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Table of contents

Table of contents....4
Mathematics Advanced 11–12 (2024)....6
Teaching advice for Stage 6....6
Teaching advice for Working with functions....6
Teaching advice for Trigonometry and measure of angles....12
Teaching advice for Trigonometric identities and equations....17
Teaching advice for Introduction to differentiation....20
Teaching advice for Exponential and logarithmic functions....25
Teaching advice for Graph transformations....29
Teaching advice for Probability and data....32
Teaching advice for Further graph transformations and modelling....36
Teaching advice for Sequences and series....39
Teaching advice for Differential calculus....42
Teaching advice for Integral calculus....46
Teaching advice for Applications of calculus....51
Teaching advice for Random variables....55
Teaching advice for Financial mathematics....62
Mathematics Extension 1 11–12 (2024)....64
Teaching advice for Stage 6....64
Teaching advice for Further work with functions....64
Teaching advice for Polynomials....70
Teaching advice for Further Trigonometry....73
Teaching advice for Permutations and combinations....77
Teaching advice for The binomial theorem....81
Teaching advice for Proof by mathematical induction....85
Teaching advice for Introduction to vectors....88
Teaching advice for Inverse trigonometric functions....94
Teaching advice for Further calculus skills....98
Teaching advice for Further applications of calculus....101
Teaching advice for The binomial distribution and sampling distribution of the mean....106
Mathematics Extension 2 11–12 (2024)....110
Teaching advice for Stage 6....110

Teaching advice for The nature of proof....110
Teaching advice for Further work with vectors....118
Teaching advice for Introduction to complex numbers....122
Teaching advice for Further integration....127
Teaching advice for Applications of calculus to mechanics....132
Glossary....137
Mathematics Advanced 11–12 (2024) Terms....137
Mathematics Extension 1 11–12 (2024) Terms....165
Mathematics Extension 2 11–12 (2024) Terms....183
Global Terms....195

Mathematics Advanced 11–12 (2024)

Teaching advice for Stage 6

Teaching advice for Working with functions

Content

§ Working with functions

Teaching advice

Purpose

The study of functions is important in developing students' ability to find connections and patterns, to communicate concisely and precisely, to use algebraic techniques and manipulations, to analyse, model and solve problems, and to predict future outcomes in areas such as finance, economics, data analysis, marketing and weather.

Key terms

- absolute value
- asymptote
- axis of symmetry
- break-even point
- completing the square
- composite function
- conjugate
- constant
- continuous function
- cost
- denominator
- dependent variable
- direct variation
- discontinuity
- discriminant
- distinct
- domain
- element
- even function
- function
- general form (of a straight line)
- gradient
- gradient–intercept form
- hyperbola
- independent variable
- inequality
- intercept
- interval notation
- inverse variation
- linear
- linear equation

- linear function
- magnitude
- model
- odd function
- parabola
- parallel
- perpendicular
- piecewise-defined function
- point–gradient form
- Pythagoras' theorem
- quadratic formula
- quadratic function
- range
- rational number
- real number
- relation
- revenue
- rotation
- root
- set
- simultaneous equations
- substitution
- surd
- variable (algebra)
- vertex
- vertical line test
- zero (of a function)

Related Stage 5 Path content

- Algebraic techniques B
- Algebraic techniques C
- Indices B
- Indices C
- Equations B
- Equations C
- Linear relationships C
- Variation and rates of change A
- Non-linear relationships C
- Functions and other graphs

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Students can determine the domain and range of oblique, horizontal and vertical lines, state whether the line is a function, find which linear functions are even or odd, and what symmetry vertical lines have.	Mathematics Stage 5 Linear relationships A
The midpoint of the interval joining two x-intercepts of a parabola $y = a x^2 + b x + c$ can be used to determine its axis of symmetry.	Mathematics Stage 5 Linear relationships C
Linear, quadratic and cubic functions are polynomials.	Mathematics Stage 5 Polynomials Mathematics Extension 1 Year 11 Polynomials
Set notation can be used to express domains and ranges of functions and relations and solutions to inequalities. Real functions are associations between *non-empty* sets.	Mathematics Advanced Year 11 Probability and data
Indices are connected to logarithmic and exponential functions and the binomial theorem.	Mathematics Advanced Year 11 Exponential and logarithmic functions Mathematics Extension 1 Year 11 The binomial theorem
Linear functions are used when finding tangents and normals to curves.	Mathematics Advanced Year 11 Introduction to differentiation
Algebraic expressions, functions and formulas are used in mathematical modelling.	Mathematics Advanced Year 12 ▪ Further graph transformations and modelling ▪ Year 12 Applications of calculus Mathematics Extension 1 Year 12 Further applications of calculus
Piecewise-defined functions are used in probability density functions and cumulative distribution functions.	Mathematics Advanced Year 12 Random variables
The concept of odd and even functions can assist in curve sketching and evaluating integrals.	Mathematics Advanced Year 12 ▪ Applications of calculus ▪ Integral calculus
Cubic functions can be used to model	Mathematics Advanced Year 12 Applications

Absolute value is related to the magnitude of a vector and the modulus of a complex number.	Mathematics Extension 1 Year 12 Introduction to vectors Mathematics Extension 2 Year 12 ▪ Further work with vectors ▪ Introduction to complex numbers
The Cartesian equation of the path of an object may be a straight line, parabola or circle.	Mathematics Extension 1 Year 12 Introduction to vectors
Complex numbers are used when solving quadratic equations of the form $x^2+k=0$ where k is a positive real number, and in expressing the roots of quadratic equations of the form $a\,x^2+b\,x+c=0$, where a, b and c are real numbers and the discriminant $\Delta=b^2-4\,a\,c<0$.	Mathematics Extension 2 Year 12 Introduction to complex numbers
Operations with conjugate surds are connected to operations with conjugate complex numbers.	Mathematics Extension 2 Year 12 Introduction to complex numbers
Equations of straight lines and circles, graphs of rays and regions associated with lines, rays and circles can be defined using complex numbers.	Mathematics Extension 2 Year 12 Introduction to complex numbers
Inequalities can be proven using algebraic and geometric properties.	Mathematics Extension 2 Year 12 The nature of proof
Simultaneous equations and inequalities are used in the solution of problems found in scientific, industrial, design, health, business and everyday contexts. Piecewise-defined linear functions are used in machine learning. Quadratic functions can be used to model satellite dishes and bridges. Boyle's law states that the volume of a given mass of gas varies inversely with the pressure when the temperature is constant. Charles's law states that the volume of a given mass of gas is directly proportional to the temperature at constant pressure.	Real-world applications

Considerations

- A review of surds may be needed by some students, particularly the concepts that $\sqrt{x}$ is undefined for $x<0$, $\sqrt{x}=0$ for $x=0$, and $\sqrt{x}$ is the positive square root of x when $x>0$.
- Students should take care to use the correct language associated with functions, i.e. functions have zeroes and equations have roots.
- Students could show that the zeroes, or x-intercepts, of the general quadratic $y=ax^2+bx+c$ are $x=\frac{-b+\sqrt{\Delta}}{2a}$ and $x=\frac{-b-\sqrt{\Delta}}{2a}$.
- Students should develop fluency in the use of variables other than x and y and notations other than $f(x)$, such as $g(t), y(x), h_1(x), A(n)$.
- Commonly used interval notation:

Domain of $f(x)$	**Interval notation**
$-1<x<4$	$(-1,4)$
$-2\le x\le 3$	$[-2,3)$
all real numbers	$(-\infty,\infty)$
$x>1$	$(1,\infty)$
$x\le -2$	$(-\infty,-2]$

- To avoid common misconceptions students should recognise that:
  $(a+b)^2\neq a^2+b^2$
  $\frac{1}{a+b}\neq\frac{1}{a}+\frac{1}{b}$
  $\sqrt{a+b}\neq\sqrt{a}+\sqrt{b}$.
- A geometric interpretation of the inequality $|x)>1$ is that the distance of x from zero is greater than 1. This leads to the solution $x<-1$ or $x>1$. Similarly, the solution of $|x)<1$ is $-1<x<1$.
- Tables of values may be used to draw the graph of a function, given its function rule, or to solve problems if students are not able to find an appropriate formula.
- Students could describe a function as a set of ordered pairs (x,y) such that for each x-value in the set, there is only one y-value, for example: $J=\{(1,5),(3,3),(2,3),(4,2)\}$ is a function and $K=\{(1,5),(4,2),(2,3),(3,3),(1,6)\}$ is not a function but is a relation.

- It may be useful for students to recognise the notation $f \circ g$ for $f(g(x))$ and $f(f(x)) = (f \circ f)(x) = f^2(x)$ when using other resources.
- Students may choose to use the intercept form of a straight line $\frac{x}{a} + \frac{y}{b} = 1$, where a is the x-intercept and b is the y-intercept but this is not essential content.
- Some students may benefit from recognising the perpendicular distance formula, $d = \frac{|a x_1 + b y_1 + c|}{\sqrt{a^2 + b^2}}$, to find the shortest distance between a point (x_1, y_1) and a line $a x + b y + c = 0$ but this is not essential content.
- Students may choose to use limit notation when determining asymptotes, such as $\lim_{x \to \infty} \frac{1}{x} = 0$ but this is not essential content.
- Through practice in recognising different types of functions and their algebraic and graphical representations students should gain fluency in moving flexibly between representations of functions such as graphs, equations and tables. This should deepen understanding of concepts and aid in solving problems.
- Applications of simultaneous equations and inequalities in the solution of practical problems could include problems derived from scientific, industrial, design, health, business and everyday contexts.

Diversity of learners

- Teachers should consider the accessibility of tools and resources. Considerations for tools such as spreadsheets, calculators and applications may include compatibility with screen readers and other assistive technologies including alternative input devices and the use of accessible calculators (see approved calculators). Considerations for resources may include the language demands, readability of and long descriptions for images and providing alternative text for graphs and diagrams.
- Students with disability may require adjustments including access to visual or digital aids to communicate and demonstrate knowledge, understanding and skills.
- Students for whom English is an additional language or dialect (EAL/D) may require additional support and targeted strategies in learning language specific to mathematics.
- The pace and complexity of learning may need to be adjusted for students who progress at different rates.

Teaching advice for Trigonometry and measure of angles

Content

§ Trigonometry and measure of angles

Teaching advice

Purpose

The study of trigonometric functions involves using periodic functions in geometric, algebraic, numerical and graphical representations and is important in developing students' understanding of periodic behaviour. A knowledge of trigonometric functions enables the solving of practical problems involving triangles or periodic graphs. Using the periodicity of trigonometric functions, mathematical models have been developed that describe the behaviour of many naturally occurring periodic phenomena, such as vibrations or waves, as well as oscillatory behaviour found in pendulums, electric currents and radio signals.

Key terms

- acute angle
- ambiguous case of the sine rule
- amplitude
- angle of depression
- angle of elevation
- arc length
- area of a triangle
- asymptote
- Cartesian plane
- compass bearing
- cosine ratio
- cosine rule
- degree (geometry)
- domain
- even function
- function
- magnitude
- minute (geometry)
- odd function
- period
- Pythagoras' theorem
- radian
- radian measure
- range
- sector
- segment
- sine ratio
- sine rule
- subtend
- tangent ratio
- trigonometric ratio
- true bearing

Related Stage 5 Path content

- Properties of geometrical figures B
- Properties of geometrical figures C
- Trigonometry C
- Trigonometry D
- Circle geometry

Sequencing advice

The content in the focus area Trigonometry and measure of angles builds on content in:

- Mathematics Advanced Year 11 Working with functions.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Problems involving right-angled triangles may require the use of Pythagoras' theorem.	Mathematics Stage 4 Right-angled triangles (Pythagoras' theorem)
The concept of similarity is useful to explain the constancy of trigonometric ratios for triangles of the same shape but different sizes.	Mathematics Stage 5 ▪ Properties of geometrical figures A ▪ Properties of geometrical figures B ▪ Properties of geometrical figures C
The definition of sine, cosine and tangent ratios can be developed using a circle centred at the origin.	Mathematics Stage 5 Circle geometry
The sine rule can be proven using similar triangles and circle geometry.	Mathematics Stage 5 ▪ Properties of geometrical figures B ▪ Circle geometry
The concept of radian measure is built upon angle measurement and different properties of circle geometry.	Mathematics Stage 4 Angle relationships Mathematics Stage 5 Circle geometry
The properties of sine, cosine and tangent functions are built upon the knowledge of functions.	Mathematics Advanced Year 11 Working with functions
Sine, cosine and tangent functions are used in graph transformations of trigonometric graphs. They are also used in modelling and solving problems.	Mathematics Advanced Year 12 Further graph transformations and modelling
Calculus can be applied to trigonometric functions.	Mathematics Advanced Year 12 ▪ Differential calculus ▪ Integral calculus Mathematics Extension 1 Year 12 Further calculus skills Mathematics Extension 2 Year 12 Further integration
Sine, cosine and tangent ratios are used in solving three-dimensional geometry problems and in proving trigonometric identities and solving trigonometric equations.	Mathematics Advanced Year 11 Trigonometric identities and equations

	Mathematics Extension 1 Year 11 Further trigonometry Mathematics Extension 2 Further integration
Sine, cosine and tangent functions are used to define inverse trigonometric functions and their graphs.	Mathematics Extension 1 Year 12 Inverse trigonometric functions
The cosine ratio is used in the geometric expression of the scalar product of two non-zero vectors and in finding the angle between them.	Mathematics Extension 1 Year 12 Introduction to vectors
Statements involving the sine, cosine and tangent functions can be proven using mathematical induction.	Mathematics Extension 2 Year 12 The nature of proof
Sine and cosine functions are used in the calculation of the argument, the polar form and de Moivre's theorem for complex numbers.	Mathematics Extension 2 Year 12 Introduction to complex numbers
Sine and cosine functions can be used in parametric equations and are used in the vector equation of a circle in two dimensions.	Mathematics Extension 1 Year 11 Further work with functions Mathematics Extension 2 Year 12 Further work with vectors
Trigonometric functions provide a powerful tool for modelling quantities that vary in a cyclical way such as tides, seasons, demand for resources and alternating current. Sine and cosine functions are used in simple harmonic motion.	Mathematics Advanced Year 12 Further graph transformations and modelling Mathematics Extension 1 Year 11 Further trigonometry Mathematics Extension 2 Year 12 Applications of calculus to mechanics

Considerations

- Some students may need to review Pythagoras' theorem and its proof in preparation for applying the theorem in a wider variety of situations.
- Some students may benefit from a review of the sine, cosine and tangent ratios for angles in right-angled triangles. They may need to revisit the concept of similarity and use it to explain the constancy of trigonometric ratios for triangles of the same shape but different sizes.
- Students could use aids such as CAST or mnemonics such as *All Stations To Central* (ASTC) related to which quadrants of the unit circle contain positive results for $\sin\theta$, $\cos\theta$ and $\tan\theta$.
- Proofs of the sine rule, cosine rule and area rule should be examined by students. Students are not expected to reproduce these proofs.
- The sine rule may be expressed as: 'in a given triangle *ABC*, the ratio of a side to the sine of the opposite angle is a constant'.

- After the cosine rule is introduced, students could compare it with Pythagoras' theorem and conclude that Pythagoras' theorem is a special case of the cosine rule.
- When working with radians, '$\sin 3$' means the sine of an angle whose circular measure is 3, and this may be read as '$\sin 3$ radians' to distinguish it from $\sin 3°$, the sine of an angle of 3 degrees.

Diversity of learners

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Teaching advice for Trigonometric identities and equations

Content

§ Trigonometric identities and equations

Teaching advice

Purpose

The study of trigonometric functions involves using periodic functions in geometric, algebraic, numerical and graphical representations. Knowledge of trigonometric functions enables the solving of practical problems involving triangles or periodic graphs, such as waves and signals.

The study of trigonometric functions is important in developing students' understanding of periodic behaviour. Using the periodicity of trigonometric functions, mathematical models have been developed that describe the behaviour of many naturally occurring periodic phenomena, such as vibrations or waves, as well as oscillatory behaviour found in pendulums, electric currents and radio signals.

