NSW Education Standards Authority

Mathematics Extension 1 11–12 (2024)

curriculum.nsw.edu.au

Generated Sep 2026

Table of contents

Mathematics Extension 1 11–12 (2024)

Implementation from 2026

The new Mathematics Extension 1 11–12 Syllabus (2024) is to be implemented from 2026 and will replace the **Mathematics Extension 1 Stage 6 Syllabus (2017)**.

2026, Term 1

- Start teaching the new syllabus for Year 11
- Start implementing the new Year 11 school-based assessment requirements
- Continue to teach the **Mathematics Extension 1 Stage 6 Syllabus (2017)** for Year 12

2026, Term 4

- Start teaching the new syllabus for Year 12
- Start implementing the new Year 12 school-based assessment requirements

2027

- First HSC examination for the new syllabus

Overview

Syllabus overview

Organisation of Mathematics Extension 1 11–12

The organisation of outcomes and content for Mathematics Extension 1 11–12 highlights the important role Working mathematically plays across all areas of mathematics and reflects the strengthened connections between concepts. Working mathematically has been embedded in the outcomes and content of the syllabus.

Mathematics Extension 1 outcomes and their related content are organised into 7 areas of study:

- Functions
- Proof
- Vectors
- Trigonometric functions
- Combinatorics
- Calculus
- Statistical analysis.

Figure 1 gives a broad overview of the organisation of content, including the focus areas to be completed in each area of study.

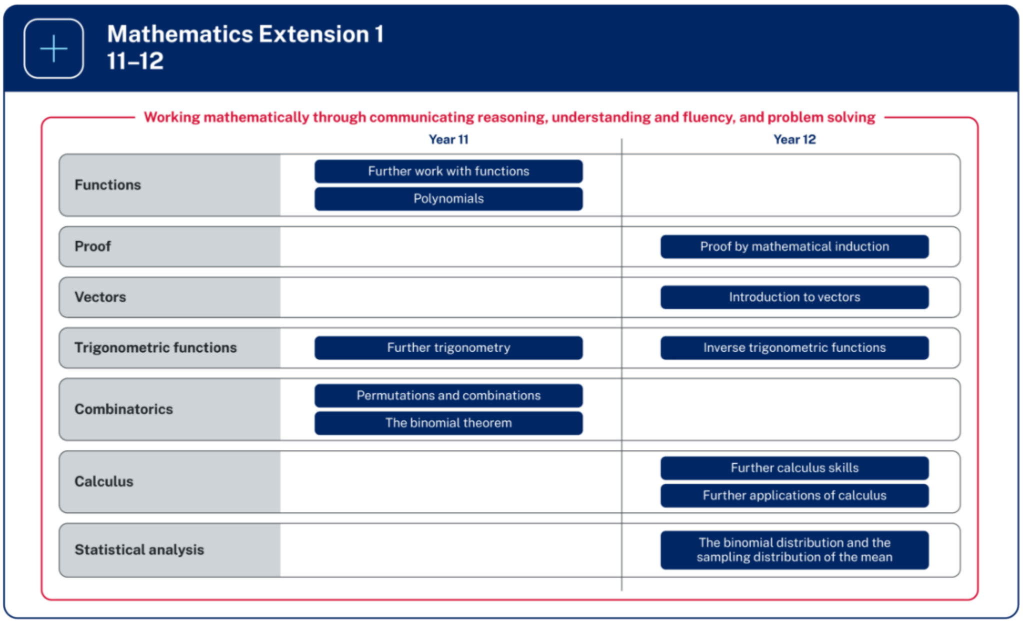


Figure 1: The organisation of Mathematics Extension 1 11–12

Image long description: Seven rows represent the areas of study for Mathematics Extension 1 11–12. Each row is divided in half, showing the focus areas for Year 11 and Year 12. The Functions area of study includes Year 11 focus areas Further work with functions and Polynomials. The Proof area of study includes the Year 12 focus area Proof by mathematical induction. The Vectors area of study includes the Year 12 focus area Introduction to vectors. The Trigonometric functions area of study includes the Year 11 focus area Further trigonometry, and the Year 12 focus area Inverse trigonometric functions. The Combinatorics area of study includes Year 11 focus areas Permutations and combinations, and The binomial theorem. The Calculus area of study includes the Year 12 focus areas Further calculus skills and Further applications of calculus. The Statistical analysis area of study includes the Year 12 focus area The binomial distribution and the sampling distribution of the mean. All content is surrounded by a box labelled with the phrase, ‘Working mathematically through communicating reasoning, understanding and fluency, and problem solving’.

Protocols for collaborating with Aboriginal and Torres Strait Islander Communities

NESA is committed to working in partnership with Aboriginal Communities and supporting teachers, schools and schooling sectors to improve educational outcomes for young people.

It is important to respect appropriate ways of interacting with Aboriginal Communities and Cultural material when teachers plan, program and implement learning experiences that focus on Aboriginal and Torres Strait Islander Priorities.

Indigenous Cultural and Intellectual Property (ICIP) protocols need to be followed. Aboriginal and Torres Strait Islander Peoples’ ICIP protocols include Cultural Knowledges, Cultural Expression and Cultural Property and documentation of Aboriginal and Torres Strait Islander Peoples’ Identities and lived experiences. It is important to recognise the diversity and complexity of different Cultural groups in NSW, as protocols may differ between local Aboriginal Communities.

Teachers should work in partnership with Elders, parents, Community members, Cultural Knowledge Holders, or a local, regional or state Aboriginal Education Consultative Group. It is important to respect Elders and the roles of men and women. Local Aboriginal Peoples should be invited to share their Cultural Knowledges with students and staff when engaging with Aboriginal histories and Cultural Practices.

Course description

Mathematics Extension 1 focuses on the development of students mathematical arguments and proofs, and use of mathematical models. The course allows students to develop a thorough knowledge and understanding of and competence in further aspects of mathematics as an extension of the Mathematics Advanced 11–12 course.

What students learn

Through the study of Mathematics Extension 1, students:

- develop thorough knowledge, understanding and skills in Working mathematically and in communicating concisely and precisely
- develop rigorous mathematical arguments and proofs, and use mathematical models extensively
- develop awareness of the interconnected nature of mathematics, its beauty and its functionality
- gain an appropriate mathematical background for future pathways that may involve mathematics and its applications.

Course structure and requirements

Year 11 course structure

Area of study	Focus area
Functions	Further work with functions Polynomials
Trigonometric functions	Further trigonometry
Combinatorics	Permutations and combinations The binomial theorem

Year 12 course structure

Area of study	Focus area
Proof	Proof by mathematical induction
Vectors	Introduction to vectors
Trigonometric functions	Inverse trigonometric functions
Calculus	Further calculus skills Further applications of calculus
Statistical analysis	The binomial distribution and sampling distribution of the mean

Safety and risk management

Schools are required to ensure they follow safety and risk management in delivering the *Mathematics Extension 1 11–12 Syllabus*.

Course enrolment details

Mathematics Extension 1 Year 11

- Course number: 11250
- Course hours: 60
- Course units: 1
- Enrolment type: Elective
- Endorsement type: Board developed

Exclusions

- Mathematics Standard (Year 11, 2 units): 11236
- Mathematics Life Skills (Year 11, 2 units): 17680

Prerequisites

- Nil

Corequisites

- Mathematics Advanced (Year 11, 2 units): 11255

Mathematics Extension 1 Year 12

- Course number: 15250
- Course hours: 60
- Course units: 1
- Enrolment type: Elective
- Endorsement type: Board developed

Exclusions

- Mathematics Standard 1 (Year 12, 2 units): 15231
- Mathematics Standard 1 (Examination) (Year 12): 15232
- Mathematics Standard 2 (Year 12, 2 units): 15236

- Mathematics Life Skills (Year 12, 2 units): 17680

Prerequisites

- Mathematics Advanced (Year 11, 2 units): 11255
- Mathematics Extension 1 (Year 11, 1 unit) 11250

Corequisites

- Mathematics Advanced (Year 12, 2 units): 15255

HSC information

Information about curriculum requirements for the HSC are available on Assessment Certification Examination (ACE).

