

--	--	--	--	--	--	--	--

CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL



2015 Trial Examination

FORM VI

GENERAL MATHEMATICS

Friday 31st July 2015

General Instructions

- Reading time — 5 minutes
- Writing time — $2\frac{1}{2}$ hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.

Total — 100 Marks

- All questions may be attempted.

Section I – 25 Marks

- Questions 1–25 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

Section II – 75 Marks

- Questions 26–30 are of equal value.
- All necessary working should be shown.
- Write all solutions on this paper in the spaces provided.

Collection

- Write your candidate number on this paper and on your multiple choice answer sheet.
- Place everything inside the question paper.

Checklist

- Multiple choice answer sheet
- Formulae sheet
- Candidature — 15 boys

Examiner

DWH

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

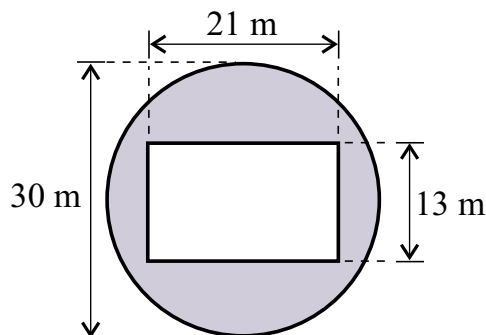
QUESTION ONE

A bank charges simple interest at a rate of 0.05226% per day on the amount owing on a credit card. What is the interest charged for 15 days on an amount of \$1200?

- (A) \$3.13
- (B) \$9.41
- (C) \$9.44
- (D) \$94.07

QUESTION TWO

A circular grass field has a diameter of 30 metres. There is a concrete rectangle in the middle with a length of 21 metres and a width of 13 metres, as shown in the diagram below.



Which of the following would correctly calculate the area of grass, in square metres, as shaded in the diagram?

- (A) $\text{Area} = 2 \times \pi \times \left(\frac{30}{2}\right) - 21 \times 13$
- (B) $\text{Area} = 2 \times \pi \times 30 - 21 \times 13$
- (C) $\text{Area} = \pi \times \left(\frac{30}{2}\right)^2 - 21 \times 13$
- (D) $\text{Area} = \pi \times 30^2 - 21 \times 13$

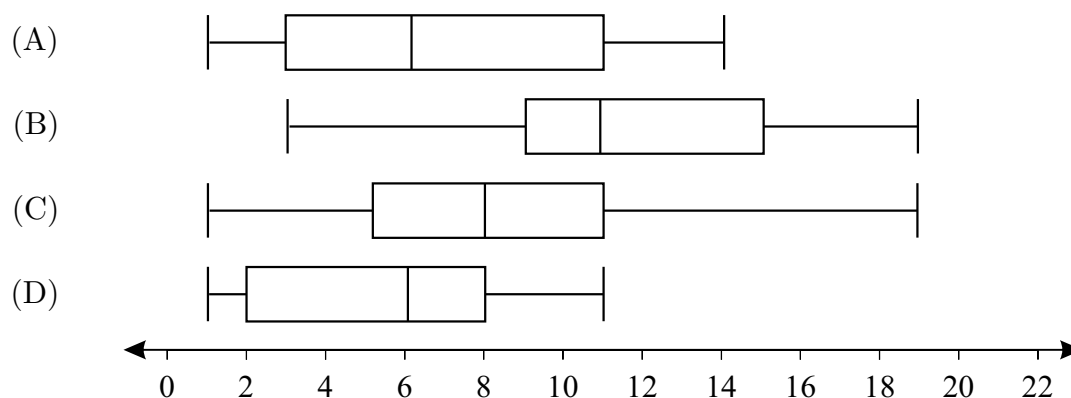
QUESTION THREE

Expand and simplify $2x(x^2 - 2) - 3x$.

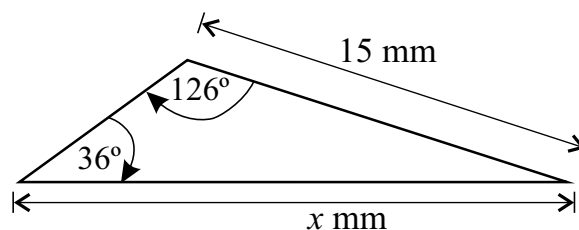
- (A) $2x^3 - 7x$
- (B) $2x^3 - x$
- (C) $2x^3 + x$
- (D) $2x^3 - 3x - 2$

QUESTION FOUR

Which of the data sets displayed in the following box-and-whisker plots has the largest interquartile range?



QUESTION FIVE



Which of the following would correctly calculate x in the above triangle?

- (A) $x = \frac{\sin 36^\circ}{15 \times \sin 126^\circ}$
- (B) $x = \frac{\sin 126^\circ}{15 \times \sin 36^\circ}$
- (C) $x = \frac{15 \times \sin 36^\circ}{\sin 126^\circ}$
- (D) $x = \frac{15 \times \sin 126^\circ}{\sin 36^\circ}$

QUESTION SIX

There will be 28 people competing in a long jump event. How many different outcomes are possible for first and second place? Assume that it is impossible to have a tied result.

- (A) $\frac{28 \times 27}{2 \times 1}$
- (B) $\frac{28^2}{2^2}$
- (C) 28^2
- (D) 28×27

QUESTION SEVEN

For the following question you should refer to the Present Value and Future Value tables attached to your formula sheet.

What is the equivalent present value of an annuity where \$1 is invested at the end of the month for 60 months at a constant interest rate of 6% per annum, compounded monthly?

- (A) \$4.21
- (B) \$51.73
- (C) \$69.77
- (D) \$533.13

QUESTION EIGHT

The data for a large population is normally distributed. Which of the following statements is correct?

- (A) The mean, the mode and the median are all approximately equal.
- (B) The mean and the mode are approximately equal but the median could be quite different.
- (C) The mean and the median are approximately equal but the mode could be quite different.
- (D) The mean, the mode and the median could all be quite different.

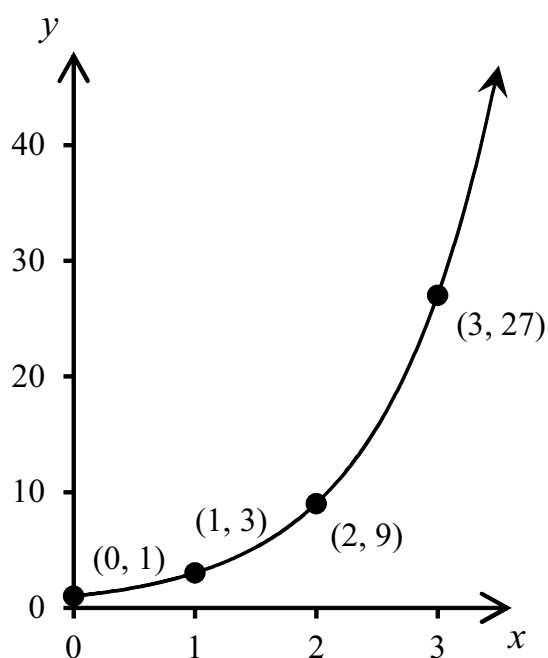
QUESTION NINE

Which statement about the linear equation $y = 9.2x + 3.8$ is true?

- (A) The gradient is 3.8 and the x -intercept is 9.2.
- (B) The gradient is 9.2 and the x -intercept is 3.8.
- (C) The gradient is 3.8 and the y -intercept is 9.2.
- (D) The gradient is 9.2 and the y -intercept is 3.8.

QUESTION TEN

The following is the graph of a relationship between two quantities x and y .



What type of function would accurately model this data?

- (A) hyperbolic
- (B) exponential
- (C) quadratic
- (D) cubic

QUESTION ELEVEN

The coordinates of our classroom are ($33^{\circ}52'30''\text{S}$, $151^{\circ}12'46''\text{E}$). The coordinates of Brackenfell Primary School in Capetown South Africa are ($33^{\circ}52'30''\text{S}$, $18^{\circ}41'05''\text{E}$). Which of the following would correctly calculate the precise time difference between the two places?

- (A) Time difference = $\frac{151^{\circ}12'46'' + 18^{\circ}41'05''}{360^{\circ}} \times 24 \text{ hours}$
- (B) Time difference = $\frac{151^{\circ}12'46'' + 18^{\circ}41'05''}{360^{\circ}} \times 12 \text{ hours}$
- (C) Time difference = $\frac{151^{\circ}12'46'' - 18^{\circ}41'05''}{360^{\circ}} \times 24 \text{ hours}$
- (D) Time difference = $\frac{151^{\circ}12'46'' - 18^{\circ}41'05''}{360^{\circ}} \times 12 \text{ hours}$

QUESTION TWELVE

Which word best fits in the blank?

“A sample occurs when all items in the population have an equal chance of being included in the sample.”

- (A) stratified
- (B) random
- (C) capture–recapture
- (D) normal

QUESTION THIRTEEN

A large data set compares the age of salmon x in years with their weight y in kilograms.

The ages have a mean of $\bar{x} \doteq 6.3$ and a standard deviation of $s_x \doteq 3.4$.

The weights have a mean of $\bar{y} \doteq 22.8$ and a standard deviation of $s_y \doteq 12.8$.

The correlation coefficient is $r \doteq 0.873$.

What is the gradient of the least-squares line of best fit?

- (A) gradient $\doteq 0.873 \times \frac{3.4}{12.8}$
- (B) gradient $\doteq 0.873 \times \frac{12.8}{3.4}$
- (C) gradient $\doteq 0.873 \times \frac{22.8}{6.3}$
- (D) gradient $\doteq 0.873 \times \frac{6.3}{22.8}$

QUESTION FOURTEEN

Hunter uses a 1.6 kilowatt dishwasher for $5\frac{1}{2}$ hours per week. He is charged at a rate of 29.845 cents per kilowatt hour. How much does his dishwasher cost him per week?

- (A) $1.6 \times 5\frac{1}{2} \times \0.29845
 (B) $1.6 \div 5\frac{1}{2} \times \0.29845
 (C) $1.6 \times 5\frac{1}{2} \times \29.845
 (D) $1.6 \div 5\frac{1}{2} \times \29.845

QUESTION FIFTEEN

A home loan of \$582 000 is taken out at an interest rate of 3.28% per annum. The interest is calculated monthly and is reducible. The following table shows the progress of the first two months of the loan.

Months	Principal(P)	Interest(I)	$P + I$	Balance = $P + I - R$
1	\$ 582 000.00	\$ 1590.80	\$ 583 590.80	\$ 580 280.85
2	\$ 580 280.85	\$ 1586.10	\$ 581 866.95	\$ 578 557.00

What is the regular monthly repayment R ?

- (A) \$1721.50
 (B) \$1723.85
 (C) \$3176.90
 (D) \$3309.95

QUESTION SIXTEEN

The volume V of a cone with base radius r and perpendicular height h is given by which formula?

- (A) $V = \pi r^2 \times h$
 (B) $V = 2\pi r \times h$
 (C) $V = \frac{1}{3} \times \pi r^2 \times h$
 (D) $V = \frac{1}{3} \times 2\pi r \times h$

QUESTION SEVENTEEN

The formula for the area of a triangle is:

$$A = \frac{1}{2}ab \sin C$$

Which option correctly expresses the same formula with b as the subject?

