2025 **MOCK HSC EXAM**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–4)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 5–27)

- Attempt Questions 11–31
- Allow about 2 hours 45 minutes for this section

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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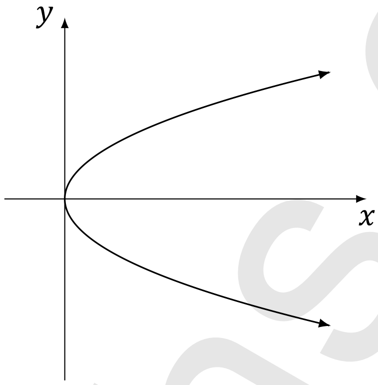
1 Consider the following relationship between x and y:



Which of the following best describes this relationship?

A. one-to-one
B. many-to-one
C. one-to-many
D. many-to-many

2 The terms

$$\log_2 x, \log_2 2, \log_2(5-x)$$

form an arithmetic sequence. Which of the following is a possible value of x?

A. 2
B. 3
C. 4
D. 5

3 What is the value of $\displaystyle\int_1^5 |x-2|\, dx$?

A. 2
B. 3
C. 4
D. 5

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4 The range of a function $f(x)$ which has domain all real x is $[-2, \infty)$.

What is the range of the function $g(x) = 3f(-2x) - 4$?

A. $(-\infty, 4]$ B. $[-10, \infty)$ C. $[-2, \infty)$ D. $(-\infty, 6]$

5 Let C and D be two events with $P(C) = 0.2, P(D) = 0.5$, and $P(C|D) = 0.3$. What is the value of $P(D|C)$?

A. 0.75 B. 0.6 C. 0.5 D. 0.3

6 The hair colour and eye colour of a class of 30 students were recorded. Students were grouped by the categories brown eyes and black hair. This is shown in the table below.

	Brown Eyes	**Not Brown Eyes**
Black Hair	12	2
Not Black Hair	10	6

A person from the class is chosen at random. Given that the person chosen has black hair, what is the probability that DO NOT have brown eyes?

A. $\frac{1}{4}$ B. $\frac{1}{6}$

C. $\frac{1}{7}$ D. $\frac{1}{15}$

7 Which of the following is an odd function?

A. $\frac{1}{x}\sin\left(\frac{1}{x}\right)$ B. $x^4 + x^2 + 1$

C. $x^5 + x^3 + x\ln x$ D. $\frac{1}{2} - \frac{1}{2^x + 1}$

8 If $f'(x) = \frac{4x}{4 + x^2}$ and $f(2) = f'(2)$, the value of $f(0)$ is:

A. $2\ln 2 + 1$ B. $4\ln 2 - 1$ C. $1 - 2\ln 2$ D. $1 - 4\ln 2$

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9 The graph of $y = x^3$ is dilated horizontally by a factor of $\frac{1}{h}$ and dilated vertically by a factor of k to $y = 1000x^3$.

Which of the following are possible values of h and k?

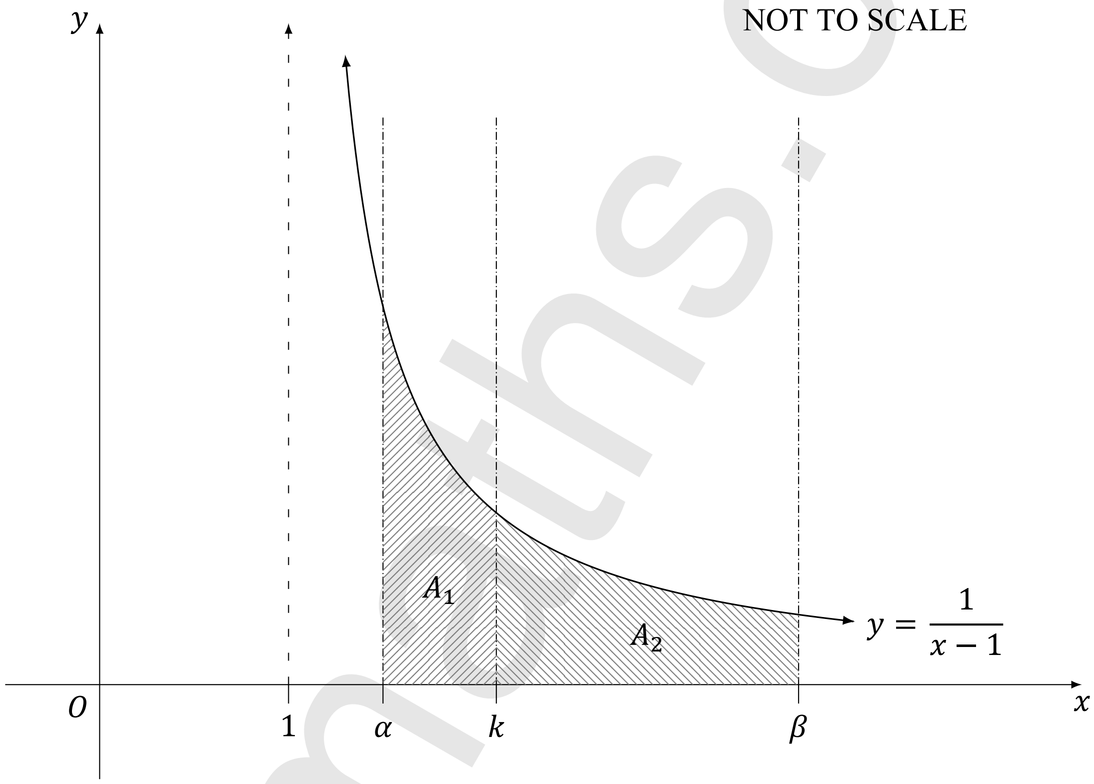
A. $h = 5, k = 8$

B. $h = 8, k = 5$

C. $h = 5, k = 50$

D. $h = 50, k = 5$

10 Youssef sketches the following curve and three vertical lines, where $1 < \alpha < k < \beta$.



It is known that the shaded areas A_1 and A_2 are equal. Which of the following must be true?

A. $k = 1 + \frac{\alpha + \beta}{2}$

B. $k(k-2) = \alpha\beta + \alpha + \beta$

C. $k = 1 + \sqrt{(\alpha - 1)(\beta - 1)}$

D. $k = \frac{\alpha + \beta}{2}$

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Section II

90 marks
Attempt Questions 11–31
Allow about 2 hours 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Evaluate $\int x\sqrt{x^2+4}\,dx$. **2**

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Question 12 (3 marks)

If $f(x) = \dfrac{1}{e^x \sin x}$, find the equation of the tangent at $x = \dfrac{\pi}{2}$. **3**

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Question 13 (2 marks)

Differentiate $y = \ln\left(\frac{2x+1}{x^2+1}\right)$. **2**

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Question 14 (3 marks)

Sketch the graph of $y = \frac{2x+1}{x-2}$, showing all important features. **3**

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Question 15 (6 marks)

Let $f(x) = e^{2x} - 4e^{x}$.

(a) Find the x–intercepts of $y = f(x)$. **1**

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(b) Find all the turning points of $f(x)$, and determine their nature. **3**

You may use the fact that $f'(x) = 2e^{2x} - 4e^{x}$ and $f''(x) = 4e^{2x} - 4e^{x}$ without proof.

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(c) Sketch $y = f(x)$, showing all important features. **2**

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Question 16 (3 marks)

Let $y = \dfrac{\ln x}{x}$.

(a) Find $\dfrac{dy}{dx}$. **1**

(b) Hence, evaluate $\displaystyle\int \frac{\ln x}{x^2}\,dx$. **2**

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Question 17 (4 marks)

Let $f(x) = \dfrac{\sec x - \cos x}{\sec x + 1}$.

(a) Show that $\dfrac{\sec x - \cos x}{\sec x + 1} = 1 - \cos x$. **2**

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(b) Hence, graph $y = f(x)$ for $0 \leq x \leq 2\pi$. **2**

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Question 18 (5 marks)

Bob has won a small lottery and has been given two withdrawal options.

Option 1: Receive a lump sum of $\$1\,000\,000$.

Option 2: Receive a lump sum of $\$500\,000$, and then receive annual payments of $\$60\,000$ at the end of the year (including the first year) for ten years.

Bob will immediately deposit any payments into a bank account that earns 1.2% interest compounded monthly.

(a) What is the value of Bob's earnings after ten years under Option 1? **1**

(b) What is the difference in the future earnings after ten years using Option 1 or Option 2? **3**

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(c) What is the *present value* of Option 2? **1**

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Question 19 (4 marks)

NSW Mathematics is reviewing the punctuality of their students. On any given day, there is a 60% chance that the weather condition is 'good'. If the weather is 'good' then the probability that any given student arrives on time is 90%. If the weather is not 'good' then this probability drops to 60%.

Let G and A denote the events where the weather is 'good' and a given student arrives on time respectively.