Rationale

Mathematics in the Stage 6 Curriculum

Mathematical ideas have evolved and continue to develop across cultures and have been practised in Australia by Aboriginal and Torres Strait Islander Peoples for thousands of years.

The utility of mathematics extends from counting and measuring to explaining real-world phenomena and its inherent beauty. It has evolved in sophisticated ways to become the language used to describe and understand much of the modern world.

The creative application of mathematical concepts is integral to scientific and technological advancement. The symbolic nature of mathematics provides a precise means of communication.

The Mathematics Stage 6 syllabuses are designed to offer students opportunities to think mathematically. Mathematical thinking is supported through questioning, communicating, reasoning, reflecting and the use of appropriate technology, and is encouraged through providing students with opportunities to generalise, challenge, find connections and to think critically and creatively. Using mathematical thinking, students apply their knowledge and skills to deepen their understanding of the world.

Mathematics Extension 1

Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 form a continuum to provide students with opportunities to acquire knowledge, understanding and skills in mathematical concepts at progressively higher levels and with applications in an increasing number of contexts. Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 provide students with opportunities to learn about the interconnected nature of mathematics, its beauty and its functionality. The concepts and techniques of differential and integral calculus are the basis of the courses.

Mathematics Extension 1 enables students to develop a thorough knowledge and understanding of and competence in further aspects of mathematics as an extension of the Mathematics Advanced course. The course provides opportunities to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively.

Mathematics Extension 1 supports students in tertiary study in mathematics and related fields.

Aim

The aim of Mathematics Extension 1 is for students to extend their knowledge and understanding of Working mathematically from Mathematics Advanced and Stage 5, further their understanding of the relationship between real-world problems and mathematical models, make connections within mathematics, and enhance their skills in using the language of mathematics to communicate in a concise and systematic manner.

Table of outcomes

MAO-WM-01 Working mathematically

develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly

This outcome is aligned to all content in each Stage.

Year 11	Year 12
ME1-11-01 solves problems involving inequalities, functions and their inverses, graphical relationships between functions, and parametric equations	**ME1-12-01** uses mathematical induction to prove results involving sums and divisibility
ME1-11-02 applies the remainder and factor theorem and sums and products of zeroes to solve problems involving polynomials	**ME1-12-02** operates with 2D and 3D vectors and uses 2D vectors to solve problems involving motion in two dimensions
ME1-11-03 solves problems in three dimensions using trigonometry and simplifies expressions, proves results and solves problems involving compound angles using trigonometric identities	**ME1-12-03** solves problems involving inverse trigonometric functions
ME1-11-04 uses permutations and combinations to solve problems involving counting, ordering and probability	**ME1-12-04** selects and applies differentiation and integration techniques to solve problems
ME1-11-05 uses the binomial theorem to solve problems and prove identities	**ME1-12-05** applies calculus to solve problems involving polynomials, further rates of change, areas and volumes and differential equations
	ME1-12-06 solves problems involving binomial distributions, sampling distribution of the mean and the central limit theorem

Outcomes and content for Year 11

Further work with functions

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- solves problems involving inequalities, functions and their inverses, graphical relationships between functions, and parametric equations **ME1-11-01**

Content

Graphical relationships

- Examine the relationship between the graph of $y = f(x)$ and the graph of $y = \frac{1}{f(x)}$ using graphing applications, and graph $y = \frac{1}{f(x)}$ given $y = f(x)$ in algebraic or graphical form, identifying any vertical and horizontal asymptotes of $y = f(x)$ and $y = \frac{1}{f(x)}$
- Graph $y = \sec x$, $y = \operatorname{cosec} x$ and $y = \cot x$ in both radians and degrees, identifying key properties including asymptotes, period, domain, range and symmetry, and compare each graph with the graph of its reciprocal
- Examine the relationship between the graph of $y = f(x)$ and the graphs of $y = |f(x)|$ and $y = f(|x|)$ using graphing applications, and graph $y = |f(x)|$ and $y = f(|x|)$ given $y = f(x)$ in algebraic or graphical form
- Examine the relationship between the graphs of $y = f(x)$ and $y = g(x)$ and the graphs of $y = f(x) + g(x)$ and $y = f(x) - g(x)$ using graphing applications, and graph $y = f(x) + g(x)$ and $y = f(x) - g(x)$ given $y = f(x)$ and $y = g(x)$ in algebraic or graphical form
- Determine the domains and ranges of the sum and difference of functions where possible, and, if appropriate, verify them using a graphing application
- Apply knowledge of graphical relationships to solve problems involving graphs of functions, justifying conclusions in the context of the problem where appropriate

Inverse functions

- Describe a function as one-to-one if every element in the range of the function corresponds to exactly one element of the domain
- Establish that the reflection of a point in the line $y = x$ reverses the coordinates of the point
- Define an inverse function f^{-1} informally as a function that reverses or undoes the effect of the function f
- Recognise that inverse functions exist for one-to-one functions
- Determine the equation for the inverse function $f^{-1}(x)$ of a given one-to-one function $y = f(x)$ by interchanging the variables x and y in $y = f(x)$
- Compare the graphs of a function and its inverse function using graphing applications, and recognise that the two graphs are reflections of each other in the line $y = x$
- Establish that the domain of $f^{-1}(x)$ is the range of $f(x)$, and the range of $f^{-1}(x)$ is the domain of $f(x)$, and use this relationship to solve problems
- Graph the inverse function of a given one-to-one function

- Explain that the reflection in the line $y = x$ exchanges horizontal and vertical lines and recognise that the horizontal line test can therefore be applied to the graph of $y = f(x)$ to determine whether its reflection in the line $y = x$ is a function
- Apply restrictions to the domain of a function, if it is not one-to-one, to obtain an inverse function
- Define formally $f^{-1}(x)$ to be the inverse function of $f(x)$ if the relationships $f\left(f^{-1}(x)\right) = x$ and $f^{-1}\left(f(x)\right) = x$ hold, and use this definition to solve problems
- Solve problems based on the relationship between a function and its inverse function using algebraic and graphical techniques, including determining the points of intersection of a function and its inverse, where they exist

Parametric form of a function or relation

- Recognise that a curve may be represented by two parametric equations that give x and y as functions of a parameter and that the curve may also have a Cartesian equation in x and y
- Express linear and quadratic functions and circles in parametric form

Example(s):

$x = 2\cos t$, $y = 2\sin t$ are the parametric equations of the circle $x^2 + y^2 = 4$.

- Convert linear and quadratic functions and circles from parametric form to Cartesian form
- Graph linear and quadratic functions and circles expressed in parametric form

Inequalities

- Solve cubic inequalities where the cubic is expressed as a product of linear factors
- Solve inequalities involving rational expressions with variables in the denominator

Example(s):

Find the value(s) of x for which $\frac{x-2}{x+3} > -2$.

- Solve absolute value inequalities of the form $|ax + b| \geq k$, $|ax + b| \leq k$, $|ax + b| < k$, $|ax + b| > k$, where a, b and k are constants, using algebraic and graphical methods or the characterisation of $|x|$ as the distance of x from the origin

Polynomials

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- applies the remainder and factor theorem and sums and products of zeroes to solve problems involving polynomials **ME1-11-02**

Content

Language and graphs of polynomials

- Define a polynomial function $P(x)$ of degree n, where n is a non-negative integer, to be a function that can be expressed in the form $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$, for real $a_n, \ldots, a_0$ and $a_n \neq 0$
- Define the leading term of $P(x)$ to be the term of highest degree and define the leading coefficient of $P(x)$ to be the coefficient of the leading term
- Define the constant term of $P(x)$ to be a_0
- Define a polynomial to be monic if its leading coefficient is 1
- Define the zero polynomial to be $P(x) = 0$ to be the polynomial with all the coefficients equal to zero and recognise that the zero polynomial has no leading term, no leading coefficient, no degree and constant term 0
- Determine the degree of $P(x) + Q(x)$ when two non-zero polynomials $P(x)$ and $Q(x)$, of degrees n and m respectively, are added
- Explain how the leading coefficient and the degree determine whether $y \to \infty$ or $y \to -\infty$ as $x \to \infty$ and as $x \to -\infty$
- Define the zeroes of $P(x)$ to be the numbers α such that $P(\alpha) = 0$, and define the roots of the polynomial equation $P(x) = 0$ to be its solutions, and recognise that every real number is a zero of the zero polynomial
- Define α as a repeated zero or multiple zero of a non-zero polynomial $P(x) = (x - \alpha)Q(x)$ when $(x - \alpha)$ is a factor of $Q(x)$, and α as a single zero of $P(x)$ when $(x - \alpha)$ is not a factor of $Q(x)$, for $Q(x) \neq 0$
- Define α as a zero of $P(x)$ of multiplicity m if $P(x) = (x - \alpha)^m Q(x)$, where m is a positive integer and $Q(\alpha) \neq 0$
- State the multiplicity of each root of a polynomial equation given in factored form

  Example(s):

  The polynomial equation $P(x) = 0$ is said to have one root of multiplicity 3 when $P(x) = (x - 1)^3$.