Key terms

- complementary angle
- cosecant ratio
- cosine ratio
- cotangent ratio
- radian
- secant ratio
- sine ratio
- tangent ratio

Related Stage 5 Path content

- Functions and other graphs
- Trigonometry C
- Trigonometry D

Sequencing advice

The content in the focus area Trigonometric identities and equations builds on content in:

- Mathematics Advanced Year 11 Working with functions
- Mathematics Advanced Year 11 Trigonometry and measure of angles.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Solving trigonometric equations may involve equations reducible to quadratic equations.	Mathematics Stage 5 Equations C
Proving trigonometric identities requires the process of proof.	Mathematics Stage 5 Properties of geometrical figures C
The definition of secant, cosecant and cotangent ratios can be developed using a circle centred at the origin.	Mathematics Advanced Year 11 Working with functions
Secant, cosecant and cotangent functions can be used in differentiation. The secant function is used in integration.	Mathematics Advanced Year 12 ▪ Differential calculus ▪ Integral calculus
Secant, cosecant and cotangent functions can be used in solving three-dimensional geometry problems, proving trigonometric identities and solving trigonometric equations.	Mathematics Extension 1 Year 11 Further trigonometry
Graphs of secant, cosecant and cotangent functions can be considered as graphs of reciprocal functions.	Mathematics Extension 1 Year 11 Further work with functions
Complementary identities are used in proving identities involving inverse trigonometric functions and Pythagorean identities are used in proving the double angle identities.	Mathematics Extension 1 Year 12 ▪ Inverse trigonometric functions ▪ Further calculus skills
Trigonometric equations and identities are used when modelling quantities that vary in a cyclical way such as tides, seasons, demand for resources and alternating current.	Mathematics Advanced Year 12 Further graph transformations and modelling Mathematics Extension 1 Year 11 Further trigonometry Mathematics Extension 2 Year 12 Applications of calculus to mechanics
Proof by mathematical induction can be used to prove trigonometric identities.	Mathematics Extension 2 Year 12 The nature of proof

Considerations

- The domains and ranges of $sec\,\theta, cosec\,\theta$ and $\cot\theta$ need to be carefully considered when solving equations involving these functions to ensure any values for which the functions are undefined are not included.

- Students use the notation $sin^n x$ to represent $(\sin x)^n$ where n is a positive real number. The notation $sin^{-1} x$ is used to represent the inverse trigonometric function and cannot be equated with $(\sin x)^{-1}$.
- When solving trigonometric equations students should be supported to evaluate the solutions of trigonometric equations and remove the values outside any given domains.

Diversity of learners

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Teaching advice for Introduction to differentiation

Content

§ Introduction to differentiation

Teaching advice

Purpose

In differential calculus, the idea of ‘gradient as a rate of change’ is used to find the rates of change of functions with non-constant rates of change. These functions are represented geometrically by curves. The process of finding these rates of change is known as differentiation. This process is of fundamental importance in mathematics and has applications in all quantitative fields of study, including physics, chemistry, medicine, engineering, computing, statistics, business, finance and economics.

Key terms

- angle of inclination
- average rate of change
- chain rule
- constant
- cubic function
- decreasing function
- derivative
- differentiable
- differentiation
- displacement
- distance
- first principles
- function
- gradient
- increasing function
- instantaneous rate of change
- limit
- linear function
- magnitude
- normal
- product rule
- quadratic function
- quotient rule
- secant
- speed
- stationary
- tangent
- velocity

Related Stage 5 Path content

- Variation and rates of change B
- Indices B
- Indices C
- Algebraic techniques B

- Algebraic techniques C
- Linear relationships C
- Non-linear relationships C
- Functions and other graphs

Sequencing advice

The content in the focus area Introduction to differentiation builds on content in:

- Mathematics Advanced Year 11 Working with functions.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Negative values have significance for displacement and velocity. They indicate direction from the original position.	Mathematics Stage 4 Computation with integers
Determining the gradient between points is important for finding the gradient of a secant.	Mathematics Advanced Year 11 Working with functions
Finding the tangent or normal to a curve involves the use of the point–gradient form of a straight line.	Mathematics Advanced Year 11 Working with functions
Function notation is important when considering derivatives.	Mathematics Stage 5 Functions and other graphs Mathematics Advanced Year 11 Working with functions
The nature of stationary points can be determined by considering the second derivative.	Mathematics Advanced Year 12 Applications of calculus
Finding the distance travelled by a particle involves considering the nature of stationary points.	Mathematics Advanced Year 12 Applications of calculus
The probability density function can be determined by finding the derivative of the cumulative distribution function.	Mathematics Advanced Year 12 Random variables
The derivative can be used to determine the multiplicity of roots of polynomials.	Mathematics Extension 1 Year 11 Polynomials
The uniqueness of Euler's number, e, can be shown using differentiation.	Mathematics Advanced Year 11 Exponential and logarithmic functions
Derivatives can be used to find the velocity, acceleration and momentum of a moving body. Change in concentration of an element involved in a chemical reaction can be predicted using differentiation. Derivatives can be used to predict the	Real-world applications

results of different investment possibilities and to find the rate of blood flow, the rate of dissolution of drugs into the bloodstream, shrinkage of the number of cells in a cancerous tumour and the growth of bacteria. Applications of calculus are prevalent in the field of engineering.	

Considerations

- For differentiation by first principles Δx is sometimes used in place of h, and the alternate definition of differentiation by first principles is sometimes used: $f'(c)=\lim\limits_{x \to c} \dfrac{f(x)-f(c)}{x-c}$.
- The terms 'maximum magnitude of velocity' or 'maximum speed' should be used when discussing velocity rather than ambiguous language such as 'maximum velocity'.
- Note that the limit $f'(x)=lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$ may fail to exist even for a continuous curve: the curve may have a vertical tangent (infinite slope) as in $f(x)=x^{\frac{1}{3}}$ at $x=0$, or a sharp bend, as in $y=|x|$ at $x=0$.
- Some students may benefit from extension that involves finding the derivative of $f(x)=x^3$, $f(x)=\sqrt{x}$ and $f(x)=\dfrac{1}{x}$ by first principles but this is not essential content.
- Some students may benefit from extension that involves consideration of concepts such as:
  - whether a function is increasing or decreasing on an interval: a function $f(x)$ is said to be *increasing* on an interval if for all $a \le b$ in the interval, $f(a) \le f(b)$, and *decreasing* on an interval if for all $a \le b$ in the interval, $f(a) \ge f(b)$.
  - whether a function is strictly increasing if $f'(x)>0$ over an interval or strictly decreasing if $f'(x)<0$ on an interval.
- Some students may benefit from extension that involves deriving the following results, though this is not essential content: if u and v are differentiable functions of x, then:
  if $y=ku$, $y'=ku'$ where k is a constant
  if $y=u \pm v$, $y'=u' \pm v'$
  if $y=f(u)$, $y'=f'(u)u'$ (for any differentiable function f)
  if $y=uv$, $y'=uv'+u'v$
  if $y=\dfrac{u}{v}$, $y'=\dfrac{vu'-uv'}{v^2}$, $(v \neq 0)$.

Diversity of learners

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Teaching advice for Exponential and logarithmic functions

Content

§ Exponential and logarithmic functions

Teaching advice

Purpose

A knowledge of exponential and logarithmic functions enables an understanding of practical applications, such as exponential growth and decay. The study of exponential and logarithmic functions is important in developing students' ability to solve practical problems involving rates of change in contexts such as population growth and compound interest.

Key terms

- asymptote
- base
- change of base rule
- derivative
- domain
- equations
- Euler's number
- exponential
- exponential function
- gradient
- intercept
- logarithm
- logarithmic function
- natural logarithm
- range
- tangent

Related Stage 5 Path content

- Indices B
- Indices C
- Non-linear relationships C
- Logarithms
- Functions and other graphs

Making connections

Connections	Course/Stage/Focus area/Post-school paths
The graphs of the exponential functions $y=k\left(a^{x}\right)$ and $y=k\left(a^{-x}\right)$ are reflections of each other over the y-axis.	Mathematics Advanced Year 11 Graph transformations
e^{x} is its own derivative.	Mathematics Advanced Year 11 Introduction to differentiation
A logarithm is an index and the logarithmic laws are derived from the index laws.	Mathematics Advanced Year 11 Working with functions
The nth term of a geometric series can be modelled as an exponential function of n.	Mathematics Advanced Year 12 Sequences and series
Financial problems involving compound interest, depreciation, annuities and loans can be modelled and solved using exponentials and logarithms.	Mathematics Stage 5 Financial mathematics B Mathematics Advanced Year 12 Financial mathematics
The graphs of $y=a^{x}$ and $y=\log_{a} x$ are reflections of each other in the line $y=x$ and are inverse functions. This reflective property can be used when solving integration problems involving logarithms and exponentials.	Mathematics Extension 1 Year 11 Further work with functions Mathematics Advanced Year 12 Integral calculus
Logarithmic scales are useful to represent a variable that ranges over a large set of values and are used to model and solve problems involving decibels (dB) in acoustics, seismic scales for earthquakes, magnitudes of stars and the pH scale in chemistry. Logarithmic scales are based on the concept of logarithm, and the process of solving problems involving logarithmic scales involves using exponentials.	Mathematics Stage 5 Logarithms Mathematics Advanced Year 12 Further graph transformations and modelling
Calculus can be applied to exponential functions and logarithmic functions.	Mathematics Advanced Year 12 ▪ Differential calculus ▪ Integral calculus
The probability density function for the normal distribution is an exponential function	Mathematics Advanced Year 12 Random

The integral of a function of the form $\frac{a}{x}$ is a logarithmic function.	Mathematics Advanced Year 12 Integral calculus Mathematics Extension 1 Year 12 Further calculus skills Mathematics Extension 2 Year 12 Further integration
Growth and decay situations can be modelled using exponentials. Logistic growth can be modelled by a differential equation and the logistic equation involving the exponential function e^{-x}.	Mathematics Advanced Year 12 ▪ Sequences and series ▪ Applications of calculus Mathematics Extension 1 Year 12 Further applications of calculus
Results involving logarithmic or exponential expressions can be proven using mathematical induction.	Mathematics Extension 2 Year 12 The nature of proof
Problems involving resisted motion can be solved using logarithms.	Mathematics Extension 2 Year 12 Applications of calculus to mechanics
The surge function, $C = a t e^{-bt}$ where a and b are constants, is a model for drug concentration over time and is used in medicine. The sigmoid function, $p = \frac{1}{1+e^{-x}}$, used in data science returns a value between 0 and 1 for any input x.	Real-world applications

Considerations

- Students could use graphing applications to observe that the value of the base a determines the properties of the graph of an exponential function $y = a^x$.
- Students could describe the properties of the graph of $y = a^x$ for $a > 1$ and $0 < a < 1$, whether the graph is increasing or decreasing, and how changing the value of a changes the steepness of the graph.
- Students could use graphing applications to observe that the value of the base a determines the properties of the graph of a logarithmic function $y = log_a x$.
- Students could describe the properties of the graph of $y = log_a x$ for $a > 1$ and $0 < a < 1$, whether the graph is increasing or decreasing, and how changing the value of a changes the steepness of the graph.
- Students should understand the difference between power functions $y = x^n$ and exponential functions $y = a^x$.
- Use of the correct language associated with logarithms should be emphasised.
- Students should be aware that on many calculators the log key is used to evaluate $\log_{10} x$, and they should take care when performing calculations involving logarithms.

- Students should be encouraged to use mental techniques to evaluate simple logarithms, for example, $\log_5 625$.

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Teaching advice for Graph transformations

Content

§ Graph transformations

Teaching advice

Purpose

The study of functions and their transformations is important in developing students' ability to find connections and patterns, to communicate concisely and precisely, to use algebraic techniques and manipulations, to analyse, model and solve problems, and to predict future outcomes in areas such as finance, economics, data analysis, marketing and weather.

Key terms

- circle
- dilation factor
- discontinuity
- domain
- enlargement
- function
- horizontal dilation
- horizontal translation
- range
- reduction
- reflection
- transformation
- translation
- vertical dilation
- vertical translation

Related Stage 5 Path content

- Algebraic techniques B
- Algebraic techniques C
- Linear relationships C
- Non-linear relationships C
- Logarithms
- Functions and other graphs

Sequencing advice

The content in the focus area Graph transformations builds on content in:

- Mathematics Advanced Year 11 Working with functions
- Mathematics Advanced Year 11 Exponential and logarithmic functions.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Transformations are related to the concept of commutativity. Any two translations commute. Any two dilations commute. A translation and a dilation commute if one is vertical and the other is horizontal.	Mathematics Stage 4 Algebraic techniques
Reflections and translations are congruence transformations. Enlargements and reductions are similarity transformations.	Mathematics Stage 5 ▪ Properties of geometrical figures A ▪ Properties of geometrical figures B ▪ Properties of geometrical figures C
Two reflections, one horizontal and one vertical, in either order, result in a rotation of $180°$, and this is the reason for the two minus signs in the algebraic test $f(-x)=-f(x)$ for oddness.	Mathematics Advanced Year 11 Working with functions
Symmetry properties should be used in sketching functions, including the oddness and evenness of functions.	Mathematics Advanced Year 11 Working with functions
Transformations can be applied to trigonometric functions and transformations of functions can be used to model and solve problems.	Mathematics Advanced Year 12 Further graph transformations and modelling
The probability density function for the standard normal distribution can be transformed to obtain the non-standard normal distribution.	Mathematics Advanced Year 12 Random variables
Transformations can be applied in statistics such that when a constant, c, is added to every item in a distribution, then c is added to the mean but the variance remains unchanged. This is known as the transformation rule in statistics.	Mathematics Advanced Year 12 Random variables
A function is reflected in the line $y=x$ to obtain an inverse relation or function.	Mathematics Extension 1 Year 11 Further work with functions Mathematics Extension 1 Year 12 Inverse trigonometric functions
The graph of the absolute value function involves a reflection of the line	Mathematics Advanced Year 11 Working with

Vectors, complex numbers and matrices can be used to represent transformations.	Mathematics Extension 1 Year 12 Introduction to vectors Mathematics Extension 2 ▪ Further work with vectors ▪ Introduction to complex numbers Tertiary mathematics
Data transformation functions are used to increase data quality to make accurate predictions in machine learning.	Real-world applications

Considerations

- Students could use graphing applications to confirm that reflections in the y-axis and x-axis are respectively horizontal and vertical dilations with factor -1.
- Students should understand that a dilation or enlargement with a negative factor can be affected by a reflection or $180°$ rotation, followed by a dilation or enlargement with the opposite factor.
- The transformation $y=-f(-x)$ rotates the graph $180°$ about the origin. Students could use a suitable object to show that a reflection in the y-axis and a reflection in the x-axis, in either order, is a rotation of $180°$ about the origin.
- Students could find all reflection and rotation symmetries of the circle $x^2+y^2=r^2$ but this is not essential content.
- Students could examine how rearrangement of the algebraic representation of a function can change the process of creating its graph. For example, the graph of $y=f\left(\frac{x+6}{2}\right)$ can be rearranged to $y=f\left(\frac{x}{2}+3\right)$ and consequently the same graph can be created using appropriate dilations and translations but in the reverse order.

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Teaching advice for Probability and data

Content

§ Probability and data

Teaching advice

Purpose

The study of probability provides a powerful and versatile toolset for understanding and navigating uncertainty and informing decisions in a wide range of disciplines.

The collection and statistical analysis of data is of fundamental importance in society. Conclusions drawn from the statistical analysis of data are used to inform decisions made in government, science, business and industry. It is important that students have a broad understanding of how data and statistics are used, as well as how statistics are misused, resulting in the misrepresentation of information.

Key terms

- array
- complement
- conditional probability
- continuous random variable
- cumulative frequency
- discrete random variable
- disjoint sets
- element
- finite
- frequency
- frequency histogram
- frequency polygon
- independent events
- intersection
- median
- mode
- multistage events
- mutually exclusive events
- outcome
- probability
- random experiment
- random variable
- relative frequency
- sample space
- set
- subset
- tree diagram
- trial
- two-way table
- union
- Venn diagram

Related Stage 5 Path content

- Probability B

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Set notation can be used to express domains and ranges of functions and relations and solutions to inequalities. Real functions are associations between two *non-empty* sets.	Mathematics Advanced Year 11 Working with functions
Probability concepts can be applied in situations involving permutations and combinations, and when working with probability distributions.	Mathematics Year 11 Extension 1 Permutations and combinations Mathematics Year 12 Advanced Random variables Mathematics Year 12 Extension 1 The binomial distribution and sampling distribution of the mean
Probability distributions show how probabilities are distributed over the values of a random variable.	Mathematics Year 12 Advanced Random variables
The area under a probability density function represents the probability of a continuous random variable falling within a particular range.	Mathematics Year 12 Advanced Random variables
Cumulative frequency histograms and polygons are related to frequency histograms and cumulative distribution functions.	Mathematics Stage 5 Data analysis A Mathematics Year 12 Advanced Random variables
Probabilities are used to forecast, predict results and evaluate risk in applications such as the weather, medicine, the stock market and insurance. Probabilities are used when analysing risk for insurance calculations. For example, insurance premiums that are designed for young adult males who have a higher probability of accidents and insurance claims, and insurance premiums affected by the likelihood of natural disasters such as flooding and bushfires. Probabilities are applied in politics to navigate uncertainty, inform decision-making, strategise campaigns and assess risks.	Real-world applications
The law of total probability is used in situations where it is easier to calculate conditional probabilities rather than find the probability of the event directly, such as finding the	Real-world applications

probability of a person having a disease using a medical test.	

Considerations

- When calculating probabilities for multistage events students should be able to identify situations and problems involving both replacement and non-replacement.
- Students should focus on limiting the sample space as the entry point for conditional probability.

Diversity of learners

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Teaching advice for Further graph transformations and modelling

Content

§ Further graph transformations and modelling

Teaching advice

Purpose

The study of functions and their transformations is important in developing students' ability to find connections and patterns and use algebraic techniques and manipulations. Functions and their transformations can be used to communicate concisely and precisely, to analyse, model and solve problems, and to predict future outcomes in areas such as finance, economics, data analysis, marketing and weather. Using the periodicity of trigonometric functions, mathematical models have been developed that describe the behaviour of many naturally occurring periodic phenomena, such as vibrations or waves, as well as oscillatory behaviour found in pendulums, electric currents and radio signals.