(a) Draw a tree diagram to represent this situation. **1**

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(b) Show that the probability that a given student arrives on time is 0.78. **2**

(c) Given that a randomly chosen student arrived on time, determine the probability that the weather was good. **1**

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Question 20 (3 marks)

Let $f(x) = a\sin x + b\cos x$, where a and b are constants. If $f\left(\frac{\pi}{4}\right) = 3$ and $f'\left(\frac{\pi}{4}\right) = 1$, solve the equation $f(x) = 0$, for $0 \le x \le 2\pi$, correct to two significant figures. **3**

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Question 21 (3 marks)

The function $f(x) = e^{x^2}$ is continuous on the interval $[-1, 1]$. Use the trapezoidal rule with four subintervals to estimate the value of $\int_{-1}^{1} e^{x^2}\, dx$. **3**

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Question 22 (4 marks)

oneeightseven records the number of hours spent training per week and the resting heart rate (in beats per minute) of six participants.

Training Hours (T)	2	3	4	6	7	10
Resting Heart Rate (H)	85	82	80	76	74	70

(a) Find the equation of the least-squares regression line, rounding coefficients to the nearest integer, and use the equation to estimate the resting heart rate of someone who trains for 5 hours a week. **3**

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(b) oneeightseven trains for 25 hours a week to prepare for a marathon. Explain why the regression equation from part (a) may not be appropriate for estimating oneeightseven's resting heart rate. **1**

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Question 23 (5 marks)

Consider the two curves $y = \ln x + 8x$ and $y = \ln x + x^2 + 7$.

(a) Find the x value(s) at which the two curves intersect. **2**

(b) Hence, find the area bound between the two curves. **3**

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Question 24 (5 marks)

A discrete random variable X has the following probability distribution table

x	0	1	2	3	4
$P(X = x)$	a	$2a$	b	0.2	c

where a, b and c are constants.

It is known that $E(X) = 2$.

(a) By finding expressions for b and c in terms of a, show that $0.05 \leq a \leq 0.18$. **3**

(b) Hence deduce the maximum possible value for the variance of X. **2**

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Question 25 (7 marks)

Abdullah wishes to purchase an item which she can resell for one dollar. Benjamin is selling this item and believes that the item is worth X dollars where X is a random variable with probability density

$$f(x) = \begin{cases} \alpha + \beta(2x - 1) & : 0 \leq x \leq 1 \\ 0 & : \text{otherwise} \end{cases}$$

where α and β are constants.

(a) Show that $\alpha = 1$ and find the possible values of β for which $f(x)$ is a valid probability density function. **2**

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(b) Abdullah places a bid b for the item and will *gain* the item only if her bid is *above* Benjamin's valuation of the item. Otherwise she will *lose* her bid. **1**

Show that her expected profit, E, is given by

$$E = F(b) - b$$

where $F(x)$ is the cumulative density function (cdf) of X.

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(c) Suppose $\beta > 0$. Explain why Abdullah should either bid $b = 0$ or $b = 1$. **2**

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(d) Suppose $\beta < 0$. Find the value of b that maximises Abdullah's expected profit and find the expected profit in terms of β. **2**

You do NOT need to show that this is a maximum.

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Question 26 (4 marks)

Show that **4**

$$2 + 22 + 222 + 2222 + \cdots + \underbrace{2222\ldots2}_{n\text{ 2's}} = \frac{2}{81}(10^{n+1} - 9n - 10).$$

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Question 27 (4 marks)

Let k be a real number. For what values of k does the equation $kx^2 + 3(k-2)x = 9k - 9$ have two real roots for x? **4**

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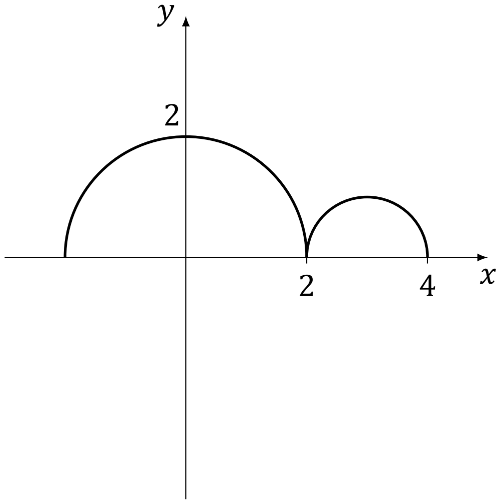
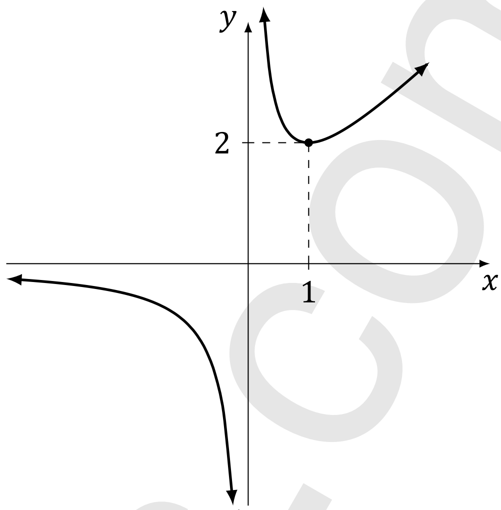
Question 28 (4 marks)

The graphs of $y = f(x)$ and $y = g(x)$ are given below. **4**



$y = f(x)$



$$y = g(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ x + \dfrac{1}{x} & \text{if } x > 0 \end{cases}$$

Find the domain and range of $f\big(g(x)\big)$.

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Question 29 (5 marks)

Consider a random variable X that is normally distributed with mean 15 and standard deviation σ. It is known that $P(X < 10) = a$ and $P(X > 25) = b$.

(a) Find $P(20 < X < 25)$ in terms of a and b. **2**

(b) Consider a random variable Y that is normally distributed with mean 25 and standard deviation 3σ. Find two pairs (m, n) such that $P(m < Y < n) = 1 - a - b$. **3**

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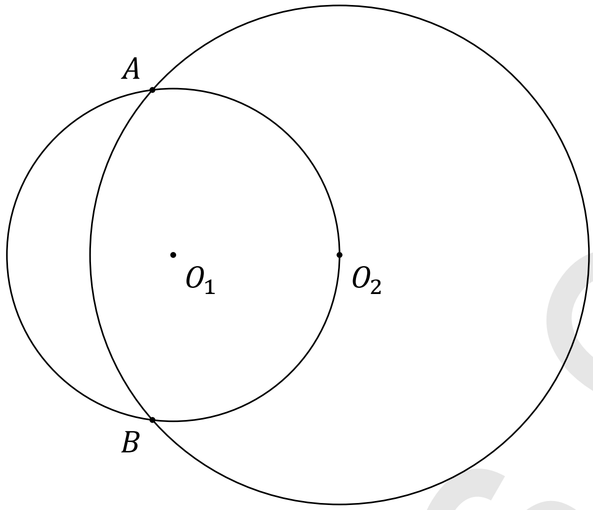
Question 30 (5 marks)

In the figure below, the graphs of $C_1 : x^2 + y^2 = 4$ with centre O_1 and $C_2 : (x-2)^2 + y^2 = 9$ with centre O_2 are shown without the coordinate axes. Their intersections are labelled A and B.



(a) Find the value of $\cos \angle AO_1O_2$. **1**

Question 30 continues on page 24

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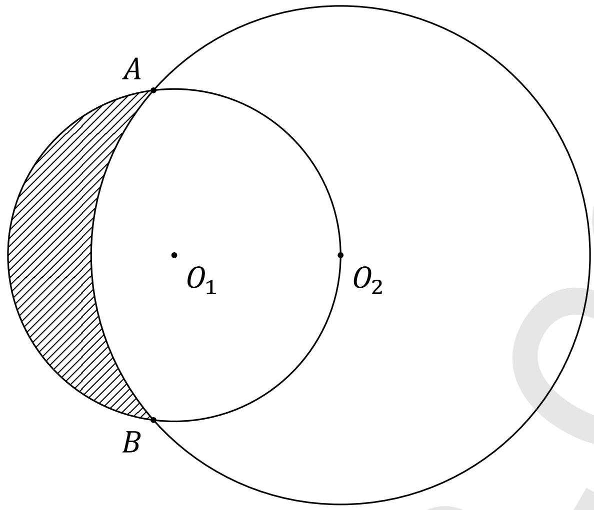
Question 30 (continued)

(b) Hence, or otherwise, find the shaded area shown below. **4**



Give your answer correct to 2 decimal places.

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Question 31 (9 marks)

In the year 2050, NSWMaths manufactures and sells Ruiace sip stickers. Suppose

$$C(q) = \frac{7q^3 - 1000q^2 + 126000q}{100000} + 84$$

is the cost in millions of dollars to produce and sell q million units (you may assume this is the only cost). It is also known that the price each sticker can be sold for can be modelled by $D(q) = b - aq$, where a and b are positive constants. Define $P(q)$ to be the profit in millions of dollars associated with selling q million stickers.

(a) From historical data, when 200 million units were produced, each unit sold for $\$1.50$, and when 50 million units were produced and sold, a profit of $\$44.25$ million was made.