(A) $b = \frac{2a}{A \sin C}$

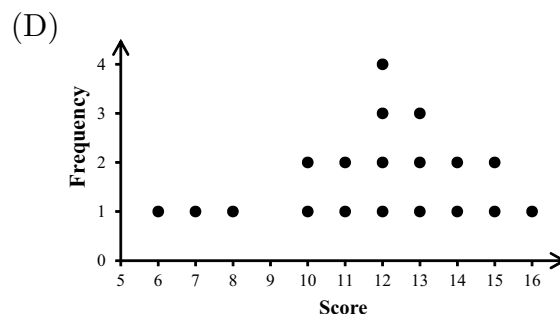
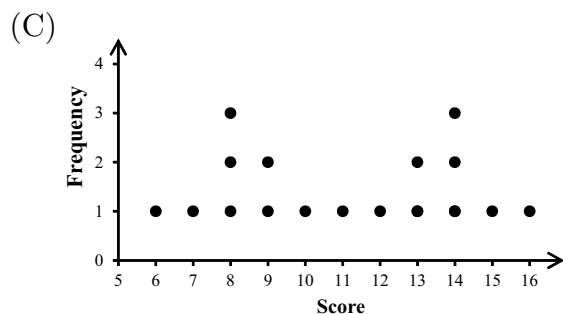
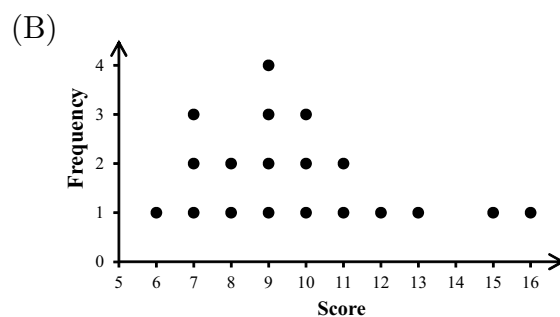
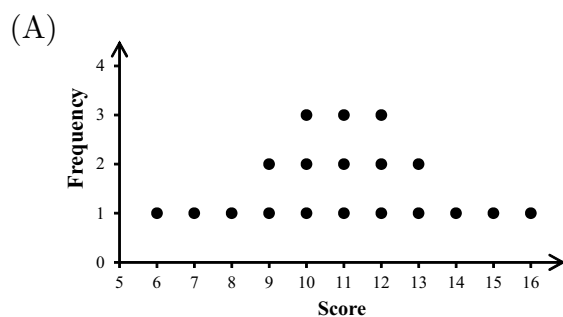
(B) $b = \frac{2a \sin C}{A}$

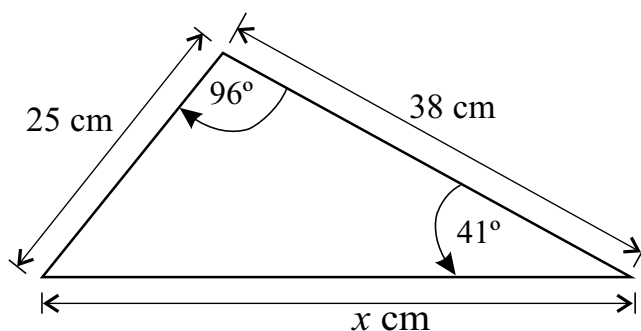
(C) $b = \frac{2A \sin C}{a}$

(D) $b = \frac{2A}{a \sin C}$

QUESTION EIGHTEEN

Which dot plot represents a data set with positive skew?



QUESTION NINETEEN

Which of the following would correctly calculate x in the above triangle?

- (A) $x = \frac{\sin 96^\circ}{25 \times \sin 41^\circ}$
- (B) $x = \frac{25 \times \sin 41^\circ}{\sin 96^\circ}$
- (C) $x = 25^2 + 38^2 - 2 \times 25 \times 38 \times \cos 96^\circ$
- (D) $x = \sqrt{25^2 + 38^2 - 2 \times 25 \times 38 \times \cos 96^\circ}$

QUESTION TWENTY

A coin is tossed 100 times and the result is recorded each time. Which of the following are pairs of complementary events?

P	Less than 30 heads
Q	More than 30 heads
R	An odd number of tails
S	An even number of tails

- (A) P and Q only
- (B) R and S only
- (C) P and Q ; and also R and S
- (D) None of these

Exam continues overleaf ...

QUESTION TWENTY ONE

Which is the correct explanation of the meaning of the formula below?

$$PV = \frac{FV}{(1 + r)^n}$$

- (A) Depreciating the Future Value FV of an annuity at a constant rate r for a certain number of periods n will give the equivalent Present Value PV .
- (B) Appreciating the Present Value PV of an annuity at a constant rate r for a certain number of periods n will give the equivalent Future Value FV .
- (C) Depreciating the Full Value of a reducible balance loan FV at a constant rate r for a certain number of periods n will give the monthly Payment Value PV .
- (D) Appreciating the monthly Payment Value PV of a reducible balance loan at a constant rate r for a certain number of periods n will give the Full Value FV .

QUESTION TWENTY TWO

The heights of 18-year-old males are normally distributed with a mean of 175 centimetres and a standard deviation of 9 centimetres. What percentage of 18-year-old males are taller than 193 centimetres?

- (A) 0.15%
- (B) 0.3%
- (C) 2.5%
- (D) 5.0%

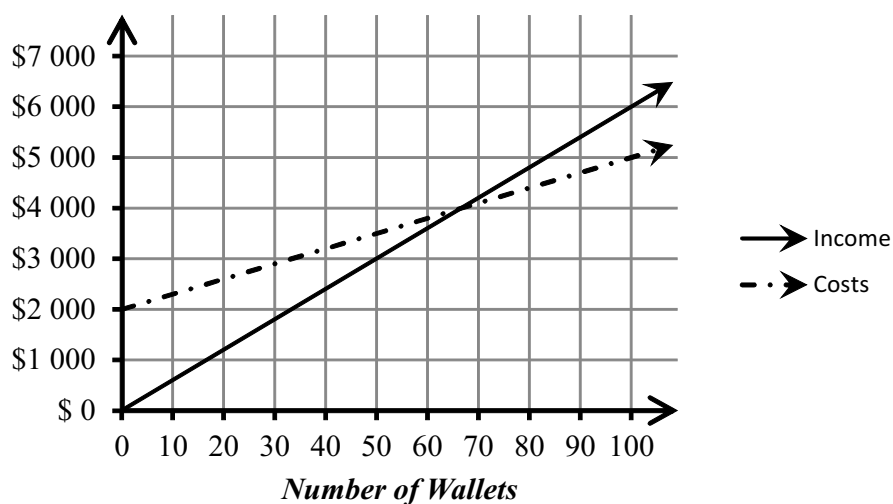
QUESTION TWENTY THREE

Two points on the earth are antipodal if they are directly opposite each other on the globe. For example, the north pole is the antipodal point to the south pole. The coordinates of Sydney Grammar School are ($33^{\circ}52'S$, $151^{\circ}13'E$). What are the correct coordinates of the point antipodal to Sydney Grammar School?

- (A) ($33^{\circ}52'N$, $28^{\circ}47'W$)
- (B) ($56^{\circ}08'N$, $28^{\circ}47'W$)
- (C) ($33^{\circ}52'N$, $151^{\circ}13'W$)
- (D) ($56^{\circ}08'N$, $151^{\circ}13'W$)

QUESTION TWENTY FOUR

Peter manufactures leather wallets. The following graph models the cost and income for his business.



Which of the following is NOT correct?

- (A) Peter's set-up costs are \$2 000.
- (B) Once he has set up his business, each wallet costs \$30 to manufacture.
- (C) He sells each wallet for \$60.
- (D) Peter will make a profit if he manufactures 60 wallets.

QUESTION TWENTY FIVE

A sample of three players is to be taken from a soccer team of eleven players. How many samples are possible?

- (A) $\frac{11^3}{3^3}$
- (B) $\frac{11!}{3!}$
- (C) $\frac{11 \times 10 \times 9}{3 \times 2 \times 1}$
- (D) $11 \times 10 \times 9$

_____ End of Section I _____

Exam continues overleaf ...

SECTION II - Written Response

Answers for this section should be recorded in the space provided in this paper.

Show all necessary working.

QUESTION TWENTY SIX (15 marks)**Marks**

- (a) The annual gross income of four people is shown below.

Name	Benjamin	Hannah	Ryan	Sienna
Income	\$82 582	\$89 642	\$100 387	\$117 597

- (i) Calculate the mean income of this group of people.

1

Solution

- (ii) List all the possible samples of size $n = 3$ from the above group of people.

2

Solution

- (iii) What is the mean of the sample means?

1

Solution

QUESTION TWENTY SIX (Continued)

(b) Two six-sided dice are thrown and the numbers on the uppermost faces are added.

(i) What is the probability of a sum equal to 9?

2

Solution

(ii) What is the probability of a sum of less than 9?

2

Solution

(iii) Isaac rolls a pair of dice, three times in a row.

(α) What is the probability of Isaac getting a sum of less than 9 all three times?

1

Solution

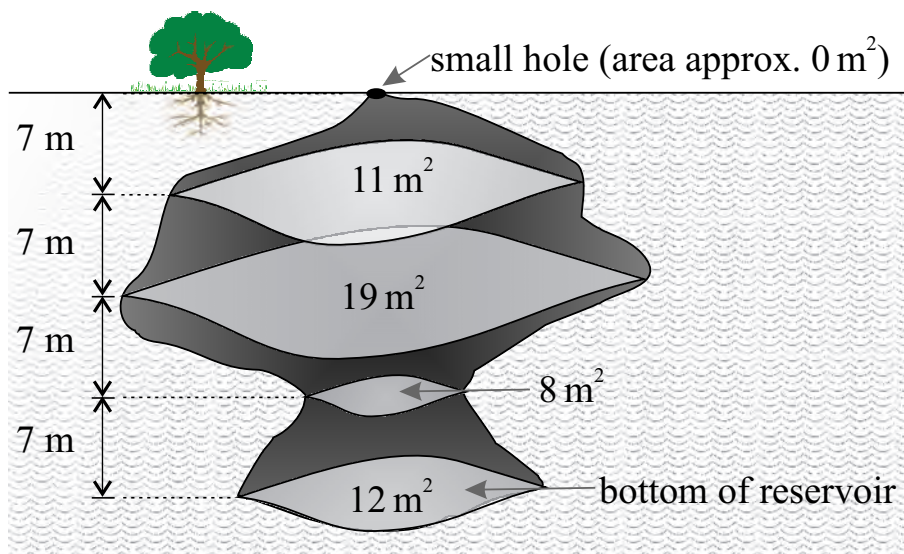
(β) What is the probability of Isaac getting a sum greater than or equal to 9 at least once?

1

Solution

QUESTION TWENTY SIX (Continued)

- (c) Lucy has discovered an underground water reservoir on her farm. The areas of horizontal cross-sections, 7 metres apart, are given in the diagram below.



- (i) Use two applications of Simpson's Rule to find the approximate volume of the reservoir. 2

Solution

- (ii) What is the approximate capacity of the reservoir in litres? 1

Solution

- (iii) Lucy can access this water for her livestock for free. The alternative is to truck water in, costing \$185 per 14 kilolitres. Assuming the reservoir is full, how much money could she save by using the water from her reservoir? 2

Solution

QUESTION TWENTY SEVEN (15 marks)

Marks

- (a) Solve the following equation.

2

$$\frac{x}{3} + \frac{x}{2} = 35$$

Solution

- (b) Solve the following pair of simultaneous equations.

3

$$2A - 3B = -12$$

$$4A + 2B = -8$$

Solution

QUESTION TWENTY SEVEN (Continued)

- (c) Scarlett collected some data from her classmates, specifically the earliest age in years they remember using email.

Frequency Distribution Table

Score x (years)	Frequency f	Cumulative Frequency cf	$f \times x$
8	2	2	16
9	6	8	54
10	8		
11	11		
12	0		
13	1		

$$\Sigma f = 28$$

$$\Sigma(f \times x) = 284$$

- (i) Complete the frequency distribution table above.

2

- (ii) What is the mean of this data?

1*Solution*

- (iii) What is the median of this data?

1*Solution*

- (iv) Describe the skewness of the distribution of this data.

1*Solution*

QUESTION TWENTY SEVEN (Continued)

- (d) The following table compares data for a five countries. Listed are the birth rate x per 1000 people, and the life expectancy at birth y in years.

Country	Birth Rate x	Life Expectancy y
Australia	1.92	82.2
Cambodia	2.86	71.8
Japan	1.43	93.3
New Zealand	1.95	81.4
Thailand	1.40	74.4

Data from the World Bank, accessed through Google Public Data.