Key terms

- amplitude
- dilation
- domain
- equation
- function
- logarithmic scale
- model
- period
- range
- dilation factor
- transformation
- translation

Related Stage 5 Path content

- Functions and other graphs
- Trigonometry C
- Trigonometry D

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Trigonometric identities and equations can be used in analysing the transformations of trigonometric functions.	Mathematics Advanced Year 11 Trigonometric identities and equations
The principles of transformations are used in working with trigonometric functions and solving graphing problems.	Mathematics Advanced Year 11 Graph transformations
Logarithmic scales are based on the concept of the logarithm, and the process of solving problems involving logarithmic scales involves using exponentials.	Mathematics Advanced Year 11 Exponential and logarithmic functions
Transformations of trigonometric functions are used in examining and applying the representations of $a\cos x+b\sin x$ as $R\cos(x\pm\alpha)$ or $R\sin(x\pm\alpha)$, interpretating trigonometric graphs and modelling and solving related practical problems.	Mathematics Extension 1 Year 11 Further trigonometry
Transformations of trigonometric functions are used to graph and analyse simple harmonic motion of the form $x=A\cos(nt+\alpha)+c$ or $x=A\sin(nt+\alpha)+c$.	Mathematics Extension 2 Year 12 Applications of calculus to mechanics

Considerations

- Both radian measure and degrees should be used for the transformations of trigonometric functions.
- The initial stage of transformations of trigonometric functions should be considered one transformation at a time, for example:
  - sketch the functions $f_1(x)=\sin x$ and $f_2(x)=\sin 3x$ on the same axes
  - sketch the functions $g(x)=\cos x$ and $h(x)=2+\cos x$ on the same axes.
- The order of transformations should be considered when multiple transformations are applied to functions. This can be investigated using graphing applications and explained by the principles of transformations.
- It is important to recognise $\cos x=\sin\left(x+\frac{\pi}{2}\right)$. That is, the graph of $f(x)=\cos x$ can be obtained by shifting the graph of $f(x)=\sin x$ to the left $\frac{\pi}{2}$ units.
- For graph transformations of trigonometric functions in the form $y=kf(a(x+b))+c$, when $f(x)=\sin x$ or $f(x)=\cos x$ the period is $\frac{2\pi}{a}$ and when $f(x)=\tan x$ the period is $\frac{\pi}{a}$.

- The terms sinusoidal, midline and frequency can be introduced to students who may benefit from extending their knowledge in graphs of trigonometric functions and their transformations, but they are not essential content.
- Practical problems such as vibrations, tidal changes, light waves, sound waves, electric currents, radio signals and the oscillation of a simple pendulum could be explored as applications of trigonometric functions.
- Given a graph showing changes in a real-world phenomenon and its equation, students could describe the oscillation in words.
- Having a side-by-side comparison between a linear scale and a logarithmic scale as an introduction may help students to recognise the non-linear feature of a logarithmic scale and understand its usefulness.

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Teaching advice for Sequences and series

Content

§ Sequences and series

Teaching advice

Purpose

Sequences are used to model mathematical patterns. In arithmetic sequences the pattern is established by addition and in geometric sequences the pattern is established by multiplication. Series are formed when the terms in a sequence are added together. Sequences and series are useful in the study of limits, convergence and divergence and have applications in fields such as number theory, finance, physics, engineering and statistics.

Key terms

- arithmetic progression (AP)
- arithmetic sequence
- common difference
- common ratio
- exponential growth and decay
- exponential function
- finite
- geometric progression (GP)
- geometric sequence
- geometric series
- infinite
- integer
- linear function
- partial sum
- sequence
- series
- summation notation
- whole number

Related Stage 5 Path content

- Algebraic techniques B
- Algebraic techniques C
- Indices B
- Equations B
- Equations C
- Non-linear relationships C

Making connections

Connections	Course/Stage/Focus area/Post-school paths
The nth term of an arithmetic sequence can be modelled as a linear function of n. Linear growth or decay problems can be modelled as an arithmetic sequence.	Mathematics Advanced Year 11 Working with functions
The nth term of a geometric sequence can be modelled as an exponential function of n. Exponential growth and decay situations can be modelled as geometric sequences.	Mathematics Advanced Year 11 Exponential and logarithmic functions
Solving sequences and series problems may involve the use of inequalities and logarithms.	Mathematics Advanced Year 11 ▪ Working with functions ▪ Exponential and logarithmic functions
Summation notation can be used to express results for the expected value and variance of discrete random variables, for the binomial expansion of $(a+b)^n$, for the sample mean of a sample taken from a population and results to be proved by mathematical induction.	Mathematics Extension 1 Year 11 The binomial theorem Mathematics Advanced Year 12 Random variables Mathematics Extension 1 Year 12 ▪ The binomial distribution and sampling distribution of the mean ▪ Proof by mathematical induction
A recursive formula, or recurrence relation, is a formula that defines each term of a sequence using a preceding term. Its results can be proved by mathematical induction.	Mathematics Extension 2 Year 12 The nature of proof
The Fibonacci sequence is a sequence where each term is the sum of the two preceding terms and has many applications including computer algorithms and biology.	Real-world applications
The representation of numbers in binary form for computer processing is connected to the geometric sequence 1, 2, 4, 8, …	Real-world applications
The large increase in the sum of a geometric series with $\|r\| \geq 1$ demonstrates why pyramid schemes for making money are not sustainable and why such schemes are illegal in many countries.	Real-world applications
Sequences and series can be used when	

The functions $\cos x$, $\sin x$ and e^x can be expressed using a Maclaurin series. $\ln x$ can be expressed using a Taylor series.	Tertiary mathematics

Considerations

- Sequences that are neither arithmetic nor geometric could be considered to help students develop an understanding of sequences. For example, what is the nth term of the sequence 1, 4, 9, 16, …?
- The formula $S_n = \frac{n}{2}(a + a_n)$ could be developed from examples. The formula $S_n = \frac{n}{2}[2a + (n-1)d]$ can then be developed from $S_n = \frac{n}{2}(a + a_n)$.
- The formula $S_n = \frac{a(1-r^n)}{1-r}$ can be developed from the identity $(x-1)(x^{n-1} + x^{n-2} + \ldots + x^2 + x + 1) = x^n - 1$ for whole numbers $n \geq 1$. Students can then find a formula for S_n when $r = 1$.
- The formula $S_n = \frac{a(r^n-1)}{r-1}$ is equivalent to $S_n = \frac{a(1-r^n)}{1-r}$ which is often used when $r > 1$.
- To assist students to understand the concept of the limiting sum of a geometric series, students could use a spreadsheet or a graphing application to plot and list the sequence of partial sums of a geometric series and explore the values of r^n or $\frac{a(1-r^n)}{1-r}$ as $n \to \infty$. An example to start with could be the series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$
- Summation notation can be used to represent sums of terms of APs, for example $\sum_{n=1}^{5}(2+3n)$, and GPs, for example $\sum_{n=1}^{4} 3 \times 2^n$ and $\sum_{n=3}^{6} 2^{6-n} \times 10^n$.

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Teaching advice for Differential calculus

Content

§ Differential calculus

Teaching advice

Purpose

Calculus involves the study of how things change and provides a framework for developing quantitative models of change and deducing their consequences. It explores the development of analytic and numeric integration techniques and the use of these techniques in solving problems.

The study of calculus is important in developing students' knowledge, understanding and capacity to operate with and model situations involving change. It also supports students to use algebraic and graphical techniques to describe and solve problems, and to predict future outcomes with relevance to science, engineering, finance, economics and the construction industry.

Key terms

- chain rule
- constant
- derivative
- differentiation
- exponential function
- identity
- logarithmic function
- natural logarithm
- normal
- product rule
- quotient rule
- tangent

Related Stage 5 Path content

- Linear relationships C
- Logarithms
- Functions and other graphs

Making connections

Connections	Course/Stage/Focus area/Post-school paths
The logarithm laws are used to simplify the process of finding the derivatives of logarithmic functions.	Mathematics Stage 5 Logarithms Mathematics Advanced Year 11 Exponential and logarithmic functions
Exponential functions and logarithmic functions are used in differentiation.	Mathematics Advanced Year 11 Exponential and logarithmic functions
The value of Euler's number, e, can be derived using the squeeze theorem.	Mathematics Extension 2 Year 12 The nature of proof
Proving the result $\frac{d}{dx}(a^x)$ requires the identity $a=e^{\ln a}$.	Mathematics Advanced Year 11 Exponential and logarithmic functions
The identity $e^{\ln x}=x$, which is used in proving the formula $\frac{d}{dx}(\ln x)=\frac{1}{x}$, can be established by considering $y=\ln x$ and $y=e^x$ as inverse functions.	Mathematics Extension 1 Year 11 Further work with functions
The proof of $\frac{d}{dx}(log_a x)=\frac{1}{x\ln a}$ uses the change of base rule.	Mathematics Advanced Year 11 Exponential and logarithmic functions
Trigonometric functions and identities are used in differentiation with trigonometric functions.	Mathematics Advanced Year 11 ▪ Trigonometry and measure of angles ▪ Trigonometric identities and equations Mathematics Advanced Year 12 Further graph transformations and modelling
The chain rule is used in proving many rules of differentiation.	Mathematics Advanced Year 11 Introduction to differentiation
Equations of tangents and normals can be used in solving problems involving differential calculus.	Mathematics Advanced Year 11 Introduction to differentiation
The pattern of results for higher derivatives of $y=e^x$, $y=e^{kx}$, $y=\sin x$ and $y=\cos x$ can be determined using rules for derivatives of	Mathematics Advanced Year 12 Applications of calculus

exponential and trigonometric functions.	
Differentiation is used in solving problems relating to the motion of a particle moving in a straight line and rates of change. Differentiation of exponential functions is used in problems involving exponential growth and decay.	Mathematics Advanced Year 12 Applications of calculus Mathematics Extension 1 Further applications of calculus
Differentiation is used in analysing two-dimensional motion using vectors.	Mathematics Extension 1 Year 12 Introduction to vectors
The derivatives of $cosec\, x$, $sec\, x$ and $\cot x$ can be useful in finding integrals.	Mathematics Extension 2 Year 12 Further integration
Differentiation of trigonometric functions is used in solving simple harmonic motion problems.	Mathematics Extension 2 Year 12 Applications of calculus to mechanics
Differential calculus is used to calculate marginal cost and to predict maximum profit.	Real-world applications

Considerations

- While differentiating $\sin x$, $\cos x$ and $\tan x$ is an essential part of the course, the derivatives of $cosec\, x$, $sec\, x$ and $\cot x$ do not need to be memorised.
- Students should recognise that use of radians is fundamental to the calculus of trigonometric functions.
- Once the key rules of differentiating exponential, logarithmic and trigonometric functions are established, students are encouraged to differentiate functions involving combinations of these functions to encourage fluency in applying the rules of differentiation in different contexts. For example, $\frac{d}{dx}\left(\frac{e^{\sin x}}{\ln x}\right)$ requires students to apply the quotient rule and then the chain rule.
- Some students may benefit from being extended by exploring the following, however this is not essential content:
  - determining $\frac{d}{dx}\left(a^x\right)$ using differentiation by first principles, i.e. $\frac{d}{dx}\left(a^x\right)=\lim_{h\to 0}\frac{a^{x+h}-a^x}{h}=\left(\lim_{h\to 0}\frac{a^h-1}{h}\right)\times a^x$, then using technology to explore this limit and determining that it is a constant
  - considering the values of $\frac{\sin x}{x}$, $\frac{\tan x}{x}$ and $\cos x$ as $x\to 0$
  - using areas of sectors and triangles in a circle to prove geometrically that $\lim_{x\to 0}\frac{\sin x}{x}=1$, and hence proving $\frac{d}{dx}\left(\sin x\right)=\cos x$ and $\frac{d}{dx}\left(\cos x\right)=-\sin x$ using the result $\sin\left(A+B\right)=\sin A\cos B+\cos A\sin B$.

- using graphing applications to demonstrate that $\frac{d}{dx}(\sin x) = \cos x$ using differentiation by first principles, by plotting the graph of $y = \frac{\sin(x+a) - \sin a}{a}$ and analysing the behaviour of the graph as $a \to 0$.
- considering Euler's number, e, as the sum of the infinite series $1 + \frac{1}{1} + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \ldots$

Diversity of learners

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Teaching advice for Integral calculus

Content

§ Integral calculus

Teaching advice

Purpose

Calculus involves the study of how things change and provides a framework for developing quantitative models of change and deducing their consequences. It explores the development of analytic and numeric integration techniques and the use of these techniques in solving problems.

The study of calculus is important in developing students' knowledge, understanding and capacity to operate with and model situations involving change. It also supports students to use algebraic and graphical techniques to describe and solve problems, and to predict future outcomes with relevance to science, engineering, finance, economics and the construction industry.

Key terms

- constant
- constant function
- continuous function
- definite integral
- derivative
- differentiation
- even function
- exponential function
- function
- Fundamental Theorem of Calculus
- identity
- indefinite integral
- integral
- integration
- logarithmic function
- odd function
- primitive
- rate of change
- real number
- reflection
- Trapezoidal rule

Related Stage 5 Path content

- Algebraic techniques C
- Functions and other graphs

Sequencing advice

The content in the focus area Integral calculus builds on content in:

- Mathematics Advanced Year 12 Differential calculus.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
The area under a curve is related to the area of composite shapes.	Mathematics Stage 5 Area and surface area A
When a curve lies below the x-axis, the area between the curve and the x-axis can be found using the absolute value.	Mathematics Advanced Year 11 Working with functions
The reflection property of the graphs of $y = e^x$ and $y = \ln x$ can be used in solving integration problems related to exponential or logarithmic functions.	Mathematics Advanced Year 11 Exponential and logarithmic functions
The symmetry properties of odd and even functions can be used in simplifying and solving integration problems.	Mathematics Advanced Year 11 Working with functions
Finding the distance travelled over a given time interval from a velocity–time graph is related to the definite integral.	Mathematics Advanced Year 11 Introduction to differentiation
The area under the curve $y = \frac{1}{x}$ between $x = 1$ and $x = k$ is the natural logarithm of a number $k > 1$ in which case e is the value of k for which this area equals 1.	Mathematics Advanced Year 11 Exponential and logarithmic functions
Displacement or distance travelled can be found by integrating a velocity function with respect to time. Velocity can be found by integrating an acceleration function with respect to time.	Mathematics Advanced Year 12 Applications of calculus
Finding the probability of a continuous random variable involves finding the area under a curve.	Mathematics Advanced Year 12 Random variables
The Trapezoidal rule can be used to approximate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$ for continuous random variables.	Mathematics Advanced Year 12 Random variables
The volume of a solid of revolution can be found using integration and the Trapezoidal rule can be used to approximate volumes of revolution.	Mathematics Extension 1 Year 12 Further applications of calculus

Differential equations can be solved using integration.	
Integration is used in analysing two-dimensional motion using vectors.	Mathematics Extension 1 Year 12 Introduction to vectors
Integration is used in solving simple harmonic motion problems.	Mathematics Extension 2 Year 12 Applications of calculus to mechanics
To calculate the Gini coefficient, which is a measure of the distribution of income across a population, the area under the Lorenz curve needs to be calculated using integration or by approximation.	Real-world applications
Integration is used to find the average value of a function, the length of a curve or the surface area of a solid. Simpson's rule uses a quadratic equation through three given points to estimate the area under a curve.	Tertiary mathematics
Integration is used in engineering to determine the geometry of a building. Integration is used in physics to calculate work and the centre of mass.	Real-world applications

Considerations

- Students should note that for $\int_a^b f(x)\,dx$ the function $f(x)$ must be continuous on the closed interval $a \le x \le b$, and that it is not possible to integrate across a discontinuity, including discontinuities at the endpoints of the domain.
- It is appropriate for students to derive general results for integration by reversing the process of differentiation. Alternative derivations are not required in this course.
- The method of integration by substitution is not required in this course. Students will be expected to establish results such as $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b)+c$ by differentiating a suitable function. Students should practise the process of integrating $\frac{f'(x)}{f(x)}$ by inspection, without a formal change of variable. Further techniques of integration are covered in the Mathematics Extension courses.
- To derive the formula $\int \frac{1}{x}\,dx = ln|x|+C$ where $x \neq 0$ students could:

  Prove by differentiation that for $x>0$, $\int \frac{1}{x}\,dx = \ln x + C$, for some constant C, and explain why this formula does not work for $x<0$.

Prove by differentiation that for $x<0$, $\int \frac{1}{x} dx = \ln(-x) + C$, for some constant C, and explain why this formula does not work for $x>0$.

Combine the results above to yield the result $\int \frac{1}{x} dx = \ln|x| + C$, for some constant C, where $x \neq 0$, and recognise that no definite integral of $\frac{1}{x}$ can be formed on any closed interval that includes the discontinuity at $x=0$.

