NSW GOVERNMENT

NSW Education Standards Authority

Centre Number

Student Number

This sample HSC examination paper is designed to show one way the Mathematics Advanced 11–12 Syllabus (2024) could be examined. It reflects the layout of a formatted HSC examination paper.

Sample HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 3–8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9–36)

- Attempt Questions 11–35
- Allow about 2 hours and 45 minutes for this section

The first HSC examination for the Mathematics Advanced 11–12 Syllabus (2024) will be held in 2027. The Mathematics Advanced examination does not include items that are common with other Mathematics HSC examinations. Where they are still relevant, past HSC questions have been used to illustrate different types of questions and how the syllabus can be examined.

The HSC Mathematics Advanced examination specifications can be found in the assessment information for the Mathematics Advanced 11–12 Syllabus (2024).

This sample examination provides examples of some types of questions that may be found in HSC examinations for Mathematics Advanced. Each question has been mapped to show how the question relates to syllabus outcomes and content.

Answers for the objective-response questions (Section I) and sample answers and marking guidelines for all other questions (Section II) are provided. The marking guidelines indicate the criteria for each mark.

In the examination, students will record their answers to Section I on a multiple-choice answer sheet and their answers to Section II in the spaces provided on the examination paper.

The sample questions, marking criteria, sample answers and annotations provide teachers and students with guidance as to the types of questions to expect and how they may be marked. They are not meant to be prescriptive. Each year the structure of the examination may differ in the number and types of questions, or focus on different syllabus outcomes and content.

Note:

Comments in coloured boxes are annotations that provide information and guidance.

Section I

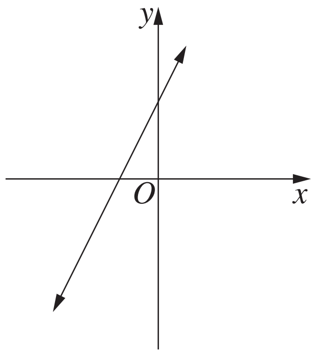
10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

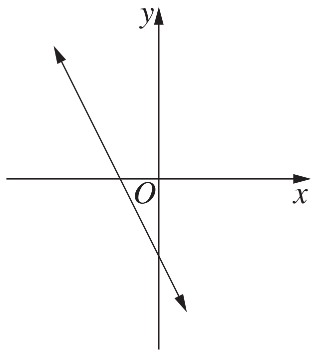
Past HSC examinations provide other examples of questions that may be asked in this section.

1 Which of the following could be the graph of $y = -2x + 2$?

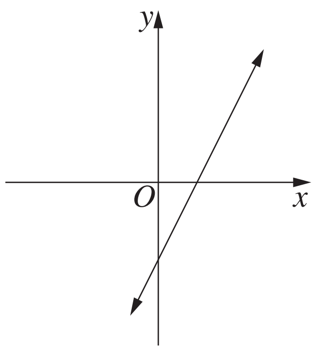
A.



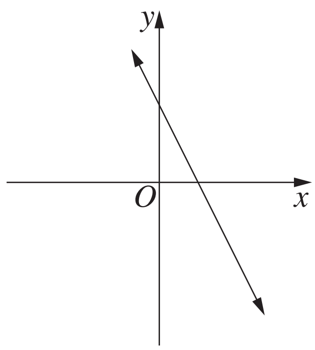
B.



C.



D.



Please turn over

2 The probability distribution for the random variable X which takes values of 0, 1, 2 and 3 is shown.

x	0	1	2	3
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{4}$

What is $\mathrm{E}(X)$?

A. 1

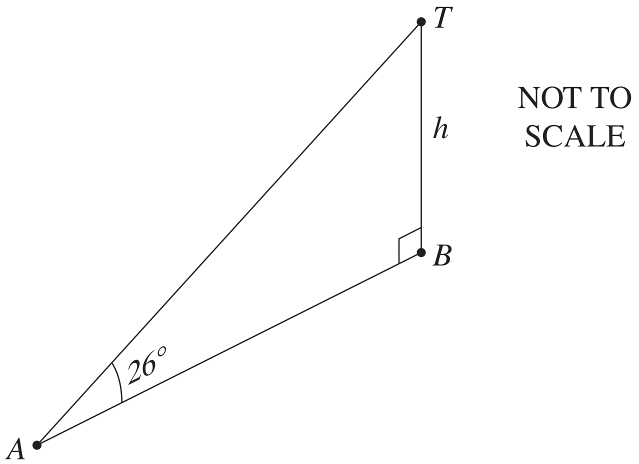
B. $\frac{3}{2}$

C. $\frac{13}{8}$

D. $\frac{15}{8}$

3 A tower BT has height h metres.

From point A, the angle of elevation to the top of the tower is 26° as shown.



Which of the following is the correct expression for the length of AB?

A. $h \tan 26°$

B. $h \cot 26°$

C. $h \sin 26°$

D. $h \operatorname{cosec} 26°$

4 Which of the following is the range of the function $f(x) = x^2 - 1$?

A. $[-1, \infty)$

B. $(-\infty, 1]$

C. $[-1, 1]$

D. $(-\infty, \infty)$

5 Let $h(x) = \dfrac{f(x)}{g(x)}$, where

$$f(1) = 2 \qquad f'(1) = 4$$
$$g(1) = 8 \qquad g'(1) = 12.$$

What is the gradient of the tangent to the graph of $y = h(x)$ at $x = 1$?

A. -8

B. 8

C. $-\dfrac{1}{8}$

D. $\dfrac{1}{8}$

6 What is $\displaystyle\int \frac{1}{(2x+1)^2}\,dx$?

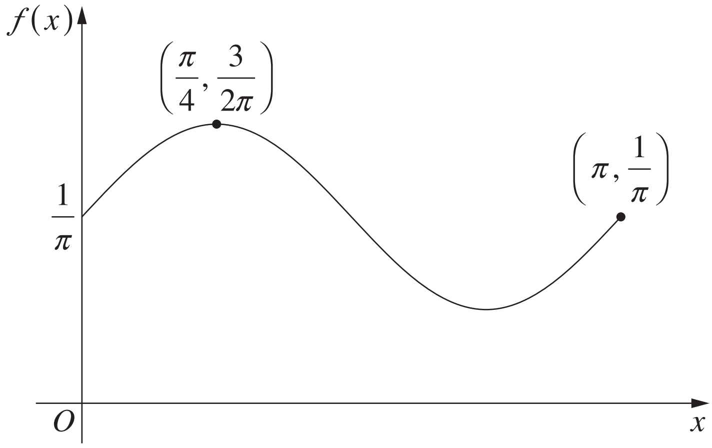
A. $\dfrac{-2}{2x+1} + C$

B. $\dfrac{-1}{2(2x+1)} + C$

C. $2\ln(2x+1) + C$

D. $\dfrac{1}{2}\ln(2x+1) + C$

7 Consider the following graph of a probability density function $f(x)$.



NOT
TO
SCALE

What is the value of the mode?

