times 123 \times \sin 63^\circ$ ✓
 $= 4712.53 \text{ m}^2$ ✓
 ① if 0 wrong



(iii) Let $\overline{AB} = c$.

$$\therefore c^2 = a^2 + b^2 - 2ab \cos C$$

$$c^2 = 123^2 + 86^2 - 2 \times 123 \times 86 \times \cos 63^\circ$$
 ✓

$$c^2 = 12920.3769$$

$$c = 113.6678$$

$$c \approx 113.67 \text{ m}$$

① if C wrong

$\frac{1}{2}$ $\frac{1}{2}$ if round too early ②

(b)

(i) $40^\circ + 32^\circ = 72^\circ$ ✓ $\frac{1}{2}$

working

①

(ii) $72^\circ \times 60 \text{ nautical miles}$ ✓

$= 4320 \text{ nautical miles (M)}$ ✓

OR $l = \frac{\theta}{360} \times 2\pi r$ where $r = 6400 \text{ km}$
 $= 8042 \text{ km} = 4343 \text{ M}$

$1^\circ = 60 \text{ nautical miles}$

$1 \text{ nm} = 1.852 \text{ km}$ ②

(iii) B and P time are same $\frac{1}{2}$

$\therefore$ The plane landed at Perth

at 10:30 pm + 11 hours $\frac{1}{2}$

is. 9:30 am on Tuesday. $\frac{1}{2}$ $\frac{1}{2}$

②

Solution

Q. 26 continued

(c) $\triangle ABD$

$$\tan 28^\circ = \frac{BD}{AD} \quad \frac{1}{2}$$
$$BD = AD \cdot \tan 28^\circ \quad \frac{1}{2}$$
$$= 26 \times \tan 28^\circ$$
$$= 13.82 \text{ m} \quad \checkmark$$

now $\triangle BCD$

$$BC^2 = BD^2 + CD^2 - 2 \times BD \times CD \times \cos 30^\circ \quad \frac{1}{2}$$

$$d^2 = 13.82^2 + 18^2 - 2 \times 13.82 \times 18 \times \cos 30^\circ$$

$$d^2 = 84.127 \quad d^2 = 84.112$$

$$d = 9.172 \quad d = 9.17 \text{ m} \quad \frac{1}{2}$$
$$\approx 9.17 \text{ m}$$

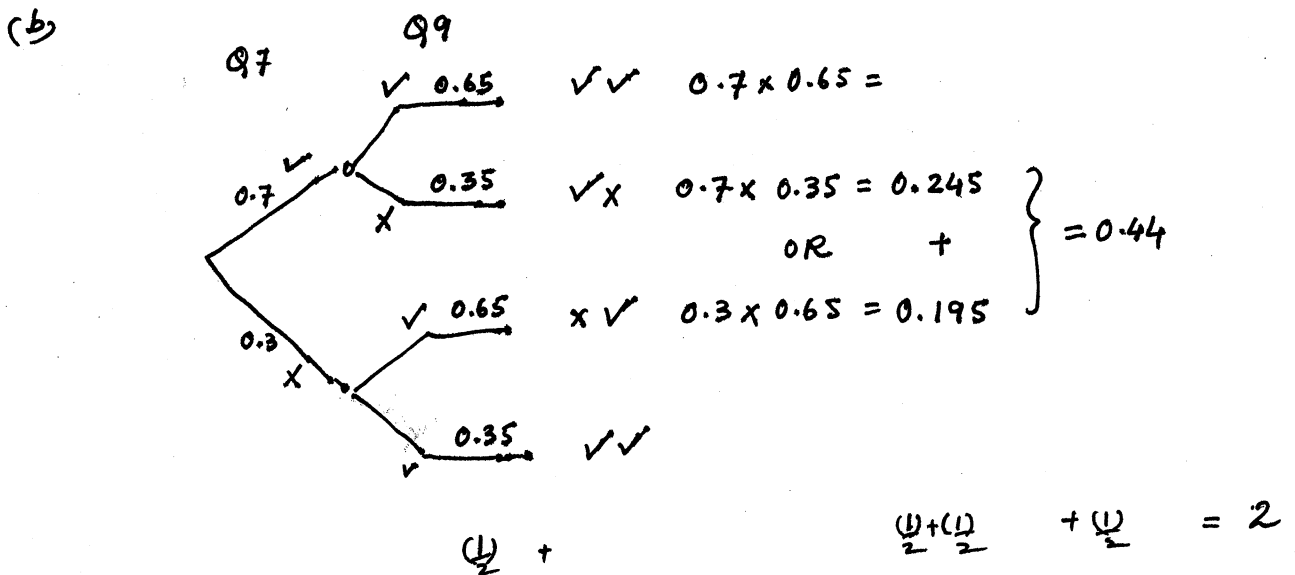
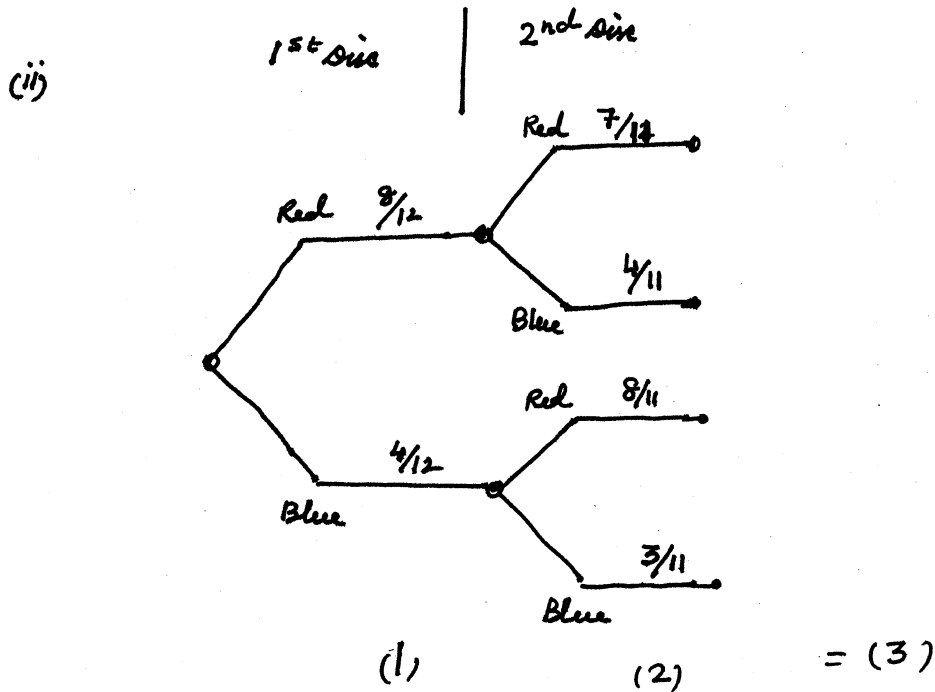
$-\frac{1}{2}$ rounded too soon