- If the curve $y=f(x)$ is approximated on the interval $a \leq x \leq b$ by the straight line passing through $(a, f(a))$ and $(b, f(b))$, the area under the curve may be estimated by the area under the line as shown in Figure 1. This approximates $\int_a^b f(x)\,dx$ by the area of a trapezium, which may be calculated using $\frac{b-a}{2}(f(a)+f(b))$. By dividing the base into two equal sub-intervals (see Figure 2), and using a linear approximation of $f(x)$ on each sub-interval, the area is approximated by two trapeziums. The method extends to dividing the interval into n equal sub-intervals of width $h=\frac{b-a}{n}$, and using a 'trapezoidal approximation' on each sub-interval.

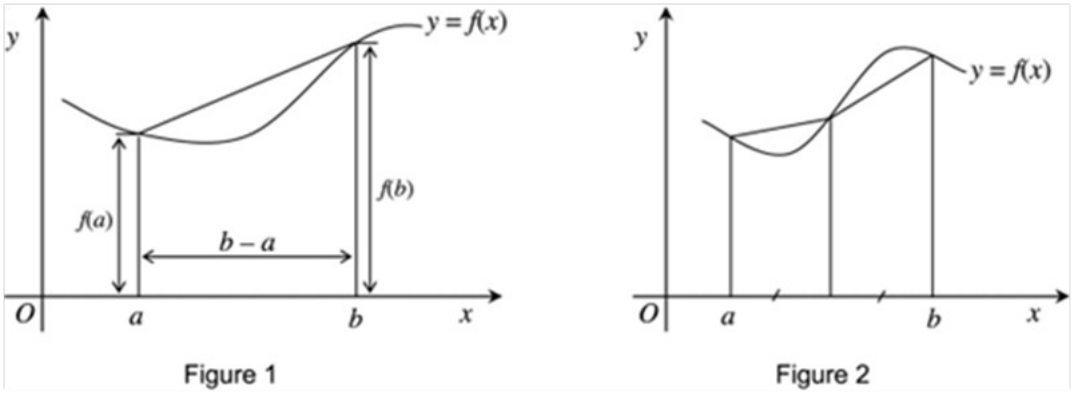

Image long description: Two graphs are shown. Figure 1 on the left, shows a curve $y=f(x)$ and marks a and b on the x axis and marks the distance between them as $b-a$. Two vertical lines are marked: $f(a)$ at a, and $f(b)$at b. In Figure 2 on the right, the same diagram is drawn as before, but there is also a vertical line in the middle, which splits the trapezium into two trapezia.

- Students may choose to use a general form of the Trapezoidal rule when there are n sub-intervals such as: $\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(a)+f(b)+2\left(f(x_1)+\ldots+f(x_{n-1})\right)\right]$ where $a=x_0$ and $b=x_n$, and the values of $x_0, x_1, x_2, \ldots, x_n$ are found by dividing the interval $a \leq x \leq b$ into n equal sub-intervals.
- When applying the Trapezoidal rule, students are not restricted to applying a formula when setting out their solution. Using tables or simply considering the areas of different trapezia with common width are appropriate methods for approximating definite integrals using the Trapezoidal rule.

- Some students may benefit from being extended by modelling the function $F(x)=\int_a^x f(t)\,dt$ using graphing applications, and examining the rate of change of the area as x varies, leading to the result $F'(x)=\frac{d}{dx}\left[\int_a^x f(t)\,dt\right)=f(x)$ but this is not essential content.
- The approximation of a region's area by adding up multiple simple shapes is called a Riemann sum.
- Students could explore different methods of approximating the area under a curve, including using different shapes.
- Some students may benefit from using graphing applications to explore areas of unbounded regions that have limiting areas in preparation for working with probability density functions, for example: finding that $\int_0^a e^{-x}\,dx \to 1$ as $a \to \infty$, but that $\int_1^a \frac{1}{x}\,dx \to \infty$ as $a \to \infty$ shows that diagrams sometimes do not pick up huge differences in behaviour. This is not essential content
- Students may use notations such as $\int_{-\infty}^{0}$ and $\int_{0}^{\infty}$ when representing unbounded regions but this is not essential content.

Diversity of learners

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Teaching advice for Applications of calculus

Content

§ Applications of calculus

Teaching advice

Purpose

Calculus involves the study of how things change and provides a framework for developing quantitative models of change and deducing their consequences. It explores the development of analytic and numeric integration techniques and the use of these techniques in solving problems.

The study of calculus is important in developing students' knowledge, understanding and capacity to operate with and model situations involving change. It also supports students to use algebraic and graphical techniques to describe and solve problems and to predict future outcomes with relevance to science, engineering, finance, economics and the construction industry.

Key terms

- acceleration
- asymptote
- concave down
- concave up
- concavity
- continuous
- derivative
- differentiable
- differentiation
- discontinuity
- displacement
- domain
- even function
- exponential growth and decay
- function
- global maximum
- global minimum
- gradient
- integration
- local maximum
- local minimum
- odd function
- point of inflection
- rate of change
- stationary point
- tangent
- velocity
- zero (of a function)

Related Stage 5 Path content

- Algebraic techniques B
- Area and surface area B

- Volume B
- Variation and rates of change A
- Variation and rates of change B
- Non-linear relationships C
- Logarithms
- Properties of geometrical figures C

Sequencing advice

The content in the focus area Applications of calculus builds on content in:

- Mathematics Advanced Year 12 Differential calculus
- Mathematics Advanced Year 12 Integral calculus.

Making connections

Connections	**Course/Stage/Focus area/Post-school paths**
Displacement, velocity and acceleration may be expressed in terms of non-linear functions when a particle is moving in a straight line.	Mathematics Stage 5 ▪ Non-linear relationships A ▪ Non-linear relationships B ▪ Non-linear relationships C Mathematics Advanced Year 11 Working with functions
Graphical features such as global maximum and points of inflection are used to describe a normal curve.	Mathematics Advanced Year 12 Random variables
Graphical features such as turning points and points of inflection are used to describe the graph of a polynomial function.	Mathematics Stage 5 Polynomials Mathematics Extension 1 Year 11 Polynomials Mathematics Extension 1 Year 12 Further applications of calculus
Integration of rates is related to initial value problems in differential equations.	Mathematics Extension 1 Year 12 Further applications of calculus
Finding the maximum height reached by a projectile involves using differential calculus.	Mathematics Extension 1 Year 12 Introduction to vectors
The motion of a particle can be represented using vectors.	Mathematics Extension 1 Year 12 Introduction to vectors
The motion of a particle can be modelled and analysed using calculus.	Mathematics Extension 2 Year 12 Applications of calculus to mechanics
The time for peak concentration of a drug in medicine can be found using the first derivative of the surge	Real-world applications

function. Calculus is important for understanding activation functions, like the Sigmoid function, used in machine learning. Calculus is used in engineering to determine rates of change, areas and the lengths of power cables. Optimisation is used in finance and business.	

Considerations

- Some students may benefit from a review of the geometrical significance of the sign $f'(x)$.
- A useful way to represent and set out the first-derivative test is using a table showing the x-value of the point to be tested, and a value close to x on either side of x, and then calculating the value of the derivative at each x-value.
- The geometrical significance of the sign of the second derivative of a function can be explored using graphing applications and observing the rate of change of the gradient of the tangent to the function within an interval.
- It may be useful to use the language ‘necessary and sufficient conditions’. For example, ‘What are the necessary and sufficient conditions for a turning point to exist at a given point for a differentiable function?’
- Students should examine functions that have a point where the second derivative is 0, the sign of the second derivative does not change and there is no point of inflection. For example, examine the behaviour of $y = 4x - x^4$ at $x = 0$.
- To avoid ambiguity, when the largest magnitude of displacement, velocity or acceleration is required, the wording ‘greatest magnitude’ should be used.
- In exponential decay, the time taken for the initial quantity to halve is called the ‘half-life’.
- When the population has reached twice its size, $2A = Ae^{kt}$, leading to $t = \frac{\ln 2}{k}$. This is called the ‘doubling-time’. The population will double again, to $4A$, when another doubling-time has passed, and so on. This property can be used to sketch the function.
- To support student understanding, the limitations of the exponential growth model in real-world situations could be explored.
- While the concept of exponential growth and decay is often applied to populations, it could equally be applied to the depletion of natural resources, industrial production or inflation.
- When solving optimisation problems it may be necessary to assume a continuous function can be used to model discrete variables. Adjustments to the solution may be necessary. For example: A tourism company constructs tours for groups of 20 to 50 people. For groups of 20, the price is $380 per person. The price per person is reduced by $12 for each additional person over 20. What size group will produce the largest revenue for the company?
- Graphs of Lorenz curves can be explored and the Gini coefficient calculated. A Lorenz curve can be used to illustrate the distribution of resources in a community or country. If the graph of $f(x)$ is a Lorenz curve, the value of $f(x)$ is the fraction of the total wealth of a population that is held by the poorest $x\%$ of the population. If $f(x) = x$, then the poorest 10% hold 10% of the wealth, the poorest 50% hold 50% of the wealth, and so on. This can be called the *line of complete equality*. For most societies, the Lorenz curve sits below the line of complete equality and shows that the poorest people hold a disproportionately small amount of the wealth. The

Gini coefficient is a value between 0 (complete equity) and 1 ('perfect' inequity, where one person owns everything) and is an indication of the distribution of wealth within a population.

Diversity of learners

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Teaching advice for Random variables

Content

§ Random variables

Teaching advice

Purpose

Statistical analysis involves the exploration, display, analysis and interpretation of random variables and their distributions to identify and communicate key information. The study of statistical analysis is important in developing students' ability to recognise, describe and apply statistical techniques to analyse current situations or to predict future outcomes. It also develops an awareness of how conclusions drawn from data can be used to inform decisions made by groups, such as scientific investigators, businesspeople and policymakers. Concepts related to normally distributed data are important in statistical analysis and making informed decisions based on the analysis.

Key terms

- continuous random variable
- cumulative distribution function (CDF)
- discrete probability distribution
- discrete random variable
- domain
- empirical rule for the normal distribution
- expected value
- function
- global maximum
- mean
- measure of centre
- measure of spread
- median
- mode
- normal distribution
- normal random variable
- point of inflection
- probability density function (PDF)
- probability distribution
- quartile
- experiment (random)
- random variable
- relative frequency
- standard deviation
- summation notation
- symmetrical
- uniform distribution
- variance
- z-score

Related Stage 5 Path content

- Data analysis C

Sequencing advice

The content in the focus area Random variables builds on content in:

- Mathematics Advanced Year 12 Differential calculus
- Mathematics Advanced Year 12 Integral calculus
- Mathematics Advanced Year 12 Applications of calculus.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
z-scores are calculated using the mean and standard deviation.	Mathematics Stage 4 Data analysis Mathematics Stage 5 Data analysis A
Relative frequencies can be used for estimates of probabilities.	Mathematics Stage 4 Probability Mathematics Advanced Year 11 Probability and data
Histograms can be used to represent discrete probability distributions.	Mathematics Stage 5 Data analysis A Mathematics Advanced Year 11 Probability and data
Cumulative frequency histograms and polygons are related to the graphical representation of the cumulative distribution function. Problems involving discrete and continuous probability distributions may involve conditional probability.	Mathematics Advanced Year 11 Probability and data
Probability density functions and cumulative distribution functions are usually defined piecewise.	Mathematics Advanced Year 11 Working with functions
Summation notation can be used to express the rules for expected value and variance of discrete random variables.	Mathematics Advanced Year 12 Sequences and series
The probability density function is the derivative of the cumulative distribution function.	Mathematics Advanced Year 11 Introduction to differentiation Mathematics Advanced Year 12 Differential calculus
The area under a probability density function for a continuous random variable is represented by an integral and the total area bounded by the curve and the horizontal axis is 1. The cumulative distribution function is an integral which represents a probability as an	Mathematics Advanced Year 12 Integral calculus

area. The expected value and the variance of continuous random variables can be expressed as integrals.	
The principles of transformations can be observed algebraically and by using graphing applications when given the probability density function of the standard normal distribution $f(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$ with mean 0 and standard deviation 1, and transforming it to the probability density function for the non-standard normal distribution $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ with mean μ and standard deviation σ. The inbuilt normal distribution function within various graphing applications can be used to explore graph transformations.	Mathematics Advanced Year 11 Graph transformations Mathematics Advanced Year 12 Further graph transformations and modelling
Probabilities associated with the normal distribution with mean μ and standard deviation σ can be represented using integrals, which may extend to $\pm\infty$, when given the probability density function $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. The results concerning areas under the normal curve can be verified using the Trapezoidal rule. The inbuilt normal distribution function within various graphing applications can be used to explore probabilities associated with areas of regions under the normal curve.	Mathematics Advanced Year 12 Integral calculus
The graphical features of the normal curve, including its points of inflection and global maximum, can be found using calculus when given the probability density function for the normal distribution $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ with mean μ and standard deviation σ.	Mathematics Advanced Year 12 Applications of calculus

Calculations involving the expected value and variance of a continuous random variable often require further integration techniques such as integration by parts.	Mathematics Extension 2 Further integration
Discrete random variables are used in: ▪ statistics, probability theory, queuing theory and decision analysis in operations research ▪ game theory and market behaviour in economics ▪ modelling stock movements and interest rates in finance ▪ algorithm analysis and cryptography in computer science ▪ quality control in engineering ▪ signal processing and error analysis in telecommunications ▪ wildlife populations in environmental science ▪ modelling disease spread, polling and behavioural studies, to predict insurance claims and determine premium rates by actuaries ▪ to analyse student performance in education.	Real-world applications
The uniform distribution is not particularly useful in describing much of the randomness we see in the natural world but is very useful in random number generation. A continuous random variable X with a uniform distribution can be denoted $U[a,b)$. Approximate values of the $U(0,1)$ distribution can be simulated on most computers using a random number generator. The generated numbers can then be used to randomly assign people to treatments in experimental studies, or to randomly select individuals for participation in a survey.	Real-world applications

The normal distribution often describes natural phenomena, such as the distribution of heights, weights and IQ scores in a population. Scientists use the normal distribution and z-scores to assess the significance of experimental results and make conclusions about hypotheses. The normal distribution is used in psychology, particularly in psychological testing and assessments. The normal distribution is applied in healthcare and medicine. Engineers use the normal distribution in quality control and manufacturing processes.	Real-world applications
The normal distribution and z-scores are used in advanced data analysis, including hypothesis testing, confidence intervals and statistical inference.	Real-world applications
Online computational applications are used in tertiary mathematics and statistics for statistical and probability calculations.	Tertiary mathematics and statistics

Considerations

- The probability distribution of a discrete random variable X lists the values X can take and their probabilities where the probabilities p_k must satisfy two requirements: Every probability p_k is a number between 0 and 1 and $p_1 + p_2 + \ldots + p_k = 1$. The distribution can be displayed in a table such as:

Value of X	X	X	X	...	X
Probability	p	p	p	…	p

- Students should be able to give examples of discrete probability distributions, including the discrete variables used to generate datasets, calculate discrete probability distributions, and graph them.
- The mean or expected value of a discrete random variable, X, can be described as its weighted average. Each value of X is weighted by its probability. To find the mean of X, multiply each value of X by its probability, then add all the products.
- Students could discuss some practical examples of discrete random variables where there is no reasonable way to calculate the probabilities. Data could be obtained for one of these examples, and the relative frequencies, data mean and data variance calculated. This process could include a discussion of the reasonableness of taking these measures as measures of the probabilities, mean and standard deviation of the random variable.

- The cumulative distribution function $F(x)$ for a given probability density function $f(x)$ can be described as 'an accumulation of all the probabilities less than or equal to x.'
- Students could use the probability density function $f(x)=\frac{1}{b-a}$ for a continuous uniform distribution for a random variable X taking values in the interval $[a,b]$ to establish the cumulative distribution function of a uniform random variable X: $F(x)=\frac{x-a}{b-a}$. The most commonly used uniform distribution is where $a=0$ and $b=1$.
- Students could establish that the mean of a continuous uniform random variable defined in the interval $a<x<b$ is $\mu=E(X)=\frac{a+b}{2}$, and that the variance of a continuous uniform random variable defined in the interval $a<x<b$ is $\sigma^2=Var(X)=\frac{(b-a)^2}{12}$.
- Students should practise graphing the probability density function and cumulative frequency functions for a variety of continuous random variables.
- Some scientific calculators and online calculators can calculate probabilities associated with z-scores.
- Some graphing applications have inbuilt statistical and distribution functions such as the normal distribution, PDF and CDF.
- Some students may benefit from being extended to recognise the convention that the symbols Z, z and $\phi(z)$ are used for the standard normal random variable, its values, and its density function respectively and that the probability density function for the standard normal random variable is $\phi(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}$, but this is not essential content.
- Initially, students should explore z-scores from a pictorial perspective, investigating only whole number multiples of the standard deviation.
- Students could explore how standardising different distributions allows comparisons to be made between distributions using z-scores, as raw scores by themselves do not necessarily provide information about the position in a distribution.
- The advantages of using standardised scores can be explored by comparing two or more sets of scores before and after conversion to z-scores.
- Graphical representations of datasets before and after standardisation should be explored to support student understanding.