A. $\frac{1}{\pi}$

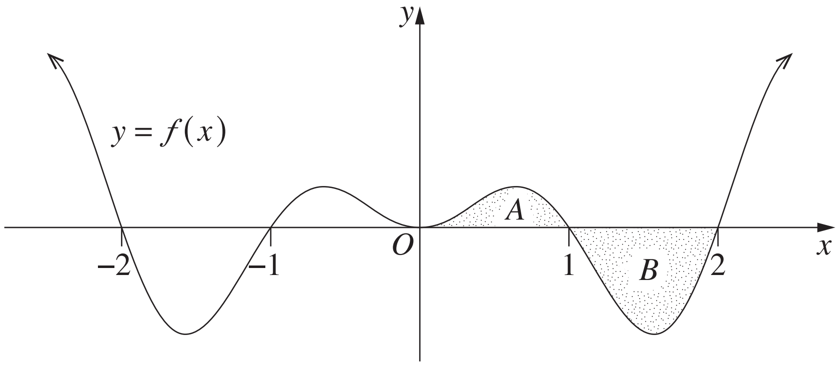
B. $\frac{3}{2\pi}$

C. $\frac{\pi}{4}$

D. π

8 The graph of the even function $y = f(x)$ is shown.

The area of the shaded region A is $\frac{1}{2}$ and the area of the shaded region B is $\frac{3}{2}$.



NOT TO SCALE

What is the value of $\displaystyle\int_{-2}^{2} f(x)\,dx$?

A. –4

B. –2

C. 2

D. 4

9 The graph of $y = x^2$ is first translated 5 units to the right. The new translated graph is then dilated horizontally by a factor of 2. The final graph produced is denoted by $y = f(x)$.

The graph of $y = f(x)$ can also be produced by dilating $y = x^2$ vertically by a factor of a and then translating the resultant graph b units to the right.

What are the values of a and b?

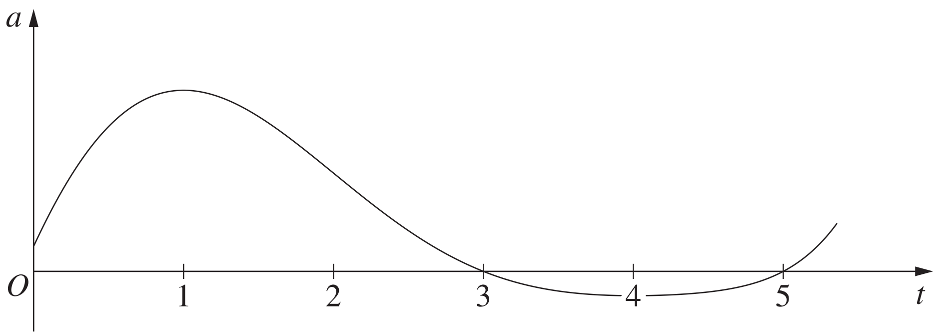
A. $a = \frac{1}{4}$ and $b = 10$

B. $a = \frac{1}{2}$ and $b = 10$

C. $a = \frac{1}{4}$ and $b = 5$

D. $a = \frac{1}{2}$ and $b = 5$

10 A particle is moving along a straight line. The graph shows the acceleration of the particle.



For what value of t is the velocity v a maximum?

A. 1

B. 2

C. 3

D. 5

Sample HIGHER SCHOOL CERTIFICATE EXAMINATION

Centre Number

Student Number

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11–35
Allow about 2 hours and 45 minutes for this section

Instructions

- Write your Centre Number and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Past HSC examinations provide other examples of questions that may be asked in this section.

Please turn over

Question 11 (1 mark)

Simplify $(3a^4)^2$. **1**

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Question 12 (2 marks)

Simplify $\dfrac{x^2}{(x-3)(x+1)} + \dfrac{1}{x+1}$. **2**

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Question 13 (2 marks)

Evaluate $\displaystyle\sum_{r=4}^{7}\left(2^r + 3r\right)$. **2**

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Question 14 (2 marks)

The quantity y varies inversely with the square of x. **2**

When $x = 4$, $y = 10$.

Find the value of y when $x = 5$.

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Question 15 (4 marks)

Consider the function $y = f(x)$ where

$$f(x) = \begin{cases} x^2 + 6, & \text{for } x \le 0 \\ 6, & \text{for } 0 < x \le 3 \\ 2^x, & \text{for } x > 3. \end{cases}$$

(a) Sketch $y = f(x)$. **3**

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(b) For what value of x is the function $y = f(x)$ NOT continuous? **1**

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Question 16 (3 marks)

Consider the universal set $U = \{1,2,3,4,5,6,7,8,9,10\}$.

Two sets, A and B, are given as

$$A = \{1,3,4,7,9\}$$
$$B = \{2,4,7,10\}.$$

(a) Find $A \cup B$ **1**

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(b) Find $A \cap \overline{B}$ **2**

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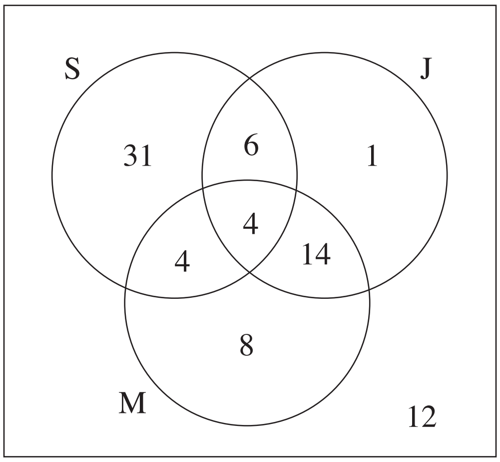
Question 17 (2 marks)

In Year 11 there are 80 students. The students may choose to study Spanish (S), Japanese (J) and Mandarin (M). **2**

The Venn diagram shows their choices.



Two of the students are selected at random.

What is the probability that both students study only Spanish?

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Question 18 (2 marks)

Use two applications of the trapezoidal rule to find an approximate value of $\int_0^2 \sqrt{1 + x^2}\, dx$. Give your answer correct to 2 decimal places. **2**

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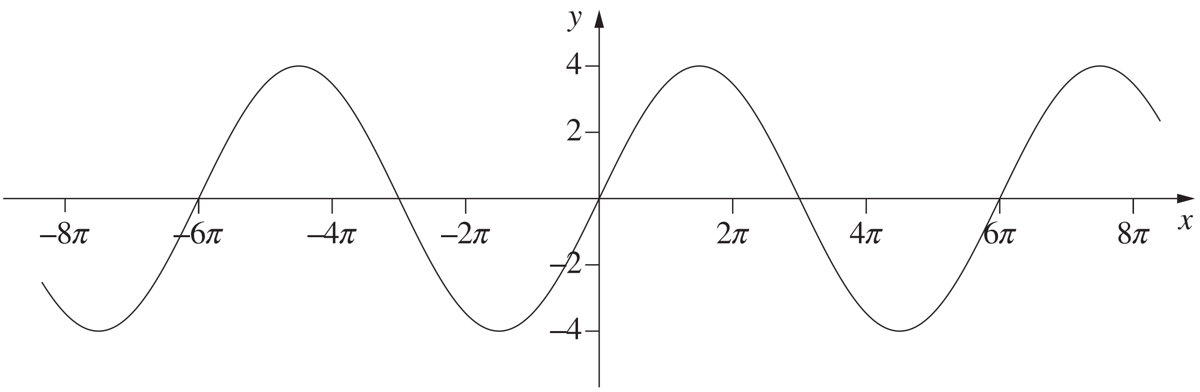
Question 19 (2 marks)

The graph of $y = k\sin(ax)$ is shown. **2**



What are the values of k and a?

