2025 MOCK HSC EXAM

Mathematics Extension 2 SL

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 81

Section I – 6 marks (pages 2–3)

- Attempt Questions 1–6
- Allow about 15 minutes for this section

Section II – 75 marks (pages 4–0)

- Attempt Questions 7–11
- Allow about 2 hours 45 minutes for this section

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Section I

6 marks
Attempt Questions 1–6
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–6.

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1 Jacob wrote six consecutive numbers on six pieces of white paper. He stuck those six pieces of paper on the front and back of three coins. He then threw the coins three times. After the first throw he drew circles around the numbers on top, which were 6, 7, and 8 were on top.

After the second throw the sum of the numbers on top was 23 and after the third throw the sum was 17. What is the sum of the three uncircled numbers?

I never wrote answers for this one, figure it out yourself.

2 In a late-night debate on the NSWMathematics server, someone posts the statement

"If you understand limits, then you can survive the university mathematics channel."

Which of the following is the contrapositive of this statement?

A. If you can survive the university mathematics channel, then you understand limits.

B. If you don't understand limits, then you cannot survive the university mathematics channel.

C. If you can survive the university mathematics channel, then you cannot understand limits.

D. If you don't survive the university mathematics channel, then you understand limits.

3 Using partial fractions, the value of $\displaystyle\sum_{k=1}^{50} \frac{1}{(2k-1)(2k+1)}$ is equal to:

A. $\dfrac{50}{101}$ B. $\dfrac{50}{99}$ C. $\dfrac{100}{101}$ D. $\dfrac{100}{99}$

4 Which of the following statements about the complex number z is always false?

A. $|z+\bar{z}| \geq 2|z+2|$ B. $|z-\bar{z}| < 2|z|$

C. $|z+\bar{z}| = 2|z-i|$ D. $|z-\bar{z}| > 2|z+1|$

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5 In the x–y plane, forces of magnitude 48 N, 55 N, and 73 N keep a mass stationary. The angle between the two largest forces is closest to:

A. 138° B. 41° C. 42° D. 131°

6 Kenneth picks his favourite nonzero two vectors and affectionately names them $\underset{\sim}{a}$ and $\underset{\sim}{b}$. Xavier notices that $|\underset{\sim}{a}|^2 = 2|\underset{\sim}{b}|^2$, and also that $\underset{\sim}{a}$ and $\underset{\sim}{a} + \underset{\sim}{b}$ are perpendicular.

Which of the following pairs of vectors must also be perpendicular?

A. $\underset{\sim}{a}$ and $\underset{\sim}{a} - 2\underset{\sim}{b}$

B. $\underset{\sim}{a}$ and $\underset{\sim}{b}$

C. $\underset{\sim}{a}$ and $\underset{\sim}{a} + \underset{\sim}{b}$

D. $\underset{\sim}{a}$ and $2\underset{\sim}{a} + \underset{\sim}{b}$

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Section II

75 marks
Attempt Questions 7–11
Allow about 2 hours 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Questions by Justin

Question 7 (10 marks) Use the Question 7 Writing Booklet

(a) Suppose that the equations of non-vertical lines ℓ_1 and ℓ_2 are given by $\begin{pmatrix} x \\ y \end{pmatrix} = \underset{\sim}{a} + \lambda \underset{\sim}{b}$ and $\begin{pmatrix} x \\ y \end{pmatrix} = \underset{\sim}{c} + \mu \underset{\sim}{d}$ respectively. Prove that if $\underset{\sim}{b} \cdot \underset{\sim}{d} = 0$, then $m_1 m_2 = -1$, where m_1 and m_2 are the gradients of lines ℓ_1 and ℓ_2, respectively. **2**

(b) By considering $z = e^{iA}$ and $w = e^{iB}$, prove that $\sin(A - B) = \sin A \cos B - \sin B \cos A$. **3**

(c) Prove that if z satisfies $4z^4 + z + 1 = 0$, then $|z| \leq 1$. **2**

(d) Let z be a complex number and suppose $w = \dfrac{az + b}{az + c}$, for real a, b, c. As w varies on the unit circle, determine the locus of z. **3**

End of Question 7

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Unused complex

Question 8 (23 marks) Use the Question 8 Writing Booklet

(a) You may assume that if a and b are real numbers, $a + b \geq 2\sqrt{ab}$.

(Do NOT prove this).

(i) Suppose that $\text{Re}(w) \neq 0$, and $\left|\text{Arg}\left(w - \dfrac{1}{w}\right)\right| = \dfrac{\pi}{2}$. Show that $|w| = 1$. **2**

(ii) Suppose that $z - \dfrac{1}{z} = \sqrt{2}i$. Prove by contradiction that $\text{Re}(z) \neq 0$. **2**

(iii) Hence, evaluate $z^{2025} + \dfrac{1}{z^{2025}}$. **2**

(b) Consider the equation $P(z) = z^5 - (z-2)^5$.

