NSW GOVERNMENT

NSW Education Standards Authority

Centre Number

Student Number

This sample HSC examination paper is designed to show one way the Mathematics Extension 1 11–12 Syllabus (2024) could be examined. It reflects the layout of a formatted HSC examination paper.

Sample HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 70

Section I – 10 marks (pages 3–8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 9–16)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

The first HSC examination for the Mathematics Extension 1 11–12 Syllabus (2024) will be held in 2027. The Mathematics Extension 1 examination does not include items that are common with other Mathematics HSC examinations. Where they are still relevant, past HSC questions have been used to illustrate different types of questions and how the syllabus can be examined.

The HSC Mathematics Extension 1 examination specifications can be found in the assessment information for the Mathematics Extension 1 11–12 Syllabus (2024).

This sample examination provides examples of some types of questions that may be found in HSC examinations for Mathematics Extension 1. Each question has been mapped to show how the question relates to syllabus outcomes and content.

Answers for the objective-response questions (Section I) and sample answers and marking guidelines for all other questions (Section II) are provided. The marking guidelines indicate the criteria for each mark.

In the examination, students will record their answers to Section I on a multiple-choice answer sheet and their answers to Section II in the appropriate writing booklets.

The sample questions, marking criteria, sample answers and annotations provide teachers and students with guidance as to the types of questions to expect and how they may be marked. They are not meant to be prescriptive. Each year the structure of the examination may differ in the number and types of questions, or focus on different syllabus outcomes and content.

Note:

Comments in coloured boxes are annotations that provide information and guidance.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

Past HSC examinations provide other examples of questions that may be asked in this section.

1 Given that $\overrightarrow{OP} = \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$, what is $\overrightarrow{PQ}$?

A. $\begin{pmatrix} 1 \\ -6 \\ 4 \end{pmatrix}$

B. $\begin{pmatrix} -1 \\ 6 \\ -4 \end{pmatrix}$

C. $\begin{pmatrix} 5 \\ 4 \\ -2 \end{pmatrix}$

D. $\begin{pmatrix} -5 \\ -4 \\ 2 \end{pmatrix}$

2 Which of the following integrals is equivalent to $\int \sin^2 3x \, dx$?

A. $\int \frac{1+\cos 6x}{2} \, dx$

B. $\int \frac{1-\cos 6x}{2} \, dx$

C. $\int \frac{1+\sin 6x}{2} \, dx$

D. $\int \frac{1-\sin 6x}{2} \, dx$

3 What is the remainder when $P(x) = -x^3 - 2x^2 - 3x + 8$ is divided by $x + 2$?

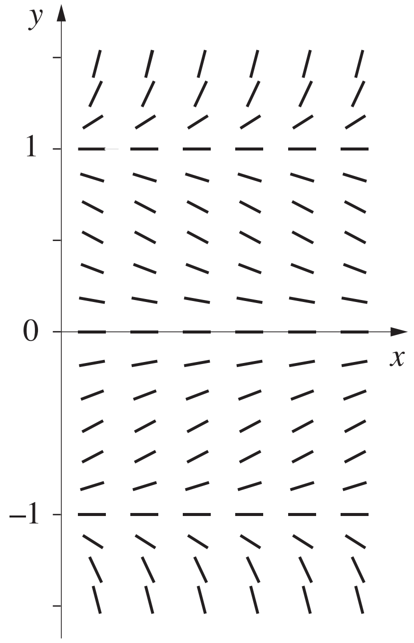
A. -14

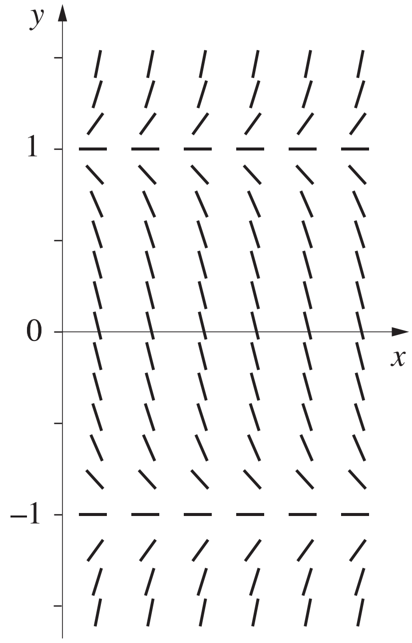
B. -2

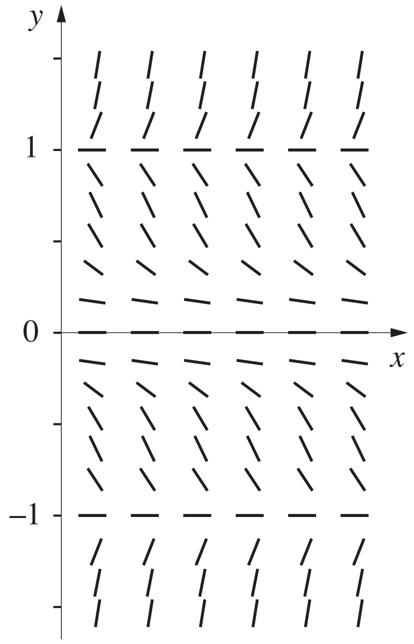
C. 2

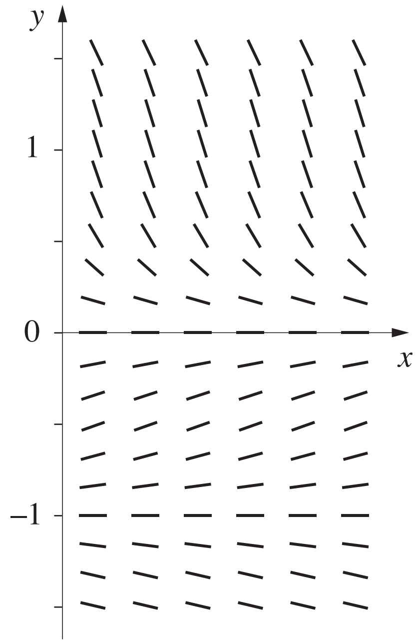
D. 14

4 Which slope field best matches the differential equation $\dfrac{dy}{dx} = -y^2\left(1 - y^2\right)$?

A.


B.


C.


D.


5 For the two vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$, it is known that

$$\overrightarrow{OA} \cdot \overrightarrow{OB} < 0.$$

Which of the following statements MUST be true?

A. Either, $\overrightarrow{OA}$ is negative and $\overrightarrow{OB}$ is positive, or, $\overrightarrow{OA}$ is positive and $\overrightarrow{OB}$ is negative.

B. The angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is obtuse.

C. The product $\left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right|$ is negative.

D. The points O, A and B are collinear.

6 The random variable X represents the number of successes in 10 independent Bernoulli trials. The probability of success is $p = 0.9$ in each trial.

Let $r = P(X \geq 1)$.

Which of the following describes the value of r?

A. $r > 0.9$

B. $r = 0.9$

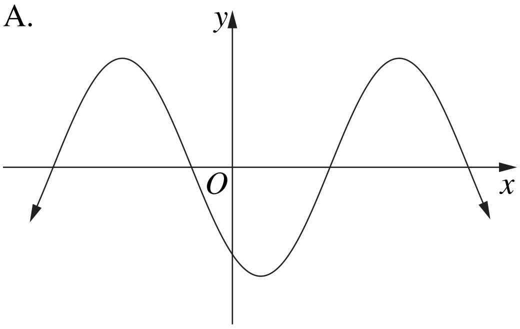
C. $0.1 < r < 0.9$

D. $r \leq 0.1$

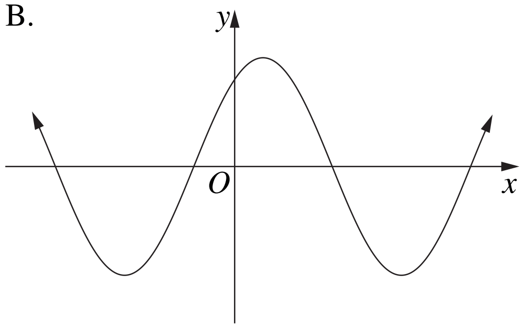
Please turn over

7 Which curve best represents the graph of the function $f(x) = -a\sin x + b\cos x$ given that the constants a and b are both positive?