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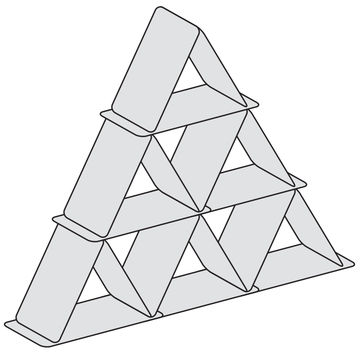
Question 20 (5 marks)

Cards are stacked to build a 'house of cards'. A house of cards with 3 rows is shown.

A house of cards requires 3 cards in the top row, 6 cards in the next row, and each successive row has 3 more cards than the previous row.



(a) Show that a house of cards with 12 rows has a total of 234 cards. **2**

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(b) Another house of cards has a total of 828 cards. **3**

How many rows are in this house of cards?

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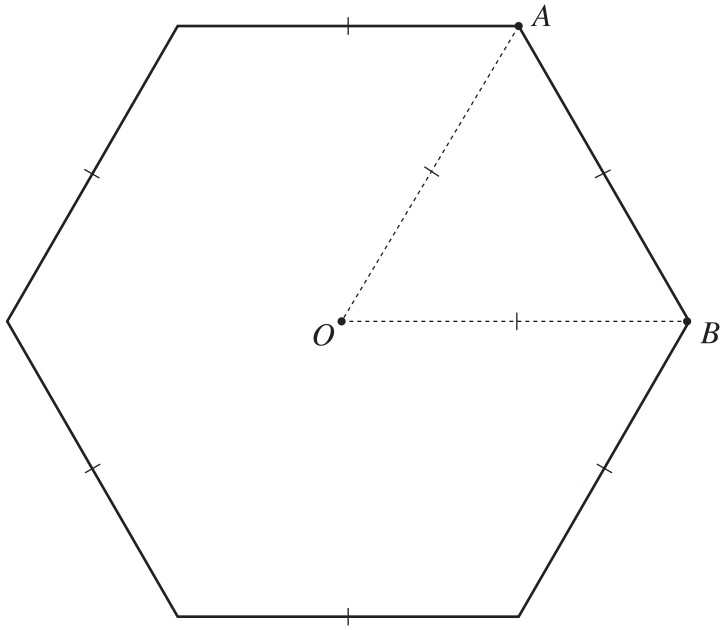
Question 21 (3 marks)

A regular hexagon with centre O is divided into six identical equilateral triangles. One such triangle is shown on the diagram. **3**



NOT TO SCALE

The area of the hexagon is $12\sqrt{3}\ \text{cm}^2$.

By considering the area of triangle OAB, calculate the exact perimeter of the hexagon.

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Question 22 (2 marks)

The table shows the probability distribution for the discrete random variable X. **2**

x	1	2	3	4
$P(X = x)$	0.3	0.4	0.2	0.1

Calculate Var(X).

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Question 23 (2 marks)

Show that $\dfrac{d}{dx}(\cot x) = -\text{cosec}^2 x$. **2**

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Question 24 (4 marks)

A quantity of radioactive material decays according to the equation

$$\frac{dM}{dt} = -kM,$$

where M is the mass of the material in kilograms, t is the time in years and k is a constant.

(a) Verify that $M = Ae^{-kt}$ is a solution to this equation, where A is a constant. **1**

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(b) The time for half of the material to decay is 300 years. The initial amount of material is 20 kg. **3**

Find the amount of material remaining after 1000 years.

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Question 25 (2 marks)

Find $\int x\left(x^2+1\right)^3 dx$. **2**

Question 26 (4 marks)

The graph of $y = x^4$ is dilated horizontally by a factor of k where $k > 0$. The normal to this dilated graph at $x = k$ intersects the y-axis at $(0, 5)$. **4**

Find the value of k.

Question 27 (6 marks)

The depth of water in a bay rises and falls with the tide. On a particular day the depth of the water, d metres, can be modelled by the equation

$$d = 1.3 - 0.6\cos\left(\frac{4\pi}{25}t\right),$$

where t is the time in hours since low tide.

(a) Find the depth of water at low tide and at high tide. **2**

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(b) What is the time interval, in hours, between two successive low tides? **1**

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(c) For how long between successive low tides will the depth of water be at least 1 metre? **3**

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Question 28 (3 marks)

Let $f(x) = \sin(2x)$. **3**

Find the value of x, for $0 < x < \pi$, for which $f'(x) = -\sqrt{3}$ AND $f''(x) = 2$.

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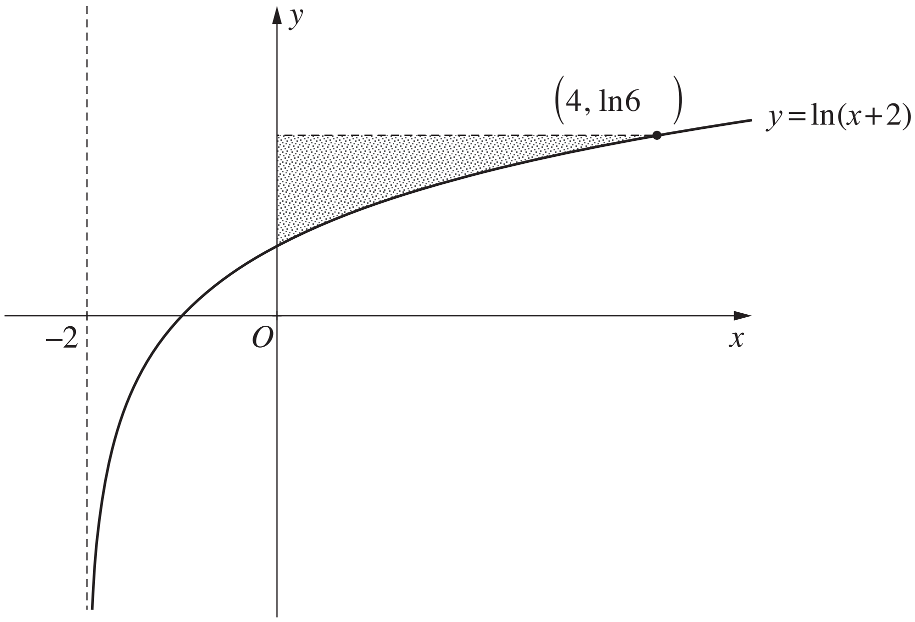
Questions 11–28 are worth 51 marks in total

Question 29 (4 marks)

The diagram shows the graph of $y = \ln(x + 2)$. **4**



Find the exact value of the shaded area bounded by the y-axis, the line $y = \ln 6$ and the curve $y = \ln(x + 2)$. Express your answer in the form $a + b \ln c$ where a, b and c are integers.