- (i) What is the correlation coefficient of this data? Express your answer correct to three significant figures. 1

Solution

- (ii) What is the mean and standard deviation of each variable? Express your answers correct to three significant figures. 2

Solution

- (iii) What is the equation of the least-squares line of best fit of the above data? Express your answer correct to three significant figures. 1

Solution

- (iv) Describe the correlation between the birth rate and life expectancy for these countries. 1

Solution

QUESTION TWENTY EIGHT (15 marks)

Marks

- (a) The coordinates of Sydney Grammar School are $(33^{\circ}52'S, 151^{\circ}13'E)$. The coordinates of a bay on the coast of Antarctica are $(75^{\circ}52'S, 28^{\circ}47'W)$.



- (i) Identify which great circle these two points lie on. Justify your answer with reference to their coordinates. 2

Solution

- (ii) Assuming that the earth is a perfect sphere with a radius of 6400 kilometres, find the shortest distance along the surface of the earth between these two places. Express your answer correct to the nearest kilometre. 3

Solution

QUESTION TWENTY EIGHT (Continued)

For the following questions you can refer to the Present Value and Future Value tables attached to your formula sheet.

- (b) Matilda is pregnant, and she is planning to invest some money at the end of every month after the birth of her child into an account in their name. In making her calculations, Matilda assumes that the interest rate will remain constant at 9.00% per annum, compounded monthly.

- (i) If she invests \$300 at the end of every month, how much will be in the account after 5 years? 2

Solution

- (ii) How much will she need to invest each month so that the balance of the account will be \$50 000 after 5 years? 1

Solution

- (iii) With a bit of research, Matilda discovers that there is a formula for the future value FV of an annuity: 2

$$FV = a \left(\frac{(1 + r)^n - 1}{r} \right)$$

where a is the contribution per period, paid at the end of the period,
 r is the interest rate per compounding period, and
 n is the number of compounding periods.

How much will Matilda need to invest at the end of each month in order for her child to have \$350 000 when they turn 18 years old?

Solution

QUESTION TWENTY EIGHT (Continued)

- (c) Ivy is an investigator for the Australian Competition and Consumer Commission, and she has received a complaint about the volumes of cartons of a certain brand of milk that are labelled “1 litre”. She obtains a systematic sample of the cartons of milk, and the volumes of the milk contained in them is recorded below. All measurements are in millilitres.

1034	1014	1028	1002	1027	1019
1021	1015	1023	1021	1022	1037

- (i) What is the mean and standard deviation of this data? Express your answer correct to the nearest 0.1 millilitre. 2

Solution

- (ii) Ivy uses this sample to make inferences about all of the cartons sold by the brand. What might be the problem with this approach? 1

Solution

- (iii) The company selling this milk claims on its website that, “Less than one in a thousand of our cartons contain less than one litre of milk.” Ignoring the possible problems you have identified above, is Ivy able to report to the public that her data supports this claim? Justify your answer with reference to the statistics, assuming that the data is normally distributed. 2

Solution

QUESTION TWENTY NINE (15 marks)**Marks**

(a) A data set of 49 scores have been collected in a table as shown below.

Row 1	23	44	59	42	52	21	57
Row 2	71	67	50	47	59	43	50
Row 3	64	31	65	22	71	69	56
Row 4	30	62	54	62	36	57	29
Row 5	45	71	46	69	11	85	44
Row 6	33	59	34	46	38	24	32
Row 7	72	55	58	54	13	45	30

- (i) A random sample is generated by taking all the values from row 3 of the table. What is the sample mean? Express your answer correct to one decimal place.

1*Solution*

- (ii) A random sample is generated by taking all of the values from row 3 and row 1 of the table. What is this sample mean? Express your answer correct to one decimal place.

1*Solution*

- (iii) The population mean is 48.1. Which sample has a mean closer to the population mean?

1*Solution*

- (iv) Explain why this was likely to be the case.

1*Solution*

QUESTION TWENTY NINE (Continued)

- (b) Xavier is buying a car with a purchase price of \$19 000, and he has to pay a 15% deposit immediately. He will take out a loan from Calcubank for the balance. They inform him he can pay off the loan with 30 monthly payments of \$613.70.

(i) What is the total amount that Xavier will end up paying for the car?

2

Solution

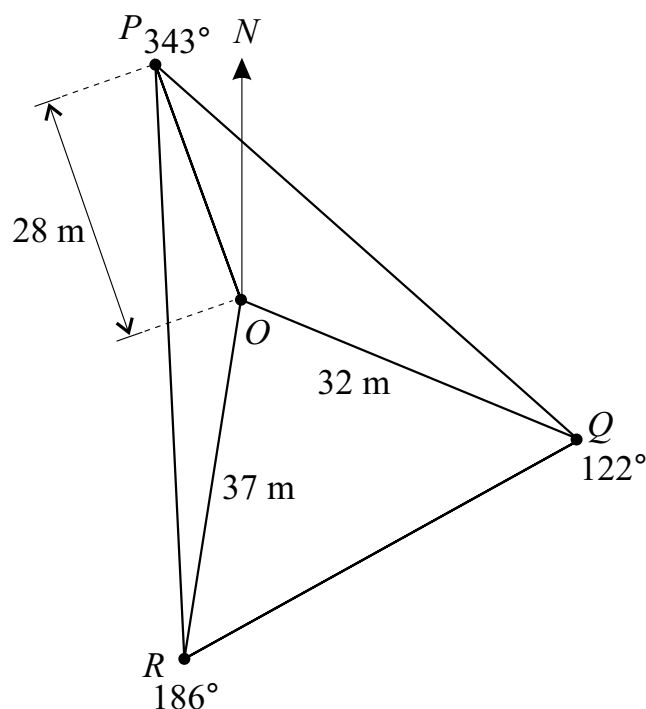
(ii) What is the annual simple interest rate that Calcubank is charging Xavier?

3

Solution

QUESTION TWENTY NINE (Continued)

(c) A radial survey of a triangular piece of land PQR is shown below.



(i) What is the size of $\angle POQ$?

1

Solution

(ii) What is the area of $\triangle POQ$? Express your answer correct to the nearest square metre.

2

Solution

(iii) What is the area of $\triangle PQR$? Express your answer correct to the nearest square metre.

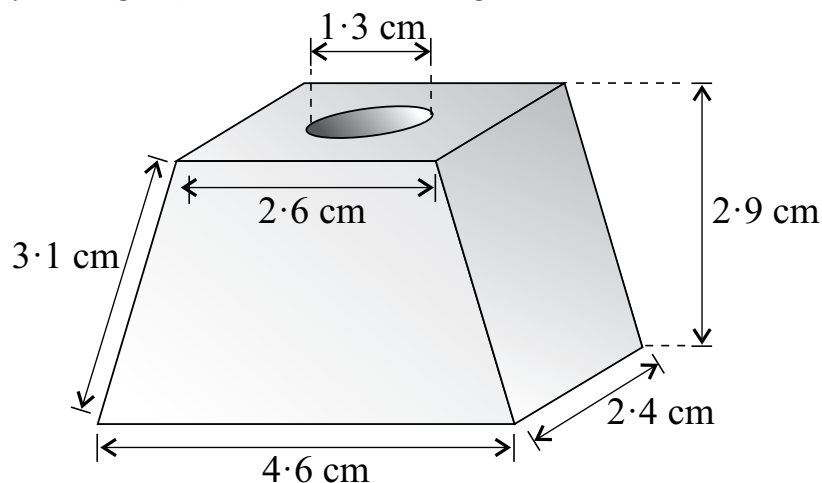
3

Solution

QUESTION THIRTY (15 marks)

Marks

- (a) A trapezoidal prism has a vertical cylindrical hole with a diameter of 1.3 centimetres drilled completely through it, as shown in the diagram.



- (i) What is the area of the trapezoidal front face of the prism?

2

Solution

- (ii) What was the volume of the trapezoidal prism before the hole was drilled through it?

1

Solution

- (iii) What is the volume of material remaining after the hole was drilled through it? Express your answer correct to 2 significant figures.

2

Solution

QUESTION THIRTY (Continued)

(b) Below is some data on the Australian population.

Year Y	1963	2013
Population P (millions)	10.95	23.13

- (i) Oscar naively assumes that the relationship between the year and population is linear. Using this assumption, what would be the estimated population in 1988, by interpolation? 2

Solution

- (ii) Willow is a bit more canny when it comes to population, so she assumes that the population grows exponentially according to the formula 2

$$P = 10.95 \times k^{(Y-1963)}.$$

Use the 2013 data to calculate k . Express your answer correct to four significant figures.

Solution

- (iii) What would be Willow's interpolation for the population in 1988? Express your answer correct to four significant figures. 1

Solution

QUESTION THIRTY (Continued)

- (c) There are 52 cards in a pack of regular playing cards. There are four suits of thirteen cards each. Each suit has nine number cards (from 2 to 10), three royal cards (King, Queen, Jack), and one Ace.

Levi takes three random cards from a shuffled pack, and places them face up on the table.

- (i) What is the probability that he has exactly three number cards?

1

Solution

- (ii) What is the probability that he has exactly two number cards?

2

Solution

QUESTION THIRTY (Continued)

- (iii) Levi invents a gambling game and invites Zara to play against him. She has to give him \$1 to play, and then she takes three random cards from a shuffled pack, as above. If Zara holds either zero or two number cards, Levi has to pay her \$2. Otherwise, Levi keeps the \$1. What is Zara's average financial expectation for each round of this game?

2

Solution

————— End of Section II —————

END OF EXAMINATION

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

B L A N K P A G E

--	--	--	--	--	--	--	--

CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL



2015
Trial Examination
FORM VI
GENERAL MATHEMATICS
Friday 31st July 2015

- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

THIS SHEET HAS TWO SIDES

Question One

A ☐ B ☐ C ☐ D ☐

Question Two

A ☐ B ☐ C ☐ D ☐

Question Three

A ☐ B ☐ C ☐ D ☐

Question Four

A ☐ B ☐ C ☐ D ☐

Question Five

A ☐ B ☐ C ☐ D ☐

Question Six

A ☐ B ☐ C ☐ D ☐

Question Seven

A ☐ B ☐ C ☐ D ☐

Question Eight

A ☐ B ☐ C ☐ D ☐

Question Nine

A ☐ B ☐ C ☐ D ☐

Question Ten

A ☐ B ☐ C ☐ D ☐

Question Eleven

A ☐ B ☐ C ☐ D ☐

Question Twelve

A ☐ B ☐ C ☐ D ☐

Question Thirteen

A ☐ B ☐ C ☐ D ☐

Question Fourteen

A ☐ B ☐ C ☐ D ☐

Question Fifteen

A ☐ B ☐ C ☐ D ☐

Question Sixteen

A ☐ B ☐ C ☐ D ☐

Question Seventeen

A ☐ B ☐ C ☐ D ☐

Question Eighteen

A ☐ B ☐ C ☐ D ☐

Question Nineteen

A ☐ B ☐ C ☐ D ☐

Question Twenty

A ☐ B ☐ C ☐ D ☐

Question Twenty One

A ☐ B ☐ C ☐ D ☐

Question Twenty Two

A ☐ B ☐ C ☐ D ☐

Question Twenty Three

A ☐ B ☐ C ☐ D ☐

Question Twenty Four

A ☐ B ☐ C ☐ D ☐

Question Twenty Five

A ☐ B ☐ C ☐ D ☐

Mathematics General 2

FORMULAE AND DATA SHEET

Financial Mathematics

Simple interest

$$I = Prn$$

P is initial amount

r is interest rate per period, expressed as a decimal

n is number of periods

Compound interest

$$A = P(1 + r)^n$$

A is final amount

P is initial amount

r is interest rate per period, expressed as a decimal

n is number of compounding periods

Present value and future value

$$PV = \frac{FV}{(1 + r)^n}, \quad FV = PV(1 + r)^n$$

r is interest rate per period, expressed as a decimal

n is number of compounding periods

Straight-line method of depreciation

$$S = V_0 - Dn$$

S is salvage value of asset after n periods

V_0 is initial value of asset

D is amount of depreciation per period

n is number of periods

Declining-balance method of depreciation

$$S = V_0(1 - r)^n$$

S is salvage value of asset after n periods

V_0 is initial value of asset

r is depreciation rate per period, expressed as a decimal

n is number of periods

Data Analysis

Mean of a sample

$$\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$$

z -score

For any score x ,

$$z = \frac{x - \bar{x}}{s}$$

$\bar{x}$ is mean

s is standard deviation

Outlier(s)

score(s) less than $Q_L - 1.5 \times IQR$

or

score(s) more than $Q_U + 1.5 \times IQR$

Q_L is lower quartile

Q_U is upper quartile

IQR is interquartile range

Least-squares line of best fit

$$y = \text{gradient} \times x + y\text{-intercept}$$

$$\text{gradient} = r \times \frac{\text{standard deviation of } y \text{ scores}}{\text{standard deviation of } x \text{ scores}}$$

$$y\text{-intercept} = \bar{y} - (\text{gradient} \times \bar{x})$$

r is correlation coefficient

$\bar{x}$ is mean of x scores

$\bar{y}$ is mean of y scores

Normal Distribution

- approximately 68% of scores have z -scores between -1 and 1
- approximately 95% of scores have z -scores between -2 and 2
- approximately 99.7% of scores have z -scores between -3 and 3