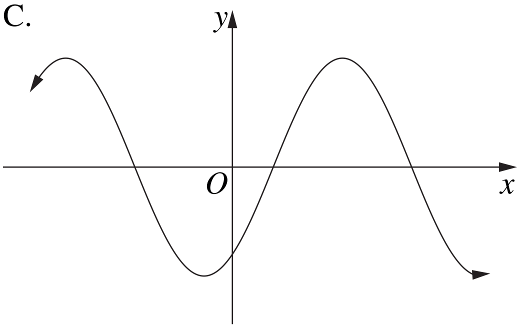
A.



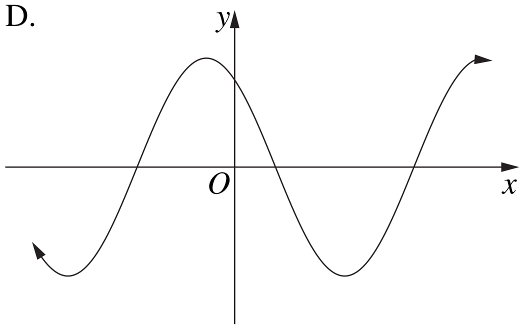
B.



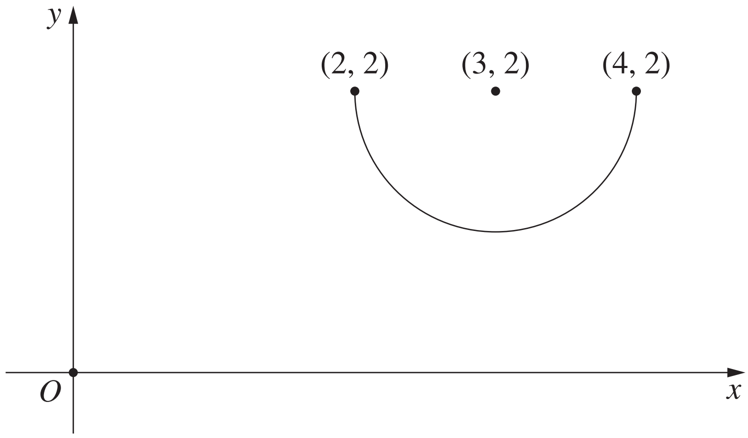
C.



D.



8 The diagram shows a semicircle.



Which pair of parametric equations represents the semicircle shown?

A. $\begin{cases} x = 3 + \sin t \\ y = 2 + \cos t \end{cases}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

B. $\begin{cases} x = 3 + \cos t \\ y = 2 + \sin t \end{cases}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

C. $\begin{cases} x = 3 - \sin t \\ y = 2 - \cos t \end{cases}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

D. $\begin{cases} x = 3 - \cos t \\ y = 2 - \sin t \end{cases}$ for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

9 Consider the differential equation $\dfrac{dy}{dx} = \dfrac{x}{y}$.

Which of the following equations best represents this relationship between x and y?

A. $y^2 = x^2 + c$

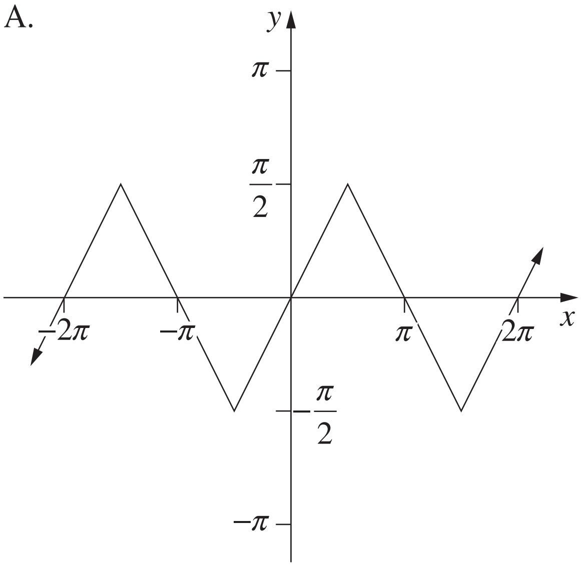
B. $y^2 = \dfrac{x^2}{2} + c$

C. $y = x \ln|y| + c$

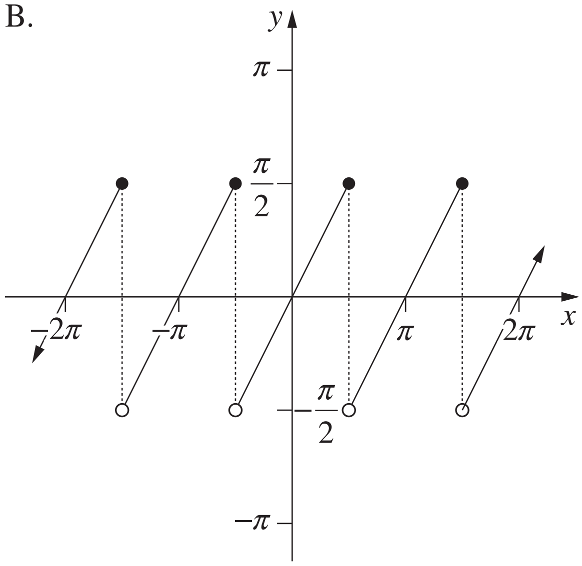
D. $y = \dfrac{x^2}{2} \ln|y| + c$

10 Which graph represents the function $y = \sin^{-1}(\sin x)$?

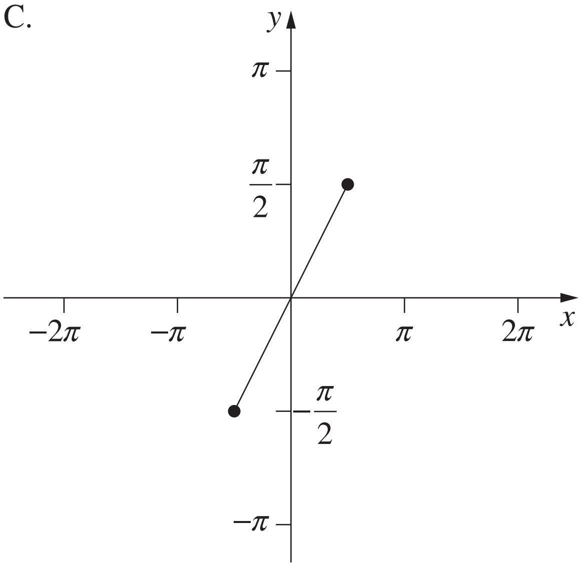
A.



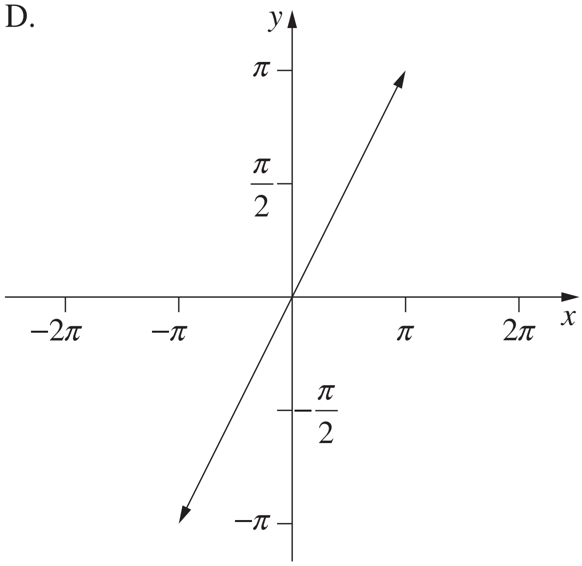
B.



C.



D.



Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Past HSC examinations provide other examples of questions that may be asked in this section.