Diversity of learners

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Teaching advice for Financial mathematics

Content

§ Financial mathematics

Teaching advice

Purpose

Knowledge of financial mathematics enables analysis and interpretation of different financial situations, the calculation of the best options for the circumstances, and the solving of financial problems. Financial mathematics involves sequences and series and their application to financial situations.

The study of financial mathematics is important in developing students' ability to make informed financial decisions, to be aware of the consequences of such decisions, and to manage personal financial resources prudently.

Key terms

- annuity
- compound interest
- future value
- geometric sequence
- investment
- loan
- reducing balance loan
- repayment
- withdrawal

Related Stage 5 Path content

- Financial mathematics A
- Financial mathematics B
- Indices A
- Logarithms

Sequencing advice

The content in the focus area Financial mathematics builds on content in:

- Mathematics Advanced Year 12 Sequences and series.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Reducing balance loans and annuities involve compound interest.	Mathematics Stage 5 Financial mathematics B
Financial problems involving annuities and loans can be solved using exponentials and logarithms.	Mathematics Stage 5 Logarithms Mathematics Advanced Year 11 Exponential and logarithmic functions
Annuities can be modelled using the terms and a partial sum of a geometric sequence.	Mathematics Advanced Year 12 Sequences and series
Financial mathematics is used to make financial decisions about, for example, home loans, savings accounts and car loans. Annuities are used when saving for a goal, retirement planning, and for plans such as sinking funds operated by clubs and societies.	Real-world applications

Considerations

- Some students may need to review the concept of compound interest in preparation for applying it to reducing balance loans and annuities.
- Students could compare current interest rates for loans from various lending institutions and use online loan calculators to explore the duration of loans, borrowing capacity and repayment amounts for different loan amounts.
- Students could use a loan spreadsheet or an online calculator to explore the effects of varying the amount borrowed, the interest rate and the repayment amount, and to determine borrowing capacity. They could determine the answers to suitable ‘what if’ questions, for example, what is the effect on the term of the loan and the amount of interest paid if: there is an interest rate rise, or more than the minimum monthly repayment is paid, or there is an interest rate rise in the case where more than the minimum monthly repayment is already being paid?

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Mathematics Extension 1 11–12 (2024)

Teaching advice for Stage 6

Teaching advice for Further work with functions

Content

§ Further work with functions

Teaching advice

Purpose

Knowledge of functions enables students to discover connections between algebraic and graphical representations. Function notation allows students to communicate concisely and precisely, to use algebraic techniques and manipulations, and to determine solutions of equations. Functions are used to model theoretical or real-life situations involving algebra, and to predict future outcomes in areas such as finance, economics, data analysis, marketing and weather.

Key terms

- absolute value
- asymptote
- denominator
- domain
- function
- inequality
- inverse function
- linear
- one-to-one function
- parameter
- parametric equations
- period
- point of intersection
- quadratic function
- radian
- range
- reciprocal
- reflection
- symmetry

Related Stage 5 Path content

- Algebraic techniques B
- Algebraic techniques C
- Equations B
- Equations C
- Linear relationships C
- Non-linear relationships C
- Functions and other graphs
- Polynomials

Sequencing advice

The content in the focus area Further work with functions builds on content in:

- Mathematics Advanced Year 11 Working with functions
- Mathematics Advanced Year 11 Exponential and logarithmic functions
- Mathematics Advanced Year 11 Trigonometric identities and equations
- Mathematics Advanced Year 11 Graph transformations.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Solving inequalities by multiplying by the denominator squared relies on the fact that the square of real numbers is non-negative.	Mathematics Stage 4 Computation with integers
Sketching the sum and difference of two functions (addition of ordinates) relies on the skill of plotting points.	Mathematics Stage 4 Linear relationships
Graphing the reciprocal of a function, the absolute value of a function and the sum and difference of functions involves understanding graphs of functions, their shape and features including intercepts and asymptotes, function notation, reflections in an axis, domain and range, and the meaning of the absolute value of a number.	Mathematics Stage 5 ▪ Linear relationships C ▪ Non-linear relationships C ▪ Functions and other graphs Mathematics Advanced Year 11 ▪ Working with functions ▪ Graph transformations ▪ Exponential and logarithmic functions
Graphing functions and their inverses involves the concept of reflection in the line $y = x$ and finding the domain and range of functions and relations.	Mathematics Stage 5 ▪ Linear relationships C ▪ Functions and other graphs Mathematics Advanced Year 11 ▪ Working with functions ▪ Graph transformations ▪ Exponential and logarithmic functions
The condition for inverse functions relies on the understanding of composite functions.	Mathematics Advanced Year 11 Working with functions
The functions $y = a^x$ and $y = \log_a x$ are inverse functions.	Mathematics Advanced Year 11 Exponential and logarithmic functions
The inverse trigonometric functions are defined by restricting the domain of the trigonometric functions $\sin\theta$, $\cos\theta$ and $\tan\theta$.	Mathematics Extension 1 Year 12 Inverse trigonometric functions
Solving cubic inequalities is related to cubic functions, polynomials, quadratic equations and quadratic inequalities.	Mathematics Stage 5 ▪ Polynomials ▪ Equations B ▪ Equations C Mathematics Advanced Year 11 Working with functions Mathematics Extension 1 Year 11 Polynomials
Solving inequalities involving rational	Mathematics Stage 5 ▪ Algebraic Techniques B

Solving absolute value inequalities requires solving linear inequalities and an understanding of the absolute value of a number.	Mathematics Stage 5 Equations B Mathematics Advanced Year 11 Working with functions
Graphing the absolute value of a function is related to piecewise functions.	Mathematics Advanced Year 11 Working with functions
Functions and relations can be defined parametrically.	Mathematics Advanced Year 11 Working with functions
Eliminating the parameter from a pair of parametric equations to obtain a Cartesian equation may involve substitution into algebraic expressions, changing the subject of an equation or using trigonometric identities.	Mathematics Stage 5 Equations C Mathematics Advanced Year 11 Trigonometric identities and equations
The derivative of the inverse function is the reciprocal of the derivative of the function. The derivative of a function defined parametrically can be found using the chain rule.	Mathematics Extension 1 Year 12 Further calculus skills
Parameters are used in statistics and vector equations of lines.	Mathematics Extension 1 Year 12 The binomial distribution and sampling distribution of the mean Mathematics Extension 2 Further work with vectors
The changing positions of an object over time can be described using parametric equations where time is the parameter.	Mathematics Extension 1 Year 12 Introduction to vectors
Curves in 2D and 3D can be defined parametrically using vector equations.	Mathematics Extension 2 Further work with vectors
Parametric equations are used to define the horizontal and vertical motion of a projectile. The Cartesian equation of the motion of a projectile can be found using its parametric equations.	Mathematics Extension 1 Year 12 Introduction to vectors Mathematics Extension 2 Applications of calculus to mechanics
The condition for two functions to be inverses is related to the general concept of the existence of an inverse element for a set defined with an operation in abstract algebra.	Pure mathematics (tertiary)

The reciprocal function is a fundamental concept in mathematics and in fields such as physics, optics, finance, engineering and acoustics. Absolute value inequalities are used to represent the tolerance of a machine part in engineering.	Real-world applications

Considerations

- Students should consider graphs of functions within the scope of the Mathematics Advanced course when graphing the reciprocal of a function, the absolute value of a function and the sum and difference of functions, as well as graphs of functions where the equation of the function is not given. These concepts may need to be revisited depending on sequencing of content.
- Where possible, students should attempt to sketch the new graph on top of the original graph of $y=f(x)$ for accuracy and to show the relationship between the graphs.
- When given the graph of $y=f(x)$ and asked to sketch graphs such as $y=\frac{1}{f(x)}$ it is important to first recognise it is the values of $f(x)$ that are being manipulated (i.e. the y-values on the given graph). A discussion on the use of graphing applications could then lead to the discovery of 'critical' points on $y=f(x)$ (e.g. points where $f(x)=0$ or $f(x)=\pm 1$) and any changes to these points for each transformation. This should be followed by a careful examination of the remaining sections of $y=f(x)$ and how the transformation changes the nature of these sections, including changes to the increasing or decreasing behaviour of the graph, and any changes to the magnitude of the ordinates. This approach may support students to develop a more intuitive understanding of the process.
- Using the definition $|x|=\begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0 \end{cases}$, the absolute value in $y=|f(x)|$ and $y=f(|x|)$ could be expressed piecewise to aid the sketching of these graphs. Recognising and showing that $f(|x|)$ is an even function could also help with understanding.
- While $\sin^2 x$ is the conventional shorthand for $(\sin x)^2$, it must be made clear that $f^{-1}(x)$ is not the same as $[f(x)]^{-1}$ (i.e. $\frac{1}{f(x)}$).
- Graphing applications could be used to draw curves from parametric equations, which gives the benefit of visualising the changing positions of a point as the parameter takes on changing values.
- Functions and relations often have restrictions when defined parametrically. The restrictions are determined by considering the domain and range of each parametric equation.
- When solving $|ax+b|=k$ (where a, b and k are constants), the step $ax+b=\pm k$ naturally follows. When solving $|ax+b|>k$ however, one cannot simply write $ax+b>\pm k$. Developing student understanding of the meaning and definition of absolute values may help to avoid these erroneous steps.

Diversity of learners

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Teaching advice for Polynomials

Content

§ Polynomials

Teaching advice

Purpose

The study of polynomials leads to further investigation of the connections between algebra and the graphical representation of functions. Polynomials are of fundamental importance in algebra and have many applications in higher mathematics. They are also significant in many other fields of study including the sciences, engineering, finance and economics.

Key terms

- coefficient
- constant
- degree
- dividend
- divisor
- factor theorem
- leading coefficient
- leading term
- monic
- multiplicity of roots
- polynomial
- quadratic
- quotient
- remainder
- remainder theorem
- root
- tangent
- zero polynomial
- zero (of a function)

Related Stage 5 Path content

- Algebraic techniques B
- Algebraic techniques C
- Equations B
- Polynomials

Sequencing advice

The content in the focus area Polynomials builds on content in:

- Mathematics Advanced Year 11 Working with functions

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Division of polynomials uses the same algorithm as long division of real numbers.	Mathematics Stage 4 Computation with integers
Working with polynomials involves simplifying, expanding and factorising algebraic expressions, and solving equations.	Mathematics Stage 5 ▪ Algebraic techniques A ▪ Algebraic techniques B ▪ Algebraic techniques C ▪ Equations B ▪ Equations C
Graphing of polynomials employs techniques involved in graphing of functions.	Mathematics Advanced Year 11 Working with functions
The polynomial $P(x)=a_n x^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\cdots+a_1 x+a_0$, where $a_0, a_1, a_2, \ldots, a_n$ are real coefficients, is defined for all real x and is a continuous and differentiable function of x.	Mathematics Advanced Year 11 ▪ Working with functions ▪ Introduction to differentiation
The multiplicity of a zero of a polynomial can be determined using differentiation.	Mathematics Extension 1 Year 12 Further applications of calculus
Turning points and points of inflection of graphs of polynomials can be found using calculus.	Mathematics Advanced Year 12 Applications of calculus Mathematics Extension 1 Year 12 Further applications of calculus
Complex numbers can be used to solve polynomial equations. Polynomials are often coupled with de Moivre's theorem to prove trigonometric identities.	Mathematics Extension 2 Introduction to complex numbers
Factorisation and division of polynomials is important for partial fraction decompositions.	Mathematics Extension 2 Further integration
The algorithm used in polynomial division is related to the Euclidian algorithm and is often used in abstract algebra. Some results involving polynomials are used	Pure mathematics (tertiary)

in the proof of the Fundamental Theorem of Algebra and its corollaries.	
Polynomials are relatively well understood functions and so are sometimes used to approximate other functions in physics and engineering, such as the trigonometric functions, by truncating their power series expansion.	Pure and applied mathematics (tertiary) Real-world applications

Considerations

- Graphing applications can be used to explore and demonstrate the graphical features of various polynomials.
- Students should recognise and use the fact that when testing for integer zeroes of a polynomial whose coefficients are integers, only divisors of the constant term need be tested.
- Similar to writing numbers in the correct place value column when performing a numerical long division, it is advisable to align terms with the same power vertically if performing a polynomial long division.
- It should be noted that a polynomial of degree n has at most n real zeroes, allowing for multiplicities. This is one implication of the Fundamental Theorem of Algebra. Students could explore and prove other algebraic and geometric implications to support further development of their understanding. For example, a multiple root obtained when solving simultaneous polynomial equations corresponds to the curves being tangential at that point.
- For polynomials of degree 3 (and similarly for higher degrees) with zeroes α, β and γ, notation such as $\Sigma\alpha$ could be used when referring to the sum of zeroes of a polynomial taken one at a time ($\alpha+\beta+\gamma$), while $\Sigma\alpha\beta$ could be used when referring to the sum of zeroes taken two at a time ($\alpha\beta+\alpha\gamma+\beta\gamma$). Abbreviations such as ‘SOR’ for ‘sum of roots’ should be avoided.

Diversity of learners

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Teaching advice for Further Trigonometry

Content

§ Further trigonometry

Teaching advice

Purpose

The study of trigonometric functions involves using periodic functions in geometric, algebraic, numerical and graphical representations. Knowledge of trigonometric functions enables the solving of practical problems that involve triangles or periodic graphs, such as waves and signals.

The study of trigonometric functions is important in developing students' understanding of periodic behaviour. Using the periodicity of trigonometric functions, mathematical models have been developed that describe the behaviour of many naturally occurring periodic phenomena, such as vibrations or waves, as well as oscillatory behaviour found in pendulums, electric currents and radio signals.

Key terms

- domain
- double angle formulas
- function
- substitution

Related Stage 5 Path content

- Functions and other graphs
- Trigonometry C
- Trigonometry D

Sequencing advice

The content in the focus area Further trigonometry builds on content in:

- Mathematics Advanced Year 11 Working with functions
- Mathematics Advanced Year 11 Trigonometry and measure of angles
- Mathematics Advanced Year 11 Trigonometric identities and equations

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Solving 3-dimensional problems using trigonometry may involve bearings and angles of elevation or depression.	Mathematics Advanced Year 11 Trigonometry and measure of angles
Trigonometric identities are sometimes used in finding a Cartesian equation from a pair of parametric equations.	Mathematics Extension 1 Year 11 Further work with functions
The ability to solve trigonometric equations is useful for finding intercepts, points of intersection between two curves and other critical points when analysing a graph or a function.	Mathematics Extension 1 Year 12 Further applications of calculus
The sum and difference expansion results can be proven using basic trigonometry or vectors.	Mathematics Advanced Year 11 Trigonometry and measure of angles Mathematics Extension 1 Year 12 Introduction to vectors
Trigonometric expressions and equations provide a powerful tool for modelling quantities that vary in a cyclical way such as tides, seasons, demand for resources and alternating current.	Mathematics Extension 1 Year 11 Further trigonometry Mathematics Advanced Year 12 Further graph transformations and modelling Physics Stage 6
Trigonometric identities can be proven using mathematical induction.	Mathematics Extension 1 Year 12 Proof by mathematical induction Mathematics Extension 2 The nature of proof
Trigonometric identities are used to simplify certain integrals and can be used to solve problems involving integration including volumes of solids of revolution.	Mathematics Extension 1 Year 12 Further calculus skills Mathematics Extension 2 Further integration
Trigonometric identities are used to solve problems in complex numbers.	Mathematics Extension 2 Introduction to complex numbers
Rewriting $a\cos x + b\sin x$ as a single trigonometric function is useful in graphing functions that model physical phenomena and can be used to simplify certain integrals.	Mathematics Extension 2 ▪ Applications of calculus to mechanics ▪ Further integration

Trigonometry is essential in Fourier analysis, particularly when summing different trigonometric functions together. This in turn is used in signal processing in engineering. Trigonometric functions are used in graphic design.	Pure mathematics (tertiary) Applied mathematics (tertiary) Real-world applications

Considerations

- 3-dimensional problems can usually be solved by considering multiple different 2-dimensional triangles within the larger problem, and students should be encouraged to draw these separately when analysing the problem.
- The domains and ranges of $sec\,\theta$, $cosec\,\theta$ and $\cot\theta$ need to be carefully considered when solving equations involving these functions.
- Students use the notation $sin^n x$ to represent $(\sin x)^n$ where n is a positive real number. The notation $sin^{-1} x$ is used to represent the inverse trigonometric function and cannot be equated with $(\sin x)^{-1}$.
- When solving trigonometric equations students should take care to evaluate the solutions of trigonometric equations and remove the values outside any given domains.
- A review of basic trigonometric identities and equations may be necessary.
- All identities and equations should be treated in degrees and radians.
- Use of technology and online applets may support a guided approach to the proofs for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$.
- After proving just one of the results for $\sin(A \pm B)$ and $\cos(A \pm B)$, the remaining results can be obtained by substitution. For example, the double angle identities can be derived from the sum and difference formulas by remapping B to A.
- The derivation of $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ and $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$, along with the generalised version for $\sin^2(nx)$ and $\cos^2(nx)$, for some constant n, should be explored when studying the double angle formulas as these identities are used in integration problems.
- A graphing application could be used to visually demonstrate why $a\sin x + b\cos x$ can be expressed in the form $R\sin(x+\alpha)$. From this, students should recognise that the sum of a sine function and cosine function with the same period yields a sinusoidal function of the same period but shifted.