Spherical Geometry

Circumference of a Circle

$$C = 2\pi r \quad \text{or} \quad C = \pi D$$

r is radius

D is diameter

Arc length of a circle

$$l = \frac{\theta}{360} 2\pi r$$

r is radius

θ is number of degrees in central angle

Radius of Earth

(taken as) 6400 km

Time differences

For calculation of time differences using longitude:
 $15^\circ = 1$ hour time difference

Area

Circle

$$A = \pi r^2$$

r is radius

Sector

$$A = \frac{\theta}{360} \pi r^2$$

r is radius

θ is number of degrees in central angle

Annulus

$$A = \pi(R^2 - r^2)$$

R is radius of outer circle

r is radius of inner circle

Trapezium

$$A = \frac{h}{2}(a + b)$$

h is perpendicular height

a and b are the lengths of the parallel sides

Area of land catchment areas

unit conversion: 1 ha=10000 m²

Surface area

Sphere

$$A = 4\pi r^2$$

r is radius

Closed Cylinder

$$A = 2\pi r^2 + 2\pi r h$$

r is radius

h is perpendicular height

Volume

Prism or cylinder

$$V = Ah$$

A is area of base

h is perpendicular height

Pyramid or cone

$$V = \frac{1}{3}Ah$$

A is area of base

h is perpendicular height

Volume and capacity

unit conversion: 1 m³ = 1000 L

Approximation Using Simpson's Rule

Area

$$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$$

h is distance between successive measurements

d_f is first measurement

d_m is middle measurement

d_l is last measurement

Volume

$$V \approx \frac{h}{3}(A_L + 4A_M + A_R)$$

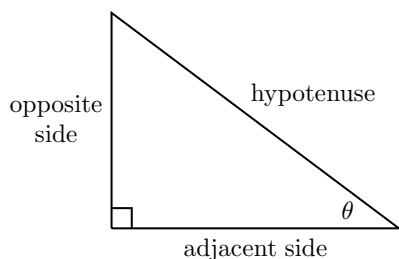
h is distance between successive measurements

A_L is area of left end

A_M is area of middle

A_R is area of right end

Trigonometric Ratios



$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Sine rule

In $\triangle ABC$,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a triangle

In $\triangle ABC$,

$$A = \frac{1}{2}ab \sin C$$

Cosine rule

In $\triangle ABC$,

$$c^2 = a^2 + b^2 - 2ab \cos C$$

or

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Units of Memory and File Size

1 byte = 8 bits

1 kilobyte = 2^{10} bytes = 1024 bytes

1 megabyte = 2^{20} bytes = 1024 kilobytes

1 gigabyte = 2^{30} bytes = 1024 megabytes

1 terabyte = 2^{40} bytes = 1024 gigabytes

Blood Alcohol Content Estimates

$$BAC_{\text{male}} = \frac{10N - 7.5H}{6.8M}$$

or

$$BAC_{\text{female}} = \frac{10N - 7.5H}{5.5M}$$

N is number of standard drinks consumed

H is number of hours of drinking

M is person's mass in kilograms

Distance, Speed, Time

$$D = ST, \quad S = \frac{D}{T}, \quad T = \frac{D}{S}$$

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$\text{stopping distance} = \left\{ \begin{array}{c} \text{reaction-time} \\ \text{distance} \end{array} \right\} + \left\{ \begin{array}{c} \text{braking} \\ \text{distance} \end{array} \right\}$$

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Straight Lines

Gradient

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient-intercept form

$$y = mx + b$$

m is gradient

b is y -intercept

The table below shows the future value of an annuity where \$1 is invested at the end of the period at the given interest rate for the given number of periods. The interest is compounded per period.

	Future value of a \$1 annuity														
Periods	0.25%	0.50%	0.75%	1.00%	1.25%	1.50%	1.75%	2.00%	2.50%	3.00%	3.50%	4.00%	5.00%	6.00%	8.00%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0025	2.0050	2.0075	2.0100	2.0125	2.0150	2.0175	2.0200	2.0250	2.0300	2.0350	2.0400	2.0500	2.0600	2.0800
3	3.0075	3.0150	3.0226	3.0301	3.0377	3.0452	3.0528	3.0604	3.0756	3.0909	3.1062	3.1216	3.1525	3.1836	3.2464
4	4.0150	4.0301	4.0452	4.0604	4.0756	4.0909	4.1062	4.1216	4.1525	4.1836	4.2149	4.2465	4.3101	4.3746	4.5061
5	5.0251	5.0503	5.0756	5.1010	5.1266	5.1523	5.1781	5.2040	5.2563	5.3091	5.3625	5.4163	5.5256	5.6371	5.8666
6	6.0376	6.0755	6.1136	6.1520	6.1907	6.2296	6.2687	6.3081	6.3877	6.4684	6.5502	6.6330	6.8019	6.9753	7.3359
7	7.0527	7.1059	7.1595	7.2135	7.2680	7.3230	7.3784	7.4343	7.5474	7.6625	7.7794	7.8983	8.1420	8.3938	8.9228
8	8.0704	8.1414	8.2132	8.2857	8.3589	8.4328	8.5075	8.5830	8.7361	8.8923	9.0517	9.2142	9.5491	9.8975	10.6366
9	9.0905	9.1821	9.2748	9.3685	9.4634	9.5593	9.6564	9.7546	9.9545	10.1591	10.3685	10.5828	11.0266	11.4913	12.4876
10	10.1133	10.2280	10.3443	10.4622	10.5817	10.7027	10.8254	10.9497	11.2034	11.4639	11.7314	12.0061	12.5779	13.1808	14.4866
11	11.1385	11.2792	11.4219	11.5668	11.7139	11.8633	12.0148	12.1687	12.4835	12.8078	13.1420	13.4864	14.2068	14.9716	16.6455
12	12.1664	12.3356	12.5076	12.6825	12.8604	13.0412	13.2251	13.4121	13.7956	14.1920	14.6020	15.0258	15.9171	16.8699	18.9771
13	13.1968	13.3972	13.6014	13.8093	14.0211	14.2368	14.4565	14.6803	15.1404	15.6178	16.1130	16.6268	17.7130	18.8821	21.4953
14	14.2298	14.4642	14.7034	14.9474	15.1964	15.4504	15.7095	15.9739	16.5190	17.0863	17.6770	18.2919	19.5986	21.0151	24.2149
15	15.2654	15.5365	15.8137	16.0969	16.3863	16.6821	16.9844	17.2934	17.9319	18.5989	19.2957	20.0236	21.5786	23.2760	27.1521
16	16.3035	16.6142	16.9323	17.2579	17.5912	17.9324	18.2817	18.6393	19.3802	20.1569	20.9710	21.8245	23.6575	25.6725	30.3243
17	17.3443	17.6973	18.0593	18.4304	18.8111	19.2014	19.6016	20.0121	20.8647	21.7616	22.7050	23.6975	25.8404	28.2129	33.7502
18	18.3876	18.7858	19.1947	19.6147	20.0462	20.4894	20.9446	21.4123	22.3863	23.4144	24.4997	25.6454	28.1324	30.9057	37.4502
19	19.4336	19.8797	20.3387	20.8109	21.2968	21.7967	22.3112	22.8406	23.9460	25.1169	26.3572	27.6712	30.5390	33.7600	41.4463
20	20.4822	20.9791	21.4912	22.0190	22.5630	23.1237	23.7016	24.2974	25.5447	26.8704	28.2797	29.7781	33.0660	36.7856	45.7620
22	22.5872	23.1944	23.8223	24.4716	25.1431	25.8376	26.5559	27.2990	28.8629	30.5368	32.3289	34.2480	38.5052	43.3923	55.4568
24	24.7028	25.4320	26.1885	26.9735	27.7881	28.6335	29.5110	30.4219	32.3490	34.4265	36.6665	39.0826	44.5020	50.8156	66.7648
26	26.8290	27.6919	28.5903	29.5256	30.4996	31.5140	32.5704	33.6709	36.0117	38.5530	41.3131	44.3117	51.1135	59.1564	79.9544
28	28.9658	29.9745	31.0282	32.1291	33.2794	34.4815	35.7379	37.0512	39.8598	42.9309	46.2906	49.9676	58.4026	68.5281	95.3388
30	31.1133	32.2800	33.5029	34.7849	36.1291	37.5387	39.0172	40.5681	43.9027	47.5754	51.6227	56.0849	66.4388	79.0582	113.2832
35	36.5292	38.1454	39.8538	41.6603	43.5709	45.5921	47.7308	49.9945	54.9282	60.4621	66.6740	73.6522	90.3203	111.4348	172.3168
40	42.0132	44.1588	46.4465	48.8864	51.4896	54.2679	57.2341	60.4020	67.4026	75.4013	84.5503	95.0255	120.7998	154.7620	259.0565
45	47.5661	50.3242	53.2901	56.4811	59.9157	63.6142	67.5986	71.8927	81.5161	92.7199	105.7817	121.0294	159.7002	212.7435	386.5056
50	53.1887	56.6452	60.3943	64.4632	68.8818	73.6828	78.9022	84.5794	97.4843	112.7969	130.9979	152.6671	209.3480	290.3359	573.7702
55	58.8819	63.1258	67.7688	72.8525	78.4225	84.5296	91.2302	98.5865	115.5509	136.0716	160.9469	191.1592	272.7126	394.1720	848.9232
60	64.6467	69.7700	75.4241	81.6697	88.5745	96.2147	104.6752	114.0515	135.9916	163.0534	196.5169	237.9907	353.5837	533.1282	1253.2133
65	70.4839	76.5821	83.3709	90.9366	99.3771	108.8028	119.3386	131.1262	159.1183	194.3328	238.7629	294.9684	456.7980	719.0829	1847.2481
70	76.3944	83.5661	91.6201	100.6763	110.8720	122.3638	135.3308	149.9779	185.2841	230.5941	288.9379	364.2905	588.5285	967.9322	2720.0801
75	82.3792	90.7265	100.1833	110.9128	123.1035	136.9728	152.7721	170.7918	214.8883	272.6309	348.5300	448.6314	756.6537	1300.9487	4002.5566
80	88.4392	98.0677	109.0725	121.6715	136.1188	152.7109	171.7938	193.7720	248.3827	321.3630	419.3068	551.2450	971.2288	1746.5999	5886.9354

The table below shows the equivalent present value of an annuity where \$1 is invested at the end of the period at the given interest rate for the given number of periods. The interest is compounded per period.