(i) Explain why $b = 1.5 + 200a$. **1**

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(ii) Write down an expression for $P(q)$ in terms of a, b, and q. **1**

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Question 31 continues on page 26

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(iii) Hence, show that **3**

$$P(q) = q\left(\frac{25}{6} - \frac{q}{75}\right) - \left(\frac{7q^3 - 1000q^2 + 126000q}{100000} + 84\right).$$

Question 31 continues on page 27

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Question 31 (continued)

(b) Find the maximum profit the company can make, to the nearest million. **4**

You must justify why your answer is a maximum.

End of paper

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I want to learn more and more to see as beautiful what is necessary in things;
then I shall be one of those who make things beautiful.

— Friedrich Nietzsche, Die fröhliche Wissenschaft.

EXAMINERS

one eight seven	Q2, 5, 20, 21, 22, 29
MokSifu	Q1, 6, 7, 9, 11, 13, 14, 17, 18, 24, 28, 30
Hugh Entwistle	Q18, 19, 24, 25
MagiicMushroom	Everything else.

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2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

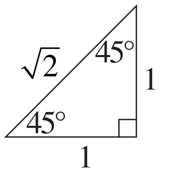
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

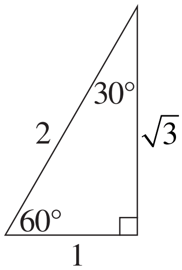


$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

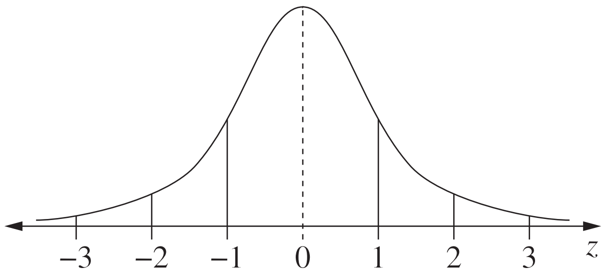
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx} dx = uv - \int v\frac{du}{dx} dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2\,,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$

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2025 **MOCK HSC EXAM**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–4)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 5–27)

- Attempt Questions 11–31
- Allow about 2 hours 45 minutes for this section

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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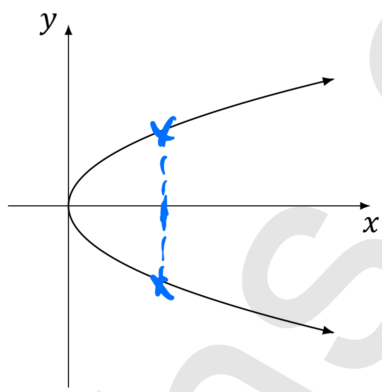
1 Consider the following relationship between x and y:



one x value
going to many x values

Which of the following best describes this relationship?

A. one-to-one
B. many-to-one
C. one-to-many
D. many-to-many

2 The terms $\log_2 2 - \log_2 x = \log_2(5-x) - \log_2 2$

$$\log_2 x, \log_2 2, \log_2(5-x)$$

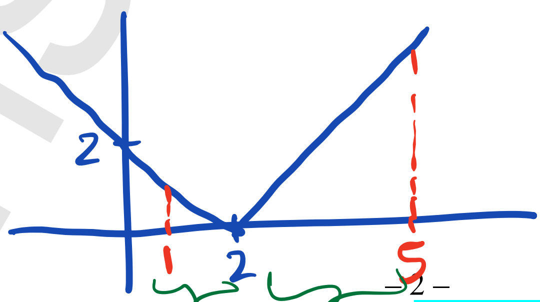
form an arithmetic sequence. Which of the following is a possible value of x?

A. 2
B. 3
C. 4
D. 5

$\Rightarrow \dfrac{x}{2} = \dfrac{2}{5-x} \Rightarrow -x^2+5x=4 \Rightarrow (x-4)(x-1)=0$

3 What is the value of $\displaystyle\int_1^5 |x-2|\,dx$?

A. 2
B. 3
C. 4
D. 5



Areas
$\frac{1}{2}\times1\times1+\frac{1}{2}\times3\times3=5$

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$[2, \infty) \xrightarrow{\text{dilate } \times 3} [-6, \infty) \xrightarrow{\text{translate } -4} [-10, \infty)$

4 The range of a function $f(x)$ which has domain all real x is $[-2, \infty)$.

What is the range of the function $g(x) = 3f(-2x) - 4$?

$-2x$ irrelevant.

A. $(-\infty, 4]$ B. $[-10, \infty)$ C. $[-2, \infty)$ D. $(-\infty, 6]$

5 Let C and D be two events with $P(C) = 0.2$, $P(D) = 0.5$, and $P(C|D) = 0.3$. What is the value of $P(D|C)$?

A. 0.75 B. 0.6 C. 0.5 D. 0.3

$$P(D|C) = \frac{P(C \cap D)}{P(C)} = \frac{P(C|D)P(D)}{P(C)} = \frac{0.3 \times 0.5}{0.2}$$

6 The hair colour and eye colour of a class of 30 students were recorded. Students were grouped by the categories brown eyes and black hair. This is shown in the table below.

	Brown Eyes	**Not Brown Eyes**
Black Hair	12	2
Not Black Hair	10	6

→ 14 total

A person from the class is chosen at random. Given that the person chosen has black hair, what is the probability that DO NOT have brown eyes?

$\frac{2}{14} = \frac{1}{7}$

A. $\frac{1}{4}$ B. $\frac{1}{6}$

C. $\frac{1}{7}$ D. $\frac{1}{15}$

7 Which of the following is an odd function?

A. $\frac{1}{x}\sin\left(\frac{1}{x}\right)$ even B. $x^4 + x^2 + 1$ even

C. $x^5 + x^3 + x\ln x$ neither D. $\frac{1}{2} - \frac{1}{2^x + 1}$ see next page for proof

8 If $f'(x) = \frac{4x}{4 + x^2}$ and $f(2) = f'(2)$, the value of $f(0)$ is:

$f'(2) = 1$

A. $2\ln 2 + 1$ B. $4\ln 2 - 1$ C. $1 - 2\ln 2$ D. $1 - 4\ln 2$

$$f(x) = 2\int \frac{2x}{4+x^2}\,dx$$
$$= 2\ln(4+x^2) + c.$$

$f(2) = 1 \Rightarrow 1 = 2\ln(8) + c$

$\Rightarrow f(x) = 2\ln(4+x^2) + 1 - 2\ln 8$

$f(0) = 2\ln 4 + 1 - 2\ln 8$

$= 1 + 2\ln\frac{1}{2}$

$= 1 - 2\ln 2$

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Let $f(x) = \frac{1}{2} - \frac{1}{2^x+1}$.

Then $f(-x) = \frac{1}{2} - \frac{1}{2^{-x}+1} \times \frac{2^x}{2^x}$

$$= \frac{1}{2} - \frac{2^x}{1+2^x}$$

$$= \frac{(1+2^x) - 2(2^x)}{2(2^x+1)}$$

$$= \frac{1-2^x}{2(2^x+1)}$$

$$= \frac{2-2^x-1}{2(2^x+1)}$$

$$= \frac{\cancel{2}}{\cancel{2}(2^x+1)} - \frac{\cancel{(2^x+1)}}{2\cancel{(2^x+1)}}$$

$$= \frac{1}{2^x+1} - \frac{1}{2}$$

$$= -f(x).$$

$\Rightarrow f(x)$ is odd.

... I assume this means $y = k(hx)^3$

9 The graph of $y = x^3$ is dilated horizontally by a factor of $\frac{1}{h}$ and dilated vertically by a factor of k to $y = 1000x^3$.

Which of the following are possible values of h and k?