- Find the zeroes of a polynomial that is expressed as a product of linear factors, determine their multiplicity and graph the polynomial

Remainder and factor theorems

- Examine the process of division of polynomials by comparing with the process of division with remainders for whole numbers, and use the terms dividend, divisor, quotient and remainder
- Express in the form $P(x) = A(x)Q(x) + R(x)$ the result of dividing $P(x)$ by a divisor $A(x)$, that is not the zero polynomial, with quotient $Q(x)$ and remainder $R(x)$, and explain why either $R(x) = 0$ or $\deg R(x) < \deg A(x)$
- Express the result of the division also in the form $\frac{P(x)}{A(x)} = Q(x) + \frac{R(x)}{A(x)}$
- Explain why division by $x - \alpha$ yields $P(x) = (x - \alpha)Q(x) + r$, where r is a constant, and why $x - \alpha$ is a factor if and only if $r = 0$
- Prove and apply the remainder theorem for polynomials: when $P(x)$ is divided by $x - \alpha$, the remainder is $P(\alpha)$, and solve related polynomial problems
- Prove and apply the factor theorem for polynomials: $P(\alpha) = 0$ if and only if $x - \alpha$ is a factor of $P(x)$, and solve related polynomial problems
- Use the factor theorem to find all factors of $P(x)$ of the form $x - \alpha$, where α is an integer, and use division to find the remaining factor of the polynomial

Sums and products of zeroes of polynomials

- Prove that if a quadratic $P(x) = ax^2 + bx + c$ has zeroes α and β then $\alpha + \beta = -\frac{b}{a}$, the sum of the zeroes, and $\alpha\beta = \frac{c}{a}$, the product of the zeroes
- Prove that if a cubic $P(x) = ax^3 + bx^2 + cx + d$ has three zeroes α, β, γ, then $\alpha + \beta + \gamma = -\frac{b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$ and $\alpha\beta\gamma = -\frac{d}{a}$
- Prove that if a quartic $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ has four zeroes α, β, γ, δ then $\alpha + \beta + \gamma + \delta = -\frac{b}{a}$, $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$, $\alpha\beta\gamma + \beta\gamma\delta + \gamma\delta\alpha + \delta\alpha\beta = -\frac{d}{a}$ and
$$\alpha\beta\gamma\delta = \frac{e}{a}$$
- Use the formulas for the sums and products of zeroes to solve problems involving zeroes and coefficients of quadratic, cubic and quartic polynomials

Further trigonometry

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- solves problems in three dimensions using trigonometry and simplifies expressions, proves results and solves problems involving compound angles using trigonometric identities **ME1-11-03**

Content

Trigonometry in three dimensions

- Interpret information about a 3-dimensional context given in diagrammatical form
- Interpret information about a 3-dimensional context given in written form
- Apply trigonometry to solve problems in three dimensions where a diagram is given

Further trigonometric identities

- Derive the sum and difference expansions for the trigonometric functions $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$, $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ and $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- Use sum and difference expansion formulas to solve problems and prove results
- Derive the double angle formulas $\sin 2A = 2 \sin A \cos A$, $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$ and $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$
- Use the double angle formulas to solve problems and prove results

Further trigonometric equations

- Solve trigonometric equations involving factorisation and/or substitution of trigonometric identities over restricted domains
- Examine the representations of $a \cos x + b \sin x$ as $R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$ with and without graphing applications, where $a \neq 0$ and $b \neq 0$
- Apply the representations of $a \cos x + b \sin x$ as $R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$ to graph functions, solve equations of the form $a\cos x + b\sin x = c$ over restricted domains, and model and solve related problems
- Interpret solutions of trigonometric equations graphically with and without graphing applications
- Model and solve practical problems using trigonometric equations and analyse their solutions in a variety of contexts

Example(s):

The current I (in amperes) in a certain circuit after t seconds is given by $I = 2\sin\left(t - \frac{\pi}{3}\right) - \cos\left(t + \frac{\pi}{2}\right)$. Find the maximum current and the earliest time that it occurs.

Permutations and combinations

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- uses permutations and combinations to solve problems involving counting, ordering and probability **ME1-11-04**

Content

- Use the notation $n!$ (read as n factorial), where $n! = n(n-1)(n-2) \times \ldots \times 3 \times 2 \times 1$ for positive integers n
- Use $n! = n \times (n-1)!$ and the convention $0! = 1$ in calculations and to simplify algebraic expressions involving factorials
- Establish and use the multiplication principle: if a selection can be made in two stages, where there are m choices for the first stage and n choices for the second stage then there are $m \times n$ choices for the selection
- Apply the multiplication principle to explain why the number of ways of ordering n distinct objects in a straight line is $n!$
- Define a permutation as an ordered selection of some or all objects from a set of distinct objects
- Use the notation ${}^nP_r$ to represent an ordered selection of r objects from n distinct objects and observe that ${}^nP_n = n!$ and ${}^nP_0 = 1$
- Use the multiplication principle to establish that the number of ordered selections of r objects from n distinct objects is $n(n-1)(n-2) \times \ldots \times (n-r+1)$ and show that ${}^nP_r = n(n-1)(n-2) \times \ldots \times (n-r+1) = \frac{n!}{(n-r)!}$
- Solve problems involving permutations, including situations where the objects are not all distinct
- Solve problems involving permutations with restrictions on the placement of one or more objects
- Explain why the number of ways to arrange n distinct objects in a circle is $(n-1)!$
- Solve problems involving circular arrangements of distinct objects with or without restrictions on the placement of one or more objects
- Define a combination and use the notation ${}^nC_r$ or $\binom{n}{r}$ to represent the number of ways of selecting a subset of r objects from n distinct objects, where order is not important
- Establish and use the formula ${}^nC_r = \frac{n!}{r!(n-r)!}$
- Show that ${}^nC_n = {}^nC_0 = 1$ and ${}^nC_1 = {}^nC_{n-1} = n$
- Show that ${}^nC_r = {}^nC_{n-r}$, for $0 \leq r \leq n$ by selecting r objects from n distinct objects for inclusions and $n-r$ objects from n distinct objects for exclusion
- Prove ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$ for $1 \leq r \leq n-1$ algebraically and using combinatorial arguments
- Solve problems involving combinations with or without restrictions on the selection of one or more objects

- Solve problems involving both permutations and combinations, including problems which require consideration of cases
- Solve probability problems involving permutations and combinations

The binomial theorem

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- uses the binomial theorem to solve problems and prove identities **ME1-11-05**

Content

- Recognise that a binomial is an expression with two terms, and that a binomial expansion is an expansion of a power of a binomial
- Examine the symmetry formed by the coefficients of decreasing powers of x in the expansion of $(x+y)^n$ for $n = 0,1,2,3,4,5$ and arrange the coefficients into Pascal's triangle
- Recognise the equivalence between the coefficient of $x^{n-r}y^r$ in the expansion of $(x+y)^n$ and ${}^nC_r$, when n is a positive integer
- Use patterns and symmetry in Pascal's triangle to confirm the identities ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$ for $1 \le r \le n-1$ and ${}^nC_r = {}^nC_{n-r}$ for $0 \le r \le n$
- Derive the binomial theorem:
  $(x+y)^n = {}^nC_0x^n + {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 + \dots + {}^nC_{n-1}xy^{n-1} + {}^nC_ny^n$, when n is a positive integer
- Apply the binomial theorem to expand and simplify expressions of the form $(x+y)^n$
- Use the binomial theorem to determine the coefficient of a term with a specific power or the constant term in a binomial expansion

Example(s):

Find the term independent of x and the coefficient of x^6 in the expansion of $\left(2x - \frac{1}{x^2}\right)^{12}$.

