

Mathematics Extension 2

REFERENCE SHEET

Mathematics Extension 2

Trigonometric identities

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1+t^2}$$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A \cos B = \frac{1}{2} [\cos(A-B) + \cos(A+B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$$

Integral calculus

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Vectors

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$$

Complex numbers

$$z = a + ib = r(\cos \theta + i \sin \theta)$$

$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$x = A \cos(nt + \alpha) + c$$

$$x = A \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

$$v^2 = n^2 [A^2 - (x - c)^2]$$

Mathematics Extension 1

Trigonometric functions

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Functions

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential calculus

Function

Derivative

$$y = \sin^{-1} f(x) \quad \frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x) \quad \frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x) \quad \frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

Integral calculus

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

Vectors

$$|\mathbf{u}| = |x\mathbf{i} + y\mathbf{j}| = \sqrt{x^2 + y^2}$$

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| |\mathbf{v}| \cos \theta = x_1 x_2 + y_1 y_2$$

$$\text{where } \mathbf{u} = x_1 \mathbf{i} + y_1 \mathbf{j}$$

$$\text{and } \mathbf{v} = x_2 \mathbf{i} + y_2 \mathbf{j}$$

$$\text{proj}_{\mathbf{b}} \mathbf{a} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|^2} \right) \mathbf{b} = (\mathbf{a} \cdot \hat{\mathbf{b}}) \hat{\mathbf{b}}$$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x + y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 + \cdots + {}^nC_{n-1} x y^{n-1} + {}^nC_n y^n$$

Mathematics Advanced

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial mathematics

$$FV = PV(1 + r)^n$$

Sequences and series

$$a_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + a_n)$$

$$a_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}, r \neq 1$$

$$S_\infty = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and exponential functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Relations

$$(x - a)^2 + (y - b)^2 = r^2$$

Trigonometric functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

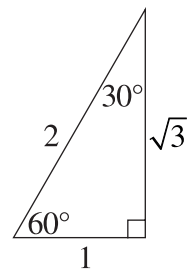
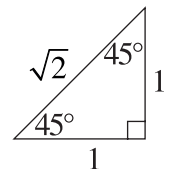
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Differential calculus

Function

Derivative

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^x$$

$$\frac{dy}{dx} = (\ln a) a^x$$

$$y = \log_a x$$

$$\frac{dy}{dx} = \frac{1}{x \ln a}$$

Integral calculus

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int_a^b f(x) dx$$

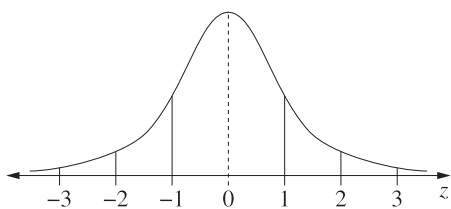
$$\approx \frac{b-a}{2n} \left\{ f(a) + f(b) + 2[f(x_1) + \dots + f(x_{n-1})] \right\}$$

where $a = x_0$ and $b = x_n$

Statistical analysis

$$z = \frac{x - \mu}{\sigma}$$

Normal distribution



- approximately 68% of scores have z-scores between -1 and 1
- approximately 95% of scores have z-scores between -2 and 2
- approximately 99.7% of scores have z-scores between -3 and 3

Discrete random variables

$$E(X) = \mu$$

$$\text{Var}(X) = E[(X - \mu)^2]$$

Continuous random variables

$$P(X \leq x) = F(x) = \int_a^x f(t) dt$$

$$P(a < X < b) = \int_a^b f(x) dx$$

$$E(X) = \mu = \int_a^b x f(x) dx$$

$$\text{Var}(X) = E(X^2) - \mu^2 = \int_a^b x^2 f(x) dx - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$