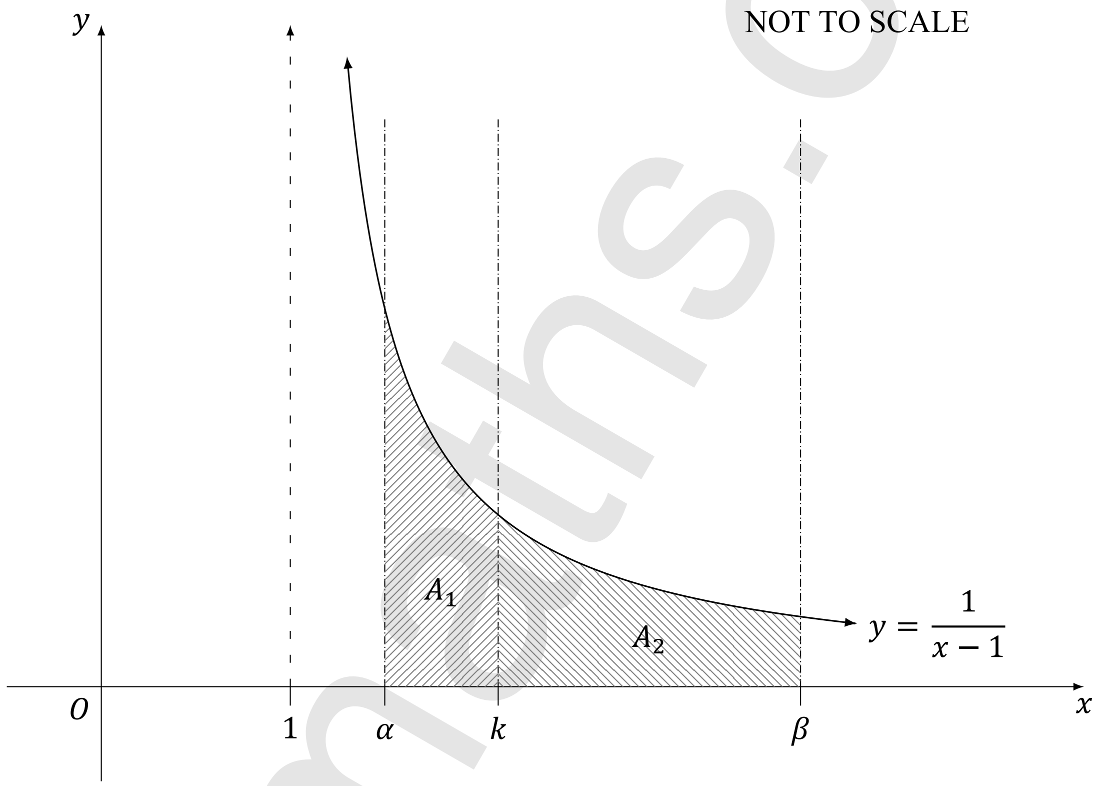
A. $h = 5, k = 8$ $5^3 \times 8 = 1000$

B. $h = 8, k = 5$

C. $h = 5, k = 50$

D. $h = 50, k = 5$

10 Youssef sketches the following curve and three vertical lines, where $1 < \alpha < k < \beta$.



It is known that the shaded areas A_1 and A_2 are equal. Which of the following must be true?

A. $k = 1 + \frac{\alpha + \beta}{2}$

B. $k(k-2) = \alpha\beta + \alpha + \beta$

C. $k = 1 + \sqrt{(\alpha - 1)(\beta - 1)}$

D. $k = \frac{\alpha + \beta}{2}$

BAIT.

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$$\int_{\alpha}^{k} \frac{1}{x-1}\,dx = \int_{k}^{\beta} \frac{1}{x-1}\,dx$$

$$\left[\ln(x-1)\right]_{\alpha}^{k} = \left[\ln(x-1)\right]_{k}^{\beta}$$

drop absolute value since we know $\beta > k > \alpha \boxed{> 1}$.

$$\ln\left(\frac{k-1}{\alpha-1}\right) = \ln\left(\frac{\beta-1}{k-1}\right)$$

log laws

$$\frac{k-1}{\alpha-1} = \frac{\beta-1}{k-1}$$

$$(k-1)^2 = (\alpha-1)(\beta-1)$$

$$k-1 = \sqrt{(\alpha-1)(\beta-1)}$$

again, $k-1>0$ is because $k>1$ given.

$$k = 1+\sqrt{(\alpha-1)(\beta-1)}$$

Section II

90 marks
Attempt Questions 11–31
Allow about 2 hours 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Evaluate $\int x\sqrt{x^2+4}\,dx$. **2**

$\frac{1}{2}\int 2x\,(x^2+4)^{\frac{1}{2}}\,dx = \frac{1}{2}\cdot\frac{(x^2+4)^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{1}{3}\sqrt{(x^2+4)^3} + c$

Should be ok to leave power as $\frac{3}{2}$.

Question 12 (3 marks)

If $f(x) = \frac{1}{e^x \sin x}$, find the equation of the tangent at $x = \frac{\pi}{2}$. **3**

$f(x) = (e^x \sin x)^{-1}$

$f'(x) = -(e^x \sin x)^{-2}\cdot(e^x \sin x + e^x \cos x).$

$f(\frac{\pi}{2}) = \frac{1}{e^{\frac{\pi}{2}}\sin\frac{\pi}{2}} = e^{-\frac{\pi}{2}}$

$f'(\frac{\pi}{2}) = -(e^{\frac{\pi}{2}}\sin\frac{\pi}{2})^{-2}\cdot(e^{\frac{\pi}{2}}\underbrace{\cancel{\sin\frac{\pi}{2}}}_{=1} + \cancel{e^{\frac{\pi}{2}}}\underbrace{\cancel{\cos\frac{\pi}{2}}}_{=0})$

$= -e^{-\frac{\pi}{2}}$

$\Rightarrow$ Tangent is $y - e^{-\frac{\pi}{2}} = -e^{-\frac{\pi}{2}}(x - \frac{\pi}{2})$

$y = -e^{-\frac{\pi}{2}}x + e^{-\frac{\pi}{2}}(1 + \frac{\pi}{2})$

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Question 13 (2 marks)

Differentiate $y = \ln\left(\dfrac{2x+1}{x^2+1}\right)$. **2**

$y = \ln(2x+1) - \ln(x^2+1)$

$\dfrac{dy}{dx} = \dfrac{2}{2x+1} - \dfrac{2x}{x^2+1}$

$= \dfrac{2(x^2+1) - 2x(2x+1)}{(2x+1)(x^2+1)}$

$= \dfrac{-2x^2 - 2x + 2}{(2x+1)(x^2+1)}$

Question 14 (3 marks)

Sketch the graph of $y = \dfrac{2x+1}{x-2}$, showing all important features. **3**

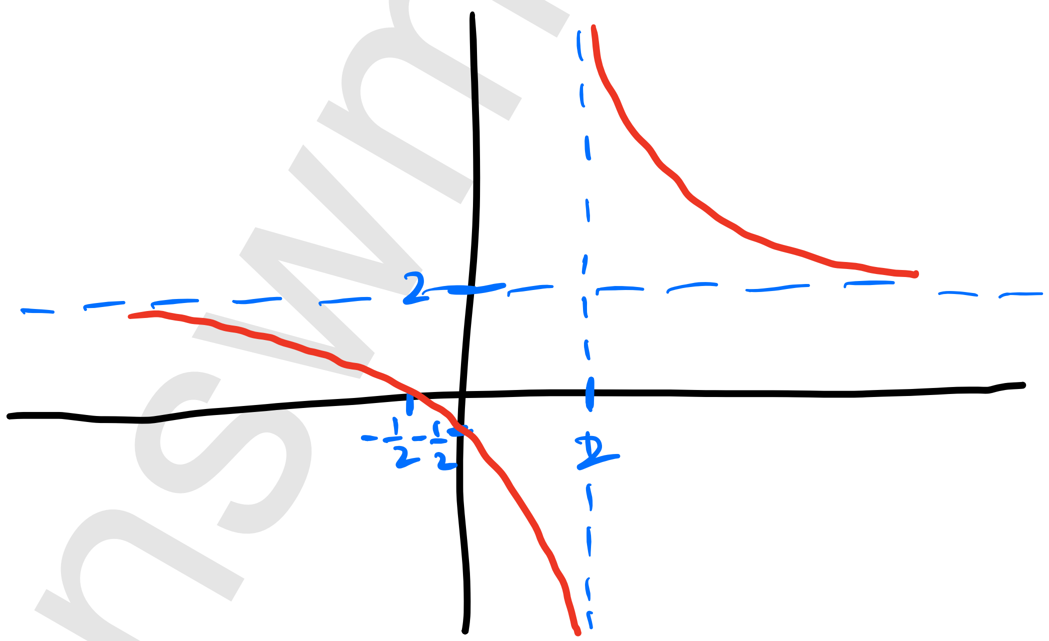
y-intercept : $y = -\frac{1}{2}$

x-intercept : $2x+1=0 \Rightarrow x = -\frac{1}{2}$

Vert. asymptote : $x-2=0 \Rightarrow x=2$

$y = \dfrac{2x-4}{x-2} + \dfrac{3}{x-2} = 2 + \dfrac{3}{x-2}$

Horizontal asymptote $y=2$.



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Question 15 (6 marks)

Let $f(x) = e^{2x} - 4e^{x}$.

(a) Find the x–intercepts of $y = f(x)$. **1**

$e^{2x} - 4e^{x} = 0$

$e^{x} - 4 = 0$ ($e^{x} = 0$ has no solution)

$x = \ln 4 = 2\ln 2$

(b) Find all the turning points of $f(x)$, and determine their nature. **3**

You may use the fact that $f'(x) = 2e^{2x} - 4e^{x}$ and $f''(x) = 4e^{2x} - 4e^{x}$ without proof.