- Use the identities ${}^nC_0 = 1$, ${}^nC_n = 1$, ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$ for $1 \le r \le n-1$ and ${}^nC_r = {}^nC_{n-r}$ for $0 \le r \le n$ to simplify expressions involving the binomial coefficients
- Prove identities involving binomial coefficients in binomial expansions by substituting values, comparing coefficients or applying a combinatorial argument to a specified context

Example(s):

Use the identity $(1+x)^4(1+x)^9 = (1+x)^{13}$ to show that ${}^4C_0\,{}^9C_4 + {}^4C_1\,{}^9C_3 + {}^4C_2\,{}^9C_2 + {}^4C_3\,{}^9C_1 + {}^4C_4\,{}^9C_0 = {}^{13}C_4$. Prove the same result by considering the number of ways a committee of 4 members can be chosen from a group of 13 people comprising 4 men and 9 women.

- Apply given or proven identities involving binomial coefficients to prove further identities, without the use of calculus

Example(s):

Use the identity ${}^{n}C_{r} = {}^{n}C_{n-r}$ to show that
${}^{2n}C_{n} = ({}^{n}C_{0})^{2} + ({}^{n}C_{1})^{2} + ({}^{n}C_{2})^{2} + \dots + ({}^{n}C_{n-1})^{2} + ({}^{n}C_{n})^{2}$.

Outcomes and content for Year 12

Proof by mathematical induction

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- uses mathematical induction to prove results involving sums and divisibility **ME1-12-01**

Content

- Examine the nature of inductive proof, including the statement to be proved, the base case and the inductive step
- Prove results for sums using mathematical induction

Example(s):

Prove $\sum_{k=1}^{n}(2k-1) = n^2$ for $n \geq 1$ and prove $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$ for $n \geq 0$.

- Prove divisibility results using mathematical induction

Example(s):

Prove $3^n + 7^n$ is divisible by 10 for all odd n.

- Identify errors in false 'proofs by induction', such as cases where only one of the two required steps of a proof by induction is true

Example(s):

Show that the inductive step can be proven for the false proposition that $1 + 2 + 3 + \ldots + n = \frac{1}{2}(n-1)(n+2)$ for integers $n \geq 1$, but that the initial case does not hold true.

Introduction to vectors

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- operates with 2D and 3D vectors and uses 2D vectors to solve problems involving motion in two dimensions **ME1-12-02**

Content

Vector representation and notation

- Define a vector as a quantity having both magnitude and direction
- Associate vectors with directed line segments and recognise that a vector may have many directed line segments associated with it
- Identify and use notation for vectors in both two dimensions and three dimensions, including **a**, $\underset{\sim}{a}$ and $\overrightarrow{AB}$, where $\overrightarrow{AB}$ is the vector with magnitude and direction those of the directed line segment from A to B

Example(s):

(upright boldface "a" used in printed text)

(undertilde notation for handwritten vectors)

Image long description: Two identical arrows representing vectors, sloping upwards from left to right. The first is labelled with '**a**' and is captioned with 'upright boldface "a" used in printed text'. The second is labelled with '*a*' and has a tilde underneath with the caption 'undertilde notation for handwritten vectors'.

- Use notations $|\mathbf{a}|$, $\left|\underset{\sim}{a}\right|$ and $|\overrightarrow{AB}|$ to represent the magnitude of a vector
- Describe a position vector as a vector with its tail at the origin
- Represent vectors graphically with and without graphing applications
- Recognise and use the fact that two vectors are equal if they have the same magnitude and direction to solve problems

Introduction to 2D and 3D vectors

- Use Cartesian coordinates to represent points in 2-dimensional (2D) and 3-dimensional (3D) space with and without graphing applications
- Use the midpoint and distance formulas in two dimensions and three dimensions

- Identify that, in three dimensions, all points on the xy-plane have a z-coordinate of 0, and deduce similar properties for points on the xz and yz-planes and thus the equations of the three coordinate planes
- Define the zero vector $\mathbf{0}$, written as $\underset{\sim}{0}$, as the vector with zero magnitude and no direction
- Define unit vectors as vectors of magnitude 1
- Recognise and use $\hat{\mathbf{a}}$ and $\underset{\sim}{\hat{a}}$ as the notation for the unit vector in the direction of $\mathbf{a}$
- Define the standard perpendicular unit vectors $\mathbf{i}$ and $\mathbf{j}$ in two dimensions and $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ in three dimensions
- Express 2D vectors in component form, $x\mathbf{i} + y\mathbf{j}$; as an ordered pair, (x,y); and in column vector notation $\begin{pmatrix} x \\ y \end{pmatrix}$
- Recognise $\overrightarrow{AB} = \begin{pmatrix} u-x \\ v-y \end{pmatrix}$ as the vector associated with the directed line segment from the point $A(x,y)$ to $B(u,v)$ in two dimensions
- Express 3D vectors in component form, $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$; as an ordered triple, (x,y,z); and in column vector notation $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$
- Recognise $\overrightarrow{AB} = \begin{pmatrix} u-x \\ v-y \\ w-z \end{pmatrix}$ as the vector associated with the directed line segment from $A(x,y,z)$ to $B(u,v,w)$ in three dimensions

Operating with vectors

- Define a scalar as a real number that is used to multiply a vector
- Represent geometrically a scalar multiple of a vector in two dimensions and three dimensions with and without graphing applications
- Perform multiplication of a vector by a scalar algebraically in component form
- Establish and identify $\mathbf{a} = k\mathbf{b}$, for a non-zero scalar k, as a condition for two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$ to be parallel to each other and determine with justification if two vectors are parallel to one another
- Identify $\begin{pmatrix} b \\ -a \end{pmatrix}$ and $\begin{pmatrix} -b \\ a \end{pmatrix}$ as vectors perpendicular to $\begin{pmatrix} a \\ b \end{pmatrix}$ and with equal magnitude
- Perform addition and subtraction of vectors algebraically in component form, and verify, with and without graphing applications, that geometrically these are obtained using the triangle law or the parallelogram law
- Establish and calculate the magnitude of a vector using $|x\mathbf{i} + y\mathbf{j}| = \sqrt{x^2 + y^2}$ for 2D vectors and $|x\mathbf{i} + y\mathbf{j} + z\mathbf{k}| = \sqrt{x^2 + y^2 + z^2}$ for 3D vectors
- Use the magnitude of a vector to find the unit vector $\hat{\mathbf{a}} = \frac{1}{|\mathbf{a}|}\mathbf{a}$ in two dimensions and three dimensions

Further operations with vectors

- Define $\mathbf{a} \cdot \mathbf{b} = x_1x_2 + y_1y_2$ as the scalar (dot) product of vectors $\mathbf{a} = x_1\mathbf{i} + y_1\mathbf{j}$ and $\mathbf{b} = x_2\mathbf{i} + y_2\mathbf{j}$ and use the scalar product to solve problems
- Define $\mathbf{a} \cdot \mathbf{b} = x_1x_2 + y_1y_2 + z_1z_2$ as the scalar product of vectors $\mathbf{a} = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ and $\mathbf{b} = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$ and use the scalar product to solve problems

- Use $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ as a geometric expression of the scalar product of non-zero vectors **a** and **b** in two dimensions and three dimensions, where θ is the angle between the vectors and $0° \le \theta \le 180°$.
- Verify the equivalence of $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ with the algebraic definition of the scalar product, $\mathbf{a} \cdot \mathbf{b} = x_1x_2 + y_1y_2$ for two dimensions and $\mathbf{a} \cdot \mathbf{b} = x_1x_2 + y_1y_2 + z_1z_2$ for three dimensions

Example(s):

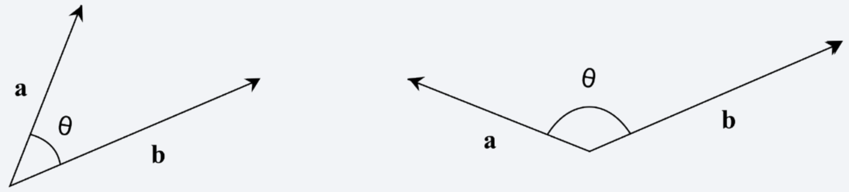


Image long description: Two vector diagrams, each represented by two arrows connected at a point. The first shows the arrows labelled a and b with an acute angle labelled theta between them. The second shows the arrows labelled a and b with an obtuse angle labelled theta between them.