Diversity of learners

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Teaching advice for Permutations and combinations

Content

§ Permutations and combinations

Teaching advice

Purpose

Knowledge of permutations and combinations is useful when considering situations and solving problems involving counting, sorting and arranging. Efficient counting methods have many applications and are used in the study of probability. The study of combinatorics is important in developing students' ability to generalise situations, explore patterns and use reasoning to consider all possible outcomes. This can be used in situations such as the arrangement of people or objects, design of surveys, and selection of a jury or committee.

Key terms

- combination
- distinct
- factorial
- integer
- multiplication principle
- permutation

Related Stage 5 Path content

- Algebraic techniques B
- Algebraic techniques C
- Indices B

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Probability problems involving permutations and combinations may require considering probabilities in multistage chance experiments, with or without replacement, and could involve conditional probability.	Mathematics Stage 5 Probability A Mathematics Advanced Year 11 Probability and data
Venn diagrams and other visual representations are often used when solving probability problems in combinatorics.	Mathematics Stage 5 Probability B Mathematics Advanced Year 11 Probability and data
Working with factorial notation involves selecting and applying appropriate algebraic techniques to operate with algebraic fractions, expanding, factorising and simplifying algebraic expressions, and using index laws when simplifying numerical expressions.	Mathematics Stage 5 ▪ Algebraic techniques C ▪ Indices B Mathematics Advanced Year 11 Working with functions
Factorials may be used in differentiation, for example, to write the third derivative of x^8 as $\frac{8!}{5!}x^5$ and the eighth derivative as $8!$.	Mathematics Advanced Year 11 Introduction to differentiation
The factorial can be defined recursively as: ▪ the function $f(n)=n\times f(n-1)$ for $n\geq 1$ with $f(0)=1$, or ▪ $0!=1$ and $n!=n\times(n-1)!$ for all whole numbers $n\geq 1$.	Mathematics Extension 2 Further integration
The factorial can be generalised to the gamma function, which extends its domain to complex and real number arguments, negative integers and non-integer values.	Mathematics Extension 2 Introduction to complex numbers Pure mathematics (tertiary) Statistics (tertiary)
Counting techniques are applied in areas of group theory, number theory, graph theory, logic, Ramsey theory, game theory and computer science. Counting techniques are used extensively in statistical analysis.	Pure mathematics (tertiary) Real-world applications

Combinatorics is used in cryptography and data mining for security purposes in computer science.	

Considerations

- Using physical manipulatives to illustrate concepts is useful for students to visualise scenarios.
- For arrangements involving a small number of items, a systematic listing of the possible arrangements to verify results is encouraged. This helps to reinforce systematic thinking and pattern recognition when more conditions are introduced in later problems. This could also serve as an introduction to the multiplication principle, establishing it as a quicker way to enumerate cases and arrangements.
- Students should develop fluency in working with factorials. Students may be supported to practise useful skills including determining how far to 'unroll' a factorial, i.e. $n! = n(n-1)! = n(n-1)(n-2)! = \cdots$, as well as the ability to recognise expressions such as $7 \times 6 \times 5$ as $\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$ and that it can be expressed as $\frac{7!}{4!}$.
- The equivalence between ${}^{n}P_{n}$ and $n!$ should be explored as this naturally leads to the development of the expression ${}^{n}P_{r} = n(n-1)(n-2) \times \cdots \times [n-(r-1)] = \frac{n!}{(n-r)!}$. It should be noted there is a natural restriction that $0 \le r \le n$.
- The term 'word' in the context of combinatorics problems refers to a string of letters. A list of all possible words will contain those that do not necessarily spell anything meaningful. To avoid ambiguity, consider the 'number of possible arrangements of the letters'.
- When working with numbers and digits, consider whether a leading 0 is allowed given the context of a problem. For example, PINs (personal identification numbers) can have a leading 0, whereas when forming numbers from given digits, leading zeroes are not allowed.
- It should be noted that arrangements involving repeated elements while sometimes referred to as 'permutations with repetition(s)' are not permutations. The development of the calculations for such problems relies on a sound understanding of the multiplication principle. Treating repeated elements first as distinct elements should aid calculation and highlight why the calculation involves a division by the factorial of how many times each repeated element reoccurs.
- A connection should be made between the ${}^{n}P_{r}$ and ${}^{n}C_{r}$ calculations. In this way, the relationship ${}^{n}C_{r} = \frac{{}^{n}P_{r}}{r!}$ should be evident.
- An alternate notation for ${}^{n}C_{r}$ is $\binom{n}{r}$, this is not to be confused with the 2D column vector notation. Ensure context is clear when using this notation.
- When working with circular arrangements of n objects, one approach is to consider all positions around the circle to be indistinguishable. Then there is only one way to place the first object, recognising that the remaining positions are now distinguishable in relation to the first object placed and hence there are $(n-1)!$ ways of arranging the remaining objects. Another approach would be to recognise that n objects can be arranged in $n!$ ways. However when counting in this manner around a circle, there are n rotations of the same table arrangement that are duplicate counts. Accounting for this duplicating counting yields the calculation $\frac{n!}{n} = (n-1)!$.

Diversity of learners

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Teaching advice for The binomial theorem

Content

§ The binomial theorem

Teaching advice

Purpose

The consideration of the expansion $(x+y)^n$, where n is an integer, draws on aspects of number theory. The material studied in this focus area provides the basis for more advanced work, where the binomial expansion is extended to probability theory.

Key terms

- binomial coefficient
- binomial expansion
- binomial expression
- binomial theorem
- coefficient
- identity
- Pascal's triangle

Related Stage 5 Path content

- Algebraic techniques B
- Algebraic techniques C
- Indices B

Sequencing advice

The content in the focus area The binomial theorem builds on content in:

- Mathematics Advanced Year 11 Working with functions
- Mathematics Extension 1 Year 11 Permutations and combinations

Connections	Course/Stage/Focus area/Post-school paths
Using the binomial theorem to expand binomial expressions involves selecting and applying appropriate algebraic techniques to operate with algebraic fractions, and expand, factorise and simplify algebraic expressions.	Mathematics Stage 5 Algebraic techniques C
Proving identities involving binomial coefficients may involve substituting into algebraic expressions, comparing coefficients or applying a combinatorial argument.	Mathematics Stage 4 Algebraic techniques Mathematics Advanced Year 11 Working with functions Mathematics Extension 1 Year 11 Permutations and combinations
The binomial theorem is often used with de Moivre's theorem to prove trigonometric identities.	Mathematics Extension 2 Introduction to complex numbers
The binomial theorem is used in the proof of the definition of Euler's constant as $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$.	Mathematics Advanced Year 11 Exponential and logarithmic functions
The expansion of $(1+x)^n$ is a polynomial in x .	Mathematics Stage 5 Polynomials Mathematics Extension 1 Year 11 Polynomials
The binomial expansion can be used to derive the differentiation rule for x^n from first principles. Alternately, calculus can be used to derive the binomial coefficient by repeated differentiation of $(1+x)^n$.	Mathematics Advanced Year 11 Introduction to differentiation
The binomial theorem is used in calculating probabilities for a binomial random variable.	Mathematics Extension 1 Year 12 The binomial distribution and sampling distribution of the mean
The binomial theorem is often used with de Moivre's theorem to prove trigonometric identities.	Mathematics Extension 2 Introduction to complex numbers
The binomial theorem is used in group	Tertiary mathematics

theory, number theory, graph theory, logic and Ramsey theory. The binomial theorem can be generalised for rational powers and multinomial expansions.	

Considerations

- Students could find the binomial expansions of $(x+y)^n$ for $n=1,2,3,4,5$ by manual expansion, for example, using $(x+y)^3=(x+y)(x+y)^2$ and using this expansion to build up to higher powers. The coefficients should be highlighted and students should be guided to connect the coefficients in the expansions with the values in Pascal's triangle.
- Students could use Pascal's triangle to observe patterns such as symmetry, number of values in each row, starting and ending values in each row, second and second last values in each row, and the sum of the values in each row.
- Students could explain why the row of coefficients in Pascal's triangle is symmetric (that is, the coefficient of $x^{n-r}y^r$ is the same as the coefficient of $x^r y^{n-r}$) and why each row begins and ends with 1.
- Students could explain by substitution why the sum of each row of coefficients is 2^n, and why the alternating sum of each row of coefficients, except the first, is 0.
- A combinatorial argument can be used to find the coefficient of $x^{n-r}y^r$. Consider $(x+y)^n=(x+y)(x+y)\ldots(x+y)$ (factor appears n times) and each term in the final expansion is obtained by selecting one term from each factor and multiplying.
- To obtain a term in $x^{n-r}y^r$, we need to select the y from r of these factors and the x from the remaining $n-r$ factors. This can be done in ${}^nC_r$ ways, yielding ${}^nC_r$ as the coefficient of $x^{n-r}y^r$. Thus, a typical or general term in the expansion of $(x+y)^n$ is ${}^nC_r x^{n-r}y^r$. Students can now use the binomial theorem to expand and simplify a variety of binomial expansions.
- The result for the general term in a binomial expansion is useful in directly determining a specific term in a given expansion without expanding.
- Students should be guided to explore and identify various patterns and identities in Pascal's triangle and to verify the results using combinatorial arguments or algebraic techniques.
- Proofs of the following Pascal's triangle relations should be established both by using a combinatorial argument and algebraically by using the factorial definition of ${}^nC_r$.
  ${}^nC_0=1, {}^nC_n=1, {}^nC_k={}^{n-1}C_{k-1}+{}^{n-1}C_k$ for $1\le k\le n-1$ and ${}^nC_k={}^nC_{n-k}$.
- Students should recognise that the size of the coefficients in the expansion of $(x+y)^n$ first increases and then decreases. However, problems that require the greatest coefficient to be found are not within the scope of this syllabus.
- The general result for the expansion of $(x+y)^n$ can be applied to the expansion of $(1+x)^n$ to obtain $(1+x)^n={}^nC_0x^0+{}^nC_1x^1+{}^nC_2x^2+\ldots+{}^nC_nx^n$. Students should recognise that ${}^nC_r$ is the coefficient of x^r in $(1+x)^n$.
- The use of a variety of notations including sigma notation should be modelled when proving statements. Examples that combine more than one identity should be covered, including those that use the Pascal's triangle identities proved previously.
- Some students may benefit from extension by proving identities involving differentiating or integrating the standard expansion and using a combinatorial argument, but this is not essential content. Students should remember that when integrating both sides of an identity,

we need to consider a constant of integration, which can be evaluated by substituting a suitable value of x.

Diversity of learners

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Teaching advice for Proof by mathematical induction

Content

§ Proof by mathematical induction

Teaching advice

Purpose

Proof involves the communication and justification of an argument for a mathematical statement in a clear, concise and precise manner. Knowledge of proof enables a level of reasoning, justification and communication that is accurate, concise and precise and lays the foundations for understanding the structure of a mathematical argument. The study of proof is important in developing students' ability to reason, justify, communicate and critique mathematical arguments and statements necessary for problem-solving and generalising patterns.

Key terms

- mathematical induction
- proof
- statement

Related Stage 5 Path content

- Algebraic techniques B
- Algebraic techniques C
- Indices B
- Indices C

Sequencing advice

The content in the focus area Proof by mathematical induction builds on content in:

- Mathematics Advanced Year 12 Sequences and series

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Algebraic proofs require manipulation of algebraic expressions, and may require index laws, surds and polynomial techniques.	Mathematics Stage 5 ▪ Algebraic techniques B ▪ Algebraic techniques C ▪ Polynomials Mathematics Advanced Year 11 Working with functions Mathematics Extension 1 Year 11 Polynomials
Summation notation (sigma notation) can be used to represent sums to be proved by mathematical induction.	Mathematics Advanced Year 12 Sequences and series
Divisibility proofs can be linked with division of polynomials.	Mathematics Extension 1 Year 11 Polynomials
Algebraic manipulation may be required to prove statements, including those involving factorials.	Mathematics Advanced Year 11 Working with functions Mathematics Extension 1 Year 11 Permutations and combinations
Mathematical induction can be used to prove results such as de Moivre's theorem, the binomial theorem and results involving functions.	Mathematics Extension 1 Year 11 The binomial theorem Mathematics Extension 2 ▪ The nature of proof ▪ Introduction to complex numbers
Mathematical induction is used to prove that an algorithm successfully solves a particular problem, which is useful in computer science and algorithm analysis. Problems can be proven using mathematical induction for optimisation of game strategies in game theory.	Real-world applications

Considerations

- As an introduction to the concept of mathematical induction, the example of knocking dominoes over can be shown either physically or on video.
- A well-written mathematical proof uses formal mathematical notation, definitions and terminologies to demonstrate concise and logical reasoning. The structure of a proof by induction should be carefully examined prior to proving results using mathematical induction.
- To improve the clarity of a proof by mathematical induction, statements such as 'Show the result is true for $n=1$' or 'Assume the result is true for $n=k$', can assist in breaking up the proof

into clear sections. It is important to remember that a proof is written for other readers, and practices that improve the readability of a proof should be encouraged.

- A convenient and common notation for the result to be proven is $P(n)$ (P for 'proposition'). A proof by induction would then consist of proving $P(a)$ is true (where the base case is when $n=a$ where a is a non-negative integer), assume $P(k)$ is true (for some k), then proving $P(k+1)$ is true using the assumption, followed with an appropriate conclusion. Care should be taken not to confuse the $P(n)$ notation with a similar notation used for polynomials, as such, some resources will use $S(n)$ (S for 'statement') as a workaround, particularly when proving results involving polynomials.
- Suitable conclusions such as:

  '$\therefore$ result is true for $n=1$' or '$\therefore P(k+1)$ is true if $P(k)$ is assumed true' at each critical step of a proof by induction should be included.
- Rewriting statements given in summation notation may simplify the problem for some students.
- When working with the base case (often where $n=1$), care should be taken to include the correct number of terms. For example, there is not one, but two terms in series $2^0+2^1+2^2+\cdots+2^n$ when $n=1$. Incorrect exclusions of terms can lead to false proofs by induction or a result that cannot be shown.
- Some results can be proven algebraically without the use of mathematical induction. Students should take care to prove the result using mathematical induction if it has been specified.

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Teaching advice for Introduction to vectors

Content

§ Introduction to vectors

Teaching advice

Purpose

Knowledge of vectors enables the understanding of the behaviour of objects in two dimensions and the ways in which this behaviour can be expressed, including the consideration of position, displacement and movement.

The study of vectors is important in developing students' understanding of an object's representation and behaviour in two dimensions using a variety of notations, and how to use these notations effectively to explore the geometry of a situation. Vectors are used in many fields of study, including engineering, structural analysis and navigation.

Key terms

- acceleration vector
- Cartesian coordinates
- column vector notation
- component
- component form (of a vector)
- coordinate planes
- constant
- directed line segment
- displacement vector
- equal
- equivalent
- force
- function
- gravitational
- magnitude
- mass
- parallel
- parallelogram law
- parametric equations
- perpendicular
- position vector
- projection (vector)
- scalar
- scalar (dot) product
- triangle law
- unit vector
- vector
- velocity vector
- zero vector

Related Stage 5 Path content

- Linear relationships C

- Non-linear relationships C
- Trigonometry C
- Trigonometry D
- Variation and rates of change A

Sequencing advice

The content in the focus area Introduction to vectors builds on content in:

Mathematics Extension 1 Year 11 Further work with functions

Making connections

Connections	Course/Stage/Focus area/Post-school paths
The use of vectors in two and three dimensions is related to deriving and applying the midpoint and distance formulas on a 2D Cartesian plane and extended into three dimensions.	Mathematics Stage 5 Linear relationships C
The path of an object can be represented by equations and graphs of parabolas, exponentials, hyperbolas, circles, other polynomial curves and their transformations.	Mathematics Stage 5 Non-linear relationships C Mathematics Advanced Year 11 ▪ Working with functions ▪ Graph transformations Mathematics Extension 1 Year 11 Polynomials Mathematics Advanced Year 12 Further graph transformations and modelling
Non-right-angled trigonometry is used to establish the equivalence between the algebraic and geometric definitions of the scalar (dot) product and to solve relative velocity vector problems.	Mathematics Stage 5 Trigonometry C Mathematics Advanced Year 11 Trigonometry and measure of angles
Bearings can be used to describe the direction of a vector.	Mathematics Stage 5 Trigonometry D Mathematics Advanced Year 11 Trigonometry and measure of angles
Time-dependent motion problems in vector form involve distance–speed–time relationships as an application of variation.	Mathematics Stage 5 Variation and rates of change A
The geometric and algebraic formulas for the scalar product are linked by the cosine rule.	Mathematics Stage 5 Trigonometry C Mathematics Advanced Year 11 Trigonometry and measure of angles
Calculating the direction of a 2-dimensional vector involves simple trigonometry.	Mathematics Stage 5 Trigonometry A Mathematics Advanced Year 11 Trigonometry and measure of angles
The Cartesian equation of the path of a projectile can be	Mathematics Extension 1 Year 11

found using its parametric equations.	Further work with functions
Lines, curves, and coordinate geometry and geometry problems can be modelled using vectors.	Mathematics Extension 2 Further work with vectors
Vectors can be used to provide elegant proofs of important results in mathematics, including: ▪ sum and difference results in trigonometry ▪ Cauchy–Schwarz inequality ▪ triangle inequality.	Mathematics Extension 2 The nature of proof
Vectors are used to represent complex numbers.	Mathematics Extension 2 Introduction to complex numbers
Vectors are useful in representing and modelling forces used to solve physical problems that are encountered in everyday life.	Mathematics Extension 1 Year 12 Introduction to vectors
Information and data are treated as vectors for calculations in computer science, statistics and actuarial studies. Vectors are essential in navigation for aircraft and ships.	Real-world applications
Vectors form the basis of vector calculus and linear algebra.	Pure and applied mathematics (tertiary)

Considerations

- It is worth noting that 3D coordinate axes are drawn in a right-handed fashion, as shown in Figure 1 below. Understanding the right-handedness of the axes may help students visualise the different orientations of the 3D coordinate axes.