$f'(x) = 0 \Rightarrow 2e^{2x} - 4e^{x} = 0$

$e^{x} - 2 = 0$ (again $e^{x} = 0$ no solution)

$x = \ln 2.$

$f(\ln 2) = e^{\ln 4} - 4e^{\ln 2} = -4$

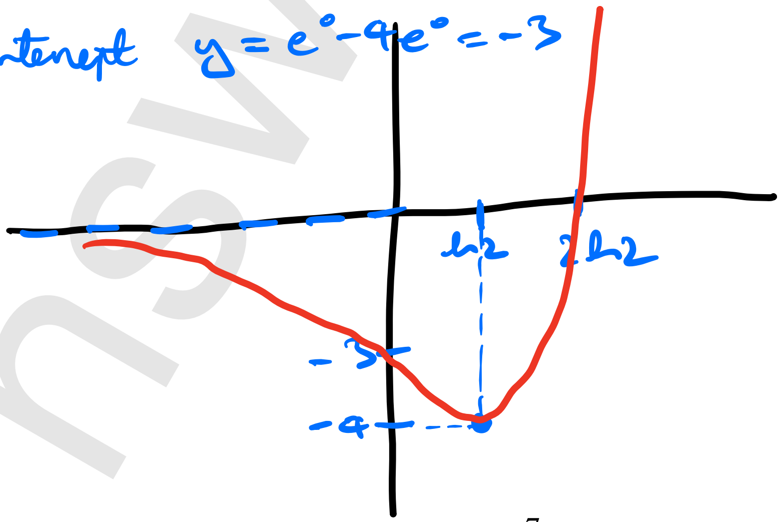
$f''(\ln 2) = 4e^{\ln 4} - 4e^{\ln 2} = 8 > 0 \Rightarrow$ concave up.

$\Rightarrow$ There is one turning point at $(\ln 2, -4)$ and it is a local minimum.

(c) Sketch $y = f(x)$, showing all important features. **2**

Note as $x \to -\infty$, $f(x) = e^{x}(e^{x} - 4) \to 0.$

y-intercept $y = e^{0} - 4e^{0} = -3$



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Question 16 (3 marks)

Let $y = \dfrac{\ln x}{x}$.

(a) Find $\dfrac{dy}{dx}$. **1**

$$\frac{dy}{dx} = \frac{x\left(\frac{1}{x}\right) - (\ln x)(1)}{x^2}$$

$$= \frac{1 - \ln x}{x^2}$$

(b) Hence, evaluate $\displaystyle\int \frac{\ln x}{x^2}\,dx$. **2**

$$-\int -\frac{\ln x}{x^2} = -\int \frac{1-\ln x}{x^2} - \frac{1}{x^2}\,dx$$

$$= -\left(\frac{\ln x}{x} - \frac{x^{-1}}{-1}\right) + C$$

$$= -\frac{\ln x + 1}{x} + C$$

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Question 17 (4 marks)

Let $f(x) = \dfrac{\sec x - \cos x}{\sec x + 1}$.

(a) Show that $\dfrac{\sec x - \cos x}{\sec x + 1} = 1 - \cos x$. **2**

$$LHS = \frac{\sec x - \cos x}{\sec x + 1} \cdot \frac{\cos x}{\cos x}$$

$$= \frac{1 - \cos^2 x}{\cos x + 1}$$

$$= \frac{(1-\cos x)\cancel{(1+\cos x)}}{\cancel{1+\cos x}}$$

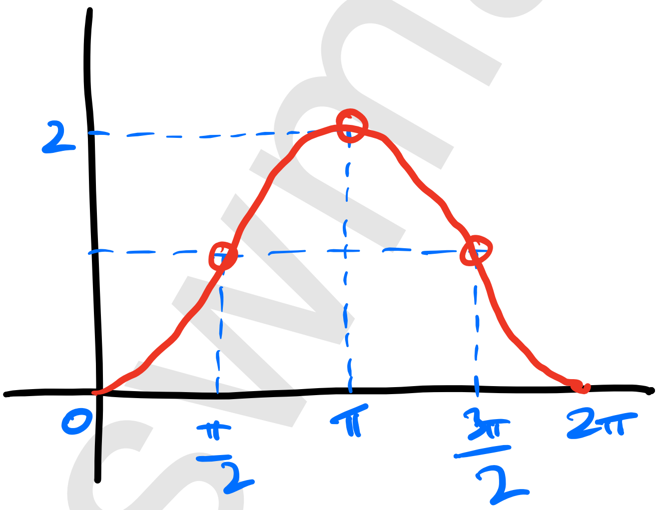
$$= 1 - \cos x = RHS.$$

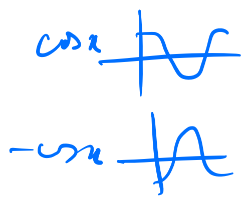
(b) Hence, graph $y = f(x)$ for $0 \leq x \leq 2\pi$. **2**

$f(x) = 1 - \cos x$ only when $\sec x + 1 \neq 0$.

Solution to $\sec x = -1 \Rightarrow \cos x = -1$ is $x = \pi$.

Also, $\sec x$ is undefined when $\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$.





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Question 18 (5 marks)

Bob has won a small lottery and has been given two withdrawal options.

Option 1: Receive a lump sum of $1 000 000.

Option 2: Receive a lump sum of $500 000, and then receive annual payments of $60 000 at the end of the year (including the first year) for ten years.

Bob will immediately deposit any payments into a bank account that earns 1.2% interest compounded monthly.

$\frac{r}{12} = \frac{0.012}{12} = 0.001$

(a) What is the value of Bob's earnings after ten years under Option 1? **1**

$1000000(1.001)^{10\times 12}$

≈ 1127429.25 dollars.

(b) What is the difference in the future earnings after ten years using Option 1 or Option 2? **3**

Let A_n = amount under Option 2 after n years.

$A_0 = 500\ 000$

$A_1 = 500\ 000(1.001)^{12} + 60\ 000$

$A_2 = A_1(1.001)^{12} + 60000$

$= 500\ 000(1.001)^{24} + 60000(1.001)^{12} + 60000$

$A_3 = A_2(1.001)^{12} + 60000$

$= 500000(1.001)^{36} + 60000(1.001)^{24} + 60000(1.001)^{12} + 60000$

$\vdots$

$A_{10} = 500000(1.001)^{120} + 60000\left(1.001^{108} + \ldots + 1.001^{12} + 1\right)$

G.P., $a=1$, $r=1.001^{12}$ write other way around

$= 500000(1.001)^{120} + 60000\left(\frac{1.001^{120}-1}{1.001^{12}-1}\right)$

≈ 1197364.15 dollars.

Option 2 worth $\approx \$69934.90$ more.

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(c) What is the *present value* of Option 2? **1**

$PV\ A_{10} = A_{10}(1.001)^{-10\times 12}$

$= 1062030.415$

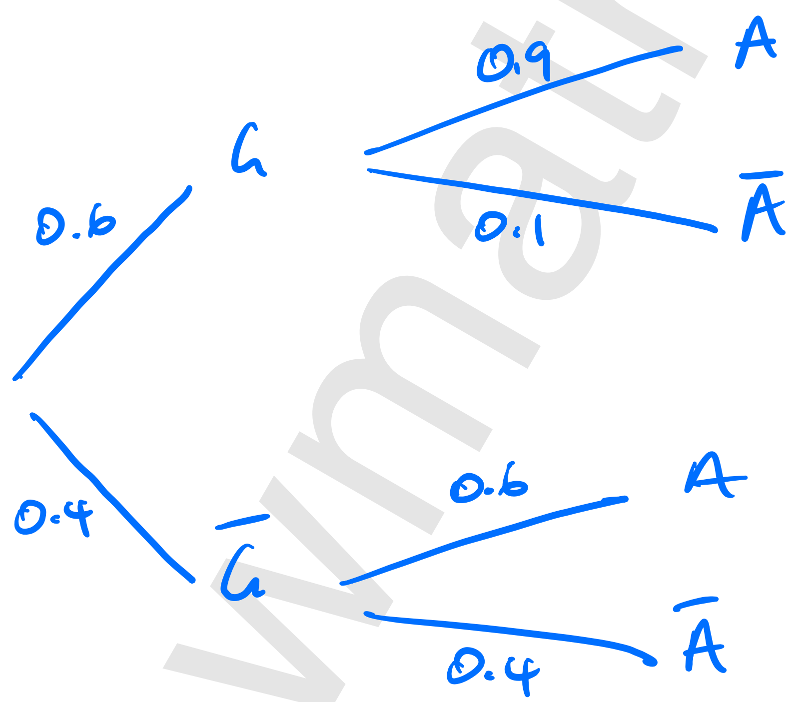
≈ 1062030.42 dollars.

Question 19 (4 marks)

NSW Mathematics is reviewing the punctuality of their students. On any given day, there is a 60% chance that the weather condition is 'good'. If the weather is 'good' then the probability that any given student arrives on time is 90%. If the weather is not 'good' then this probability drops to 60%.

Let G and A denote the events where the weather is 'good' and a given student arrives on time respectively.