- Derive and use the property $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$ to establish the scalar product definition of the magnitude of a vector ($|\mathbf{a}| = \sqrt{\mathbf{a} \cdot \mathbf{a}}$) in two dimensions and three dimensions
- Calculate the angle between two non-zero vectors **a** and **b**, in both two dimensions and three dimensions, using the scalar product by deriving and applying the relationship $\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = (\hat{\mathbf{a}} \cdot \hat{\mathbf{b}})$
- Establish $\mathbf{a} \cdot \mathbf{b} = 0$ as a condition for two non-zero vectors **a** and **b** to be perpendicular to each other and use it to determine if two vectors are perpendicular
- Establish $\mathbf{a} \cdot \mathbf{b} = \pm|\mathbf{a}||\mathbf{b}|$ as another way to determine if two non-zero vectors **a** and **b** are parallel
- Define the projection of a vector **a** onto a vector **b**, denoted by $\text{proj}_{\mathbf{b}}\mathbf{a}$, to be the vector component of **a** in the direction of vector **b**

Example(s):

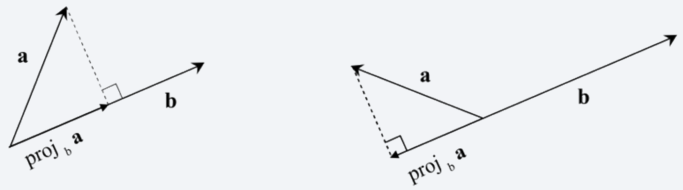


Image long description: Two vector diagrams both representing vectors, each showing two arrows connected at a point. In the first diagram, the angle between the arrows is acute. One arrow is labelled with boldface a, the other with boldface b, with the b arrow as the longer vector, pointing up and to the right. A dotted line perpendicular to arrow b joins the tip of arrow a. There is an arrow drawn on top of arrow b from the vertex, which is labelled proj b a, with a subscripted b and boldface a. In the second diagram the angle between the arrows is obtuse. One arrow is labelled with boldface a and the other with boldface b, with b as the longer vector, pointing up and to the right. A dotted line perpendicular to arrow b joins the tip of arrow a, pointing up and to the left, to an extension of arrow b, pointing down and to the left. The arrow drawn on top of arrow b from the vertex is labelled proj b a with a subscripted b and boldface a.

- Examine the proof of the formula $\text{proj}_{\mathbf{b}}\mathbf{a} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{b}|^2}\right)\mathbf{b} = (\mathbf{a}\cdot\hat{\mathbf{b}})\hat{\mathbf{b}}$ and use the formula to solve problems
- Determine that the component of a vector **a** perpendicular to another vector **b** is $\mathbf{a} - \text{proj}_{\mathbf{b}}\mathbf{a}$

Example(s):

Define and find the vector component of **a** perpendicular to **b**.

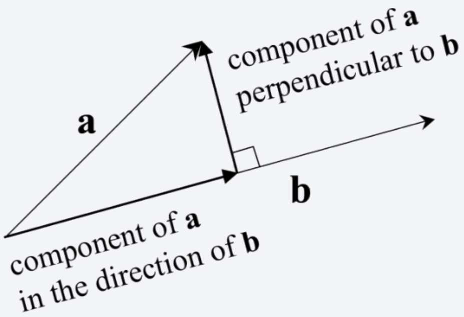


Image long description: Two arrows representing vectors, connected at a point. The angle between the arrows is acute. The arrows are labelled with boldface a and boldface b with b as the longer vector. An arrow, perpendicular to arrow b and starting from arrow b, joins the tip of arrow a and is labelled ‘component of (boldface) a perpendicular to (boldface) b’. Another arrow is drawn on top of arrow b from the vertex to the point where the perpendicular arrow starts and is labelled ‘component of (boldface) a in the direction of (boldface) b’.

Motion in vector form in two dimensions

- Describe the position of an object at a point in 2D space using a vector
- Describe the changing positions of an object by expressing its vector as a function of time using $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, or $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ where x and y are functions of time t
- Recognise that $x(t)$ and $y(t)$ form a pair of parametric equations for the path of the object
- Find the Cartesian equation of the path of an object, where the path is a straight line, parabola or circle
- Express the change in an object’s position between two points as a displacement vector and recognise the magnitude of the displacement vector as the distance between the two points
- Solve motion problems involving constant velocity using vectors
- Solve relative velocity problems involving constant crosswind/cross-current using vector diagrams, and describe the direction of a vector where required
- Find the velocity vector $\mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$ and acceleration vector $\mathbf{a} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$ of an object using differential calculus
- Find the position vector and the velocity vector of an object using integral calculus given its acceleration vector
- Solve motion problems involving non-constant velocity using vectors

Projectile motion

- Recognise that the gravitational force on a mass may be regarded as a constant acting in a downwards direction when the motion of the object is restricted to a small region near the Earth’s surface

- Model and analyse a projectile's path where the projectile is a point and air resistance is negligible, subject to only acceleration due to gravity, assuming that the projectile is moving close to the Earth's surface
- Represent the motion of a projectile using vectors
- Recognise that the horizontal and vertical components of the motion of a projectile can be represented by horizontal and vertical vectors
- Derive and use the equations of motion of a projectile in vector form by splitting 2D motion into horizontal and vertical components to solve problems on projectiles
- Find the Cartesian equation of the path of a projectile using parametric equations for the horizontal and vertical components of the displacement vector
- Determine features of the flight of a projectile, including time of flight, maximum height, range, instantaneous velocity and impact velocity
- Solve problems relating to the path of a projectile in which the initial velocity and/or angle of projection may be unknown, in a variety of contexts

Inverse trigonometric functions

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- solves problems involving inverse trigonometric functions **ME1-12-03**

Content

Definitions of inverse trigonometric functions

- Graph $y = \sin x$, $y = \cos x$ and $y = \tan x$, for $-2\pi \leq x \leq 2\pi$ using graphing applications and recognise that these three functions fail the horizontal line test
- Examine possible domain restrictions of $y = \sin x$, $y = \cos x$ and $y = \tan x$ to obtain the corresponding inverse functions using graphing applications
- Define $\sin^{-1}x$ or $\arcsin x$ to be the inverse function of $y = \sin x$ restricted to $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, and determine the domain and range of $y = \sin^{-1}x$
- Define $\cos^{-1}x$ or $\arccos x$ to be the inverse function of $y = \cos x$ restricted to $0 \leq x \leq \pi$, and determine the domain and range of $y = \cos^{-1}x$
- Define $\tan^{-1}x$ or $\arctan x$ to be the inverse function of $y = \tan x$ restricted to $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and determine the domain and range of $y = \tan^{-1}x$
- Evaluate and simplify expressions, and prove results using the definitions of $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$

Example(s):

Find the exact value of $\cos\left(\sin^{-1}\left(-\frac{1}{4}\right)\right)$.

Graphs of inverse trigonometric functions

- Graph $y = \sin x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, then use a reflection in $y = x$ to graph $y = \sin^{-1} x$ with and without graphing applications
- Graph $y = \cos x$ for $0 \leq x \leq \pi$, then use a reflection in $y = x$ to graph $y = \cos^{-1} x$ with and without graphing applications
- Graph $y = \tan x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, then use a reflection in $y = x$ to graph $y = \tan^{-1} x$ with and without graphing applications
- Classify $y = \sin^{-1} x$, $y = \cos^{-1} x$ and $y = \tan^{-1} x$ as odd, even, or neither odd nor even
- Examine the properties $\sin^{-1}(-x) = -\sin^{-1} x$, $\cos^{-1}(-x) = \pi - \cos^{-1} x$, $\tan^{-1}(-x) = -\tan^{-1} x$ and $\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$ using graphing applications and prove them algebraically

- Use the properties $\sin^{-1}(-x) = -\sin^{-1} x$, $\cos^{-1}(-x) = \pi - \cos^{-1} x$, $\tan^{-1}(-x) = -\tan^{-1} x$ and $\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$ to solve problems, simplify expressions and prove results

Example(s):

Solve $\cos^{-1} x = \sin^{-1}\left(-\frac{5}{13}\right)$ for x.