	Present value of a \$1 annuity																
Periods	0.25%	0.50%	0.75%	1.00%	1.25%	1.50%	1.75%	2.00%	2.50%	3.00%	3.50%	4.00%	5.00%	6.00%	8.00%	10.00%	12.00%
1	0.9975	0.9950	0.9926	0.9901	0.9877	0.9852	0.9828	0.9804	0.9756	0.9709	0.9662	0.9615	0.9524	0.9434	0.9259	0.9091	0.8929
2	1.9925	1.9851	1.9777	1.9704	1.9631	1.9559	1.9487	1.9416	1.9274	1.9135	1.8997	1.8861	1.8594	1.8334	1.7833	1.7355	1.6901
3	2.9851	2.9702	2.9556	2.9410	2.9265	2.9122	2.8980	2.8839	2.8560	2.8286	2.8016	2.7751	2.7232	2.6730	2.5771	2.4869	2.4018
4	3.9751	3.9505	3.9261	3.9020	3.8781	3.8544	3.8309	3.8077	3.7620	3.7171	3.6731	3.6299	3.5460	3.4651	3.3121	3.1699	3.0373
5	4.9627	4.9259	4.8894	4.8534	4.8178	4.7826	4.7479	4.7135	4.6458	4.5797	4.5151	4.4518	4.3295	4.2124	3.9927	3.7908	3.6048
6	5.9478	5.8964	5.8456	5.7955	5.7460	5.6972	5.6490	5.6014	5.5081	5.4172	5.3286	5.2421	5.0757	4.9173	4.6229	4.3553	4.1114
7	6.9305	6.8621	6.7946	6.7282	6.6627	6.5982	6.5346	6.4720	6.3494	6.2303	6.1145	6.0021	5.7864	5.5824	5.2064	4.8684	4.5638
8	7.9107	7.8230	7.7366	7.6517	7.5681	7.4859	7.4051	7.3255	7.1701	7.0197	6.8740	6.7327	6.4632	6.2098	5.7466	5.3349	4.9676
9	8.8885	8.7791	8.6716	8.5660	8.4623	8.3605	8.2605	8.1622	7.9709	7.7861	7.6077	7.4353	7.1078	6.8017	6.2469	5.7590	5.3282
10	9.8639	9.7304	9.5996	9.4713	9.3455	9.2222	9.1012	8.9826	8.7521	8.5302	8.3166	8.1109	7.7217	7.3601	6.7101	6.1446	5.6502
11	10.8368	10.6770	10.5207	10.3676	10.2178	10.0711	9.9275	9.7868	9.5142	9.2526	9.0016	8.7605	8.3064	7.8869	7.1390	6.4951	5.9377
12	11.8073	11.6189	11.4349	11.2551	11.0793	10.9075	10.7395	10.5753	10.2578	9.9540	9.6633	9.3851	8.8633	8.3838	7.5361	6.8137	6.1944
13	12.7753	12.5562	12.3423	12.1337	11.9302	11.7315	11.5376	11.3484	10.9832	10.6350	10.3027	9.9856	9.3936	8.8527	7.9038	7.1034	6.4235
14	13.7410	13.4887	13.2430	13.0037	12.7706	12.5434	12.3220	12.1062	11.6909	11.2961	10.9205	10.5631	9.8986	9.2950	8.2442	7.3667	6.6282
15	14.7042	14.4166	14.1370	13.8651	13.6005	13.3432	13.0929	12.8493	12.3814	11.9379	11.5174	11.1184	10.3797	9.7122	8.5595	7.6061	6.8109
16	15.6650	15.3399	15.0243	14.7179	14.4203	14.1313	13.8505	13.5777	13.0550	12.5611	12.0941	11.6523	10.8378	10.1059	8.8514	7.8237	6.9740
17	16.6235	16.2586	15.9050	15.5623	15.2299	14.9076	14.5951	14.2919	13.7122	13.1661	12.6513	12.1657	11.2741	10.4773	9.1216	8.0216	7.1196
18	17.5795	17.1728	16.7792	16.3983	16.0295	15.6726	15.3269	14.9920	14.3534	13.7535	13.1897	12.6593	11.6896	10.8276	9.3719	8.2014	7.2497
19	18.5332	18.0824	17.6468	17.2260	16.8193	16.4262	16.0461	15.6785	14.9789	14.3238	13.7098	13.1339	12.0853	11.1581	9.6036	8.3649	7.3658
20	19.4845	18.9874	18.5080	18.0456	17.5993	17.1686	16.7529	16.3514	15.5892	14.8775	14.2124	13.5903	12.4622	11.4699	9.8181	8.5136	7.4694
22	21.3800	20.7841	20.2112	19.6604	19.1306	18.6208	18.1303	17.6580	16.7654	15.9369	15.1671	14.4511	13.1630	12.0416	10.2007	8.7715	7.6446
24	23.2660	22.5629	21.8891	21.2434	20.6242	20.0304	19.4607	18.9139	17.8850	16.9355	16.0584	15.2470	13.7986	12.5504	10.5288	8.9847	7.7843
26	25.1426	24.3240	23.5422	22.7952	22.0813	21.3986	20.7457	20.1210	18.9506	17.8768	16.8904	15.9828	14.3752	13.0032	10.8100	9.1609	7.8957
28	27.0099	26.0677	25.1707	24.3164	23.5025	22.7267	21.9870	21.2813	19.9649	18.7641	17.6670	16.6631	14.8981	13.4062	11.0511	9.3066	7.9844
30	28.8679	27.7941	26.7751	25.8077	24.8889	24.0158	23.1858	22.3965	20.9303	19.6004	18.3920	17.2920	15.3725	13.7648	11.2578	9.4269	8.0552
35	33.4724	32.0354	30.6827	29.4086	28.2079	27.0756	26.0073	24.9986	23.1452	21.4872	20.0007	18.6646	16.3742	14.4982	11.6546	9.6442	8.1755
40	38.0199	36.1722	34.4469	32.8347	31.3269	29.9158	28.5942	27.3555	25.1028	23.1148	21.3551	19.7928	17.1591	15.0463	11.9246	9.7791	8.2438
45	42.5109	40.2072	38.0732	36.0945	34.2582	32.5523	30.9663	29.4902	26.8330	24.5187	22.4955	20.7200	17.7741	15.4558	12.1084	9.8628	8.2825
50	46.9462	44.1428	41.5664	39.1961	37.0129	34.9997	33.1412	31.4236	28.3623	25.7298	23.4556	21.4822	18.2559	15.7619	12.2335	9.9148	8.3045
55	51.3264	47.9814	44.9316	42.1472	39.6017	37.2715	35.1354	33.1748	29.7140	26.7744	24.2641	22.1086	18.6335	15.9905	12.3186	9.9471	8.3170
60	55.6524	51.7256	48.1734	44.9550	42.0346	39.3803	36.9640	34.7609	30.9087	27.6756	24.9447	22.6235	18.9293	16.1614	12.3766	9.9672	8.3240
65	59.9246	55.3775	51.2963	47.6266	44.3210	41.3378	38.6406	36.1975	31.9646	28.4529	25.5178	23.0467	19.1611	16.2891	12.4160	9.9796	8.3281
70	64.1439	58.9394	54.3046	50.1685	46.4697	43.1549	40.1779	37.4986	32.8979	29.1234	26.0004	23.3945	19.3427	16.3845	12.4428	9.9873	8.3303
75	68.3108	62.4136	57.2027	52.5871	48.4890	44.8416	41.5875	38.6771	33.7227	29.7018	26.4067	23.6804	19.4850	16.4558	12.4611	9.9921	8.3316
80	72.4260	65.8023	59.9944	54.8882	50.3867	46.4073	42.8799	39.7445	34.4518	30.2008	26.7488	23.9154	19.5965	16.5091	12.4735	9.9951	8.3324

--	--	--	--	--	--	--	--

CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL



SOLUTIONS

2015 Trial Examination

FORM VI

GENERAL MATHEMATICS

Friday 31st July 2015

General Instructions

- Reading time — 5 minutes
- Writing time — $2\frac{1}{2}$ hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.

Total — 100 Marks

- All questions may be attempted.

Section I – 25 Marks

- Questions 1–25 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

Section II – 75 Marks

- Questions 26–30 are of equal value.
- All necessary working should be shown.
- Write all solutions on this paper in the spaces provided.

Collection

- Write your candidate number on this paper and on your multiple choice answer sheet.
- Place everything inside the question paper.

Checklist

- Multiple choice answer sheet
- Formulae sheet
- Candidature — 15 boys

Examiner
DWH

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

A bank charges simple interest at a rate of 0.05226% per day on the amount owing on a credit card. What is the interest charged for 15 days on an amount of \$1200?

(A) \$3.13

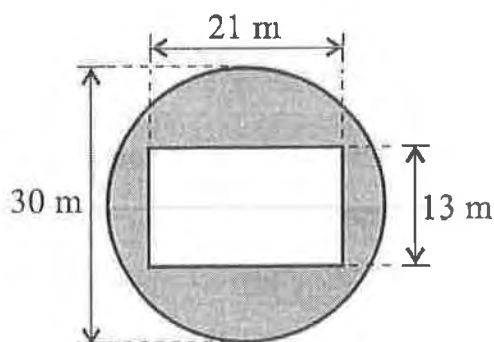
(B) \$9.41

(C) \$9.44

(D) \$94.07

QUESTION TWO

A circular grass field has a diameter of 30 metres. There is a concrete rectangle in the middle with a length of 21 metres and a width of 13 metres, as shown in the diagram below.



Which of the following would correctly calculate the area of grass, in square metres, as shaded in the diagram?

(A) $\text{Area} = 2 \times \pi \times \left(\frac{30}{2}\right) - 21 \times 13$

(B) $\text{Area} = 2 \times \pi \times 30 - 21 \times 13$

(C) $\text{Area} = \pi \times \left(\frac{30}{2}\right)^2 - 21 \times 13$

(D) $\text{Area} = \pi \times 30^2 - 21 \times 13$

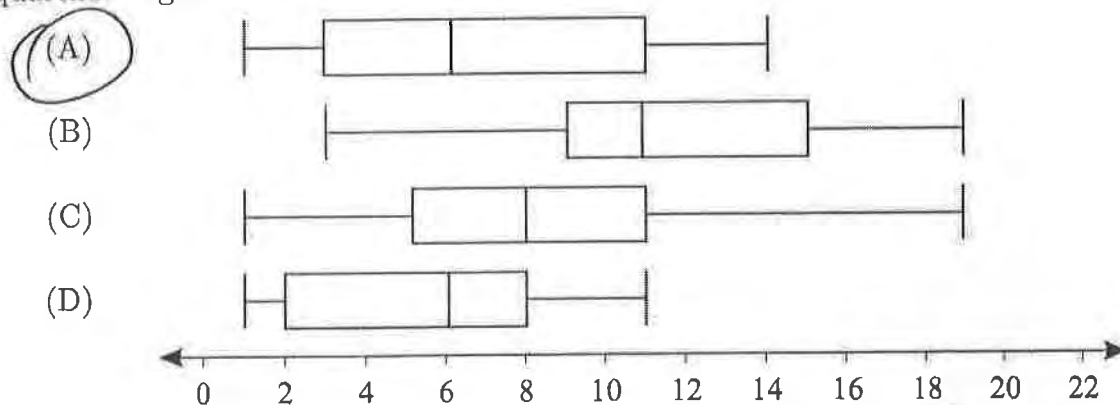
QUESTION THREE

Expand and simplify $2x(x^2 - 2) - 3x$.

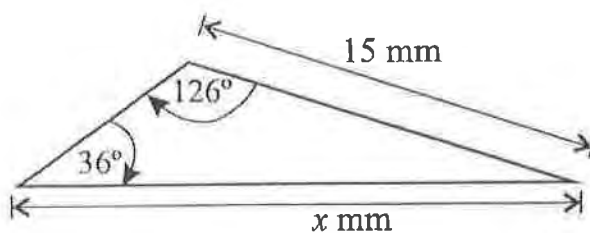
- (A) $2x^3 - 7x$
- (B) $2x^3 - x$
- (C) $2x^3 + x$
- (D) $2x^3 - 3x - 2$

QUESTION FOUR

Which of the data sets displayed in the following box-and-whisker plots has the largest interquartile range?



QUESTION FIVE



Which of the following would correctly calculate x in the above triangle?