(a) Draw a tree diagram to represent this situation. **1**



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(b) Show that the probability that a given student arrives on time is 0.78. **2**

$P(A) = P(A|G)P(G) + P(A|\bar{G})P(\bar{G})$

$= 0.9 \times 0.6 + 0.6 \times 0.4$

$= 0.78$

(c) Given that a randomly chosen student arrived on time, determine the probability that the weather was good. **1**

$P(G|A) = \dfrac{P(A|G)P(G)}{P(A)}$ (Bayes rule)

$= \dfrac{0.9 \times 0.6}{0.78}$

$= \dfrac{9}{13}$

$\simeq 0.69$ (2 d.p.)

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Question 20 (3 marks)

Let $f(x) = a\sin x + b\cos x$, where a and b are constants. If $f\left(\frac{\pi}{4}\right) = 3$ and $f'\left(\frac{\pi}{4}\right) = 1$, solve the equation $f(x) = 0$, for $0 \leq x \leq 2\pi$, correct to two significant figures. **3**

$3 = a\sin\frac{\pi}{4} + b\cos\frac{\pi}{4}$

$3 = \frac{a}{\sqrt{2}} + \frac{b}{\sqrt{2}}$

$3\sqrt{2} = a + b$ —— (1)

$f'(x) = a\cos x - b\sin x$

$1 = a\cos\frac{\pi}{4} - b\sin\frac{\pi}{4}$

$1 = \frac{a}{\sqrt{2}} - \frac{b}{\sqrt{2}}$

$\sqrt{2} = a - b$ —— (2)

(1)+(2): $4\sqrt{2} = 2a \Rightarrow a = 2\sqrt{2}$

(1)−(2): $2\sqrt{2} = 2b \Rightarrow b = \sqrt{2}$

$f(x)=0$: $2\sqrt{2}\sin x + \sqrt{2}\cos x = 0$

$2\sin x = -\cos x$

$\tan x = -\frac{1}{2}$

$x = \pi - \tan^{-1}\frac{1}{2},\ 2\pi - \tan^{-1}\frac{1}{2}$

$\approx 2.7,\ 5.8$

Question 21 (3 marks)

The function $f(x) = e^{x^2}$ is continuous on the interval $[-1, 1]$. Use the trapezoidal rule with four subintervals to estimate the value of $\int_{-1}^{1} e^{x^2}\,dx$. **3**

$\int_{-1}^{1} e^{x^2}\,dx \approx \frac{1-(-1)}{2(4)}\left[e^{1^2} + e^{(-1)^2} + 2\left(e^{(-\frac{1}{2})^2} + e^{0^2} + e^{(\frac{1}{2})^2}\right)\right]$

$= \frac{1}{4}\left(2e^1 + 4e^{\frac{1}{4}} + 2\right)$

$= \frac{1}{2}\left(e + 2e^{\frac{1}{4}} + 1\right)$

≈ 3.14316633

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Question 22 (4 marks)

oneeightseven records the number of hours spent training per week and the resting heart rate (in beats per minute) of six participants.

Training Hours (T)	2	3	4	6	7	10
Resting Heart Rate (H)	85	82	80	76	74	70

(a) Find the equation of the least-squares regression line, rounding coefficients to the nearest integer, and use the equation to estimate the resting heart rate of someone who trains for 5 hours a week. **3**

slope −1.861538462
intercept 87.76153846

From calculator, $H = 88 - 2T$

For $T = 5$, $H = 88 - 10 = 78$ beats per minute.

(b) oneeightseven trains for 25 hours a week to prepare for a marathon. Explain why the regression equation from part (a) may not be appropriate for estimating oneeightseven's resting heart rate. **1**

Extrapolation: data collected is only for 2 to 10 hours per week, and 25 hours falls outside of this range.

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Question 23 (5 marks)

Consider the two curves $y = \ln x + 8x$ and $y = \ln x + x^2 + 7$.

(a) Find the x value(s) at which the two curves intersect. **2**

$\ln x + 8x = \ln x + x^2 + 7$

$x^2 - 8x + 7 = 0$

$(x-1)(x-7) = 0$.

Hence there are two points of intersection at $x = 1$ and $x = 8$.

Note both do not violate the restriction $x > 0$ from the $\ln x$ terms.

(b) Hence, find the area bound between the two curves. **3**

Without knowing which curve lies on "top", use absolute value.

$$A = \left| \int_1^7 (\cancel{\ln x} + 8x) - (\cancel{\ln x} + x^2 + 7)\, dx \right|$$

$$= \left| \left[4x^2 - \frac{x^3}{3} - 7x \right]_1^7 \right|$$

$$= \left| \left(196 - \frac{343}{3} - 49\right) - \left(4 - \frac{1}{3} - 7\right) \right|$$

$$= 36 \text{ units}^2.$$

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Question 24 (5 marks)

A discrete random variable X has the following probability distribution table

x	0	1	2	3	4
$P(X=x)$	a	$2a$	b	0.2	c

where a, b and c are constants.

It is known that $E(X) = 2$.

(a) By finding expressions for b and c in terms of a, show that $0.05 \leq a \leq 0.18$. **3**

$a + 2a + b + 0.2 + c = 1$

$3a + b + c = 0.8 \quad —(1)$

$0a + 1(2a) + 2b + 3(0.2) + 4c = 2$

$2a + 2b + 4c = 1.4$

$a + b + 2c = 0.7 \quad —(2)$

$(1)-(2): \quad 2a - c = 0.1 \quad —(3)$

Since probablities are ≥ 0, $c \geq 0$

Hence $2a \geq 0.1 \Rightarrow a \geq 0.05$

$(1) - \frac{1}{2}\times(2): \quad \frac{5}{2}a + \frac{1}{2}b = 0.45$

$5a + b = \frac{9}{10} \quad —(4)$

Similarly $b \geq 0$

Hence $5a \leq 0.9 \Rightarrow a \leq 0.18$

Notice due to + versus −, the direction of the inequality changes.

(b) Hence deduce the maximum possible value for the variance of X. **2**

$Var(X) = E(X^2) - (E(X))^2$

$= 0^2a + 1^2(2a) + 2^2b + 3^2(0.2) + 4^2c - 2^2$

use (3) & (4) $= 2a + 4\left(\frac{9}{10} - 5a\right) + 1.8 + 16(2a - 0.1) - 4$

$= 14a - 0.2$

$\leq 14(0.18) - 0.2$

$= 2.32$

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Question 25 (7 marks)

Abdullah wishes to purchase an item which she can resell for one dollar. Benjamin is selling this item and believes that the item is worth X dollars where X is a random variable with probability density

$$f(x) = \begin{cases} \alpha + \beta(2x-1) & : 0 \le x \le 1 \\ 0 & : \text{otherwise} \end{cases}$$

where α and β are constants.

(a) Show that $\alpha = 1$ and find the possible values of β for which $f(x)$ is a valid probability density function. **2**

$1 = \int_0^1 [\alpha + \beta(2x-1)]\,dx = [\alpha x + \beta(x^2 - x)]_0^1$

$= (\alpha \cdot 1 + \beta \cdot 0) - (\alpha \cdot 0 + \beta \cdot 0)$

$= \alpha$ as required.

Now we also require $f(x) \geq 0$ for all $0 \leq x \leq 1$

Take cases on β, noting $1 + \beta(2x-1)$ is a line:

- $\beta > 0$: $f(x)$ increasing, min value at $x = 0$.
  $f(0) \geq 0 \Rightarrow 1 - \beta \geq 0 \Rightarrow \beta \leq 1$
- $\beta < 0$: $f(x)$ decreasing, min value at $x = 1$.
  $f(1) \geq 0 \Rightarrow 1 + \beta \geq 0 \Rightarrow \beta \geq -1$
- $\beta = 0$: $f(x) = 1 \geq 0$ always.

Combining all cases, all $-1 \leq \beta \leq 1$ are valid.

(b) Abdullah places a bid b for the item and will *gain* the item only if her bid is *above* Benjamin's valuation of the item. Otherwise she will *lose* her bid. **1**

Show that her expected profit, E, is given by

$$E = F(b) - b$$

where $F(x)$ is the cumulative density function (cdf) of X.

Expense $= b$, always (due to fixed bid).

Income $= 1$ if outbid, which occurs with probability $P(b > X) = P(X < b) = F(b)$.

(by def of CDF)

Otherwise income $= 0$.

$\therefore$ Expected profit $= F(b) - b$.

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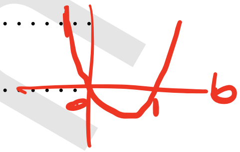
(c) Suppose $\beta > 0$. Explain why Abdullah should either bid $b = 0$ or $b = 1$. **2**

$$E = F(b) - b = \int_0^b 1+\beta(2x-1)\,dx - b$$

$$= \left[x+\beta(x^2-x)\right]_0^b - b$$

$$= \left(b+\beta(b^2-b)\right) - \left(0+\beta^2\cdot 0\right) - b$$

$$= \beta(b^2-b) = \beta b(b-1).$$



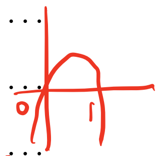
If $\beta>0$, this is a concave up parabola (as a function of the bid b). Since the intercepts are at $b=0$ & $b=1$, if $b\neq 0$ or 1, then $E<0$. Hence Abdullah should bid $b=0$ or $b=1$, with greater profit $E=0$.