- Graph the composite functions $y = \sin(\sin^{-1} x)$, $y = \cos(\cos^{-1} x)$ and $y = \tan(\tan^{-1} x)$ by considering their domains and ranges
- Examine the graphs of the composite functions $y = \sin(\sin^{-1} x)$, $y = \cos(\cos^{-1} x)$ and $y = \tan(\tan^{-1} x)$, establish the values of x for which the formulas $\sin(\sin^{-1} x) = x$, $\cos(\cos^{-1} x) = x$ and $\tan(\tan^{-1} x) = x$ are true, and apply the formulas to solve problems, simplify expressions and prove results
- Graph the composite functions $y = \sin^{-1}(\sin x)$, $y = \cos^{-1}(\cos x)$ and $y = \tan^{-1}(\tan x)$ by considering their domains and ranges
- Examine the graphs of the composite functions $y = \sin^{-1}(\sin x)$, $y = \cos^{-1}(\cos x)$ and $y = \tan^{-1}(\tan x)$, establish the values of x for which the formulas $\sin^{-1}(\sin x) = x$, $\cos^{-1}(\cos x) = x$ and $\tan^{-1}(\tan x) = x$ are true, and apply the formulas to solve problems, simplify expressions and prove results

Example(s):

Given that $\cos\left(\frac{23\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$, find the exact value of $\cos^{-1}\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$.

- Use graphing applications to explore reflections, translations and dilations of the functions $f(x) = \sin^{-1} x$, $f(x) = \cos^{-1} x$ and $f(x) = \tan^{-1} x$, and confirm that the principles of transformations hold for the inverse trigonometric functions
- Apply the principles of transformations to inverse trigonometric functions to determine the function rule and graph the function, finding the domain and range of the transformed graph and determining any intercepts and asymptotes where appropriate

Example(s):

Graph $y = -2\cos^{-1}(3x)$ and describe its graphical transformation from $y = \cos^{-1} x$.

Further calculus skills

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- selects and applies differentiation and integration techniques to solve problems **ME1-12-04**

Content

Further derivatives of functions

- Find the derivative of a function defined parametrically using the chain rule
- Solve problems involving derivatives of functions defined parametrically
- Verify using the chain rule that the derivative of the inverse function is the reciprocal of the derivative of the function, evaluated at the value of the inverse function, that is $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$
- Solve problems involving derivatives of inverse functions

Example(s):

A particular function $y = f(x)$ has an inverse function $y = f^{-1}(x)$. The tangent l to $y = f(x)$ at the point $(2,3)$ has a gradient of 1. Show that the tangent to $y = f^{-1}(x)$ at the point $(3,2)$ is parallel to l.

- Examine the proofs of the derivatives of $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$
- Use the chain rule to show that $\frac{d}{dx}[\sin^{-1} f(x)] = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$, $\frac{d}{dx}[\cos^{-1} f(x)] = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}} = -\frac{d}{dx}[\sin^{-1} f(x)]$ and $\frac{d}{dx}[\tan^{-1} f(x)] = \frac{f'(x)}{1+[f(x)]^2}$ and apply the results to solve problems involving derivatives of $\sin^{-1} f(x)$, $\cos^{-1} f(x)$ and $\tan^{-1} f(x)$
- Apply the product, quotient and chain rules to find derivatives of functions of the form $f(x)g(x), \frac{f(x)}{g(x)}$ and $f(g(x))$ where $f(x)$ and $g(x)$ are any of the functions covered in the scope of the *Mathematics Advanced 11–12 Syllabus* (2024) or inverse trigonometric functions and solve related problems

Example(s):

Find the equation of the tangent to the curve $y = \tan^{-1}(x^2)$ at the point where $x = 1$.

Techniques of integration

- Find indefinite and definite integrals involving expressions of the form $\frac{1}{\sqrt{a^2-x^2}}$ or $\frac{a}{a^2+x^2}$

Example(s):

Find $\int \frac{x^2}{x^2+4}\,dx$ and $\int \frac{dx}{\sqrt{1-9x^2}}$.

- Use integration by substitution to evaluate definite and indefinite integrals given the substitution, where the substitution is expressed as a function of the variable of integration or where the variable of integration is the subject of the substitution

Example(s):

Find $\int \frac{e^{\sqrt{x}+1}}{\sqrt{x}}\,dx$ using the substitution $u = \sqrt{x} + 1$. Find $\int_0^1 \frac{dx}{(1+x^2)^{\frac{3}{2}}}$ using the substitution $x = \tan\theta$.

- Prove and use the identities $\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$ and $\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$ to find integrals involving $\int \sin^2 nx\, dx$ and $\int \cos^2 nx\, dx$

Further applications of calculus

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- applies calculus to solve problems involving polynomials, further rates of change, areas and volumes and differential equations **ME1-12-05**

Content

Multiplicity of zeroes of polynomial functions

- Use the product rule to prove that if $P(x)$ has a zero, α, of multiplicity $m > 1$, then α is a zero of $P'(x)$ of multiplicity $m - 1$, and use the result to determine the multiplicity of a discovered zero of $P(x)$ and solve related polynomial problems

Example(s):

Let $P(x) = x^3 + ax^2 + bx - 4$ where a and b are real numbers. If $x = 2$ is a double root of $P(x) = 0$, find the values of a and b.

- Prove that if α is a zero of multiplicity m, meeting the x-axis at $A(\alpha,0)$, then if $m = 1$, the curve crosses the x-axis at A at an acute or obtuse angle; if $m > 1$ is even, the curve is tangent to the x-axis at A and does not cross it; if $m > 1$ is odd, the curve has a horizontal inflection at A
- Graph a polynomial function in factored form, identifying its turning points and points of inflection if possible, and explore its behaviour as $x \to \infty$ and $x \to -\infty$, verifying the shape of the graph using graphing applications

Example(s):

Graph the polynomials $P(x) = (x + 1)^2(2 - x)^3$ and $P(x) = (x - 1)(x^2 + 3x + 1)$

Further rates of change

- Develop models in contexts where a rate of change of a function can be expressed as a rate of change of a composition of two functions, so that the chain rule can be applied
- Solve problems involving related rates of change using the chain rule, given the required formulas for problems relating to area, surface area or volume
- Describe and examine the graphs of practical situations where the rate of change of a quantity is proportional to the amount $Q - P$ by which the quantity Q exceeds some fixed value P
- Explain that if the rate of change of a quantity Q over time t is proportional to the difference $Q - P$ at any instant, then this may be represented by the equation $\frac{dQ}{dt} = k(Q - P)$, where k is a constant
- Verify by substitution that the function $Q = P + Ae^{kt}$, where A is a constant, satisfies the relationship $\frac{dQ}{dt} = k(Q - P)$, and that $Q = P$ in the case where $A = 0$

- Graph the function $Q = P + Ae^{kt}$, where $k > 0$ and $k < 0$ and $A > 0$ and $A < 0$, with and without graphing applications, and identify any asymptotes
- Use $\frac{dQ}{dt} = k(Q - P)$, $Q = P + Ae^{kt}$ and the graph of $Q = P + Ae^{kt}$ for $t \geq 0$, where $k > 0$ or $k < 0$ and $A > 0$ or $A < 0$, to model and solve problems where a limiting value of Q exists, including Newton's Law of Cooling and ecosystems with a natural carrying capacity, and justify conclusions in the context of the problem

Areas between curves and volumes of solids of revolution

- Calculate areas of regions between curves determined by functions in both real-life and abstract contexts
- Examine a solid of revolution whose boundary is formed by rotating an arc of a function about the x-axis or y-axis with and without graphing applications
- Calculate the volume of a solid of revolution formed by rotating a region in the plane about the x-axis or y-axis in both real-life and abstract contexts
- Calculate the volume of a solid of revolution formed by rotating the region between two curves about either the x-axis or y-axis in both real-life and abstract contexts

Differential equations

- Define a differential equation as an equation involving an unknown function and one or more of its derivatives
- Define and identify the order of a differential equation as the order of the highest derivative contained within the equation

Example(s):

$x^2 \frac{dy}{dx} = \tan y$ is a first order differential equation and $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x$ is a second order differential equation.