③

Solution

Q. 27

(a) (i) $\frac{8}{12}$ or $0.\overline{66}$ ✓



(c) (i) $24x = 8x - 32$ $\frac{1}{2}$
 $\Rightarrow 24x - 8x = -32$
 $\Rightarrow 16x = -32$ $\frac{1}{2}$
 $x = -2$ $(-)\frac{1}{2}, \frac{1}{2}$

(ii) $1.05^x = 2$

$\text{Ans} = 4.2059$

Accept ans x between
 14 & 14.5 ✓
 (2)

Solution

Question 27 continued

$$(d) (i) \quad m \propto \frac{1}{t} \quad \frac{1}{2}$$

$$m = \frac{k}{t}$$

$$5 = \frac{k}{8}$$

$$k = 40$$

$$\therefore m = \frac{40}{t} \quad \frac{1}{2}$$

(1)

$$(ii) \quad t = \frac{40}{m}$$

$$t = \frac{40}{2}$$

$$t = 20 \quad \text{hours} \quad \checkmark$$

(2)

Q. 28.

(a) $A = 50x(40-x)$

$A=0 \Rightarrow 50x=0$ OR $40-x=0$
 $\Rightarrow x=40.$

(i) $m = 40.$

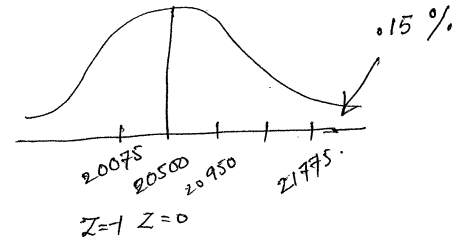
(ii) Midway $x=20$

(iii) $A_{\max} = 50 \times 20 (40-20)$
 $= 1000 (20)$
 $= 20000 \text{ meters}^2$

(iv) The curve is symmetric about 20 and 15 and 25 are equidistant from 20.

(b)

(i) A Z score of -1 represents a life expectancy of 20075 hours



(ii) Note 21975 is 3 standard deviations above the mean

$\frac{100 - 99.7}{2} \% = 0.15\%$

$\therefore 0.15\% \text{ of } 100000 \approx 150 \text{ microprocessors}$

(c)

$\sqrt{4n-3} = 9$
 $4n-3 = 9^2$
 $4n = 81+3$
 $n = \frac{84}{4}$
 $n = 21$ ✓

(iii) $3n^3 = 192$
 $n^3 = \frac{192}{3}$
 $n^3 = 64$
 $n = \sqrt[3]{64}$
 $n = 4$