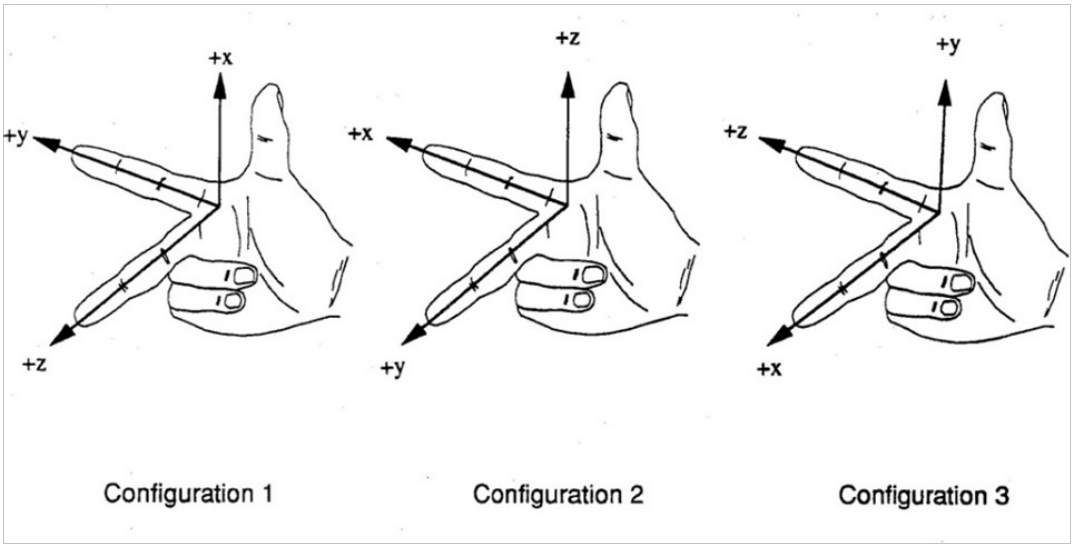

Figure 1: Configurations of 3D coordinate axes

Image long description: Three separate right hands each with the thumb pointing up and the middle and forefingers pointing to the left show the different orientations of the 3D coordinate axes. Configuration 1 shows the first hand with the $+z$-axis drawn along the extended middle finger, the $+y$-axis drawn along the extended forefinger and the $+x$-axis drawn straight up between the extended thumb and extended forefinger. Configuration 2 shows the second hand with the $+y$-axis drawn along the extended middle finger, the $+x$-axis drawn along the extended forefinger and the $+z$-axis drawn straight up between the extended thumb and extended forefinger. Configuration 3 shows the third hand with the $+x$-axis drawn along the extended middle finger, the $+z$-axis drawn along the extended forefinger and the $+y$-axis drawn straight up between the extended thumb and extended forefinger.

- Note that there are quantities with both magnitude and direction that are not considered vectors, for example, electric current.
- When describing the direction of a 2D vector, some resources will measure the angle between the vector and the positive direction of the x-axis while other texts will quote the direction as a bearing. In some examples, it may be more appropriate to describe the direction of a vector by referring to an angle appropriate to the context, for example, the angle of deviation from the intended path.
- Some resources refer to the magnitude of a as its 'norm', denoted $\|a\|$.
- The term 'anti-parallel' can be used to describe 2 vectors that are parallel to one another but point in opposite directions, but this is not essential content.
- Students must ensure the context is clear when using notation that is identical in multiple contexts, for example $(4,3)$ can refer to the point as well as the position vector $4i+3j$. Likewise, $\begin{pmatrix}4\\3\end{pmatrix}$ can refer to the same vector, or ${}^{4}\square$ in combinatorics. Clarifying with statements like 'the point $(4,3)$' or 'the vector $(4,3)$' is advisable.
- When operating with vectors, the cross-product, which is not essential content, uses the operator '$\times$', while '$\cdot$' is used for the scalar (dot) product. These notations should be used correctly and in the right contexts. Scalar multiples of vectors should simply be written as $3(2i+5j)$ not $3\times(2i+5j)$, while notation such as $3\cdot6$ to represent the multiplication 3×6 is not advisable.
- Students should consider the scalar product of a and b where one (or both) is the zero vector. In component form, the scalar product will yield a result of 0, this means that the zero vector is perpendicular to all vectors. It is not uncommon to include the zero vector when talking about the scalar product, however, the angle between a vector and the zero vector is not well defined, and therefore the use of the formula $a\cdot b=|a||b|cos\theta$ to evaluate $a\cdot b$ in this instance should be avoided.
- To determine whether two objects will collide, students should check that the objects meet at the same position (both components of the objects' position vectors are the same) at the same time.
- When working with projectile motion, the 6 equations of motion should be derived using calculus, unless given as facts.
- The path of a projectile is referred to as its trajectory. The equation of the trajectory should be derived rather than memorised.
- Students may benefit from extension through exploring the notion of linear independence and why vectors are often resolved into rectangular components but this is not essential content.
- Students may benefit from extension through exploring the cross-product as an operation between two 3D vectors but this is not essential content.

Diversity of learners

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Teaching advice for Inverse trigonometric functions

Content

§ Inverse trigonometric functions

Teaching advice

Purpose

The study of trigonometric functions involves using periodic functions in geometric, algebraic, numerical and graphical representations. Inverse trigonometric functions are important examples of inverse functions. The study of inverse trigonometric functions includes the graphs of these functions and the connections between algebraic and geometrical representations of functions. This enables a deeper understanding of the nature of periodic functions, which are used as modelling tools for any quantity that varies in a cyclical way.

Key terms

- asymptote
- composite function
- dilation
- domain
- even function
- function
- intercept
- inverse function
- odd function
- period
- range
- reflection
- transformation
- translation

Related Stage 5 Path content

- Functions and other graphs
- Trigonometry D

Sequencing advice

The content in the focus area Inverse trigonometric functions builds on content in:

- Mathematics Extension 1 Year 11 Further trigonometry
- Mathematics Advanced Year 12 Further graph transformations and modelling

Making connections

Connections	Course/Stage/Focus area/Post-school paths
The inverse trigonometric functions require knowledge of both inverse functions and trigonometric functions, including their graphs, properties and identities.	Mathematics Advanced Year 11 ▪ Trigonometry and measure of angles ▪ Trigonometric identities and equations Mathematics Extension 1 Year 11 ▪ Further work with functions ▪ Further trigonometry
Working with inverse trigonometric functions requires consideration of domain and range, composition of functions, symmetry and transformation of functions.	Mathematics Advanced Year 11 Working with functions Mathematics Advanced Year 12 Further graph transformations and modelling
Sum and difference expansions and double angle results are used when evaluating expressions such as $\tan\left(\tan^{-1}3-\tan^{-1}2\right)$ and $\sin\left(2\sin^{-1}\left(\frac{4}{5}\right)\right)$.	Mathematics Extension 1 Year 11 Further trigonometry
Calculus can be applied to the inverse trigonometric functions. The identity $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ can be proven using calculus.	Mathematics Extension 1 Year 12 Further calculus skills Mathematics Extension 2 Further integration
Inverse trigonometric functions often appear in engineering, architecture and surveying.	Real-world applications

Considerations

- Some students may benefit from a review of restricting the domain of a function or relation so that the inverse relation is a function, and applying this, and reflection in $y=x$, to quadratics, simple cubics, circles and in particular to generating the function $y=\sqrt[3]{x}$.
- Students could solve, in the interval $-2\pi\le x\le 2\pi$, the equations $\sin x=\frac{1}{2}$ and $\sin x=-\frac{1}{2}$, and explain why the restriction $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ is the most convenient when forming the inverse function.
- Students could solve, in the interval $-2\pi\le x\le 2\pi$, the equations $\cos x=\frac{1}{2}$ and $\cos x=-\frac{1}{2}$, and explain why the restriction $0\le x\le\pi$ is the most convenient when forming the inverse function.
- Students could solve, in the interval $-2\pi\le x\le 2\pi$, the equations $\tan x=1$ and $\tan x=-1$, and explain why the restriction $-\frac{\pi}{2}<x<\frac{\pi}{2}$ is the most convenient when forming the inverse function.
- Students could use a scientific calculator, set on degrees, and then on radians, to verify that the calculator's inverse trigonometric functions follow the conventions for the definitions of the inverse trigonometric functions.
- While $\sin^2 x$ represents $(\sin x)^2$, it must be noted that $\sin^{-1}x$ does not mean $(\sin x)^{-1}=\frac{1}{\sin x}$, likewise for $\cos^{-1}x$ and $\tan^{-1}x$. The notations $\arcsin(x)$, $\arccos(x)$ and $\arctan(x)$ eliminate the potential for this misunderstanding and are used by various graphing applications.
- When transforming inverse trigonometric functions, an intuitive understanding of the changes to the domain and range should be justified using algebra. For example, the graph of $y=\sin^{-1}(x-3)$ has a domain of $2\le x\le 4$ from translating the $\sin^{-1}x$ function with domain $-1\le x\le 1$ to the right 3 units. This can be justified by recognising that there is a natural domain on $\sin^{-1}(x-3)$ such that $-1\le x-3\le 1$, which yields $2\le x\le 4$. The algebraic approach may be advantageous when sketching $y=\sin^{-1}(\sin x)$ and the like.
- Graphing applications can and should be used when exploring the graphs of $y=\sin(\sin^{-1}x)$, $y=\sin^{-1}(\sin x)$, $y=\cos(\cos^{-1}x)$, $y=\cos^{-1}(\cos x)$, $y=\tan(\tan^{-1}x)$ and $y=\tan^{-1}(\tan x)$. The domain and range of these functions should be justified using algebra.
- An algebraic proof of $\sin^{-1}(-x)=-\sin^{-1}x$:

Let $\sin^{-1}(-x)=y$, i.e. $-x=\sin y$.

Therefore, $x=-\sin y$.

Now since $\sin(-y)=-\sin y$, then $x=\sin(-y)$.

i.e. $\sin^{-1}(x)=-y=-\sin^{-1}(-x)$.

Therefore, $\sin^{-1}(-x)=-\sin^{-1}(-x)$.

Diversity of learners

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Teaching advice for Further calculus skills

Content

§ Further calculus skills

Teaching advice

Purpose

Calculus involves the study of how things change and provides a framework for developing quantitative models of change and deducing their consequences. It explores the development of analytic and numeric integration techniques and the use of these techniques in solving problems.

The study of calculus is important in developing students' knowledge, understanding and capacity to operate with and model situations involving change, and to use algebraic and graphical techniques to describe and solve problems. Calculus is also used to predict future outcomes with relevance to, for example, science, engineering, finance, economics and the construction industry.

Key terms

- chain rule
- definite integral
- derivative
- function
- identity
- indefinite integral
- integration
- inverse function
- parametric equation
- product rule
- quotient rule
- reciprocal
- substitution

Related Stage 5 Path content

- Algebraic techniques C
- Linear relationships C
- Functions and other graphs

Sequencing advice

The content in the focus area Further calculus skills builds on content in:

- Mathematics Advanced Year 12 Differential calculus
- Mathematics Advanced Year 12 Integral calculus
- Mathematics Extension 1 Year 12 Inverse trigonometric functions

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Finding indefinite and definite integrals involving expressions of the form $\frac{1}{\sqrt{a^2 - x^2}}$ or $\frac{a}{a^2 + x^2}$ may involve manipulation of algebraic fractions.	Mathematics Stage 5 Algebraic techniques C Mathematics Advanced Year 11 Working with functions
Finding equations of tangents and normals to curves involves finding equations of lines.	Mathematics Stage 5 Linear relationships C Mathematics Advanced Year 11 ▪ Working with functions ▪ Introduction to differentiation
Differentiation of inverse functions and inverse trigonometric functions involves an understanding of function notation and the use of the chain rule, product rule and quotient rule.	Mathematics Stage 5 Functions and other graphs Mathematics Advanced Year 11 ▪ Working with functions ▪ Introduction to differentiation
Trigonometric substitutions for integrals often make use of the Pythagorean identities.	Mathematics Advanced Year 11 Trigonometric identities and equations
Finding the derivative of a function defined parametrically using the chain rule involves using parametric equations. Verifying the derivative of the inverse function involves an understanding that if $y = f^{-1}(x)$ then $x = f(y)$.	Mathematics Extension 1 Year 11 Further work with functions
Proofs of the derivatives of $y = \sin^{-1} x$, $y = \cos^{-1} x$ and $y = \tan^{-1} x$ make use of the results $\frac{d}{dx}(\sin x) = \cos x$, $\frac{d}{dx}(\cos x) = -\sin x$ and $\frac{d}{dx}(\tan x) = sec^2 x$, as well as the Pythagorean identities.	Mathematics Advanced Year 11 Trigonometric identities and equations Mathematics Advanced Year 12 Further graph transformations and modelling
Integrals of $\sin^2 x$ and $\cos^2 x$ can be used in the context of volumes of revolution.	Mathematics Extension 1 Year 12 Further applications of calculus

Considerations

- When evaluating a definite integral using integration by substitution, students should ensure they change the boundary limits as well as the variable of integration.

- Identities such as $\cos^{-1}(-x)=\pi-\cos^{-1}x$ can be derived using calculus by showing $\frac{d}{dx}\left[\cos^{-1}(-x)+\cos^{-1}x\right]=0$, implying $\cos^{-1}(-x)+\cos^{-1}x$ is a constant, the value of which can be evaluated by a convenient substitution such as $x=0$.
- Students should consider how integrals such as $\int\frac{dx}{\sqrt{4-9x^2}}$ can be expressed in the form $\int\frac{dx}{\sqrt{a^2-x^2}}$ before integrating, i.e. $\int\frac{dx}{\sqrt{4-9x^2}}=\frac{1}{3}\int\frac{dx}{\sqrt{\frac{4}{9}-x^2}}$.
- Students should practise using algebraic manipulation when working with integrals such as $\int\frac{x^2}{x^2+4}dx$ to obtain an integral of the form $\int\frac{a}{a^2+x^2}dx$, i.e. $\int\frac{x^2}{x^2+4}dx=\int\frac{x^2+4-4}{x^2+4}dx=\int\left(1-\frac{4}{x^2+4}\right)dx$.

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Teaching advice for Further applications of calculus

Content

§ Further applications of calculus

Teaching advice

Purpose

Calculus involves the study of how things change and provides a framework for developing quantitative models of change and deducing their consequences. It explores the development of analytic and numeric integration techniques and the use of these techniques in solving problems.

The study of calculus is important in developing students' knowledge, understanding and capacity to model and operate in situations involving change. Calculus involves using algebraic and graphical techniques to describe and solve problems and to predict future outcomes with relevance to, for example science, engineering, finance, economics and the construction industry.