Question 11 (16 marks) Use the Question 11 Writing Booklet

(a) Simplify $(\mathbf{i} + 6\mathbf{j}) + (2\mathbf{i} - 7\mathbf{j})$. **1**

(b) Use the substitution $u = x + 1$ to find $\int x\sqrt{x+1}\,dx$. **3**

(c) Find the term independent of x in the expansion of $\left(2x^3 + \frac{1}{x^4}\right)^7$. **2**

(d) A spherical bubble is moving up through a liquid. As it rises, the bubble gets bigger and its radius increases at the rate of 0.2 mm/s. **3**

At what rate is the volume of the bubble increasing when its radius reaches 0.6 mm? Express your answer in mm^3/s rounded to one decimal place.

(e) Evaluate $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4 - x^2}}\,dx$. **2**

Question 11 continues on page 10

Question 11 (continued)

(f) Given the vectors $\mathbf{a} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} + 2\mathbf{j}$, find the projection of **a** onto **b**. **2**

(g) An office has 7 printers and 5 photocopiers. On average, each printer is used 73% of the time and each photocopier is used 46% of the time.

(i) Write an expression for the probability that, at a particular time, at least one printer is in use. **1**

(ii) Write an expression for the probability that, at a particular time, at least one printer and exactly three photocopiers are in use. **2**

End of Question 11

Question 12 (16 marks) Use the Question 12 Writing Booklet

(a) Use mathematical induction to prove that **3**

$$\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\cdots+\frac{1}{n(n+1)(n+2)}=\frac{1}{4}-\frac{1}{2(n+1)(n+2)}$$

for all integers $n \geq 1$.

(b) A function is defined by $f(x)=4-\left(1-\dfrac{x}{2}\right)^2$ for x in the domain $(-\infty, 2]$.

(i) Explain why the function $f(x)$ has an inverse function. **2**

(ii) Find the equation of the inverse function, $f^{-1}(x)$, and state its domain. **3**

(iii) Sketch the graph of $y=f^{-1}(x)$. **1**

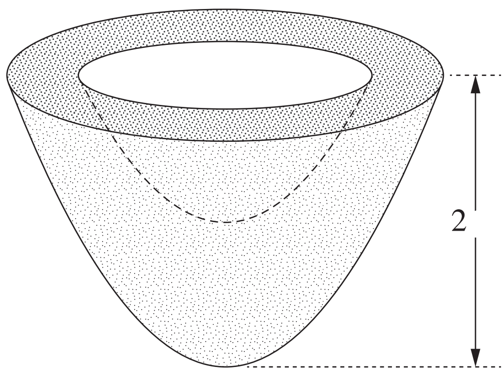
(c) A polynomial has the equation

$$P(x)=(x-1)(x-3)(x+2)^2(x^2-x-6).$$

(i) By expressing $P(x)$ as a product of its linear factors, determine the multiplicity of each of the roots of $P(x)=0$. **2**

(ii) Hence, without using calculus, draw a sketch of $y=P(x)$ showing all x-intercepts. **2**

(d) A 2-metre-high sculpture is to be made out of concrete. The sculpture is formed by rotating the region between $y=x^2$, $y=x^2+1$ and $y=2$ about the y-axis. **3**



Find the volume of concrete needed to make the sculpture.

Question 13 (14 marks) Use the Question 13 Writing Booklet

(a) A number plane is shown on page 1 of the Question 13 Writing Booklet. **3**

On the number plane provided in the writing booklet, draw $y = \cos^{-1}x + \sin^{-1}x$ by first drawing $y = \cos^{-1}x$ and $y = \sin^{-1}x$.

(b) When an object is projected from a point h metres above the origin with initial speed V m/s at an angle of $\theta°$ to the horizontal, its displacement vector, t seconds after projection, is **4**

$$\mathbf{r}(t) = \left(Vt\cos\theta\right)\mathbf{i} + \left(-5t^2 + Vt\sin\theta + h\right)\mathbf{j}.$$ (Do NOT prove this.)

A person, standing in an empty room which is 3 m high, throws a ball at the far wall of the room. The ball leaves their hand 1 m above the floor and 10 m from the far wall. The initial speed of the ball is 12 m/s at an angle of 30° to the horizontal.

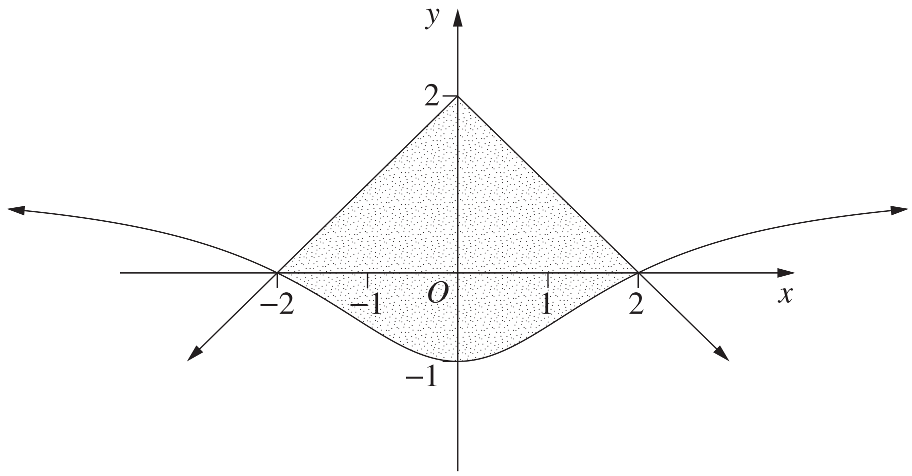
Show that the ball will NOT hit the ceiling of the room but that it will hit the far wall without hitting the floor.

(c) Consider a polynomial $y = -2x^5 - 26x^4 - x + 1$. **2**

With reference to the leading term, explain what happens to y as $x \to -\infty$.

Question 13 continues on page 13

(d) The region enclosed by $y = 2 - |x|$ and $y = 1 - \dfrac{8}{4 + x^2}$ is shaded in the diagram. **3**



Find the exact value of the area of the shaded region.

(e) An object is travelling around a circular path of radius 5 m with position vector **2**

$$\mathbf{r}(t) = 5\cos(t^2)\mathbf{i} + 5\sin(t^2)\mathbf{j} \qquad t \geq 0$$

where t is the time in seconds.

Find an expression for the speed of the object in terms of t.

Question 13 (e) is an example of solving motion problems involving non-constant velocity using a vector of the form $\mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$.

End of Question 13

Question 14 (14 marks) Use the Question 14 Writing Booklet

(a) A plane needs to travel to a destination that is on a bearing of 063°. The engine is set to fly at a constant 175 km/h. However, there is a wind from the south with a constant speed of 42 km/h. **3**

On what constant bearing, to the nearest degree, should the direction of the plane be pointed in order to reach the destination?

(b) The population of deer, P, in a certain country was estimated to be 150 000 in the year 1980. The population of deer is governed by the logistic equation $\frac{dP}{dt} = 0.1P\left(\frac{C-P}{C}\right)$ where t is the number of years after 1980 and C is the carrying capacity.

(i) Show that the maximum growth rate of the population of deer occurs when P is half of the carrying capacity. **2**

(ii) The population of deer was estimated to be 600 000 in the year 2000. **4**

Use the fact that $\frac{C}{P(C-P)} = \frac{1}{P} + \frac{1}{C-P}$ to show that the carrying capacity is approximately 1 130 000.

Question 14 (b)(ii) shows one way that a differential equation can be solved to obtain a logistic function.

Question 14 continues on page 15

Question 14 (continued)

(c) In a large school, the average amount of money spent per student per day at the canteen is \$8 with a standard deviation of 6.5. **3**

At the end of each day, 50 randomly chosen students are asked how much they spent at the canteen on that day.

Use the standard normal distribution to find the probability that the sample mean on a particular day is greater than \$10. You may use the information provided on page 16.

Question 14 (c) shows one way that students could be asked to apply the central limit theorem to estimate the probability that the sample mean lies within given bounds.

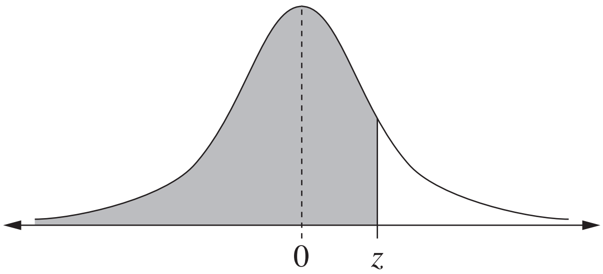
(d) The polynomial $g(x) = x^3 + 4x - 2$ passes through the point $(1, 3)$. **2**

Find the gradient of the tangent to $f(x) = xg^{-1}(x)$ at the point where $x = 3$.