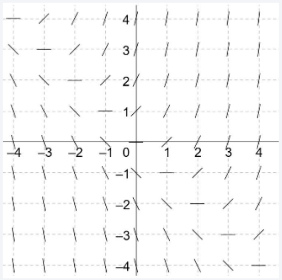
- Recognise that a solution to a first order differential equations is a function, and that there may be infinitely many functions that are solutions to a given first order differential equation
- Recognise the solutions to differential equations in the context of slope fields, and that slope fields are useful in determining the behaviour of solutions when the differential equation cannot be easily solved
- Recognise that a unique solution of a differential equation can be determined when sufficient initial conditions are given, and refer to a problem involving a differential equation and initial conditions as an initial value problem (IVP)
- Graph solutions to first order differential equations given a slope field and identify the unique solution curve that satisfies a set of initial conditions
- Explore problems given a slope field representing a practical context and justify conclusions
- Form a slope field for a first order differential equation using graphing applications

- Recognise the features of a slope field corresponding to a first order differential equation and vice versa

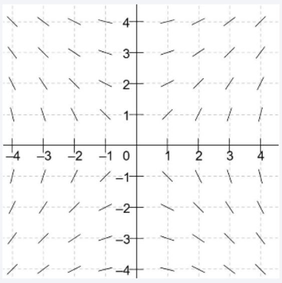
Example(s):

Which of the following slope fields best represents the differential equation $\frac{dy}{dx} = x - y$?

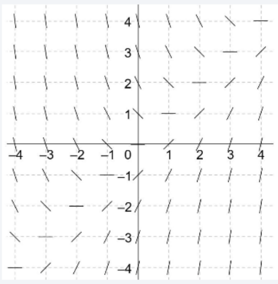
(A)



(B)



(C)



(D)

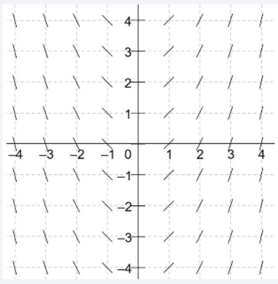


Image long description: Option A represents a slope field with all slope segments on the line $y = -x - 1$ having gradient -1. Option B represents a slope field with all slope segments on the line $y = x$ having gradient 1. Option C represents a slope field with all slope segments on the line $y = x - 1$ having gradient 1. Option D represents a slope field with all slope segments on the line $x = 0$ having 0 gradient. Image generated using Desmos.

- Solve first order differential equations of the form $\frac{dy}{dx} = f(x)$
- Solve first order differential equations of the form $\frac{dy}{dx} = g(y)$, where possible expressing the solution as a function with y as the subject

Example(s):

Solve the differential equations $\frac{dy}{dx} = 3y$ and $\frac{dy}{dx} = 20e^{-5y}$.

- Recognise and solve the first order differential equations for exponential growth and decay: $\frac{dQ}{dt} = kQ$ and $\frac{dQ}{dt} = k(Q - P)$

- Solve first order differential equations of the form $\frac{dy}{dx} = f(x)g(y)$ using separation of variables, where possible expressing the solution as a function with y as the subject

Example(s):

Given $\frac{dy}{dx} = -\frac{x}{ye^{x^2}}$, find the particular solution that goes through the point $(0,1)$.

- Graph solutions of first order differential equations using graphing applications and examine the behaviour of solutions for different values of the constant of integration and initial conditions
- Solve differential equations of the form $\frac{dP}{dt} = kP\left(1 - \frac{P}{C}\right)$ for some constants k and C, given the appropriate decomposition into partial fractions, to obtain the logistic function
- Model and solve differential equations in practical scenarios including in chemistry, biology and economics

The binomial distribution and sampling distribution of the mean

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- solves problems involving binomial distributions, sampling distribution of the mean and the central limit theorem **ME1-12-06**

Content

Bernoulli distributions

- Define a Bernoulli random variable as a model for two-outcome situations, referred to as success and failure
- Model a Bernoulli trial using a random variable, X, where the probability of success, $X = 1$, is p and the probability of failure, $X = 0$, is $q = 1 - p$
- Define a Bernoulli distribution as the discrete probability distribution of a Bernoulli random variable
- Establish and use the formulae $\mathrm{E}(X) = \mu = p$ to find the mean, and $\mathrm{Var}(X) = p(1 - p)$ to find the variance of the Bernoulli distribution with parameter p
- Identify contexts suitable to be modelled by Bernoulli random variables
- Solve practical problems involving Bernoulli random variables

Binomial distributions

- Define a binomial random variable, X, as the number of 'successes' in n independent and identically distributed Bernoulli trials, with the same probability of success p in each trial, where n is a fixed finite number
- Identify contexts suitable to be modelled using binomial distributions

Example(s):

A scenario that can be modelled using a binomial distribution: 5 cards are randomly drawn from a standard deck of playing cards with replacement. A scenario that cannot be modelled using a binomial distribution: 5 cards are randomly drawn from a standard deck of playing cards without replacement.

- Define a binomial experiment as a fixed number of Bernoulli trials
- Use the notation $X \sim \mathrm{Bin}(n,p)$ to represent a binomial random variable, X, where n is the number of trials and probability of success is p
- Recognise the significance of ${}^nC_r$ as the number of ways in which an outcome with r successes can occur in n trials
- Use the formula $P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$ to find the probability of r successes in a binomial experiment involving n trials
- Calculate the expected frequencies of the various possible outcomes from a binomial experiment, where the expected frequency of an outcome happening r times in n trials is
$$n \times P(X = r)$$

- Use the formulae $\mathrm{E}(X) = \mu = np$ to find the mean, and $\mathrm{Var}(X) = np(1 - p)$ to find the variance when $X \sim \mathrm{Bin}(n,p)$
- Solve practical problems involving binomial distributions and binomial probabilities with and without online computational applications, excluding the normal approximation to the binomial distribution

Sampling distribution of the mean and the central limit theorem

- Define a statistical population as the entire group of people or objects about which information is sought
- Define a sample as a selection of people or objects drawn from a population
- Recognise that a statistic taken from a sample will be a good approximation if the sample is random and the sample size is large enough, typically $n \geq 30$
- Recognise that, in general, the distribution of a population and a statistic, such as its mean, are unknown
- Recognise that it is likely that the distribution of a random sample will resemble the distribution of the population when the sample size, n, is large enough, typically $n \geq 30$
- Define the sample mean for a sample $X_1, X_2, \ldots, X_n$ taken from a population as $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$
- Recognise that the sample means obtained from repeated sampling may be different, even if all the samples are of the same size
- Define, for a given sample size n, the sampling distribution of the mean to be the distribution of the sample means of all samples of size n
- Recognise that sample mean, $\overline{X}$, is a random variable that estimates the population mean, μ, when the size of the sample is large enough and consider its usefulness in making predictions in a variety of contexts including politics, finance, agriculture and biology

Example(s):

Assume the weight of pumpkins in a particular farm is normally distributed with a mean of 13.5 kg and a standard deviation of 3 kg. A sample of 15 pumpkins is randomly selected from the farm. Find the expected value, variance and the standard deviation of the distribution of the sample mean. Let $X_1, X_2, X_3, \ldots, X_{15}$ denote the weights of the pumpkins in the sample.