(d) Suppose $\beta < 0$. Find the value of b that maximises Abdullah's expected profit and find the expected profit in terms of β. **2**

You do NOT need to show that this is a maximum.

If $\beta<0$ then $E=\beta(b^2-b)$ now represent a concave-up parabola.

$$\frac{dE}{db} = \beta(2b-1) = 0 \text{ when } b=\frac{1}{2}.$$



Hence bid $b=\frac{1}{2}$. The expected profit is

$$E_{max} = \beta\left(\tfrac{1}{2}\right)\left(1-\tfrac{1}{2}\right) = -\frac{\beta}{4} = \frac{|\beta|}{4}$$

Absolute value final answer optional

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Question 26 (4 marks)

Show that 4

$$2 + 22 + 222 + 2222 + \cdots + \underbrace{2222 \ldots 2}_{n\ 2\text{'s}} = \frac{2}{81}(10^{n+1} - 9n - 10).$$

$$\underbrace{2222\ldots2}_{n\ 2\text{'s}} = 2 + 20 + 200 + \cdots + 2\underbrace{00\ldots0}_{n-1\ 0\text{'s}}$$

$$= 2(1 + 10 + 100 + \cdots + 10^{n-1})$$

$$= \frac{2(10^n - 1)}{10 - 1} = \frac{2(10^n - 1)}{9}.$$

$$\therefore 2 + 22 + 222 + \cdots + \underbrace{2222\ldots2}_{n\text{'s}}$$

$$= \frac{2(10^1 - 1)}{9} + \frac{2(10^2 - 1)}{9} + \frac{2(10^3 - 1)}{9} + \cdots + \frac{2(10^n - 1)}{9}$$

$$= \frac{2}{9}\left[(10 + 10^2 + 10^3 + \cdots + 10^n) - (\underbrace{1 + 1 + 1 + \cdots + 1}_{n\text{ terms}})\right]$$

$$= \frac{2}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{2}{81}(10^{n+1} - 9n - 10).$$

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Question 27 (4 marks)

Let k be a real number. For what values of k does the equation $kx^2 + 3(k-2)x = 9k - 9$ have two real roots for x? **4**

on RHS

2 real roots $\Rightarrow \Delta > 0$.

$\Delta = [3(k-2)]^2 - 4k(-9k+9)$

$= 9k^2 - 36k + 36 + 36k^2 - 36k$

$= 9(5k^2 - 8k + 4)$.

Hence first consider

$9(5k^2 - 8k + 4) = 0$

Discriminant for this quadratic is

$(-8)^2 - 4(5)(4) = -16 < 0$.

$\therefore 9(5k^2 - 8k + 4) = 0$ has no real roots for all k.

Since $9 \cdot 5 = 45 > 0$, $9(5k^2 - 8k + 4)$ represents a concave up parabola with no roots, i.e. no x-intercept.

Hence $9(5k^2 - 8k + 4) > 0$ always holds.

$\therefore \Delta > 0$ always.

HOWEVER: if $k = 0$, original equation is not a quadratic, hence has only one root!!!

$\Rightarrow$ All real k except $k \neq 0$.

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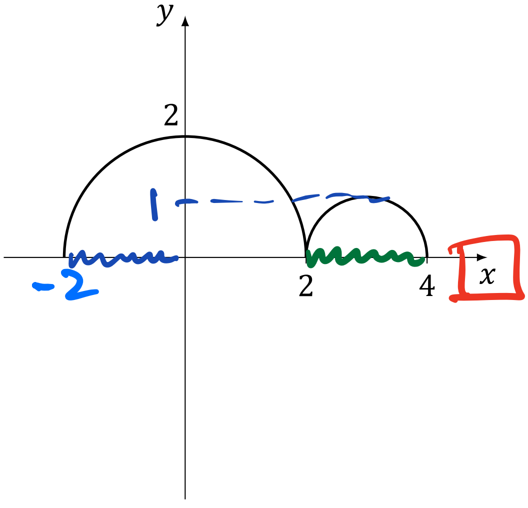
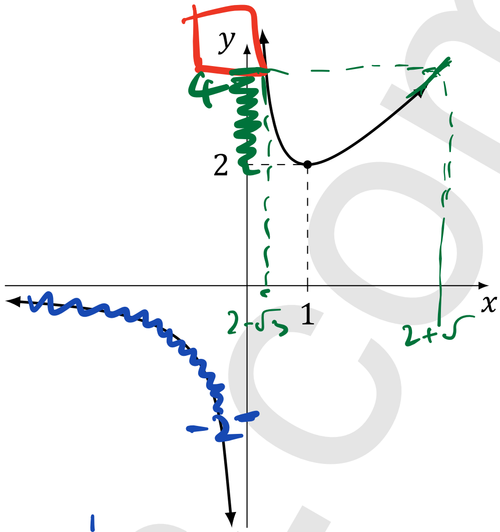
Question 28 (4 marks)

The graphs of $y = f(x)$ and $y = g(x)$ are given below. **4**





$$y = g(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ x + \dfrac{1}{x} & \text{if } x > 0 \end{cases}$$

Find the domain and range of $f(g(x))$.

$g(x) = -2 \Rightarrow \frac{1}{x} = -2 \Rightarrow x = -\frac{1}{2}$

$g(x) = 4$: $x + \frac{1}{x} = 4$

$x^2 - 4x + 1 = 0$

$(x-2)^2 = 3 \Rightarrow x = 2 \pm \sqrt{3}$

Hence from graphs:

- If $2-\sqrt{3} \le x \le 2+\sqrt{3}$, then $2 \le g(x) \le 4$

$\Rightarrow 0 \le f(g(x)) \le 1$

- If $x \le -\frac{1}{2}$ then $-2 \le g(x) < 0$ noting asymptote

$\Rightarrow 0 \le f(g(x)) < 2$.

For all other values of $g(x)$, either $g(x) > 4$ or $g(x) < -2$. Both $(4, \infty)$ and $(-\infty, 2)$ are outside the range of $f(x)$.

$\therefore f(g(x))$ domain: $(-\infty, -\frac{1}{2}] \cup [2-\sqrt{3}, 2+\sqrt{3}]$

range: $[0, 2)$

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Question 29 (5 marks)

Consider a random variable X that is normally distributed with mean 15 and standard deviation σ. It is known that $P(X < 10) = a$ and $P(X > 25) = b$.

(a) Find $P(20 < X < 25)$ in terms of a and b. **2**

$P(X>20) = P(X<10) = a$ due to symmetry of normal distribution about the mean.

Note $|20-15| = |10-15| = 5$.

Then $P(20 < X < 25) = P(X<25) - P(X \leq 20)$

$= (1-b) - (1-a)$

$= a - b$.

[Reminder: $<$ and $\leq$ give equal probabilities for continuous random variables.]

(b) Consider a random variable Y that is normally distributed with mean 25 and standard deviation 3σ. Find two pairs (m, n) such that $P(m < Y < n) = 1 - a - b$. **3**

Observe $P(10 < X < 25) = P(X<25) - P(X<10)$

$= (1-b) - a$

Let $Z \sim N(0,1)$. Then

$1-a-b = P\left(\frac{10-15}{\sigma} < \frac{X-15}{\sigma} < \frac{25-15}{\sigma}\right)$

$= P\left(-\frac{5}{\sigma} < Z < \frac{10}{\sigma}\right)$

$= P(-15 < 3\sigma Z < 30)$

$= P(10 < 25 + 3\sigma Z < 55)$

$= P(10 < Y < 55)$

$\Rightarrow$ One pair is $(m,n) = (10, 55)$.

Also $P(X<5) = P(X>25) = b$, therefore

$P(5 \leq X \leq 20) = P(X \leq 20) - P(X \leq 5) = b + (1-a)$.

$\therefore 1-a-b = P\left(\frac{5-15}{\sigma} < Z < \frac{20-15}{\sigma}\right)$

$= P(25 + 3(-10) < Y < 25 + 3(5))$

$= P(-5 < Y < 40)$.

$\Rightarrow$ Other pair is $(m,n) = (-5, 40)$.

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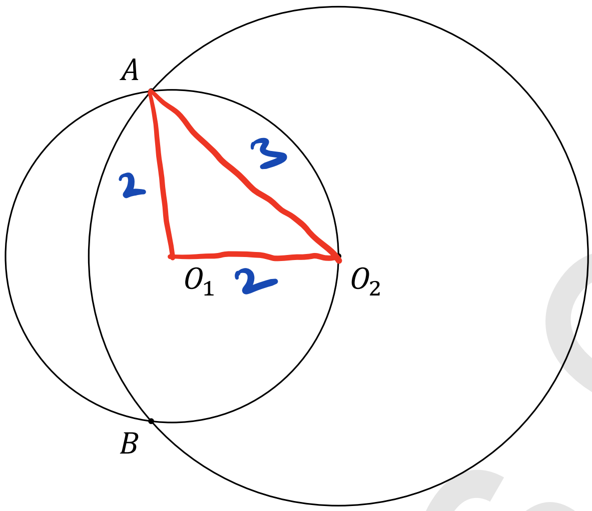
Question 30 (5 marks)

In the figure below, the graphs of $C_1 : x^2 + y^2 = 4$ with centre O_1 and $C_2 : (x-2)^2 + y^2 = 9$ with centre O_2 are shown without the coordinate axes. Their intersections are labelled A and B.