End of paper

You may use the information below to answer Question 14 (c).

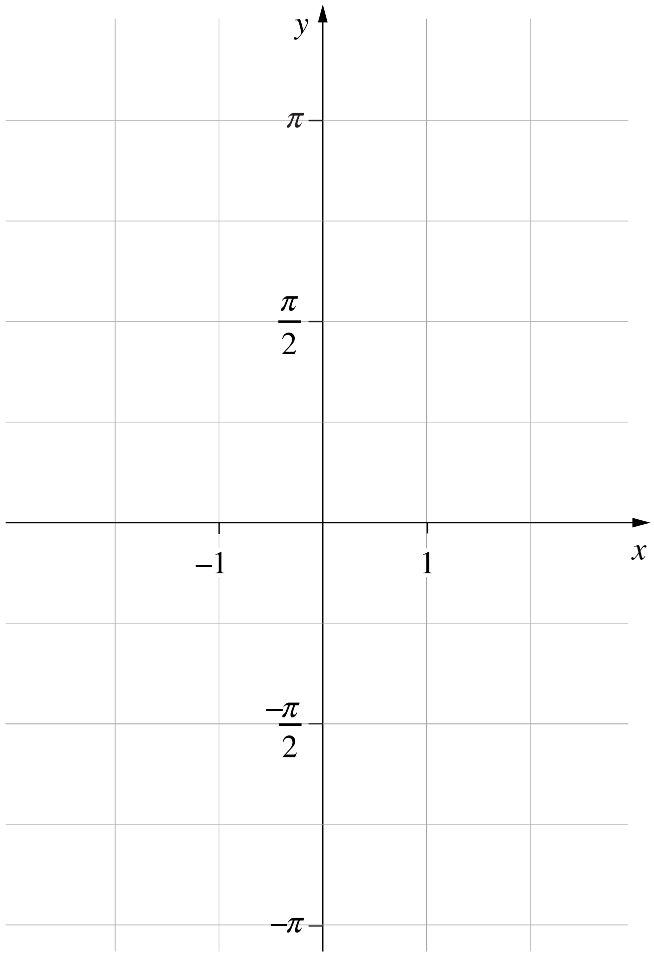
Table of values $P(Z \le z)$ for the normal distribution $N(0,1)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995

Start here for Question Number: **13**

(a) On the number plane, draw $y = \cos^{-1}x + \sin^{-1}x$ by first drawing $y = \cos^{-1}x$ and $y = \sin^{-1}x$.



This number plane has been included to illustrate how students would respond to 13 (a) in the designated writing booklet.

Mathematics Extension 1

The reference sheet has been updated. Additions have been highlighted in red boxes below.

REFERENCE SHEET

Mathematics Extension 1

Trigonometric functions

Compound angles

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\sin^2 nx = \dfrac{1}{2}(1 - \cos 2nx)$

$\cos^2 nx = \dfrac{1}{2}(1 + \cos 2nx)$

Functions

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\dfrac{d}{a}$

Binomial distribution

$P(X=r) = {}^nC_r p^r(1-p)^{n-r}$

$X \sim \mathrm{Bin}(n, p)$

$\mathrm{E}(X) = np$

$\mathrm{Var}(X) = np(1-p)$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+y)^n = {}^nC_0x^n + {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 + \cdots + {}^nC_{n-1}xy^{n-1} + {}^nC_ny^n$$

Differential calculus

Function	Derivative
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+[f(x)]^2}$

Integral calculus

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{a}{a^2 + x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

Vectors

$|\mathbf{u}| = |x\mathbf{i} + y\mathbf{j}| = \sqrt{x^2 + y^2}$

$\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos\theta = x_1x_2 + y_1y_2$

where $\mathbf{u} = x_1\mathbf{i} + y_1\mathbf{j}$

and $\mathbf{v} = x_2\mathbf{i} + y_2\mathbf{j}$

$$\boxed{\mathrm{proj}_{\mathbf{b}}\mathbf{a} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2}\right)\mathbf{b} = (\mathbf{a}\cdot\hat{\mathbf{b}})\hat{\mathbf{b}}}$$

Mathematics Advanced

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(a + b\right)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial mathematics

$$FV = PV\left(1 + r\right)^n$$

Sequences and series

$$a_n = a + \left(n - 1\right)d$$

$$S_n = \frac{n}{2}\left[2a + \left(n - 1\right)d\right] = \frac{n}{2}\left(a + a_n\right)$$

$$a_n = ar^{n-1}$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r}, \ r \neq 1$$

$$S_\infty = \frac{a}{1 - r}, \ \left|r\right| < 1$$

Logarithmic and exponential functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Relations

$$\left(x - a\right)^2 + \left(y - b\right)^2 = r^2$$

Trigonometric functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

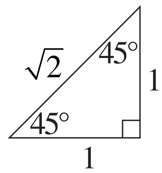
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

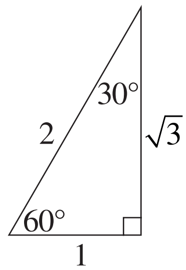
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Differential calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^x$	$\dfrac{dy}{dx} = (\ln a)a^x$
$y = \log_a x$	$\dfrac{dy}{dx} = \dfrac{1}{x\ln a}$

Integral calculus

$$\int f'(x)[f(x)]^n\,dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + c$$

$$\int_a^b f(x)\,dx$$

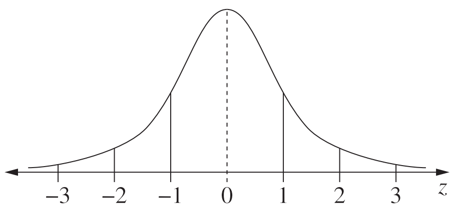
$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Statistical analysis

$$z = \frac{x-\mu}{\sigma}$$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

Discrete random variables

$$\mathrm{E}(X) = \mu$$

$$\mathrm{Var}(X) = \mathrm{E}\left[(X-\mu)^2\right]$$

Continuous random variables

$$P(X \le x) = F(x) = \int_a^x f(t)\,dt$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

$$\mathrm{E}(X) = \mu = \int_a^b x f(x)\,dx$$

$$\mathrm{Var}(X) = \mathrm{E}\left(X^2\right) - \mu^2 = \int_a^b x^2 f(x)\,dx - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

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HSC Mathematics Extension 1 Sample Examination

Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	C
2	B
3	D
4	C
5	B
6	A
7	D
8	C
9	A
10	A

Section II

Question 11 (a)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$(\mathbf{i}+6\mathbf{j})+(2\mathbf{i}-7\mathbf{j})=3\mathbf{i}-\mathbf{j}$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	3
• Obtains correct primitive in terms of u	2
• Obtains the integrand in terms of u, or equivalent merit	1

Sample answer:

$$\int x\sqrt{x+1}\ dx \qquad\qquad \begin{aligned} u &= x+1 \qquad \therefore x = u-1 \\ du &= dx \end{aligned}$$

$$= \int (u-1)\sqrt{u}\ du$$

$$= \int u^{\frac{3}{2}} - u^{\frac{1}{2}}\ du$$

$$= \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + c$$

$$= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + c$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	2
• Obtains the general term in simplest form, or equivalent merit	1

Sample answer:

$$^{7}C_r\left(2x^3\right)^{7-r}\left(\frac{1}{x^4}\right)^r$$

$$= {}^{7}C_r\, 2^{7-r}\, x^{21-7r}$$

$$\therefore x^{21-7r} = x^0$$

$$r = 3$$

$$^{7}C_3\, 2^4 = 560$$

Question 11 (d)

Criteria	Marks
• Provides correct solution	3
• Uses the chain rule $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ AND obtains $\frac{dV}{dr}$	2
• Obtains $\frac{dV}{dr}$ from the volume of a sphere, or equivalent merit	1

Sample answer:

$$\frac{dr}{dt} = 0.2 \text{ mm/s} \quad \text{and} \quad \frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$

$$V = \frac{4}{3}\pi r^3 \qquad \therefore \frac{dV}{dr} = 4\pi r^2 \text{ and } \frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi \times (0.6)^2 \times (0.2) \simeq 0.9 \text{ mm}^3\text{/s} \qquad \text{(1 decimal place)}$$

Question 11 (e)

Criteria	Marks
• Provides correct solution	2
• Integrates to obtain an inverse sin function, or equivalent merit	1

Sample answer:

$$\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}\,dx = \left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^{\sqrt{3}}$$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1} 0$$

$$= \frac{\pi}{3} - 0$$

$$= \frac{\pi}{3}$$

Question 11 (f)

Criteria	Marks
• Provides correct solution	2
• Finds $\mathbf{a} \cdot \mathbf{b}$, or equivalent merit	1

Sample answer:

Let $\mathbf{a} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} + 2\mathbf{j}$.