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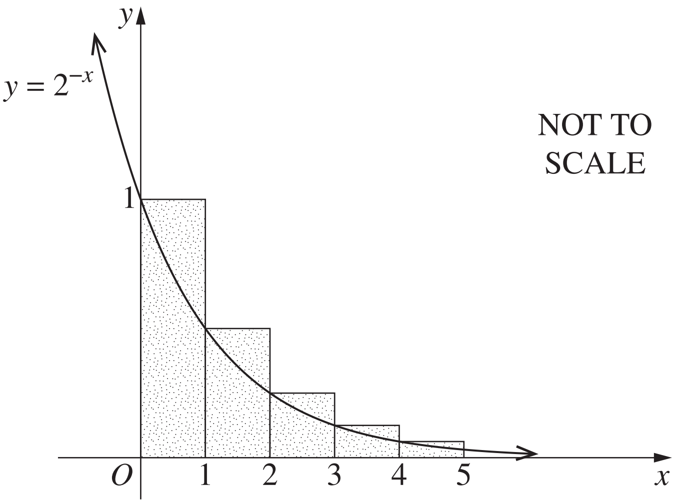
Question 30 (6 marks)

(a) Find the stationary point on the curve $y = xe^x$ and determine its nature. **3**

(b) Sketch the curve $y = xe^x$, showing the stationary point, any points of inflection and intercepts with the axes. **3**

Question 31 (6 marks)

The diagram shows the graph of $y = 2^{-x}$. Also shown on the diagram are the first 5 of an infinite number of rectangular strips of width 1 unit and height $y = 2^{-x}$ for non-negative integer values of x. For example, the second rectangle shown has width 1 and height $\frac{1}{2}$.



The sum of the areas of the rectangles forms a geometric series.

(a) Show that the limiting sum of this series is 2. **1**

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(b) (i) Show that $2^{-x} = e^{-x\ln 2}$. **1**

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Question 31 continues on page 25

Question 31 (continued)

(b) (ii) Hence, or otherwise, show that $\int_0^4 2^{-x}\,dx = \dfrac{15}{16\ln 2}$. **2**

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(c) Use parts (a) and (b)(ii) to show that $e^{15} < 2^{32}$. **2**

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End of Question 31

Question 32 (6 marks)

A continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} kx(1-x)^5, & \text{for } 0 \le x \le 1 \\ 0, & \text{for all other values of } x \end{cases}, \text{ where } k \text{ is a constant.}$$

It is given that

$$\int_0^a x(1-x)^5 \, dx = \frac{1}{42} + \frac{(1-a)^7}{7} - \frac{(1-a)^6}{6}$$

and $$\int_0^1 x^m (1-x)^5 \, dx = \frac{120}{(m+1)(m+2)(m+3)(m+4)(m+5)(m+6)}$$

where $a > 0$ and $m > 0$.

(a) Show that $k = 42$. **1**

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(b) Show that $\mathrm{E}(X) = 0.25$. **2**

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Question 32 continues on page 27

Question 32 (continued)

(c) Show that the median of X is less than the expected value of X. **3**

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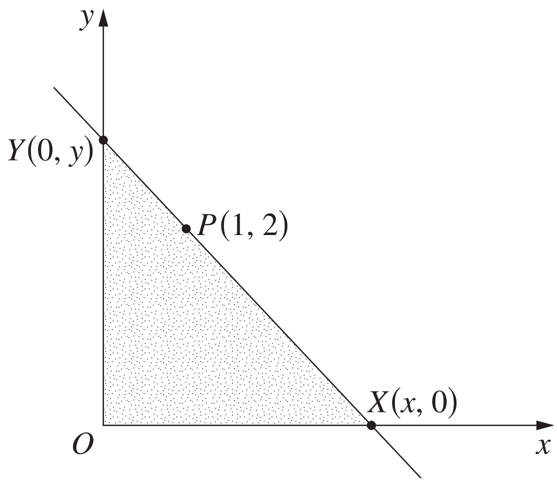
End of Question 32

Question 33 (6 marks)

A line passes through the point $P(1, 2)$ and meets the axes at $X(x, 0)$ and $Y(0, y)$, where $x > 1$.



NOT TO SCALE

(a) Show that $y = \dfrac{2x}{x-1}$. **2**

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Question 33 continues on page 29

Question 33 (continued)

(b) Find the minimum value of the area of triangle XOY. **4**

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End of Question 33

Question 34 (7 marks)

In a reducing-balance loan, an amount $\$P$ is borrowed for a period of n months at an interest rate of 0.25% per month, compounded monthly. At the end of each month, a repayment of $\$M$ is made. After the nth repayment has been made, the amount owing, $\$A_n$, is given by

$$A_n = P(1.0025)^n - M\left(1 + (1.0025)^1 + (1.0025)^2 + \cdots + (1.0025)^{n-1}\right).$$

(Do NOT prove this.)

(a) Jane borrows \$200 000 in a reducing-balance loan as described. **2**

The loan is to be repaid in 180 monthly repayments.

Show that $M = 1381.16$, when rounded to the nearest cent.

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Question 34 continues on page 31

Question 34 (continued)

(b) After 100 repayments of \$1381.16 have been made, the interest rate changes to 0.35% per month. **3**

At this stage, the amount owing to the nearest dollar is \$100 032. (Do NOT prove this.)

Jane continues to make the same monthly repayments.

For how many more months will Jane need to make full monthly payments of \$1381.16?

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(c) The final repayment will be less than \$1381.16. **2**

How much will Jane need to pay in the final payment in order to pay off the loan?

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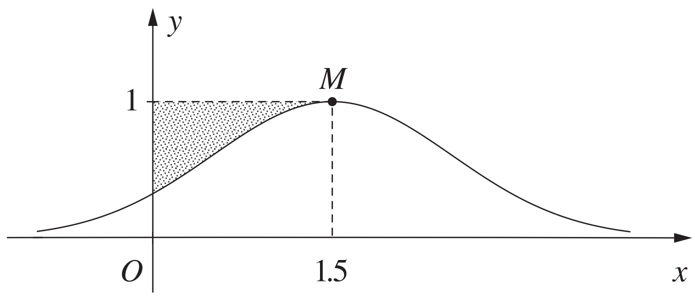
End of Question 34

Question 35 (4 marks)

The probability density function for the normal distribution with mean μ and standard deviation σ is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

The graph of $y = e^{-\frac{1}{2}(x-1.5)^2}$ is shown. The point M is a local maximum.



By using the information on page 33, calculate the area of the shaded region. Give your answer correct to three decimal places. **4**

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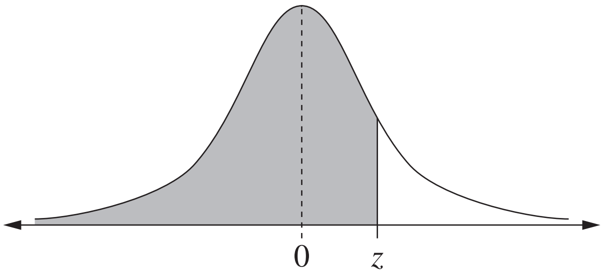
Question 35 is an example of a question that provides additional information or formulae to answer the question.

End of paper

Use the information below to answer Question 35.

Table of values $P(Z \leq z)$ for the normal distribution $N(0,1)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995

Section II extra writing space

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NSW Education Standards Authority

Sample 2027 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced

The reference sheet has been updated. Additions have been highlighted in red boxes below.