(i) Explain why if $P(z) = 0$, then z can be expressed in the form $\dfrac{2\omega^k}{\omega^k - 1}$, where $k \in \{1, 2, 3, 4\}$, and $\omega = e^{\frac{2i\pi}{5}}$ is a fifth root of unity. **3**

(ii) Show that $P(z) = 0 \implies \text{Re}(z) = 1$. **2**

(iii) Express the roots of $P(z+1) = 0$ in Cartesian form. **3**

(iv) Hence, show that $\cot\dfrac{4\pi}{5} = -\sqrt{1 + \dfrac{2}{\sqrt{5}}}$. **3**

(c) Given that z is a complex number lying on the unit circle, show that **3**

$$\frac{1}{1+z} + \frac{1}{1-z} = \frac{1 - \bar{z}\,\text{Re}(z)}{1 - (\text{Re}(z))^2}.$$

(d) If $|z| = 1$, prove that $|z^4 - 3z^2 + 11iz^2 - 3z + 1| \leq \sqrt{187}$. **3**

End of Question 8

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Unused proof

Question 9 (23 marks) Use the Question 9 Writing Booklet

(a) Suppose that a, b, and c are nonzero real numbers. It is given that both roots of the polynomial $P(z) = az^2 + bz + c$ all have negative real part.

(i) Explain why there must be a root z with $\operatorname{Re}(z)\operatorname{Im}(z) \le 0$. **2**

(ii) Prove that a, b, and c all have the same sign. **3**

(b) (i) Prove that if a and b are real numbers, then $a^2 + b^2 \ge 2ab$. **1**

(ii) Suppose the point (x, y) lies outside the unit circle. Prove that $x^2 + xy + y^2 > x + y$. **2**

(c) Prove that for primes p and $n \in \mathbb{Z}^+$, no positive integer q exists such that **3**

$$(187q + 94)^2 = n^2 + p.$$

(d) Consider the parametric vector equation defined by $\underset{\sim}{r}(t) = \begin{pmatrix} \sin t \cos t \\ \sin^2 t \\ \sin^2 t \cos t \end{pmatrix}$ for $0 \le t \le 2\pi$.

(i) Show that $\sin^4 x + \sin^2 x - \sin^6 x = 1 - (1 + \sin^2 x)\cos^4 x$. **1**

(ii) Hence, or otherwise, show that $\underset{\sim}{r}(t)$ can be fully contained within a unit sphere. **3**

(e) Suppose that P and Q are polynomials with real coefficients, and t is a real number. It is known that t is a root of degree exactly 2 to the polynomial equation $P(x)^3 = Q(x)^3$. **3**

(i) Prove by contradiction that $P(t) \ne 0$. **3**

(ii) Show that t is also a root of degree exactly 2 to the polynomial equation $P(x) = Q(x)$. **2**

Note this is DIFFERENT to 3U POTD 314!

End of Question 9

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Question 10 (13 marks) Use the Question 10 Writing Booklet

(a) (i) Suppose that $f(x)$ is a continuous function. Prove that if a is a positive real number, then **2**

$$\int_{\frac{1}{a}}^{a} f(x)\,dx = \int_{\frac{1}{a}}^{a} \frac{f(x^{-1})}{x^2}\,dx.$$

(ii) Evaluate $\displaystyle\lim_{t\to\infty}\int_{\frac{1}{t}}^{t} \frac{1}{1+x+x^2+x^3}\,dx.$ **3**

(b) Let $f(x) = x\cos x$ for $\dfrac{3\pi}{2} \le x \le 2\pi$. You may assume f has an inverse function f^{-1}. **4**

By using the substitution $x = f(u)$, evaluate $\displaystyle\int_0^{2\pi} f^{-1}(x)\,dx.$

(c) Using the substitution $x = \sin^2 2\theta$, evaluate the integral **4**

$$\int_0^1 \sqrt{x\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)}\,dx.$$

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Unused vectors

Question 11 (6 marks) Use the Question 11 Writing Booklet

(a) A set of vectors is said to be "mutually orthogonal" if any two members of the set are perpendicular. E.g, $\{\underset{\sim}{i} + \underset{\sim}{j}, \underset{\sim}{i} - \underset{\sim}{j}, \underset{\sim}{k}\}$ is a set of mutually orthogonal vectors.

Let **P** be the statement that "If $\{\underset{\sim}{x}, \underset{\sim}{y}, \underset{\sim}{z}\}$ is a set of mutually orthogonal vectors, then $|\underset{\sim}{x} + \underset{\sim}{y} + \underset{\sim}{z}|^2 = |\underset{\sim}{x}|^2 + |\underset{\sim}{y}|^2 + |\underset{\sim}{z}|^2$."

(i) Prove that **P** is true. **1**

(ii) What conditions must $\underset{\sim}{x}, \underset{\sim}{y}$, and $\underset{\sim}{z}$ satisfy for the converse of **P** to be false? **2**

(iii) If $\underset{\sim}{p} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\underset{\sim}{q} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, graph the possible position vectors of $\underset{\sim}{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ on the Cartesian if the set $\{\underset{\sim}{p}, \underset{\sim}{q}, \underset{\sim}{r}\}$ is a counterexample to the converse of **P**. **3**

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2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

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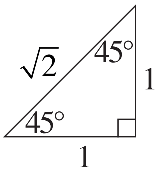
Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

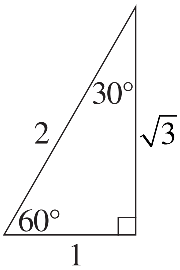
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

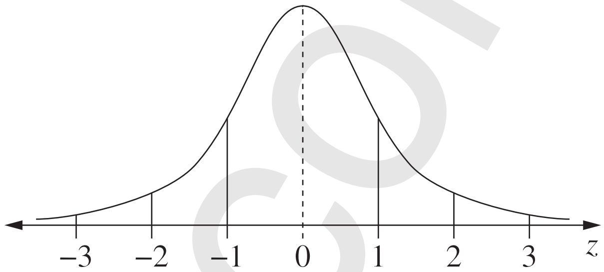
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \le r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n - r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x}p^x(1 - p)^{n - x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

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Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

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Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

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