Then $\mathbf{a} \cdot \mathbf{b} = 1 \times 4 + 3 \times 2 = 10$ and $|\mathbf{b}|^2 = 4^2 + 2^2 = 20$.

$$\begin{aligned}
\text{proj}_{\mathbf{b}}(\mathbf{a}) &= \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|^2}\right)\mathbf{b} \\
&= \left(\frac{10}{20}\right)\mathbf{b} \\
&= \frac{1}{2}(4\mathbf{i} + 2\mathbf{j}) \\
&= 2\mathbf{i} + \mathbf{j}
\end{aligned}$$

Question 11 (g)(i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\begin{aligned} 1-P(\text{none in use}) &= 1-{}^{7}C_{0}(0.27)^{7} \\ &= 1-(0.27)^{7} \end{aligned}$$

Question 11 (g)(ii)

Criteria	Marks
• Provides correct solution	2
• Obtains a correct expression for exactly three photocopies in use, or equivalent merit.	1

Sample answer:

$$\left(1-(0.27)^{7}\right)\times {}^{5}C_{3}(0.46)^{3}(0.54)^{2}$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	3
• Proves the inductive step, or equivalent merit	2
• Verifies the base case, or equivalent merit	1

Sample answer:

When $n = 1$

$$\text{LHS} = \frac{1}{1(1+1)(1+2)} = \frac{1}{1\times2\times3} = \frac{1}{6}$$

$$\text{RHS} = \frac{1}{4} - \frac{1}{2(1+1)(1+2)} = \frac{1}{4} - \frac{1}{2\times2\times3} = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$$

$\therefore$ statement is true when $n = 1$.

Assume statement is true when $n = k$, some integer $k \geq 1$, that is

$$\frac{1}{1\times2\times3} + \cdots + \frac{1}{k(k+1)(k+2)} = \frac{1}{4} - \frac{1}{2(k+1)(k+2)}$$

Consider $n = k + 1$:

$$\frac{1}{1\times2\times3} + \cdots + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+1+1)(k+1+2)}$$

$$= \frac{1}{4} - \frac{1}{2\big((k+1)+1\big)\big((k+1)+2\big)}$$

$$= \frac{1}{4} - \frac{1}{2(k+2)(k+3)}$$

$$\begin{aligned}
\text{LHS} &= \frac{1}{1\times2\times3} + \cdots + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+1+1)(k+1+2)} \\
&= \frac{1}{4} - \frac{1}{2(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)} \\
&= \frac{1}{4} - \left[\frac{1}{2(k+1)(k+2)} - \frac{1}{(k+1)(k+2)(k+3)}\right] \\
&= \frac{1}{4} - \left[\frac{k+3-2}{2(k+1)(k+2)(k+3)}\right] \\
&= \frac{1}{4} - \left[\frac{1}{2(k+2)(k+3)}\right] \\
&= \text{RHS}
\end{aligned}$$

$\therefore$ statement is true for all integers $n \geq 1$, by mathematical induction.

Question 12 (b)(i)

Criteria	Marks
• Provides correct explanation	2
• Sketches the left half of a concave-down parabola, or equivalent merit	1

Sample answer:

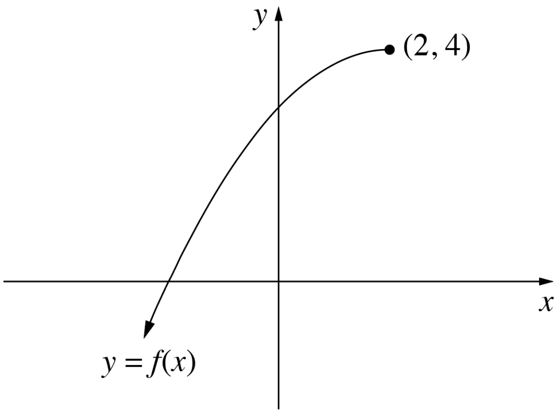
$y = 4 - \left(1 - \frac{x}{2}\right)^2$ is a parabola.

Vertex when $\left(1 - \frac{x}{2}\right)^2 = 0$ $\quad\therefore\ x = 2$

$$\begin{aligned} y &= 4 - 0 \\ &= 4 \end{aligned}$$

$\therefore$ vertex (2, 4)

Coefficient of x^2 is negative, so concave down.
Domain $(-\infty, 2]$, so left half only.



The function has an inverse because it is a one-to-one relationship (which can be seen from the graph).

Question 12 (b)(ii)

Criteria	Marks
• Provides correct solution	3
• Finds the expression for $f^{-1}(x)$, or equivalent merit	2
• Attempts to find the inverse by switching variables, or equivalent merit	1

Sample answer:

For $f(x)$, $y = 4 - \left(1 - \frac{x}{2}\right)^2$, x in the domain $(-\infty, 2]$, y in the range $(-\infty, 4]$

For inverse, $x = 4 - \left(1 - \frac{y}{2}\right)^2$

$$\left(1 - \frac{y}{2}\right)^2 = 4 - x$$

$$\left(\frac{y}{2} - 1\right)^2 = 4 - x$$

$$\frac{y}{2} - 1 = \pm\sqrt{4 - x}$$

$$y = 2 \pm 2\sqrt{4 - x}$$

Domain of $f(x)$ is $(-\infty, 2]$, so range of $f^{-1}(x)$ is $(-\infty, 2]$.

$\therefore\ f^{-1}(x) = 2 - 2\sqrt{4 - x}$ for x in the domain $(-\infty, 4]$.

Alternative solution:

$$y = 4 - \left(1 - \frac{x}{2}\right)^2 \quad \text{for} \quad x \le 2$$

For $f^{-1}(x)$, make x the subject:

$$\left(1 - \frac{x}{2}\right)^2 = 4 - y$$

$$x \le 2 \quad \therefore 1 - \frac{x}{2} \ge 0$$

$$\therefore\ 1 - \frac{x}{2} = \sqrt{4 - y}$$

$$\frac{x}{2} = 1 - \sqrt{4 - y}$$

$$x = 2 - 2\sqrt{4 - y}$$

$$\therefore f^{-1}(x) = 2 - 2\sqrt{4 - x}$$

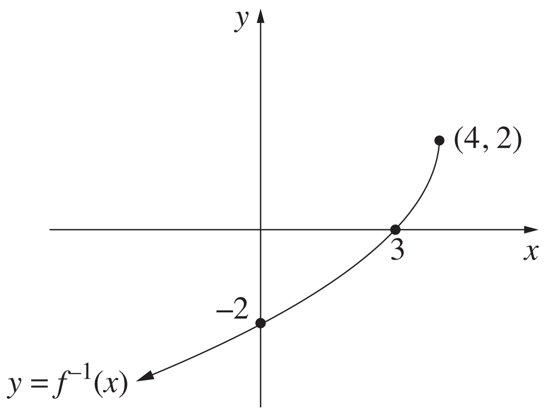
From the graph, range of $f(x)$ is $(-\infty, 4]$

$\therefore$ domain of $f^{-1}(x)$ is $(-\infty, 4]$.