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(a + b\right)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial mathematics

$$FV = PV\left(1 + r\right)^n$$

Sequences and series

$$a_n = a + \left(n - 1\right)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}\left(a + a_n\right)$$

$$a_n = ar^{n-1}$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r},\ r \neq 1$$

$$S_\infty = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and exponential functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Relations

$$\left(x - a\right)^2 + \left(y - b\right)^2 = r^2$$

Trigonometric functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

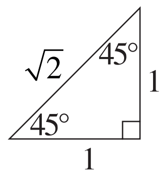
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

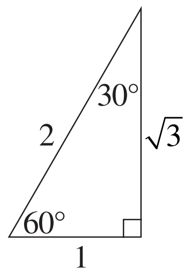
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A},\ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Differential calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^x$	$\dfrac{dy}{dx} = (\ln a)a^x$
$y = \log_a x$	$\dfrac{dy}{dx} = \dfrac{1}{x \ln a}$

Integral calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int_a^b f(x)\,dx$$

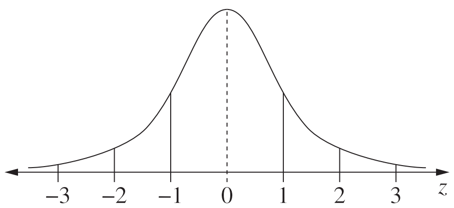
$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Statistical analysis

$$z = \frac{x - \mu}{\sigma}$$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

Discrete random variables

$$\mathrm{E}(X) = \mu$$

$$\mathrm{Var}(X) = \mathrm{E}\left[(X - \mu)^2\right]$$

Continuous random variables

$$P(X \le x) = F(x) = \int_a^x f(t)\,dt$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

$$\mathrm{E}(X) = \mu = \int_a^b x f(x)\,dx$$

$$\mathrm{Var}(X) = \mathrm{E}\left(X^2\right) - \mu^2 = \int_a^b x^2 f(x)\,dx - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

HSC Mathematics Advanced Sample Examination

Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	D
2	C
3	B
4	A
5	D
6	B
7	C
8	B
9	A
10	C

Section II

Question 11

Criteria	Marks
• Provides correct answer	1

Sample answer:

$= 9a^8$

Question 12

Criteria	Marks
• Provides correct solution	2
• Finds the lowest common denominator and attempts to combine the fractions, or equivalent merit	1

Sample answer:

$$= \frac{x^2 + x - 3}{(x-3)(x+1)}$$

Question 13

Criteria	Marks
• Provides correct solution	2
• Demonstrates some understanding of sigma notation, or equivalent merit	1

Sample answer:

$= 2^4 + 12 + 2^5 + 15 + 2^6 + 18 + 2^7 + 21$

$= 306$

Question 14

Criteria	Marks
• Provides correct answer	2
• Find the equation linking x and y, or equivalent merit	1

Sample answer:

$$y = \frac{k}{x^2}$$

$$\therefore 10 = \frac{k}{4^2}$$

$$\therefore k = 160$$

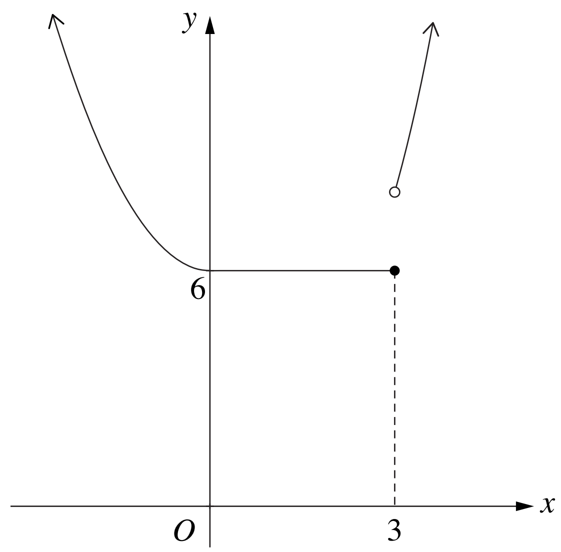
$$\text{When } x = 5,\ y = \frac{160}{5^2}$$

$$= 6.4$$

Question 15 (a)

Criteria	Marks
• Provides correct graph	3
• Provides a substantially correct graph	2
• Provides one correct component of the graph, or equivalent merit	1

Sample answer:



Question 15 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$x = 3$

Question 16 (a)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$A \cup B = \{1, 2, 3, 4, 7, 9, 10\}$

Question 16 (b)

Criteria	Marks
• Provides correct answer	2
• Finds $\overline{B}$, or equivalent merit	1

Sample answer:

$$\overline{B} = \{1, 3, 5, 6, 8, 9\}$$

$$\therefore A \cap \overline{B} = \{1, 3, 9\}$$

Question 17

Criteria	Marks
• Provides correct solution	2
• Provides correct probability for one student studying only Spanish, or equivalent merit	1

Sample answer:

$$\frac{31}{80} \times \frac{30}{79} = \frac{93}{632}$$

Question 18

Criteria	Marks
• Provides correct solution	2
• Finds function values at 0 and 2, or equivalent merit	1

Sample answer:

x	0	1	2
$\sqrt{1+x^2}$	1	1.414	2.236

$$\int_0^2 \sqrt{1+x^2}\,dx = \frac{2-0}{2\,(2)}\left[1+2.236+2(1.414)\right]$$

$$= 3.03 \text{ (2 decimal places)}$$

Question 19

Criteria	Marks
• Provides the values of k and a	2
• Finds the value of k or a, or equivalent merit	1

Sample answer:

k = amplitude = 4

Period = $\dfrac{2\pi}{a}$

$6\pi = \dfrac{2\pi}{a}$

$\therefore \quad a = \dfrac{1}{3}$

Question 20 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the values of a and d, or equivalent merit	1

Sample answer:

Number of cards $= 3 + 6 + 9 + \cdots$ (12 terms)

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$a = 3 \qquad d = 3$$

$$S_{12} = \frac{12}{2}(2 \times 3 + 11 \times 3)$$

$$= 234$$

Question 20 (b)

Criteria	Marks
• Provides correct solution	3
• Finds correct quadratic expression	2
• Correctly substitutes a, d and 828 into formula for S_n, or equivalent merit	1

Sample answer:

Let n = number of rows

$$\frac{n}{2}\left(2 \times 3 + (n-1)3\right) = 828$$

$$n(3n+3) = 1656$$

$$3n(n+1) = 1656$$

$$n^2 + n = 552$$

$$n^2 + n - 552 = 0$$

$$n = \frac{-1 \pm \sqrt{1^2 - 4 \times -552}}{2}$$

$$= \frac{-1 \pm 47}{2}$$

$= 23$ (taking positive solution)

There are 23 rows.

Question 21

Criteria	Marks
• Provides correct solution	3
• Attempts to solve the equation for the unknown side, or equivalent merit	2
• Substitutes into an appropriate formula for area of a triangle, or equivalent merit	1

Sample answer:

Area of $\Delta OAB = \dfrac{12\sqrt{3}}{6}$ cm^2

$= 2\sqrt{3}$ cm^2

Also, $A = \dfrac{1}{2} \times OA \times OB \times \sin 60°$

ie $2\sqrt{3} = \dfrac{1}{2} \times x \times x \times \sin 60°$ (where x = OA = OB)

$$2\sqrt{3} = \frac{x^2 \sin 60°}{2}$$

$$\frac{x^2\sqrt{3}}{2} = 4\sqrt{3}$$

$$x^2 = \frac{8\sqrt{3}}{\sqrt{3}}$$

$$x^2 = 8$$

$$x = \sqrt{8} = 2\sqrt{2}$$

$\therefore AB = 2\sqrt{2}$ (since ΔOAB is equilateral).