- (A) $x = \frac{\sin 36^\circ}{15 \times \sin 126^\circ}$
- (B) $x = \frac{\sin 126^\circ}{15 \times \sin 36^\circ}$
- (C) $x = \frac{15 \times \sin 36^\circ}{\sin 126^\circ}$
- (D) $x = \frac{15 \times \sin 126^\circ}{\sin 36^\circ}$

QUESTION SIX

There will be 28 people competing in a long jump event. How many different outcomes are possible for first and second place? Assume that it is impossible to have a tied result.

(A) $\frac{28 \times 27}{2 \times 1}$

(B) $\frac{28^2}{2^2}$

(C) 28^2

(D) 28×27

QUESTION SEVEN

For the following question you should refer to the Present Value and Future Value tables attached to your formula sheet.

What is the equivalent present value of an annuity where \$1 is invested at the end of the month for 60 months at a constant interest rate of 6% per annum, compounded monthly?

(A) \$4.21

(B) \$51.73

(C) \$69.77

(D) \$533.13

QUESTION EIGHT

The data for a large population is normally distributed. Which of the following statements is correct?

(A) The mean, the mode and the median are all approximately equal.

(B) The mean and the mode are approximately equal but the median could be quite different.

(C) The mean and the median are approximately equal but the mode could be quite different.

(D) The mean, the mode and the median could all be quite different.

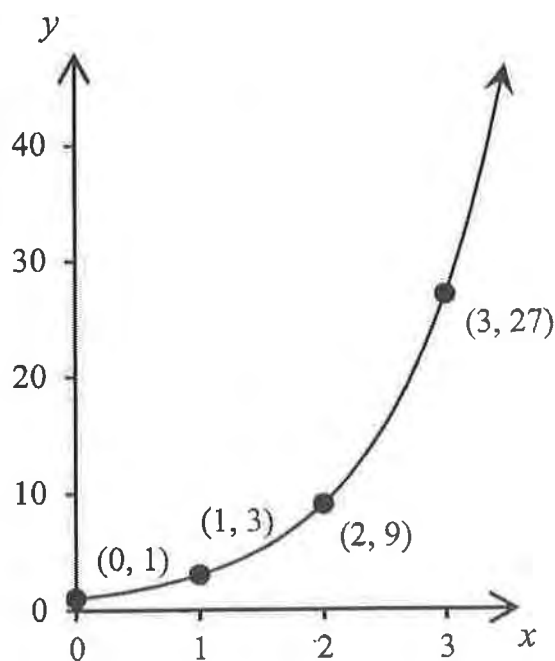
QUESTION NINE

Which statement about the linear equation $y = 9.2x + 3.8$ is true?

- (A) The gradient is 3.8 and the x -intercept is 9.2.
- (B) The gradient is 9.2 and the x -intercept is 3.8.
- (C) The gradient is 3.8 and the y -intercept is 9.2.
- ☒ (D) The gradient is 9.2 and the y -intercept is 3.8.

QUESTION TEN

The following is the graph of a relationship between two quantities x and y .



$$(y = 3^x)$$

What type of function would accurately model this data?

- (A) hyperbolic
- ☒ (B) exponential
- (C) quadratic
- (D) cubic

QUESTION ELEVEN

The coordinates of our classroom are ($33^{\circ}52'30''\text{S}$, $151^{\circ}12'46''\text{E}$). The coordinates of Brackenfell Primary School in Capetown South Africa are ($33^{\circ}52'30''\text{S}$, $18^{\circ}41'05''\text{E}$). Which of the following would correctly calculate the precise time difference between the two places?

(A) Time difference = $\frac{151^{\circ}12'46'' + 18^{\circ}41'05''}{360^{\circ}} \times 24 \text{ hours}$

(B) Time difference = $\frac{151^{\circ}12'46'' + 18^{\circ}41'05''}{360^{\circ}} \times 12 \text{ hours}$

☒ (C) Time difference = $\frac{151^{\circ}12'46'' - 18^{\circ}41'05''}{360^{\circ}} \times 24 \text{ hours}$

(D) Time difference = $\frac{151^{\circ}12'46'' - 18^{\circ}41'05''}{360^{\circ}} \times 12 \text{ hours}$

QUESTION TWELVE

Which word best fits in the blank?

"A sample occurs when all items in the population have an equal chance of being included in the sample."

(A) stratified

☒ (B) random

(C) capture-recapture

(D) normal

QUESTION THIRTEEN

A large data set compares the age of salmon x in years with their weight y in kilograms.

The ages have a mean of $\bar{x} \doteq 6.3$ and a standard deviation of $s_x \doteq 3.4$.

The weights have a mean of $\bar{y} \doteq 22.8$ and a standard deviation of $s_y \doteq 12.8$.

The correlation coefficient is $r \doteq 0.873$.

What is the gradient of the least-squares line of best fit?

(A) gradient $\doteq 0.873 \times \frac{3.4}{12.8}$

☒ (B) gradient $\doteq 0.873 \times \frac{12.8}{3.4}$

(C) gradient $\doteq 0.873 \times \frac{22.8}{6.3}$

(D) gradient $\doteq 0.873 \times \frac{6.3}{22.8}$

QUESTION FOURTEEN

Hunter uses a 1.6 kilowatt dishwasher for $5\frac{1}{2}$ hours per week. He is charged at a rate of 29.845 cents per kilowatt hour. How much does his dishwasher cost him per week?

- (A) $1.6 \times 5\frac{1}{2} \times \0.29845
 (B) $1.6 \div 5\frac{1}{2} \times \0.29845
 (C) $1.6 \times 5\frac{1}{2} \times \29.845
 (D) $1.6 \div 5\frac{1}{2} \times \29.845

QUESTION FIFTEEN

A home loan of \$582 000 is taken out at an interest rate of 3.28% per annum. The interest is calculated monthly and is reducible. The following table shows the progress of the first two months of the loan.

Months	Principal(P)	Interest(I)	$P + I$	Balance = $P + I - R$
1	\$582 000.00	\$1590.80	\$583 590.80	\$580 280.85
2	\$580 280.85	\$1586.10	\$581 866.95	\$578 557.00

What is the regular monthly repayment R ?

- (A) \$1721.50
 (B) \$1723.85
 (C) \$3176.90
 (D) \$3309.95

QUESTION SIXTEEN

The volume V of a cone with base radius r and perpendicular height h is given by which formula?

- (A) $V = \pi r^2 \times h$
 (B) $V = 2\pi r \times h$
 (C) $V = \frac{1}{3} \times \pi r^2 \times h$
 (D) $V = \frac{1}{3} \times 2\pi r \times h$

QUESTION SEVENTEEN

The formula for the area of a triangle is:

$$A = \frac{1}{2}ab \sin C$$

Which option correctly expresses the same formula with b as the subject?

(A) $b = \frac{2a}{A \sin C}$

(B) $b = \frac{2a \sin C}{A}$

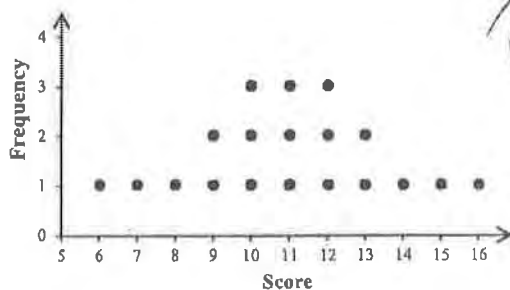
(C) $b = \frac{2A \sin C}{a}$

(D) $b = \frac{2A}{a \sin C}$

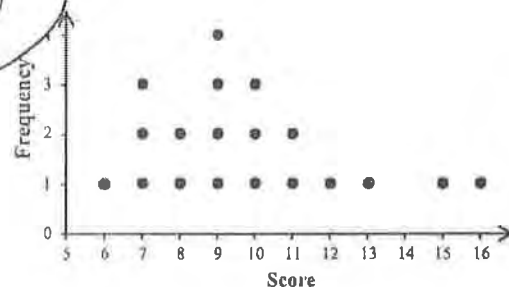
QUESTION EIGHTEEN

Which dot plot represents a data set with positive skew?

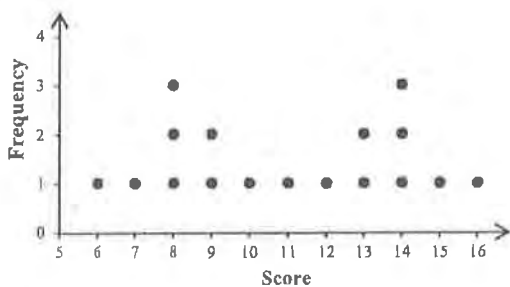
(A)



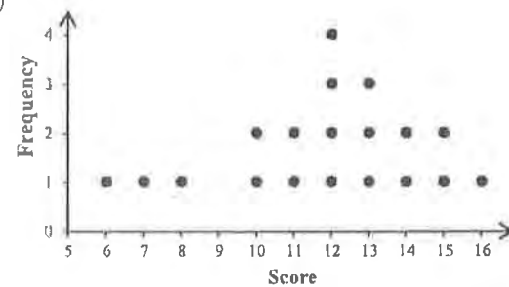
(B)



(C)

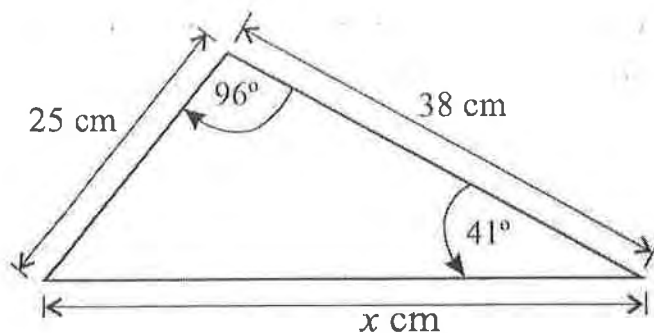


(D)



Exam continues next page ...

QUESTION NINETEEN



Which of the following would correctly calculate x in the above triangle?

(A) $x = \frac{\sin 96^\circ}{25 \times \sin 41^\circ}$

(B) $x = \frac{25 \times \sin 41^\circ}{\sin 96^\circ}$

(C) $x = 25^2 + 38^2 - 2 \times 25 \times 38 \times \cos 96^\circ$

(D) $x = \sqrt{25^2 + 38^2 - 2 \times 25 \times 38 \times \cos 96^\circ}$

QUESTION TWENTY

A coin is tossed 100 times and the result is recorded each time. Which of the following are pairs of complementary events?

P	Less than 30 heads
Q	More than 30 heads
R	An odd number of tails
S	An even number of tails

(A) P and Q only

(B) R and S only

(C) P and Q ; and also R and S

(D) None of these

QUESTION TWENTY ONE

Which is the correct explanation of the meaning of the formula below?

$$PV = \frac{FV}{(1 + r)^n}$$

- (A) Depreciating the Future Value FV of an annuity at a constant rate r for a certain number of periods n will give the equivalent Present Value PV .
- ☒ (B) Appreciating the Present Value PV of an annuity at a constant rate r for a certain number of periods n will give the equivalent Future Value FV .
- (C) Depreciating the Full Value of a reducible balance loan FV at a constant rate r for a certain number of periods n will give the monthly Payment Value PV .
- (D) Appreciating the monthly Payment Value PV of a reducible balance loan at a constant rate r for a certain number of periods n will give the Full Value FV .

QUESTION TWENTY TWO

The heights of 18-year-old males are normally distributed with a mean of 175 centimetres and a standard deviation of 9 centimetres. What percentage of 18-year-old males are taller than 193 centimetres?