(a) Find the value of $\cos \angle AO_1O_2$. **1**

$O_2A = 3$ (radius of C_2)

$O_1A = 2$ (radius of C_1)

$O_1O_2 = 2$ (distance between $(0,0)$ and $(0,2)$; entirely along x-axis)

$$\Rightarrow \cos\angle AO_1O_2 = \frac{O_1A^2 + O_1O_2^2 - O_2A^2}{2\, O_1A \cdot O_1O_2}$$

$$= \frac{4+4-9}{2\cdot 2\cdot 2} = -\frac{1}{8}$$

from cosine rule in $\triangle AO_1O_2$

Question 30 continues on page 24

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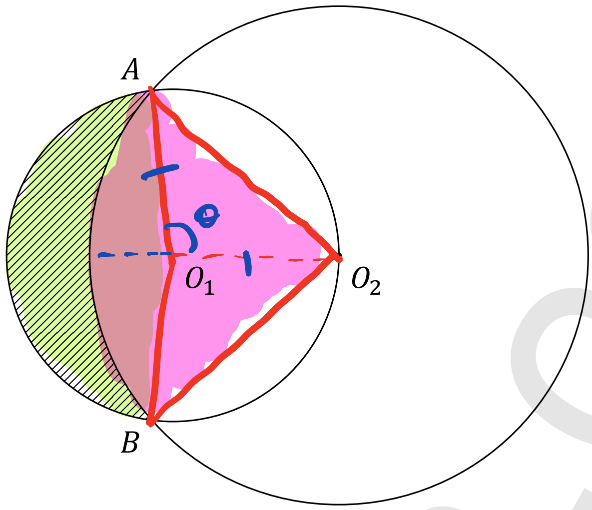
(b) Hence, or otherwise, find the shaded area shown below. **4**



Give your answer correct to 2 decimal places.

Let $\angle AO_1O_2 = \theta = \cos^{-1}\left(-\frac{1}{8}\right)$ from part a).

Reflex $\angle AO_1B = 2\theta$ from symmetry

$\therefore$ Obtuse $\angle AO_1B = 2\pi - 2\theta$.

$\triangle AO_1O_2$ isosceles due to equal radii $AO_1 = O_1O_2$

$\therefore \angle AO_2O_1 = \frac{1}{2}(\pi - \theta)$ from base angles of isosceles triangle.

$\therefore \angle AO_2B = \pi - \theta$ from symmetry

Required area

= Area of sector AO_1B in C_1

− Area of sector AO_2B in C_2

+ 2 × Area of $\triangle AO_1O_2$

Can be faster if you use areas of segments instead

$= \frac{1}{2}(2)^2(2\pi - 2\theta) - \frac{1}{2}(3)^2(\pi - \theta)$

$+ 2 \times \frac{1}{2}(2)(2)\sin\theta$

$= 3.24589\ldots$

≈ 3.246 units^2

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Question 31 (9 marks)

WAIT, WHAT?!

In the year 2050, NSWMaths manufactures and sells Ruiace sip stickers. Suppose

$$C(q) = \frac{7q^3 - 1000q^2 + 126000q}{100000} + 84$$

is the cost in millions of dollars to produce and sell q million units (you may assume this is the only cost). It is also known that the price each sticker can be sold for can be modelled by $D(q) = b - aq$, where a and b are positive constants. Define $P(q)$ to be the profit in millions of dollars associated with selling q million stickers.

(a) From historical data, when 200 million units were produced, each unit sold for \$1.50, and when 50 million units were produced and sold, a profit of \$44.25 million was made.

(i) Explain why $b = 1.5 + 200a$. **1**

Question provides that $D(200) = 1.5$

$\Rightarrow \quad 1.5 = b - 200a$

$\therefore \quad b = 1.5 + 200a$

(ii) Write down an expression for $P(q)$ in terms of a, b, and q. **1**

Income = qty × per-unit price = $q\,D(q)$

$\therefore \; P(q)$

$= q\,D(q) - C(q)$

$= q(b - aq) - \left(\dfrac{7q^3 - 1000q^2 + 126000q}{100000} + 84\right)$

Question 31 continues on page 26

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(iii) Hence, show that **3**

$$P(q) = q\left(\frac{25}{6} - \frac{q}{75}\right) - \left(\frac{7q^3 - 1000q^2 + 126000q}{100000} + 84\right).$$

$P(q) = q(1.5 + 200a - aq) - \left(\frac{7q^3 - 1000q^2 + 126000q}{100000} + 84\right)$

from (i).

$P(50) = 44.25$, therefore

$44.25 = 50(1.5 + 200a - 50a) - 46.75 - 84$

$100 = 7500a$

$a = \frac{1}{75}$

$\frac{3}{2} + 200\left(\frac{1}{75}\right) = \frac{25}{6}$

$\therefore P(q) = q\left(\frac{25}{6} - \frac{q}{75}\right)$

$- \left(\frac{7q^3 - 1000q^2 + 126000q}{100000} + 84\right)$

Question 31 continues on page 27

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Question 31 (continued)

(b) Find the maximum profit the company can make, to the nearest million. **4**

You must justify why your answer is a maximum.

$q\left(\frac{25}{6}-\frac{q}{75}\right)=\frac{25q}{6}-\frac{q^2}{75}$

$P'(q)=\frac{25}{6}-\frac{2q}{75}-\left(\frac{21q^2-2000q+126000}{100000}\right)$

$=-\frac{21q^2}{100000}-\frac{q}{150}+\frac{218}{75}$

$P''(q)=-\frac{21q}{50000}-\frac{1}{150}$ which for any $q>0$ is clearly obviously negative, hence $P'(q)=0$ solves to give the maximum profit.

$P'(q)=0 \Rightarrow$ by quadratic formula,

$q=\dfrac{\frac{1}{150}\pm\sqrt{\left(\frac{1}{150}\right)^2-4\left(\frac{-21}{100000}\right)\left(\frac{218}{75}\right)}}{2\left(-\frac{21}{100000}\right)}$

$=102.841863\ldots$ from negative case

(reject +'ve case as this gives negative q, and $q>0$ for quantities.

$P(102.841863\ldots)=103.53306704\,5$

hence max approx \$103 533 067.05

$\approx$ \$104 M.

End of paper

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I want to learn more and more to see as beautiful what is necessary in things;
then I shall be one of those who make things beautiful.

— Friedrich Nietzsche, Die fröhliche Wissenschaft.

EXAMINERS

one eight seven	Q2, 5, 20, 21, 22, 29
MokSifu	Q1, 6, 7, 9, 11, 13, 14, 17, 18, 24, 28, 30
Hugh Entwistle	Q18, 19, 24, 25
MagiicMushroom	Everything else.

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NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

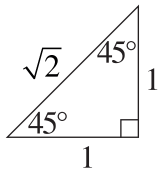
$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \dfrac{\text{opp}}{\text{hyp}}, \quad \cos A = \dfrac{\text{adj}}{\text{hyp}}, \quad \tan A = \dfrac{\text{opp}}{\text{adj}}$

$A = \dfrac{1}{2}ab\sin C$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

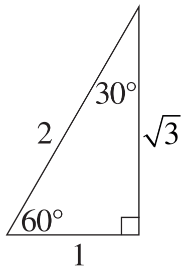


$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \dfrac{1}{2}r^2\theta$



Trigonometric identities

$\sec A = \dfrac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \dfrac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \dfrac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1+t^2}$

$\cos A = \dfrac{1-t^2}{1+t^2}$

$\tan A = \dfrac{2t}{1-t^2}$

$\cos A\cos B = \dfrac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \dfrac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \dfrac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \dfrac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \dfrac{1}{2}(1 - \cos 2nx)$

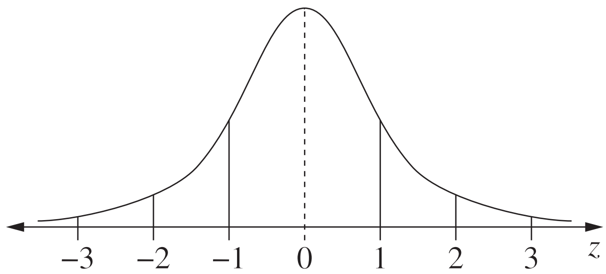
$\cos^2 nx = \dfrac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \dfrac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq r) = \displaystyle\int_a^r f(x)\,dx$

$P(a < X < b) = \displaystyle\int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$

$X \sim \operatorname{Bin}(n, p)$

$\Rightarrow \quad P(X = x)$

$= \dbinom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2\,,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

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