Perimeter of hexagon $= 6 \times 2\sqrt{2}$ cm

$= 12\sqrt{2}$ cm

Question 22

Criteria	Marks
• Provides correct answer	2
• Provides correct expression for $E(X)$, or equivalent merit	1

Sample answer:

$$
\begin{aligned}
Var(X) &= E\left(X^2\right) - \mu^2 \\
&= 1^2(0.3) + 2^2(0.4) + 3^2(0.2) + 4^2(0.1) - \left[1(0.3) + 2(0.4) + 3(0.2) + 4(0.1)\right]^2 \\
&= 5.3 - 2.1^2 \\
&= 0.89
\end{aligned}
$$

Question 23

Criteria	Marks
• Provides correct solution	2
• Attempts to use the quotient rule, or equivalent merit	1

Sample answer:

$$\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$$

$$= \frac{-\sin x \sin x - \cos x \cos x}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= -\left(\frac{1}{\sin^2 x}\right)$$

$$= -\operatorname{cosec}^2 x$$

Question 24 (a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$
\begin{aligned}
M &= Ae^{-kt} \\
\frac{dM}{dt} &= -kAe^{-kt} \\
&= -kM \ \text{ (since } M = Ae^{-kt}\text{)}
\end{aligned}
$$

Question 24 (b)

Criteria	Marks
• Provides correct solution	3
• Finds the value of k, or equivalent merit	2
• Finds the value of A, or equivalent merit	1

Sample answer:

When $t = 0,\ M = 20$

$$20 = Ae^{-k(0)} = A$$

$$\therefore M = 20e^{-kt}$$

When $t = 300,\ M = 10$

$$10 = 20e^{-300k}$$

$$\frac{1}{2} = e^{-300k}$$

$$-300k = \ln\frac{1}{2} = \ln 2$$

$$k = \frac{\ln 2}{300}$$

When $t = 1000$,

$$M = 20e^{-1000k}$$

$$= 20e^{-\frac{10}{3}\ln 2}$$

$$= 1.984\ldots$$

$\therefore$ The amount remaining after 1000 years is approximately 1.98 kg

Question 25

Criteria	Marks
• Provides correct solution	2
• Includes $2x$ in the integrand, or equivalent merit	1

Sample answer:

$$\int x\left(x^2+1\right)^3 dx$$

$$=\frac{1}{2}\int 2x\left(x^2+1\right)^3 dx$$

$$=\frac{1}{2}\times\frac{1}{4}\left(x^2+1\right)^4+c$$

$$=\frac{1}{8}\left(x^2+1\right)^4+c$$

Question 26

Criteria	Marks
• Provides correct solution	4
• Finds the y-intercept of the normal, or equivalent merit	3
• Finds the gradient of the normal, or equivalent merit	2
• Provides equation of transformed graph, or equivalent merit	1

Sample answer:

Transformed graph has equation $y = \dfrac{x^4}{k^4}$

$\dfrac{dy}{dx} = \dfrac{4x^3}{k^4}$

Gradient of normal at $x = k$ is $-\dfrac{k^4}{4k^3} = -\dfrac{k}{4}$

Equation of normal at $x = k$ is $y - 1 = -\dfrac{k}{4}(x - k)$

y-intercept of normal is at $\left(0,\ 1 + \dfrac{k^2}{4}\right)$.

$\therefore\ 1 + \dfrac{k^2}{4} = 5$

$\therefore\ k^2 = 16$

Since $k > 0$, $k = 4$.

Question 27 (a)

Criteria	Marks
• Provides correct solution	2
• Finds the depth of water at either high or low tide, or equivalent merit	1

Sample answer:

$d = 1.3 - 0.6\cos\left(\dfrac{4\pi}{25}t\right)$

Maximum depth (*high tide*) $= 1.3 + 0.6 = 1.9$ m

Minimum depth (*low tide*) $= 1.3 - 0.6 = 0.7$ m

Question 27 (b)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\text{Period} = \frac{2\pi}{\left(\frac{4\pi}{25}\right)} \text{ hours}$$

$$= 12.5 \text{ hours}$$

Question 27 (c)

Criteria	Marks
• Provides correct solution	3
• Finds one value of t for $d = 1$, or equivalent merit	2
• Attempts to solve the given equation when $d = 1$	1

Sample answer:

$$d = 1.3 - 0.6\cos\left(\frac{4\pi}{25}t\right)$$

$$1 = 1.3 - 0.6\cos\left(\frac{4\pi}{25}t\right), \qquad \text{when } d = 1$$

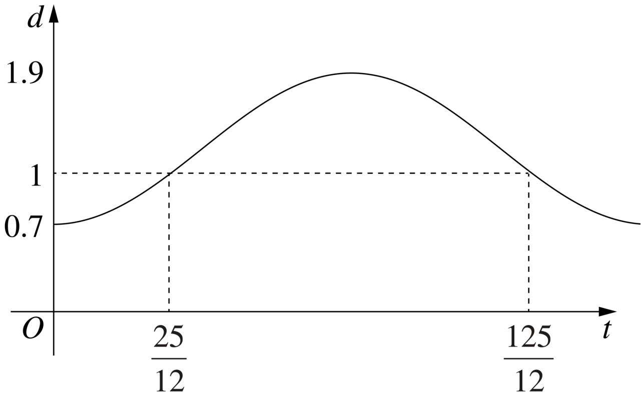
$$-0.3 = -0.6\cos\left(\frac{4\pi}{25}t\right)$$

$$\frac{1}{2} = \cos\left(\frac{4\pi}{25}t\right)$$

$$\frac{4\pi}{25}t = \frac{\pi}{3}, \ \frac{5\pi}{3}$$

$$t = \frac{\pi}{3} \times \frac{25}{4\pi}, \ \frac{5\pi}{3} \times \frac{25}{4\pi}$$

$$= \frac{25}{12}, \ \frac{125}{12}$$



$$\text{Time interval } = \frac{125}{12} - \frac{25}{12}$$

$$= 8\frac{1}{3} \text{ hours}$$

Question 28

Criteria	Marks
• Provides correct solution	3
• Finds the values of x, $0 < x < \pi$, for each derivative equation	2
• Attempts to find a value for x that solves $f'(x)$ or $f''(x)$, or equivalent merit	1

Sample answer:

$$f(x) = \sin 2x$$

$$f'(x) = 2\cos 2x$$

$$f''(x) = -4\sin 2x$$

$$f'(x) = -\sqrt{3} \quad \Rightarrow \quad 2\cos 2x = -\sqrt{3}$$

$$\cos 2x = \frac{-\sqrt{3}}{2} \qquad 0 < 2x < 2\pi$$

$$2x = \frac{5\pi}{6}, \frac{7\pi}{6}$$

$$x = \frac{5\pi}{12}, \frac{7\pi}{12}$$

$$f''(x) = 2 \quad \Rightarrow \quad -4\sin 2x = 2$$

$$\sin 2x = \frac{-1}{2} \qquad 0 < 2x < 2\pi$$

$$2x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{7\pi}{12}, \frac{11\pi}{12}$$

$$\therefore \ x = \frac{7\pi}{12} \text{ for } 0 < x < \pi$$

Question 29

Criteria	Marks
• Provides correct solution	4
• Performs correct integration, or equivalent merit	3
• Provides correct integral, or equivalent merit	2
• Provides correct expression for x, or equivalent merit	1