- (A) 0.15%
- (B) 0.3%
- ☒ (C) 2.5%
- (D) 5.0%

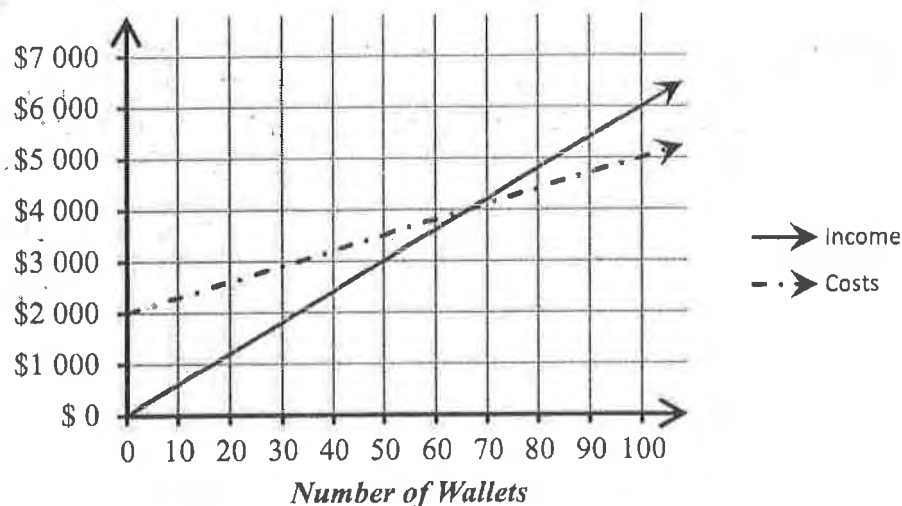
QUESTION TWENTY THREE

Two points on the earth are antipodal if they are directly opposite each other on the globe. For example, the north pole is the antipodal point to the south pole. The coordinates of Sydney Grammar School are ($33^{\circ}52'S$, $151^{\circ}13'E$). What are the correct coordinates of the point antipodal to Sydney Grammar School?

- ☒ (A) ($33^{\circ}52'N$, $28^{\circ}47'W$)
- (B) ($56^{\circ}08'N$, $28^{\circ}47'W$)
- (C) ($33^{\circ}52'N$, $151^{\circ}13'W$)
- (D) ($56^{\circ}08'N$, $151^{\circ}13'W$)

QUESTION TWENTY FOUR

Peter manufactures leather wallets. The following graph models the cost and income for his business.



Which of the following is NOT correct?

- (A) Peter's set-up costs are \$2 000.
- (B) Once he has set up his business, each wallet costs \$30 to manufacture.
- (C) He sells each wallet for \$60.
- (D) Peter will make a profit if he manufactures 60 wallets.

QUESTION TWENTY FIVE

A sample of three players is to be taken from a soccer team of eleven players. How many samples are possible?

- (A) $\frac{11^3}{3^3}$
- (B) $\frac{11!}{3!}$
- (C) $\frac{11 \times 10 \times 9}{3 \times 2 \times 1}$
- (D) $11 \times 10 \times 9$

End of Section I

Exam continues overleaf ...

SECTION II - Written Response

Answers for this section should be recorded in the space provided in this paper.

Show all necessary working.

QUESTION TWENTY SIX (15 marks)

Marks

(a) The annual gross income of four people is shown below.

Name	Benjamin	Hannah	Ryan	Sienna
Income	\$82 582	\$89 642	\$100 387	\$117 597

(i) Calculate the mean income of this group of people.

1

Solution

$$\text{Mean} = \frac{390168}{4} = \$97552$$

(ii) List all the possible samples of size $n = 3$ from the above group of people.

2

Solution

Ben, Hannah, Ryan
 Ben, Hannah, Sienna
 Ben, Ryan, Sienna
 Hannah, Ryan, Sienna

(iii) What is the mean of the sample means?

1

Solution

$= \text{pop}^n \text{mean} = \97552
 There is a long way but it's unnecessary.

QUESTION TWENTY SIX (Continued)

(b) Two six-sided dice are thrown and the numbers on the uppermost faces are added.

(i) What is the probability of a sum equal to 9? 2

Solution

$$P(9) = \frac{4}{36}$$

$$= \frac{1}{9}$$

$$E = \{(6,3), (5,4), (4,5), (3,6)\}$$

$$N(E) = 4$$

$$N(S) = 36$$

(ii) What is the probability of a sum of less than 9? 1

Solution

$$P(\text{less than } 9) = \frac{26}{36} = \frac{13}{18}$$

(iii) Isaac rolls a pair of dice, three times in a row.

(\alpha) What is the probability of Isaac getting a sum of less than 9 all three times? 1

Solution

$$P(< 9, \text{ three times}) = \left(\frac{13}{18}\right)^3 = \frac{2197}{5832} \left(\approx 38\%\right)$$

(\beta) What is the probability of Isaac getting a sum greater than or equal to 9 at least once? 1

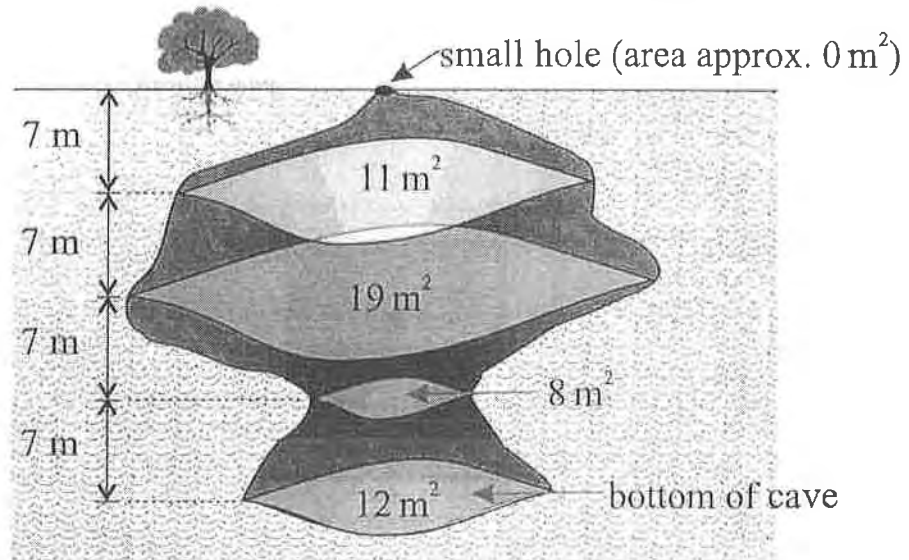
Solution

$$P(\geq 9, \text{ at least one}) = 1 - \left(\frac{13}{18}\right)^3$$

$$= \frac{3635}{5832} \left(\approx 62\%\right)$$

QUESTION TWENTY SIX (Continued)

- (c) Lucy has discovered an underground water reservoir on her farm. The areas of horizontal cross-sections, 7 metres apart, are given in the diagram below.



- (i) Use two applications of Simpson's Rule to find the approximate volume of the reservoir. 2

Solution

$$V \approx \frac{7}{3} \left((0 + 4 \times 11 + 19) + (19 + 4 \times 8 + 12) \right) \\ \approx 294 \text{ m}^3$$

- (ii) What is the approximate capacity of the reservoir in litres? 1

Solution

$$294\,000 \text{ L}$$

- (iii) Lucy can access this water for her livestock for free. The alternative is to truck water in, costing \$185 per 14 kilolitres. Assuming the reservoir is full, how much money could she save by using the water from her reservoir? 2

Solution

$$\frac{294\,000}{14\,000} \times \$185 = \$3885$$

QUESTION TWENTY SEVEN (15 marks)

Marks

(a) Solve the following equation.

2

$$\frac{x}{3} + \frac{x}{2} = 35$$

Solution

$$\frac{2x + 3x}{6} = 35$$

$$5x = 210$$

$$\underline{x = 42}$$

(b) Solve the following pair of simultaneous equations.

3

$$2A - 3B = -12 \quad \dots (1)$$

$$4A + 2B = -8 \quad \dots (2)$$

Solution

$$2 \times (1) \quad 4A - 6B = -24$$

$$(2) - 2 \times (1) \quad 0 + 8B = 16$$

$$B = 2$$

$$\text{Sub in } (1) \quad 2A - 6 = -12$$

$$2A = -6$$

$$\underline{A = -3}$$

$$(A, B) = (-3, 2)$$

QUESTION TWENTY SEVEN (Continued)

- (c) Scarlett collected some data from her classmates, specifically the earliest age in years they remember using email.

Frequency Distribution Table

Score (x) (years)	Frequency (f)	Cumulative Frequency (cf)	$f \times x$
8	2	2	16
9	6	8	54
10	8	16	80
11	11	27	121
12	0	27	0
13	1	28	13

$$\Sigma f = 28$$

$$\Sigma(f \times x) = 284$$

- (i) Complete the frequency distribution table above.
- (ii) What is the mean of this data?

2

1

Solution

$$\bar{x} = \frac{284}{28} = 10\frac{1}{7}$$

- (iii) What is the median of this data?

1

Solution

$$\text{Median} = \frac{14^{\text{th}} + 15^{\text{th}} \text{ scores}}{2} = \frac{10 + 10}{2} = 10$$

- (iv) Describe the skewness of the distribution of this data.

1

Solution

negative skew. There is a long tail towards the lower end of the data

QUESTION TWENTY SEVEN (Continued)

- (d) The following table compares data for a five countries. Listed are the birth rate x per 1000 people, and the life expectancy at birth y in years.

Country	Birth Rate x	Life Expectancy y
Australia	1.92	82.2
Cambodia	2.86	71.8
Japan	1.43	93.3
New Zealand	1.95	81.4
Thailand	1.40	74.4

Data from the World Bank, accessed through Google Public Data.

- (i) What is the correlation coefficient of this data? Express your answer correct to three significant figures. 1

Solution

$$r \approx -0.569$$

- (ii) What is the mean and standard deviation of each variable? Express your answers correct to three significant figures. 2

Solution

$$\bar{x} \approx 1.91$$

$$\bar{y} \approx 80.6$$

$$\sigma_x \approx 0.528$$

$$\sigma_y \approx 7.49$$

- (iii) What is the equation of the least-squares line of best fit of the above data? Express your answer correct to three significant figures. 1

Solution

$$y \approx -8.06x + 96.0$$

- (iv) Describe the correlation between the birth rate and life expectancy for these countries. 1

Solution

weak/moderate negative correlation

QUESTION TWENTY EIGHT (15 marks)**Marks**

- (a) The coordinates of Sydney Grammar School are ($33^{\circ}52'S$, $151^{\circ}13'E$). The coordinates of a bay on the coast of Antarctica are ($75^{\circ}52'S$, $28^{\circ}47'W$).



- (i) Identify which great circle these two points lie on. Justify your answer with reference to their coordinates. 2

Solution

They lie on the same longitudinal meridian
 since $151^{\circ}13'E$ and $28^{\circ}47'W$ are on the same meridian,
 since
$$\begin{array}{r} 151^{\circ}13' \\ 28^{\circ}47' + \\ \hline 180^{\circ}0' \end{array}$$

- (ii) Assuming that the earth is a perfect sphere with a radius of 6400 kilometres, find the shortest distance along the surface of the earth between these two places. Express your answer correct to the nearest kilometre. 3

Solution

$$\begin{aligned} \text{Distance} &= (\text{SGS to South Pole}) + (\text{South Pole to Bay}) \\ &= \frac{(90^{\circ} - 33^{\circ}52') + (90^{\circ} - 75^{\circ}52')}{360^{\circ}} \times 2 \times \pi \times 6400 \text{ km} \\ &= \frac{70^{\circ}16'}{360^{\circ}} \times 2 \times \pi \times 6400 \text{ km} \approx \underline{\underline{7849 \text{ km}}} \end{aligned}$$

QUESTION TWENTY EIGHT (Continued)

For the following questions you can refer to the Present Value and Future Value tables attached to your formula sheet.

- (b) Matilda is pregnant, and she is planning to invest some money at the end of every month after the birth of her child into an account in their name. In making her calculations, Matilda assumes that the interest rate will remain constant at 9.00% per annum, compounded monthly.