Question 12 (b)(iii)

Criteria	Marks
• Provides correct sketch	1

Sample answer:



Question 12 (c)(i)

Criteria	Marks
• Provides correct solution	2
• Finds $P(x)$ as a product of linear factors, or equivalent merit	1

Sample answer:

$$P(x) = (x-1)(x-3)(x+2)^2(x^2-x-6)$$

$$= (x-1)(x-3)(x+2)^2(x-3)(x+2)$$

$$= (x-1)(x-3)^2(x+2)^3$$

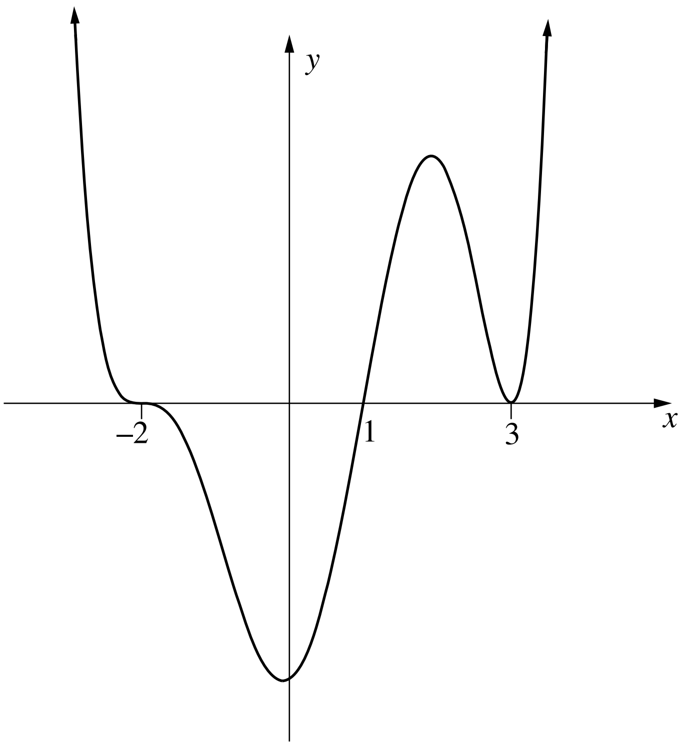
So, $P(x) = 0$ has a root at $x = 1$ of multiplicity 1, a root at $x = -2$ of multiplicity 3, and a root at $x = 3$ of multiplicity 2.

Question 12 (c)(ii)

Criteria	Marks
• Provides correct sketch	2
• Demonstrates some understanding of the geometric nature of the roots, or equivalent merit	1

Sample answer:

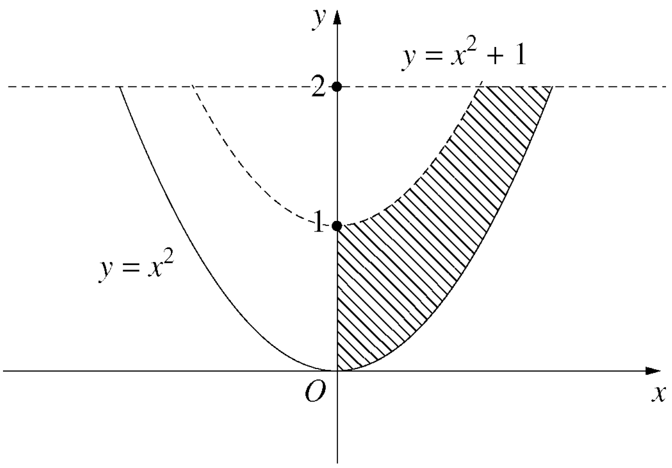
Since $y = P(x)$ has a leading term of x^6, it starts and ends above the x-axis.



Question 12 (d)

Criteria	Marks
• Provides correct solution	3
• Evaluates one volume OR • Obtains a correct expression for the volume as a difference of two integrals that are only in terms of y	2
• Obtains an integral for one relevant volume OR • Writes the required volume as the difference of two relevant volumes OR • Finds the limits of integration for both volumes	1

Sample answer:



The volume of the garden sculpture is the difference between the outer volume and the inner one.

$$\text{Outer volume} = \pi \int_0^2 x^2\,dy$$

$$= \pi \int_0^2 y\,dy \quad \text{since } y = x^2$$

$$\text{Inner volume} = \pi \int_1^2 x^2\,dy$$

$$= \pi \int_1^2 (y-1)\,dy \quad \text{since } y = x^2 + 1$$

$$\therefore V = \pi \int_0^2 y\,dy - \pi \int_1^2 (y-1)\,dy$$

$$V = \pi\left[\frac{y^2}{2}\right]_0^2 - \pi\left[\frac{y^2}{2} - y\right]_1^2$$

$$= \pi\left(\frac{4}{2} - 0\right) - \pi\left(\left[\frac{4}{2} - 2\right] - \left[\frac{1}{2} - 1\right]\right)$$

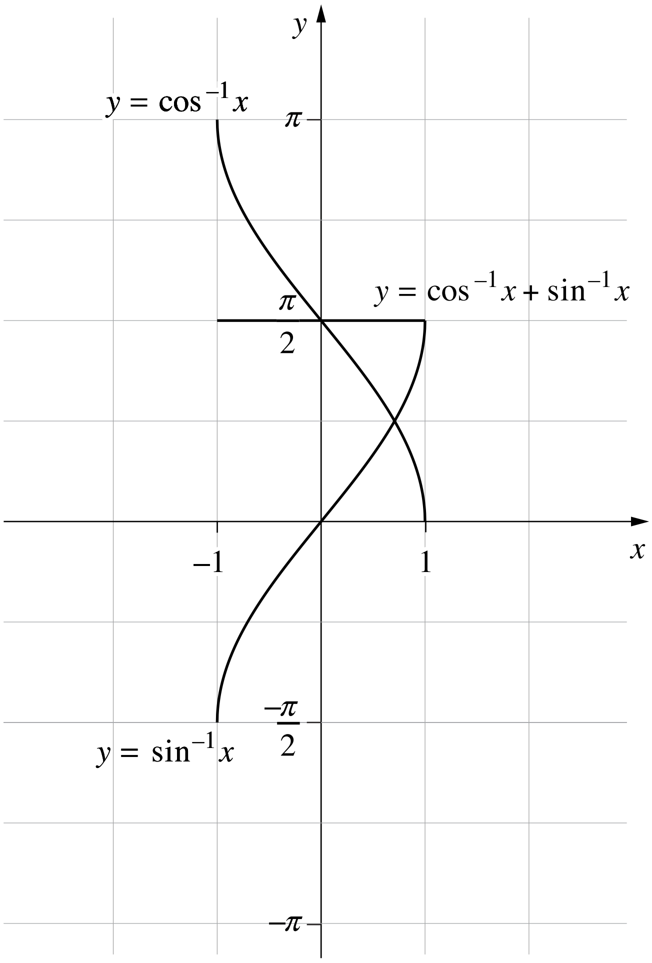
$$= \frac{3\pi}{2}$$

The volume of the sculpture is $\frac{3\pi}{2}$ m^3.

Question 13 (a)

Criteria	Marks
• Provides correct solution	3
• Draws TWO correct graphs	2
• Draws ONE correct graph	1

Sample answer:



Question 13 (b)

Criteria	Marks
• Provides correct solution	4
• Finds the maximum height and the time of flight, or equivalent merit	3
• Finds the maximum height, or equivalent merit	2
• Finds an expression for the vertical velocity, or equivalent merit	1

Sample answer:

$$\mathbf{v}(t) = V\cos\theta\mathbf{i} + (-10t + V\sin\theta)\,\mathbf{j}$$

The height is greatest when $\dot{y} = 0$ ie when $-10t + V\sin\theta = 0$

$$t = \frac{V\sin\theta}{10} = \frac{12\sin 30^\circ}{10} = \frac{6}{10} = 0.6\ \text{s}$$

$$\therefore\ y = -5 \times (0.6)^2 + 12 \times (0.6) \times \sin 30^\circ + 1$$

$$= 2.8\ \text{m}$$

The room is 3 metres high and maximum height reached by the object is 2.8 m so the object will not hit the ceiling.