Sample answer:

$$x + 2 = e^y$$

$$x = e^y - 2$$

$$A = \int_{\ln 2}^{\ln 6} \left(e^y - 2\right) dy$$

$$= \left[e^y - 2y\right]_{\ln 2}^{\ln 6}$$

$$= e^{\ln 6} - 2\ln 6 - e^{\ln 2} + 2\ln 2$$

$$= 6 - 2 - 2\left(\ln \frac{6}{2}\right)$$

$$= (4 - 2\ln 3)\,\text{units}^2$$

Question 30 (a)

Criteria	Marks
• Provides correct solution	3
• Provides correct stationary point and attempts to find its nature, or equivalent merit	2
• Provides correct derivative, or equivalent merit	1

Sample answer:

$$y' = xe^x + e^x$$

$$= e^x (x + 1)$$

$$y' = 0 \Rightarrow x = -1$$

$\therefore$ Stationary point at $\left(-1, -\frac{1}{e}\right)$.

$$y'' = e^x + (x + 1)\,e^x$$

$$= e^x (x + 2)$$

When $x = -1$, $y'' = \frac{1}{e} > 0$

$\therefore$ Minimum turning point at $\left(-1, -\frac{1}{e}\right)$.

Question 30 (b)

Criteria	Marks
• Provides correct solution	3
• Finds the point of inflection and the x-intercept, or equivalent merit	2
• Attempts to find point of inflection, or equivalent merit	1

Sample answer:

$y = xe^x$

To find x-intercepts, let $y = 0$

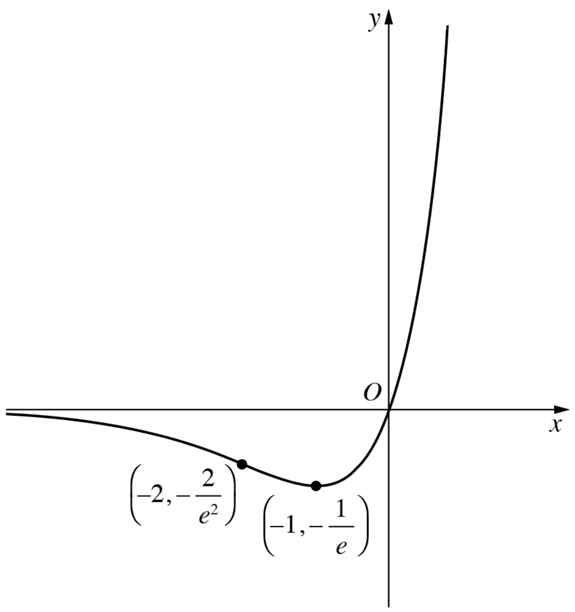
$xe^x = 0$ when $x = 0$, ie at $(0,0)$

$y'' = e^x(x + 2)$

To find point of inflection, let $y'' = 0$

$y'' = 0$ when $x = -2$, $y = -\frac{2}{e^2}$

$\therefore$ point of inflection at $\left(-2, -\frac{2}{e^2}\right)$ since there are no other x-intercepts.



Question 31 (a)

Criteria	Marks
• Shows the limiting sum	1

Sample answer:

Limiting sum = $2^0 + 2^{-1} + 2^{-2} + \cdots$

$$S = \frac{a}{1-r}, \quad \text{where } a = 1 \text{ and } r = \frac{1}{2}$$

$$= \frac{1}{1-\frac{1}{2}}$$

$$= 2$$

Question 31 (b)(i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$2^{-x} = e^{\ln 2^{-x}} \qquad \left(\text{using } x = a^{\log_a x}\right)$$

$$= e^{-x\ln 2}$$

Question 31 (b)(ii)

Criteria	Marks
• Provides correct solution	2
• Substitutes correctly into the primitive function, or equivalent merit	1

Sample answer:

$$\int_0^4 2^{-x}\,dx = \int_0^4 e^{-x\ln 2}\,dx$$

$$= \frac{-1}{\ln 2}\left[e^{-x\ln 2}\right]_0^4$$

$$= \frac{-1}{\ln 2}\left[e^{-4\ln 2} - e^0\right]$$

$$= \frac{-1}{\ln 2}\left[e^{\ln\frac{1}{16}} - 1\right]$$

$$= -\frac{1}{\ln 2}\left(\frac{1}{16} - 1\right)$$

$$= \frac{-1}{\ln 2}\left(\frac{-15}{16}\right)$$

$$= \frac{15}{16\ln 2}$$

Question 31 (c)

Criteria	Marks
• Provides correct solution	2
• Sets up correct inequality, or equivalent merit	1

Sample answer:

$\frac{15}{16\ln 2} < 2$ since area under the curve between 0 and 4 < limiting sum of rectangles

$\therefore \quad 15 < 32\ln 2$

$\therefore \quad 15 < \ln\left(2^{32}\right)$

$\therefore \quad e^{15} < 2^{32}$

Question 32(a)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$k\int_0^1 x(1-x)^5\,dx = 1$$

$$\therefore k\left[\frac{1}{42}+\frac{(1-1)^7}{7}-\frac{(1-1)^6}{6}\right] = 1$$

$$\frac{k}{42} = 1$$

$$k = 42$$

Question 32 (b)

Criteria	Marks
• Provides correct solution	2
• Writes a correct integral expression for $E(X)$, or equivalent merit	1

Sample answer:

$$\mathrm{E}(X) = \int_0^1 42x^2(1-x)^5\,dx$$

$$= 42\times\frac{120}{3\times4\times5\times6\times7\times8}$$

$$= 0.25$$

Question 32 (c)

Criteria	Marks
• Provides correct solution	3
• Evaluates area to the left of $\mathrm{E}(X)$, or equivalent merit	2
• Shows understanding of calculating median from a probability density function, or equivalent merit	1

Sample answer:

$$\int_0^{0.25} 42x(1-x)^5\,dx = 42\left[\frac{1}{42}+\frac{(1-0.25)^7}{7}-\frac{(1-0.25)^6}{6}\right]$$

$$= 0.555\ldots(3dp)$$

$\therefore \int_0^{m} 42x(1-x)^5\,dx = 0.5$ requires the median to be less than 0.25.

Therefore, the median of X is less than the expected value of X.