- (i) If she invests \$300 at the end of every month, how much will be in the account after 5 years? 2

Solution

$$r = \frac{9\%}{12} = 0.75\% \text{ per month, } n = 60 \text{ months} \Rightarrow \$75.4241$$

$$\text{So } \$75.4241/\$1 \times \$300 = \$22\,627.23$$

- (ii) How much will she need to invest each month so that the balance of the account will be \$50 000 after 5 years? 1

Solution

$$\frac{\$50\,000}{75.4241} \approx \$662.92$$

- (iii) With a bit of research, Matilda discovers that there is a formula for the future value FV of an annuity: 2

$$FV = a \left(\frac{(1+r)^n - 1}{r} \right)$$

where a is the contribution per period, paid at the end of the period,
 r is the interest rate per compounding period, and
 n is the number of compounding periods.

How much will Matilda need to invest at the end of each month in order for her child to have \$350 000 when they turn 18 years old?

Solution

$$n = 18 \times 12 = 216$$

$$\$350\,000 = a \left(\frac{(1+0.75\%)^{216} - 1}{0.75\%} \right)$$

$$a \approx \$652.56$$

- (c) Ivy is an investigator for the Australian Competition and Consumer Commission, and she has received a complaint about the volumes of cartons of a certain brand of milk that are labelled "1 litre". She obtains a systematic sample of the cartons of milk, and the volumes of the milk contained in them is recorded below. All measurements are in millilitres.

1034	1014	1028	1002	1027	1019
1021	1015	1023	1021	1022	1037

- (i) What is the mean and standard deviation of this data? Express your answer correct to the nearest 0.1 millilitre. 2

Solution

$$\bar{x} \approx 1021.9 \text{ mL}$$

$$s \approx 8.9 \text{ mL}$$

- (ii) Ivy uses this sample to make inferences about all of the cartons sold by the brand. What might be the problem with this approach? 1

Solution

Sample, even though systematic, is very small
 $\therefore$ could easily be unreliable / skewed.

- (iii) The company selling this milk claims on its website that, "Less than one in a thousand of our cartons contain less than one litre of milk." Ignoring the possible problems you have identified above, is Ivy able to report to the public that her data supports this claim? Justify your answer with reference to the statistics, assuming that the data is normally distributed. 2

Solution

$$\text{if } z = -3 \Rightarrow \text{volume} \approx 995.2 \text{ mL}$$

$\therefore 0.15\%$ of cartons contain $< 995.2 \text{ mL}$

$\therefore$ CANNOT conclude that $< 0.1\%$ of
 cartons contain less than 1L of milk

QUESTION TWENTY NINE (15 marks)

Marks

(a) A data set of 49 scores have been collected in a table as shown below.

Row 1	23	44	59	42	52	21	57
Row 2	71	67	50	47	59	43	50
Row 3	64	31	65	22	71	69	56
Row 4	30	62	54	62	36	57	29
Row 5	45	71	46	69	11	85	44
Row 6	33	59	34	46	38	24	32
Row 7	72	55	58	54	13	45	30

- (i) A random sample is generated by taking all the values from row 3 of the table. What is the sample mean? Express your answer correct to one decimal place. 1

Solution

$$\text{Sample mean} = 54.0$$

- (ii) A random sample is generated by taking all of the values from row 3 and row 1 of the table. What is this sample mean? Express your answer correct to one decimal place. 1

Solution

$$\text{Sample mean} = 48.3$$

- (iii) The population mean is 48.1. Which sample has a mean closer to the population mean? 1

Solution

Sample mean from Row 1 and Row 3

- (iv) Explain why this was likely to be the case. 1

Solution

larger sample is likely to give a closer approximation to the mean of the population.

QUESTION TWENTY NINE (Continued)

- (b) Xavier is buying a car with a purchase price of \$19 000, and he has to pay a 15% deposit immediately. He will take out a loan from Calcubank for the balance. They inform him he can pay off the loan with 30 monthly payments of \$613.70.

(i) What is the total amount that Xavier will end up paying for the car?

2

Solution

$$\begin{aligned} \$19\,000 \times 15\% + \$613.70 \times 30 \\ = \underline{\underline{\$21\,261}} \end{aligned}$$

(ii) What is the annual simple interest rate that Calcubank is charging Xavier?

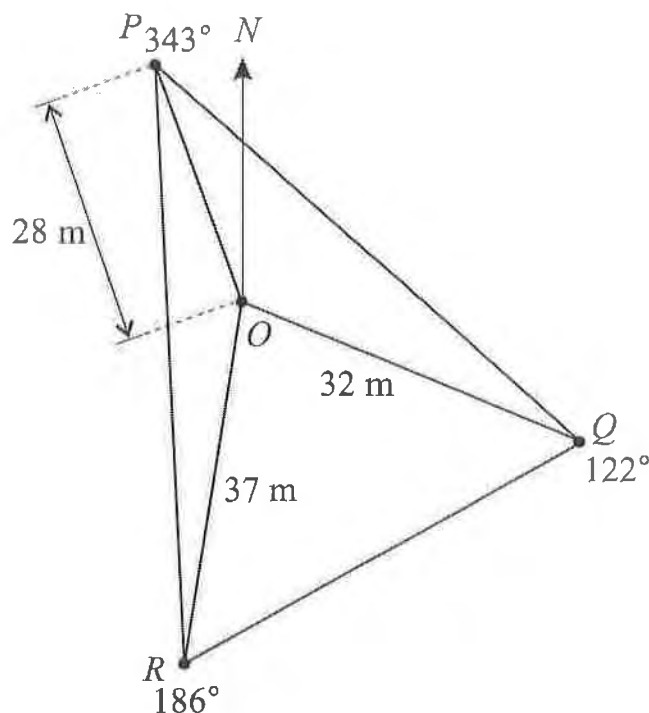
3

Solution

$$\begin{aligned} \text{Amount borrowed} &= 85\% \times 19\,000 \\ &= 16\,150 \\ \text{Amount repaid} &= 613.7 \times 30 = 18\,411 \\ \therefore \text{Interest paid} &= 18\,411 - 16\,150 \\ &= 2\,261 \\ I &= PRN \\ 2\,261 &= 16\,150 \times R \times 2\frac{1}{2} \text{ years} \\ R &= 5.6\% \text{ per annum.} \end{aligned}$$

QUESTION TWENTY NINE (Continued)

(c) A radial survey of a triangular piece of land PQR is shown below.



(i) What is the size of $\angle POQ$?

1

Solution

$$360^\circ - 343^\circ = 17^\circ$$

$$17^\circ + 122^\circ = 139^\circ$$

(ii) What is the area of $\triangle POQ$? Express your answer correct to the nearest square metre.

2

Solution

$$A = \frac{1}{2} \times 28 \times 32 \times \sin 139^\circ$$

$$\approx 294 \text{ m}^2$$

(iii) What is the area of $\triangle PQR$? Express your answer correct to the nearest square metre.

3

Solution

$$A(\triangle POQ) = \frac{1}{2} \times 32 \times 37 \times \sin (186^\circ - 122^\circ)$$

$$\approx 532 \text{ m}^2$$

$$A(\triangle ROP) = \frac{1}{2} \times 37 \times 28 \times \sin (343^\circ - 186^\circ)$$

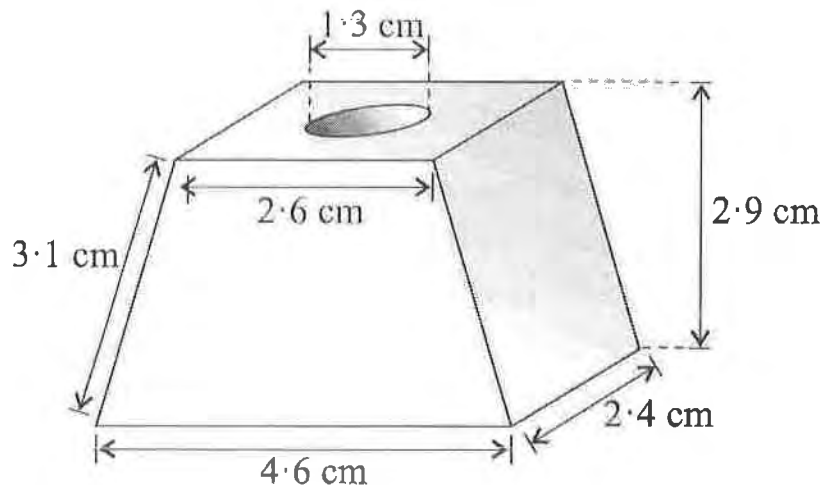
$$\approx 202 \text{ m}^2$$

$$\text{Total Area (PQR)} \approx 294 + 532 + 202 \approx 1028 \text{ m}^2$$

QUESTION THIRTY (15 marks)

Marks

- (a) A trapezoidal prism has a cylindrical hole with a diameter of 1.3 centimetres drilled vertically through it, as shown in the diagram.



- (i) What is the area of the trapezoidal front face of the prism?

2

Solution

$$A = \frac{1}{2} \times 2.9 \times (2.6 + 4.6) = 10.44 \text{ cm}^2$$

- (ii) What was the volume of the trapezoidal prism before the hole was drilled through it?

1

Solution

$$V = 10.44 \times 2.4 = 25.056 \text{ cm}^3$$

- (iii) What is the volume of material remaining after the hole was drilled through it? Express your answer correct to 2 significant figures.

2

Solution

$$V = 25.056 - \left(\pi \times \left(\frac{1.3}{2} \right)^2 \times 2.9 \right)$$

$$\approx 21 \text{ cm}^3$$

QUESTION THIRTY (Continued)

(b) Following is some data on the Australian population.

Year Y	1963	2013
Population P (millions)	10.95	23.13

- (i) Oscar naively assumes that the relationship between the year and population is linear. Using this assumption, what would be the estimated population in 1988, by interpolation? 2

Solution

1988 is halfway b/w 1963 and 2013

$$\therefore \text{Interpolate} = \frac{10.95 + 23.13}{2} = 17.04 \text{ million}$$

- (ii) Willow is a bit more canny when it comes to population, so she assumes that the population grows exponentially according to the formula 2

$$P = 10.95 \times k^{(Y-1963)}.$$

Use the 2013 data to calculate k . Express your answer correct to four significant figures.

Solution

$$23.13 = 10.95 \times k^{(2013-1963)}$$

$$k^{50} = \frac{23.13}{10.95}$$

$$k = \left(\frac{23.13}{10.95} \right)^{\frac{1}{50}} \approx 1.015$$

- (iii) What would be Willow's interpolation for the population in 1988? Express your answer correct to four significant figures. 1

Solution

$$P = 10.95 \times 1.015^{(1988-1963)}$$

$$\approx \underline{15.89 \text{ million}}$$

QUESTION THIRTY (Continued)

- (c) There are 52 cards in a pack of regular playing cards. There are four suits of thirteen cards each. Each suit has nine number cards (from 2 to 10), three royal cards (King, Queen, Jack), and one Ace.

Levi takes three random cards from a shuffled pack, and places them face up on the table.

- (i) What is the probability that he has exactly three number cards?

1

Solution

$$\frac{36}{52} \times \frac{35}{51} \times \frac{34}{50} = \frac{21}{65}$$

- (ii) What is the probability that he has exactly two number cards?

2

Solution

$$\left(\frac{36}{52} \times \frac{35}{51} \times \frac{16}{50} \right) \times 3$$

$$= \frac{504}{1105}$$

QUESTION THIRTY (Continued)

- (iii) Levi invents a gambling game and invites Zara to play against him. She has to give him \$1 to play, and then she takes three random cards from a shuffled pack, as above. If Zara then holds either zero or two number cards, Levi has to pay her \$2. Otherwise, Levi keeps the \$1. What is Zara's average financial expectation for each round of this game? 2

Solution

$$P(\text{Zara win}) = P(0) + P(2)$$

$$= \frac{16}{52} \times \frac{15}{51} \times \frac{14}{50} + \frac{504}{1105}$$

$$= \frac{532}{1105}$$

$$\therefore \text{Zara's Expectation} = (+\$1) \times P_{\text{win}} + (-\$1) \times P_{\text{loss}}$$

$$= 1 \times \frac{532}{1105} - 1 \times \frac{573}{1105}$$

$$= -\frac{41}{1105} \approx -\$0.04$$

$\therefore$ She expects to lose an average of 4¢ per game.

————— End of Section II —————

END OF EXAMINATION