- State the central limit theorem as: for a population with mean μ and variance σ^2, provided the sample size n is large enough ($n \geq 30$), the sampling distribution is approximately normal with mean μ and variance $\frac{\sigma^2}{n}$; that is, $\overline{X}$ is approximately $N\left(\mu, \frac{\sigma^2}{n}\right)$
- Recognise the significance of the central limit theorem for populations that are not necessarily normally distributed; that, irrespective of the population distribution, for a sufficiently large sample size, the sampling distribution of the mean is approximately normal
- Use the formulas $E(\overline{X}) = \mu$ and $\mathrm{Var}(\overline{X}) = \frac{\sigma^2}{n}$ as the mean and the variance of the sampling distribution of the mean for samples of size n drawn from a population with mean μ and variance σ^2
- Examine the effect of the sample size on the variance of sample means with digital tools

- Apply the central limit theorem to estimate the probability that the sample mean lies within given bounds

Example(s):

A group of researchers conducted a random sampling study on the sleep time of Year 12 students. The population has an average of 7.4 hours of sleep time with a standard deviation of 2.42 hours. If a sample of 100 students is taken, what is the probability that their sample mean will be less than 7 hours?

Assessment

Course standards

Course standards are comprised of syllabus standards and performance standards. Syllabus outcomes and content form the syllabus standards that describe what students are expected to know, understand and do. Performance standards, including the Common Grade Scale for Preliminary courses, HSC Performance Band Descriptions and Achievement Level Descriptions (for HSC courses with grades only) describe how well students have achieved in relation to syllabus standards. Performance standards are used to report student achievement on NESA credentials.

Common Grade Scale for Preliminary courses

Stage 6 – Year 11

The Common Grade Scale for Preliminary courses provides generic, holistic descriptions for performance at each of the 5 grade levels. It is used to report student achievement in Year 11 courses in all NSW schools.

Performance Band Descriptions for Mathematics Extension 1

Stage 6

HSC performance band descriptions summarise the knowledge, understanding and skills typically demonstrated by students at each band level in a particular course. NESA uses these holistic descriptions to report a student's overall level of achievement in an HSC course.

Band E4

A student who performs at this level typically:

- demonstrates extensive knowledge, understanding and skills appropriate to the course
- synthesises mathematical concepts, techniques and results to efficiently solve problems
- demonstrates sophisticated multi-step logic to solve problems in familiar and unfamiliar situations
- constructs, analyses and applies mathematical models effectively to solve problems in a wide variety of situations
- communicates complex reasoning, justification and arguments effectively, using appropriate mathematical language, notation, diagrams and graphs.

Band E3

A student who performs at this level typically:

- demonstrates thorough knowledge, understanding and skills appropriate to the course
- synthesises mathematical concepts, techniques and results to solve problems
- demonstrates well-developed multi-step logic to solve problems in familiar and unfamiliar situations
- constructs, analyses and applies mathematical models to solve problems in a variety of situations
- communicates reasoning and justification effectively, using appropriate mathematical language, notation, diagrams and graphs.

Band E2

A student who performs at this level typically:

- demonstrates sound knowledge, understanding and skills appropriate to the course
- uses mathematical concepts, techniques and results to solve problems
- constructs and applies mathematical models to solve problems in a range of situations
- communicates reasoning and justification using mathematical language, notation, diagrams and graphs.

Band E1

Performance reported as Band E1 indicates that the minimum standard expected was not demonstrated.

School-based assessment

Schools are required to develop an assessment program for each Year 11 and Year 12 course. NESA provides information about the responsibilities of schools in developing assessment programs in course-specific assessment requirements and in the Assessment Certification Examination (ACE) rules and requirements.

Year 11 Mathematics Extension 1 school-based assessment

Stage 6 – Year 11

Schools are required to submit to NESA a grade for each student based on their achievement at the end of the course.

Teachers use professional, on-balance judgement to allocate grades based on the Common Grade Scale for Preliminary courses.

Teachers consider all available assessment information, including formal and informal assessment, to determine the grade that best matches each student's achievement at the end of the course.

See Assessment Certification Examination (ACE) rules and requirements for further information.

Assessment programs must reflect course components and weightings

The course components and component weightings for Year 11 are mandatory.

Course component	Weighting %
Knowledge and understanding of course content	50
Skills in Working mathematically	50

Schools may determine specific elements of their assessment program

Schools have authority to determine the number, type of task and the weighting allocated to an assessment task. Schools may also follow the sample assessment programs provided by NESA.

Sample assessment program

NESA's sample Year 11 formal school-based assessment program for Mathematics Extension 1 is:

- 3 assessment tasks, including:
  - a formal written examination

- a weighting for any individual task of 20% to 40%.

Some students with disability may require adjustments in order to access assessment opportunities and demonstrate achievement of outcomes.

Year 12 Mathematics Extension 1 school-based assessment

Stage 6 – Year 12

NESA requires schools to submit a school-based assessment mark for each Year 12 student who completes this course. Formal school-based assessment tasks should reflect the syllabus outcomes and content. The mark submitted by the school provides a summation of each student's achievement measured at several points throughout the course.

A school's program of school-based assessment includes both mandatory and non-mandatory elements.

See Assessment Certification Examination (ACE) rules and requirements for further information.

Assessment programs must reflect course components and weightings

The course components and component weightings for Year 12 are mandatory.

Course component	Weighting %
Knowledge and understanding of course content	50
Skills in Working mathematically	50

Schools may determine specific elements of their assessment program

Schools have authority to determine the number, type of task and the weighting allocated to an assessment task. Schools may also follow the sample assessment programs provided by NESA.

Sample assessment program

NESA's sample Year 12 formal school-based assessment program for Mathematics Extension 1 is:

- 4 assessment tasks, including:
  - a formal written examination
  - a weighting for any individual task of 10% to 40%.

Some students with disability may require adjustments in order to access assessment opportunities and demonstrate achievement of outcomes.

Formal school-based assessment in this course should focus on the Year 12 outcomes. The Year 11 course is assumed knowledge and may be assessed.

Sample assessment schedules

Stage 6

Sample assessment schedules and tasks are being updated to align with new ACE Rules. More information: New limit on take-home assessment tasks from Term 4 2026.

Sample assessment task notifications

Stage 6

Sample assessment schedules and tasks are being updated to align with new ACE Rules. More information: New limit on take-home assessment tasks from Term 4 2026.

Further information

Stage 6 – Year 12

- Assessment in Stage 6
- Principles of effective assessment
- Assessment in practice
- School-based assessment requirements
- Adjustments to assessment for students with disability
- Life Skills
- Assessment Certification Examination (ACE)

HSC examinations

The external HSC examination measures student achievement in a range of syllabus outcomes.

The external examination and its marking relate to the syllabus by:

- providing clear links to syllabus outcomes
- enabling students to demonstrate the levels of achievement outlined in the Performance Band Descriptions
- applying marking guidelines based on criteria that relate to the quality of the response
- aligning performance in the examination each year to the standards established for the course.

Examination questions may require students to integrate knowledge, understanding and skills developed through studying the course.

The Year 11 course is assumed knowledge for the Year 12 course.

Some students with disability may be eligible for HSC exam provisions when completing the HSC examination.

The examination will be based on the Mathematics Extension 1 Year 12 course and will focus on the Year 12 outcomes. The Mathematics Advanced course and the Mathematics Extension 1 Year 11 course will be assumed knowledge for this examination. The Mathematics Extension 1 Year 11 course may be examined.

Students will also be required to complete either the Mathematics Advanced examination paper or the Mathematics Extension 2 examination paper, in addition to the Mathematics Extension 1 paper, based on their course enrolment.

Mathematics Extension 1 HSC examination specifications

Stage 6 – Year 12

The examination will consist of a written paper worth 70 marks.

Time allowed: 2 hours plus 10 minutes reading time.

A reference sheet will be provided.

NESA-approved calculators may be used.

The paper will consist of two sections.

Section I (10 marks)

- There will be objective-response questions to the value of 10 marks.

Section II (60 marks)

- Questions may contain parts.
- There will be 23 to 28 items.
- At least one item will be worth 4 or 5 marks.

HSC Mathematics Extension 1 – Sample examination materials

Stage 6 – Year 12

- HSC Mathematics Extension 1 – annotated sample examination materials
- Mathematics Extension 1 – HSC reference sheet
- Sample HSC examinations: A4 guide for teachers

Further information

Stage 6 – Year 12

- Assessment in Stage 6
- Principles of effective assessment
- Assessment in practice
- School-based assessment requirements
- Adjustments to assessment for students with disability
- Life Skills
- Assessment Certification Examination (ACE)