To know if the object hits the far wall or not, it is enough to determine if the unrestricted horizontal range would be more or less than 10 metres.

$$y = 0 \text{ if } -5t^2 + Vt\sin\theta + h = 0$$

$$-5t^2 + 12 \times \frac{1}{2}t + 1 = 0$$

$$-5t^2 + 6t + 1 = 0$$

$$\triangle = 6^2 - 4(-5) = 56$$

$$\therefore\ t = \frac{-6 + \sqrt{56}}{-10} < 0 \quad \text{or} \quad \frac{-6 - \sqrt{56}}{-10} > 0$$

We need $t > 0$, so the object, in the absence of a far wall, would hit the floor after $t = \dfrac{6 + \sqrt{56}}{10} \simeq 1.348$ s.

Substitute in $x(t)$:

$$x(t) = Vt\cos\theta = 12 \times 1.348\cos 30^\circ$$

$$\approx 14\ \text{m}$$

Since the far wall is only 10 m away, the object will hit the wall.

Alternative solution for second part

When $x = 10$ $\quad 12t\cos 30° = 10$

$$6\sqrt{3}t = 10$$

$$t = \frac{10}{6\sqrt{3}} = 0.9622 \text{ seconds}$$

When $t = 0.9622$

$$y = -5(0.9622\ldots)^2 + 12(0.9622\ldots)\sin 30° + 1$$
$$= 2.144 \text{ m}$$

$\therefore$ Ball is still above the floor when $x = 10$ m and so will hit the far wall without hitting the floor.

Question 13 (c)

Criteria	Marks
• Provides correct solution	2
• Uses the leading coefficient or the degree to explain what happens to y as $x \to -\infty$	1

Sample answer:

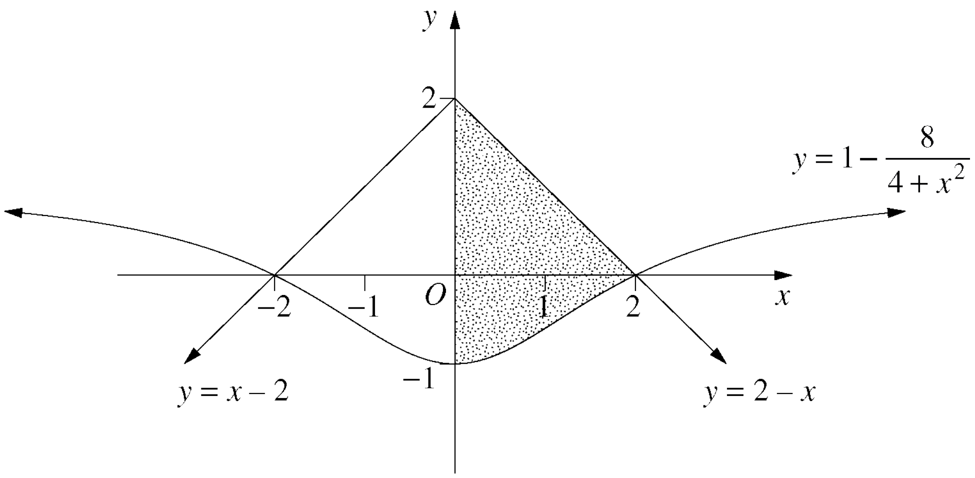
The leading term $-2x^5$ has an odd power, so x^5 approaches $-\infty$ as $x \to -\infty$.

Since the leading coefficient is negative we can conclude that y approaches ∞ as $x \to -\infty$.

Question 13 (d)

Criteria	Marks
• Provides correct solution	3
• Obtains the (signed) area OR the area of either side of the y-axis between the x-axis and the curve $y = 1 - \dfrac{8}{4+x^2}$, or equivalent merit	2
• Finds an integral expression for the area, or equivalent merit OR • Recognises that the total area is twice the area on one side of the y-axis OR • Finds the area of a relevant triangle	1

Sample answer:



By symmetry, area required is double the shaded region above

$$\frac{\text{Area}}{2} = \int_0^2 (2-x) - \left(1 - \frac{8}{4+x^2}\right) dx$$

$$= \int_0^2 1 - x + \frac{8}{4+x^2}\, dx$$

$$= \left[x - \frac{x^2}{2} + \frac{8}{2}\tan^{-1}\left(\frac{x}{2}\right) \right]_0^2$$

$$= \left(2 - 2 + 4\tan^{-1}1\right) - 0$$

$$= \pi$$

So Area $= 2\pi$ square units

Question 13 (e)

Criteria	Marks
• Provides correct solution	2
• Obtains an expression for the **i** or **j** component of the velocity, or equivalent merit	1

Sample answer:

The velocity is given by

$$\mathbf{v}(t) = \frac{d}{dt}\mathbf{r}(t) = -10t\sin\left(t^2\right)\mathbf{i} + 10t\cos\left(t^2\right)\mathbf{j}$$

The speed is the magnitude of the velocity

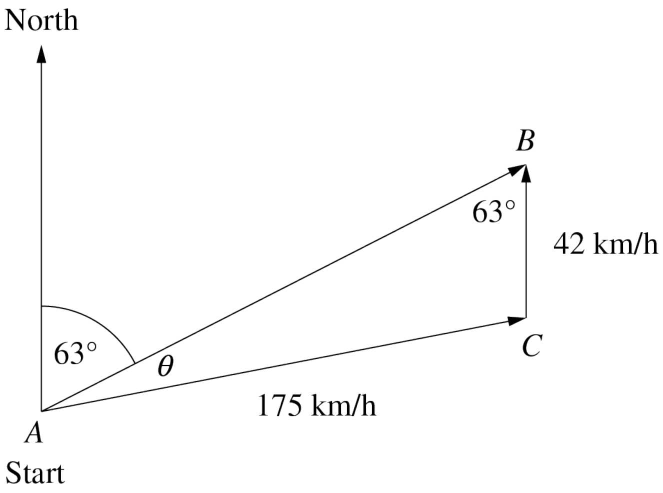
$$\begin{aligned} |\,\mathbf{v}(t)| &= \sqrt{100t^2\sin^2\left(t^2\right) + 100t^2\cos^2\left(t^2\right)} \\ &= 10t\sqrt{\sin^2\left(t^2\right) + \cos^2\left(t^2\right)} \\ &= 10t \text{ m/s} \end{aligned}$$

Question 14 (a)

Criteria	Marks
• Provides correct solution	3
• Attempts to use the sine rule in the correct triangle, or equivalent merit	2
• Sketches a suitable diagram, or equivalent merit	1

Sample answer:

Drawing a vector diagram



Using alternate angles on parallel lines

$$\angle ABC = 63°$$

Let $\angle BAC = \theta$

$$\therefore \quad \frac{\sin\theta}{42} = \frac{\sin 63°}{175}$$

$$\sin\theta = 0.2138\ldots$$

$$\theta = 12.34\ldots$$

$$\approx 12°$$

$\therefore$ required bearing $= 63 + 12$

$= 075°$

Question 14 (b)(i)

Criteria	Marks
• Provides correct solution	2
• Recognises the maximum growth rate occurs when $\frac{d}{dP}\left(\frac{dP}{dt}\right)=0$, or equivalent merit	1

Sample answer:

The maximum growth rate occurs when $\frac{d}{dP}\left(\frac{dP}{dt}\right)=0$.

$$\begin{aligned}\frac{d}{dP}\left(\frac{dP}{dt}\right) &= \frac{d}{dP}\left(\frac{0.1P(C-P)}{C}\right)\\ &= \frac{0.1}{C}\frac{d}{dP}\left(PC-P^2\right)\\ &= \frac{0.1}{C}(C-2P)\end{aligned}$$

So $\frac{d}{dP}\left(\frac{dP}{dt}\right)=0$ when $P=\frac{C}{2}$. This is a maximum because

$$\frac{d^2}{dP^2}\left(\frac{dP}{dt}\right)=\frac{d}{dP}\left(\frac{0.1}{C}(C-2P)\right)=\frac{0.1}{C}(-2)<0$$

Alternative solution:

The growth rate is given by $\frac{dP}{dt}=0.1P\left(\frac{C-P}{C}\right)$.

The expression $0.1P\left(\frac{C-P}{C}\right)$ is a concave down parabola with zeroes at $P = 0$ and $P = C$.