Question 33 (a)

Criteria	Marks
• Provides correct solution	2
• Equates the two expressions for the gradients, or equivalent merit	1

Sample answer:

gradient XP = gradient PY

$$\frac{0-2}{x-1}=\frac{y-2}{0-1}$$

$$(x-1)(y-2)=2$$

$$y-2=\frac{2}{x-1}$$

$$y=\frac{2}{x-1}+2$$

$$=\frac{2+2x-2}{x-1}$$

$$=\frac{2x}{x-1}$$

Question 33 (b)

Criteria	Marks
• Provides correct solution	4
• Solves $\frac{dA}{dx} = 0$ and determines the nature of stationary point at $x = 2$	3
• Finds $\frac{dA}{dx}$, or equivalent merit	2
• Attempts to find the area in terms of x	1

Sample answer:

$$A = \frac{x^2}{x-1} \qquad \left[\text{from } A = \frac{1}{2}xy \text{ and } y = \frac{2x}{x-1}\right]$$

$$\begin{aligned}\frac{dA}{dx} &= \frac{(x-1)2x - x^2(1)}{(x-1)^2} \\ &= \frac{2x^2 - 2x - x^2}{(x-1)^2} \\ &= \frac{x^2 - 2x}{(x-1)^2} \\ &= \frac{x(x-2)}{(x-1)^2}\end{aligned}$$

$\frac{dA}{dx} = 0$ when $x = 0$ or $x = 2$

discard $x = 0$ since $x > 1$. (from question)

x	1.5	2	3
$\frac{dA}{dx}$	-3	0	$\frac{3}{4}$

Slope changes from negative to positive so

$x = 2$ is a minimum.

$$A = \frac{4}{2-1}$$

$\therefore$ Minimum area = 4 units^2

Question 34 (a)

Criteria	Marks
• Provides correct solution	2
• Uses the sum for a geometric progression	1

Sample answer:

$A_{180} = 0$

$$\therefore\ 200\,000(1.0025)^{180} = M\left[1+(1.0025)^{1}+\cdots+(1.0025)^{179}\right]$$

$$200\,000(1.0025)^{180} = M\left[\frac{(1.0025)^{180}-1}{1.0025-1}\right]$$

$$M = \frac{200\,000(1.0025)^{180}\times 0.0025}{(1.0025)^{180}-1}$$

$$= 1381.163281$$

$$= 1381.16 \text{ when rounded down}$$

Question 34 (b)

Criteria	Marks
• Provides correct solution	3
• Writes the equation in simplest exponential form	2
• Writes the equation in terms of n using the new interest rate and the new principal	1

Sample answer:

$$100\,032(1.0035)^{n} = 1381.16\left[\frac{(1.0035)^{n}-1}{1.0035-1}\right]$$

$$100\,032\times 0.0035(1.0035)^{n} = 1381.16(1.0035)^{n} - 1381.16$$

$$1031.048(1.0035)^{n} = 1381.16$$

$$(1.0035)^{n} = 1.33956906$$

$$n = \frac{\log(1.33956906)}{\log(1.0035)}$$

$$= 83.67407889$$

$$\therefore\ 83 \text{ full payments}$$

Question 34 (c)

Criteria	Marks
• Provides correct solution	2
• Substitutes n = 83 into amount owing, or equivalent merit	1

Sample answer:

After 83 payments

$$\text{Amount owing} = 100\,032(1.0035)^{83} - 1381.16\left[\frac{(1.0035)^{83}-1}{1.0035-1}\right]$$

$$= 928.29$$

Final payment (84th month) = 928.29 × 1.0035 (1-month interest)

$$= 931.54$$

Question 35

Criteria	Marks
• Calculates area under normal curve and subtracts this from 1.5, or equivalent merit	3
• Provides correct solution	4
• Calculates area under normal curve = 0.4332, or equivalent merit	2
• Calculates area of rectangle, or equivalent merit	1

Sample answer:

The area under the curve bounded by the x-axis, the y-axis and the line $x = 1.5$ is

$\sqrt{2\pi}\,(0.9332 - 0.5)$

$= 1.08587...$

Shaded area $= 1.5 - 1.08587...$

$= 0.414$ units^2 (to three decimal places)

HSC Mathematics Advanced Sample Examination

Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes	Targeted performance bands
1	1	Working with functions – Linear functions	MAV-11-02	2–3
2	1	Random variables – Discrete random variables	MAV-12-07	2–3
3	1	Trigonometry and measure of angles – Trigonometry with acute angles	MAV-11-04	3–4
4	1	Working with functions – Properties of functions, relations and graphs	MAV-11-02	3–4
5	1	Differential calculus – Using derivatives	MAV-12-04	3–4
6	1	Integral calculus – Indefinite integrals	MAV-12-05	4–5
7	1	Random variables – Continuous random variables	MAV-12-07	4–5
8	1	Integral calculus – Areas and the definite integral	MAV-12-05	4–5
9	1	Graph transformations	MAV-11-03	5–6
10	1	Applications of calculus – Rates of change	MAV-12-06	5–6

Section II

Question	Marks	Content	Syllabus outcomes	Targeted performance bands
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12	2	Working with functions – Algebraic techniques	MAV-11-01	2–3
13	2	Sequences and series – Sequences and series	MAV-12-03	2–3
14	2	Working with functions – Direct and inverse variation	MAV-11-02	2–4
15 (a)	3	Working with functions – Piecewise – defined functions	MAV-11-02	2–4
15 (b)	1	Working with functions – Piecewise – defined functions	MAV-11-02	3–4
16 (a)	1	Probability and data – Sets and set notation	MAV-11-09	2–3
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17	2	Probability and data – Probability	MAV-11-09	3–5
18	2	Integral Calculus – Areas and the definite integral	MAV-12-05	3–4
19	2	Further graph transformations and modelling – Transformations of trigonometric functions	MAV-12-01	3–4
20 (a)	2	Sequences and series – Arithmetic sequences and series	MAV-12-03	2–4
20 (b)	3	Sequences and series – Arithmetic sequences and series	MAV-12-03	3–5

Question	Marks	Content	Syllabus outcomes	Targeted performance bands
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22	2	Random variables – Discrete random variables	MAV-12-07	3–5
23	2	Differential calculus – Differentiation with trigonometric functions	MAV-12-04	3–4
24 (a)	1	Applications of calculus – Rates of change	MAV-12-06	3–4
24 (b)	3	Applications of calculus – Rates of change	MAV-12-06	3–5
25	2	Integral calculus – Indefinite integrals	MAV-12-05	3–4
26	4	Differential calculus – Using derivatives	MAV-12-04	3–6
27 (a)	2	Further graph transformations and modelling – Modelling with functions	MAV-12-02	3–5
27(b)	1	Further graph transformations and modelling – Modelling with functions	MAV-12-02	3–4
27 (c)	3	Further graph transformations and modelling – Modelling with function	MAV-12-02	3–6
28	3	Applications of calculus – Turning points, inflections and curve-sketching	MAV-12-06	3–5
29	4	Integral calculus – Areas and the definite integral	MAV-12-05	3–5
30 (a)	3	Applications of calculus - Turning points, inflections and curve-sketching	MAV-12-06	3–5
30 (b)	3	Applications of calculus - Turning points, inflections and curve-sketching	MAV-12-06	3–6
31 (a)	1	Sequences and series page – Geometric sequences and series	MAV-12-03	4–5
31 (b)(i)	1	Exponential and logarithmic functions – Logarithmic functions	MAV-11-07	5–6
31 (b)(ii)	2	Integral calculus – Integration with exponential functions	MAV-12-05	3–5
31 (c)	2	Integral calculus – Areas and the definite integral	MAV-12-05	4–6
32 (a)	1	Random variables – Continuous random variables	MAV-12-07	4–5
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33 (a)	2	Working with functions – Linear functions	MAV-11-02	3–5
33 (b)	4	Applications of calculus – Optimisation	MAV-12-06	3–6
34 (a)	2	Financial mathematics – Reducing balance loans	MAV-12-08	3–5
34 (b)	3	Financial mathematics – Reducing balance loans	MAV-12-08	4–6
34 (c)	2	Financial mathematics – Reducing balance loans	MAV-12-08	5–6
35	4	Random variables – The normal distribution	MAV-12-07	4–6