Hence the maximum occurs when $P=\left(\frac{0+C}{2}\right)=\frac{C}{2}$.

Question 14 (b)(ii)

Criteria	Marks
• Provides correct solution	4
• Uses the given information to obtain two equations in A and C, or equivalent merit	3
• Integrates both sides correctly, or equivalent merit	2
• Separates the variables in the differential equation, or equivalent merit	1

Sample answer:

$$\frac{dP}{dt} = 0.1P\left(\frac{C-P}{C}\right)$$

$$\int \frac{C}{P(C-P)}\,dP = \int 0.1\,dt$$

$$\int \left(\frac{1}{P} + \frac{1}{C-P}\right)dP = \int 0.1\,dt$$

$\ln|P| - \ln|C-P| = 0.1t + k$ where k is a constant

$$\ln\left|\frac{P}{C-P}\right| = 0.1t + k$$

$\dfrac{P}{C-P} = Ae^{0.1t}$ where $A = e^k$ is a constant

When $t = 0$, $P = 150\,000$ so $\dfrac{150\,000}{C - 150\,000} = A$ ①

When $t = 20$, $P = 600\,000$ so $\dfrac{600\,000}{C - 600\,000} = Ae^2$ ②

Substituting ① into ②: $\dfrac{600\,000}{C - 600\,000} = \dfrac{150\,000}{C - 150\,000}e^2$

Taking the reciprocal of both sides

$$\frac{C - 600\,000}{600\,000} = \frac{C - 150\,000}{150\,000}e^{-2}$$

$$150\,000(C - 600\,000) = 600\,000(C - 150\,000)e^{-2}$$

$$C\left(150\,000 - 600\,000e^{-2}\right) = 150\,000 \times 600\,000\left(1 - e^{-2}\right)$$

$$C = \frac{150\,000 \times 600\,000\left(1 - e^{-2}\right)}{150\,000 - 600\,000e^{-2}} \simeq 1\,131\,121$$

$$\simeq 1\,130\,000$$

Question 14 (c)

Criteria	Marks
• Provides correct solution	3
• Recognises $\dfrac{\overline{X}-8}{\sigma} \sim N(0,1)$ and determines the value for the z-score, or equivalent merit	2
• Recognises $n \geq 30$ allows use of the central limit theorem, or equivalent merit OR • Finds σ in terms of n, or equivalent merit	1

Sample answer:

As the sample size is ≥ 30, the central limit theorem can be applied.

The mean $\overline{X}$ of 50 random samples is normally distributed with $\mu = 8$ and $\sigma = \dfrac{6.5}{\sqrt{50}}$.

By the central limit theorem

$$Z = \frac{\overline{X} - 8}{\frac{6.5}{\sqrt{50}}} \sim N(0,1)$$

$$Z = \frac{10 - 8}{\frac{6.5}{\sqrt{50}}} = 2.18 \text{ (to two decimal places)}$$

(Using the information provided)

$$P\left(\overline{X} \geq 10\right) = 1 - P(Z \leq 2.18)$$
$$= 1 - 0.9854 = 1.46\%$$

Question 14 (d)

Criteria	Marks
• Provides correct solution	2
• Obtains an expression for the derivative of $g^{-1}(x)$ in terms of x, or equivalent merit	1

Sample answer:

$g(1) = 3 \quad \therefore \; g^{-1}(3) = 1$

Using product rule:

$$f'(x) = g^{-1}(x).1 + x.\frac{d}{dx}\left(g^{-1}(x)\right)$$

Now if $g^{-1}(x) = y$ then $x = g(y)$

$$\therefore \quad \frac{dx}{dy} = g'(y)$$

and $\dfrac{dy}{dx} = \dfrac{1}{g'(y)}$

$$\therefore \quad \frac{d}{dx}\left(g^{-1}(x)\right) = \frac{1}{g'\left(g^{-1}(x)\right)}$$

$$\therefore \quad f'(x) = g^{-1}(x) + \frac{x}{g'\left(g^{-1}(x)\right)}$$

$$\therefore \quad f'(3) = g^{-1}(3) + \frac{3}{g'\left(g^{-1}(3)\right)}$$

$$= 1 + \frac{3}{g'(1)}$$

Now, $g(x) = x^3 + 4x - 2$

$$\therefore \quad g'(x) = 3x^2 + 4$$

$$\therefore \quad f'(3) = 1 + \frac{3}{3(1)^2 + 4}$$

$\therefore$ gradient of tangent $= \dfrac{10}{7}$.

HSC Mathematics Extension 1 Sample Examination

Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes	Targeted performance bands
1	1	Introduction to vectors – Introduction to 2D and 3D vectors	ME1-12-02	E2–E3
2	1	Further calculus skills – Techniques of Integration	ME1-12-04	E2–E3
3	1	Polynomials – Remainder and factor theorems	ME1-11-02	E2–E3
4	1	Further applications of calculus – Differential equations	ME1-12-05	E2–E3
5	1	Introduction to vectors – Further operations with vectors	ME1-12-02	E3–E4
6	1	The binomial distribution and sampling distribution of the mean – Bernoulli distributions	ME1-12-06	E3–E4
7	1	Further trigonometry – Further trigonometric equations	ME1-11-03	E3–E4
8	1	Further work with functions – Parametric form of a function or relation	ME1-11-03	E3–E4
9	1	Further applications of calculus – Differential equations	ME1-12-05	E3–E4
10	1	Inverse trigonometric functions – Graphs of inverse trigonometric functions	ME1-12-03	E3–E4

Section II

Question	Marks	Content	Syllabus outcomes	Targeted performance bands
11(a)	1	Introduction to vectors – Operating with vectors	ME1-12-02	E2–E3
11(b)	3	Further calculus skills – Techniques of integration	ME1-12-04	E2–E3
11(c)	2	The binomial theorem	ME1-11-05	E2–E3
11(d)	3	Further applications of calculus – Further rates of change	ME1-12-05	E2–E3
11(e)	2	Further calculus skills – Techniques of integration	ME1-12-04	E2–E3
11(f)	2	Introduction to vectors – Further operations with vectors	ME1-12-02	E2–E3
11(g)(i)	1	The binomial distribution and sampling distribution of the mean – Binomial distributions	ME1-12-06	E2–E3
11(g)(ii)	2	The binomial distribution and sampling distribution of the mean – Binomial distributions	ME1-12-06	E2–E3
12(a)	3	Proof by mathematical induction	ME1-12-01	E2–E4

Question	Marks	Content	Syllabus outcomes	Targeted performance bands
12(b)(i)	2	Further work with functions – Inverse functions	ME1-11-01	E2–E3
12(b)(ii)	3	Further work with functions – Inverse functions	ME1-11-01	E2–E4
12(b)(iii)	1	Further work with functions – Inverse functions	ME1-11-01	E2–E3
12(c)(i)	2	Polynomials – Language and graphs of polynomials	ME1-11-02	E2–E3
12(c)(ii)	2	Polynomials – Language and graphs of polynomials	ME1-11-02	E2–E3
12(d)	3	Further applications of calculus – Areas between curves and volumes of solids of revolution	ME1-12-05	E2–E3
13(a)	3	Inverse trigonometric functions – Graphs of inverse trigonometric functions	ME1-12-03	E2–E3
13(b)	4	Introduction to vectors – Projectile motion	ME1-12-02	E2–E4
13(c)	2	Polynomials – Language and graphs of polynomials	ME-11-02	E2–E3
13(d)	3	Further applications of calculus - Areas between curves and volumes of solids of revolution	ME1-12-05	E2–E3
13(e)	2	Introduction to vectors – Motion in vector form in two dimensions	ME1-12-02	E2–E3
14(a)	3	Introduction to vectors – Motion in vector form in two dimensions	ME1-12-02	E2–E3
14(b)(i)	2	Further applications of calculus – Differential equations	ME1-12-05	E3–E4
14(b)(ii)	4	Further applications of calculus – Differential equations	ME1-12-05	E2–E4
14(c)	3	The binomial distribution and sampling distribution of the mean – Sampling distribution of the mean and the central limit theorem	ME1-12-06	E2–E4
14(d)	2	Further applications skills – Further derivatives of functions	ME1-12-04	E3–E4