Key terms

- arc
- asymptote
- chain rule
- constant
- derivative
- differential equation
- exponential decay
- exponential growth
- function
- infinite
- integration
- logistic function
- point of inflection
- logistic function
- model
- multiplicity of a zero
- partial fractions
- point of inflection
- polynomial
- product rule
- rate of change
- slope field
- tangent
- turning point
- zero (of a function)

Related Stage 5 Path content

- Algebraic techniques B
- Area and surface area B
- Variation and rates of change A
- Variation and rates of change B
- Indices B

- Non-linear relationships C
- Logarithms
- Polynomials
- Volume B
- Properties of geometrical figures C

Sequencing advice

The content in the focus area Further applications of calculus builds on content in:

- Mathematics Advanced Year 12 Differential calculus
- Mathematics Advanced Year 12 Integral calculus
- Mathematics Advanced Year 12 Applications of calculus
- Mathematics Extension 1 Year 12 Inverse trigonometric functions
- Mathematics Extension 1 Year 12 Further calculus skills

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Algebraic fractions may require manipulation when separating variables in differential equations.	Mathematics Stage 5 Algebraic techniques B
Formulas for volumes of cylinders, cones and spheres can be proven using solids of revolution.	Mathematics Stage 5 ▪ Volume A ▪ Volume B
Graphing of polynomials employs graphing techniques for functions and may require calculus.	Mathematics Stage 5 Polynomials Mathematics Advanced Year 11 ▪ Working with functions ▪ Introduction to differentiation Mathematics Extension 1 Year 11 Polynomials Mathematics Advanced Year 12 Applications of calculus
Differentiation can be used to determine the multiplicity of a zero of a polynomial.	Mathematics Advanced Year 11 Introduction to differentiation
Variation and rates of change can be represented algebraically and geometrically.	Mathematics Stage 5 ▪ Variation and rates of change A ▪ Variation and rates of change B Mathematics Advanced Year 11 Introduction to differentiation Mathematics Advanced Year 12 Applications of calculus
Problems involving related rates of change may require properties of shapes, Pythagoras' theorem or formulas for perimeter, arc length, areas of plane shapes, surface area and volume.	Mathematics Stage 4 ▪ Right-angled triangles (Pythagoras' theorem) ▪ Area Mathematics Stage 5 ▪ Area and surface area A ▪ Area and surface area B ▪ Volume A ▪ Volume B Mathematics Advanced Year 11 Trigonometry and measure of angles
Exponential growth and decay can be represented using exponential functions and graphs. Exponential growth and decay problems may require the use of logarithms in their solutions.	Mathematics Stage 5 ▪ Non-linear relationships C ▪ Logarithms Mathematics Advanced Year 11 Exponential and logarithmic functions

Slope fields can be used to model first order differential equations encountered in resisted motion problems.	Mathematics Extension 2 Applications of calculus to mechanics
A logistic function such as $P=\frac{1}{1+e^{-x}}$ with the property that $\frac{dP}{dx}=P(1-P)$ returns a value between 0 and 1 for all input x and has applications in machine learning, data science, computer science and signal processing.	Real-world applications

Considerations

- In related rates problems, when a diagram is constructed as part of solving a problem, care needs to be taken to correctly identify the fixed and varying quantities, and label these accordingly.
- The constant k in exponential growth and decay models is sometimes referred to as the 'growth constant' in growth problems, and 'decay constant' in decay problems, however, this usage is not standard and can be confused with the instantaneous rate of change. As such, care should be taken when using these terms to describe population change. For example, if a population increases by 10% per year, the population can be modelled by the equation $N(t)=A\times 1.1^t$, which is equivalent to $N(t)=Ae^{(\ln 1.1)t}$. Here, the growth constant is $k=\ln 1.1\approx 0.095$.
- Exponential growth and decay models can be used to solve problems involving doubling time and half-life.
- It is important to recognise that using exponential growth and decay equations to model real-world situations has limitations. An understanding of these limitations can contribute to the development of more suitable and accurate models.
- When modelling with $\frac{dQ}{dt}=k(Q-P)$, the role of the parameter P should be considered as this often represents a natural carrying capacity of an ecosystem, the maximum population that the ecosystem can sustain indefinitely. The logistic model of population change also considers the carrying capacity of an ecosystem.
- Note that when $P>0$, $k<0$ and $A<0$, the initial value of Q (which is $P+A$) will be less than P. The graph of Q approaches the limiting value of P from below.
- It should be noted that Newton's Law of Cooling, which states that the rate at which an object cools is proportional to the difference in temperature between the object and the object's surroundings, also applies to bodies that are placed in surroundings that are warmer than the initial temperature of the body, where the body's temperature rises.
- When solving integration problems involving area or volume, a neat sketch of the area in question or the region to be rotated is recommended. Graphing applications may aid the visualisation of solids of revolution.
- When evaluating areas between curves and finding volumes of solids of revolution formed by rotating a region between two curves around a coordinate axis, students should carefully consider any points of intersection of the given curves.
- When calculating the area of a region that is bounded by the graphs of $y=f(x)$, $y=g(x)$, and the lines $x=a$ and $x=b$, when $f(x)\geq g(x)$ throughout the interval $a\leq x\leq b$, the

result is always given by $\int_a^b [f(x) - g(x)]\,dx$, whether or not either or both graphs cross the x-axis or lie below the x-axis. This is because both graphs can be translated vertically by the same amount, until the required region between them lies completely above the x-axis.

The translated region has area:

$$\int_a^b [(f(x)+k) - (g(x)+k)]\,dx = \int_a^b [f(x) - g(x)]\,dx.$$

- The calculation of volumes may involve the addition to, or subtraction from, volumes of simple solids such as cones and cylinders. A review of the formulas for these solids may be necessary.
- When using graphing applications that are capable of drawing slope fields, students should be aware that points where there are vertical tangents may not display properly. Online applets are available that will draw slope fields after inputting a differential equation.
- It should be emphasised that a slope field is a useful tool to model various differential equations and visualise their solutions, particularly in the case where the differential equation cannot be easily solved.
- The differential equation $\frac{dy}{dx} = ky(P - y)$ used to model logistic population growth can appear in different forms with different pronumerals representing the carrying capacity. Students should take care to distinguish between the pronumerals used to represent the dependent and independent variables.
- Students may benefit from understanding descriptors such as homogeneous and autonomous when working with differential equations but this is not essential content.
- Students may benefit from understanding the notion of isoclines and nullclines on a slope field as these may be useful terms when working with slope fields but this is not essential content.

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Teaching advice for The binomial distribution and sampling distribution of the mean

Content

§ The binomial distribution and sampling distribution of the mean

Teaching advice

Purpose

The study of probability provides a powerful and versatile toolset for understanding and navigating uncertainty and informing decisions in a wide range of disciplines.

The collection and statistical analysis of data is of fundamental importance in society. Conclusions drawn from the statistical analysis of data are used to inform decisions made in government, science, business and industry. It is important that students have a broad understanding of how data and statistics are used, as well as how statistics are misused, resulting in the misrepresentation of information.

Key terms

- Bernoulli distribution
- Bernoulli random variable
- Bernoulli trial
- Binomial coefficient
- binomial distribution
- binomial experiment
- binomial random variable
- central limit theorem (CLT)
- discrete probability distribution
- expected value
- expected frequency
- experiment
- finite
- mean
- outcome
- parameter
- population
- random variable
- sample
- sample mean
- sampling distribution
- trial
- variance

Related Stage 5 Path content

- Probability B

Sequencing advice

The content in the focus area The binomial distribution and sampling distribution of the mean builds on content in:

- Mathematics Advanced Year 12 Random variables

Making connections

Connections	Course/Stage/Focus area/Post-school paths
The distribution of a Bernoulli random variable can be expressed piecewise.	Mathematics Advanced Year 11 Working with functions
Binomial probabilities are related to combinatorics and the binomial theorem.	Mathematics Extension 1 Year 11 ▪ Permutations and combinations ▪ The binomial theorem
If we know that a sampling distribution is approximately normal, we can use the rules of probability to make statistical inferences including the use of the empirical rule for the normal distribution and z-scores.	Mathematics Advanced Year 12 Random variables
The central limit theorem allows the application of the properties of the normal distribution to non-normal distributions.	Mathematics Advanced Year 12 Random variables
The binomial distribution is used to model probabilities in business, medicine, genetics technology and the natural world.	Real-world applications
The central limit theorem is used in various fields to inform decisions including in finance to make investment decisions, in clinical trials to determine effectiveness of health treatments and in quality control in manufacturing.	Real-world applications
In computer science algorithms can use probabilistic models to make decisions. Machine learning algorithms use probabilities to classify data.	Real-world applications

Considerations

- A Bernoulli trial is sometimes referred to as a Bernoulli experiment.
- When working with Bernoulli trials, and subsequently with the Bernoulli distribution and binomial distributions, it is conventional to use the parameter p to represent the probability of success. Some resources may use q to represent the probability of failure, making $q = 1 - p$.
- The distribution of a Bernoulli random variable can be expressed piecewise as:

$$X \sim Bernoulli(p) \text{ if } X = \begin{cases} 0 \; with \; P(X=0) = q \\ 1 \; with \; P(X=1) = p \end{cases}$$

where $q = 1 - p$, and $0 < p < 1$ and where $X \sim Bernoulli(p)$ is spoken as 'X follows a Bernoulli distribution with probability p'.

- An expression for the mean (or expected value) of a Bernoulli random variable can be established by considering each possible value of X and how likely it is to occur, giving:

$$E(X) = (0 \times (1 - p)) + (1 \times p) = p$$

- The variance of a Bernoulli random variable can be established by considering the formula met in the Mathematics Advanced course: $Var(X) = E(X^2) - [E(X)]^2$, giving:

$$Var(X) = (0^2(1 - p) + 1^2(p)) - p^2 = p - p^2 = p(1 - p)$$

- The formula for the mean of a binomial random variable can be derived from the formula for the mean of a Bernoulli random variable but this is not essential. If $X \sim Bin(n, p)$ and X consists of n Bernoulli trials, $X_1, X_2, \ldots, X_n$, then: $E(X) = E(X_1 + X_2 + \ldots + X_n)$. As the trials are independent, then:

$$E(X) = E(X_1) + E(X_2) + \ldots + E(X_n) = p + p + \ldots + p = np$$

- The formula for the variance of a binomial random variable can be derived from the formula for the mean of a Bernoulli random variable. If $X \sim Bin(n, p)$ and X consists of n Bernoulli trials, $X_1, X_2, \ldots, X_n$, then $Var(X) = Var(X_1 + X_2 + \ldots + X_n)$. As the trials are independent, then:

$$Var(X) = Var(X_1) + Var(X_2) + \ldots + Var(X_n) = p(1 - p) + p(1 - p) + \ldots + p(1 - p)$$

$$= np(1 - p)$$

- The binomial distribution can be used to model repeated independent trials of any experiment or observation with precisely two outcomes. The probability may be known in advance, for example when sampling with replacement from a box containing 2 red and 3 white balls, if success represents drawing a red ball then $p = \frac{2}{5}$; or p may be estimated from observations or experimental data, for example, guessing the gender of a baby from birth data.
- Binomial distributions can be approximated by the normal distribution; however, the use of computational applications has made this process unnecessary as these applications can work with large numbers of trials.
- Simulations could be used to model binomial distributions, sampling distributions of sample means and the central limit theorem.
- Each of the terms: population distribution, sample distribution and sampling distribution should be used carefully. For example, the distribution of heights of the entire population of students at a school forms a population distribution (of heights). If samples were taken from this population and the distribution of heights of the members in this sample was recorded, the distribution formed is a sample distribution. Sample statistics help to determine a population characteristic, for example the mean height of a population. To do this, the sample mean would be taken from many samples, the distribution of these sample means would form a sampling distribution and statistics for this distribution can then be calculated.
- When working with sampling distributions of sample means, it is important to understand and distinguish similar notation such as $\acute{x}$ and $\overline{X}$. The notation $\acute{x}$ refers to the mean of one sample. Different samples of equal size may yield different values of $\acute{x}$. After recording all the different values of $\acute{x}$ and noting its distribution, then $\overline{X}$ would be used to represent the random variable whose values are the different values of the sample mean, $\acute{x}$.

⋮

Diversity of learners

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Mathematics Extension 2 11–12 (2024)

Teaching advice for Stage 6

Teaching advice for The nature of proof

Content

§ The nature of proof

Teaching advice

Purpose

Proof involves the communication and justification of an argument for a mathematical statement in a clear, concise and precise manner. A knowledge of proof enables a level of reasoning, justification and communication that is accurate, concise and precise and lays the foundations for understanding the structure of a mathematical argument. The study of proof is important in developing students' ability to reason, justify, communicate and critique mathematical arguments and statements necessary for problem-solving and generalising patterns.

Key terms

- arithmetic mean
- binomial theorem
- contrapositive
- converse
- counterexample
- equivalent
- geometric mean
- implication
- inequality
- integer
- mathematical induction
- negation
- proof
- proof by contradiction
- proposition
- quantifier
- rational number
- recurrence relation/recursive formula
- squeeze theorem
- statement
- triangle inequality

Related Stage 5 Path content

- Algebraic techniques B
- Algebraic techniques C
- Indices B
- Indices C
- Polynomials
- Logarithms

- Functions and other graphs
- Trigonometry C
- Trigonometry D
- Properties of geometrical figures B
- Properties of geometrical figures C
- Circle geometry

Sequencing advice

The content in the focus area The nature of proof builds on content in:

- Mathematics Extension 1 Year 12 Proof by mathematical induction.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Number proofs regarding integers may involve prime numbers, properties of integers and factors.	Mathematics Stage 3 Multiplicative relations A Mathematics Stage 3 Represents numbers B Mathematics Stage 4 Computation with integers
Algebraic proofs require manipulating algebraic expressions, and may require index laws, surds, laws of logarithms and polynomial techniques.	Mathematics Stage 5 Algebraic techniques B Mathematics Stage 5 Algebraic techniques C Mathematics Stage 5 Polynomials Mathematics Stage 5 Logarithms Mathematics Advanced Year 11 ▪ Working with functions ▪ Exponential and logarithmic functions Mathematics Extension 1 Further work with functions
Geometry results, such as the sum of interior angles of a polygon, can be proved using induction. 'If-and-only-if' statements appear frequently in geometric properties.	Mathematics Stage 5 Properties of geometrical figures B Mathematics Stage 5 Properties of geometrical figures C
Trigonometric identities can be proven using mathematical induction.	Mathematics Advanced Year 11 Trigonometric identities and equations Mathematics Extension 1 Year 11 Further trigonometry
The binomial theorem can be proven by mathematical induction.	Mathematics Extension 1 Year 11 The binomial theorem

The squeeze theorem requires an understanding of limits and can be related to differentiation by first principles.	Mathematics Advanced Year 11 Introduction to differentiation
De Morgan's law, involving unions and intersections is connected to negation concepts in proof. De Morgan's law: $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$ and $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$	Mathematics Advanced Year 11 Probability and data Tertiary mathematics
Proof of inequalities by the squeeze theorem (or sandwiching) is connected to areas under curves and optimisation.	Mathematics Advanced Year 12 ▪ Integral calculus ▪ Applications of calculus
Results involving complex numbers including de Moivre's theorem and results for arguments and moduli can be proved using mathematical induction. For example: $\lvert z_1 + z_2 + z_3 + \ldots + z_{n-1} + z_n \rvert \leq$ $\lvert z_1 \rvert + \lvert z_2 \rvert + \lvert z_3 \rvert + \ldots + \lvert z_{n-1} \rvert + \lvert z_n \rvert$ and $\arg(z_1 z_2 z_3 \cdots z_n) = \arg(z_1) + \arg(z_2) + \arg(z_3) + \cdots$	Mathematics Extension 2 Introduction to complex numbers
Statements can be quantified with the set of complex numbers.	Mathematics Extension 2 Introduction to complex numbers

Complex number, vector and calculus proofs all involve the use of sound logic and employ different methods of proof.	Mathematics Extension 2 Introduction to complex numbers Mathematics Extension 2 Further work with vectors Mathematics Extension 2 Further integration
The triangle inequality can be proven for real numbers, complex numbers and vectors.	Mathematics Extension 2 Further work with vectors
Sequences can be defined using recurrence relations and financial mathematics is a practical use of recursive formulas.	Mathematics Advanced Year 12 ▪ Sequences and series ▪ Financial mathematics
Recursive functions are used in computer science, in biology to model cell division, and in game theory to model repeated interaction between players.	Real-world applications
Truth values and implications are used in propositional and Boolean logic. The language of proof are heavily relied on to define, describe and prove mathematical ideas. The AM–GM inequality, $\frac{a+b}{2} \geq \sqrt{ab}$, can be linked to further inequalities such as inequalities between the power mean and the Cauchy–Schwarz inequality.	Pure mathematics (Tertiary) Philosophy Logic Applied mathematics

Proof by strong induction is required for some statements, and recursive formulas of order 2 or higher can be proven by induction. Forward-backward induction (or Cauchy induction) can be used to prove the AM–GM inequality for n non-negative terms $a_1, a_2, a_3, \ldots, a_n$.	Pure mathematics
The flow of a computer program requires the construction of correct logical statement. Inequalities are used for constraints on variables in optimisation problems, such as in financial mathematics. Many important results are established through the use of inequalities. Inequalities are important in many applications of mathematics, where they are used, for example, to obtain estimates of the accuracy of approximations in the field of numerical analysis, allowing computer software programs that solve equations numerically to be used with confidence.	Real-world applications

Considerations

- A well-written mathematical proof uses formal mathematical notation, definitions and terminologies to demonstrate concise and logical reasoning. There are many elegant results that have been proven, some of which could be explored while studying this focus area.
- The nuances between statements used in proofs and sentences used in everyday contexts require attention. For example, the sentence ‘cats are fluffy’, may be ambiguous as a logical statement when not expressed explicitly as ‘all cats are fluffy’, or as ‘there exist some cats that are fluffy’. The use of quantifiers aids the correct negation of statements. The correct negation of ‘all cats are fluffy’, is ‘not all cats are fluffy’, meaning ‘there is the existence of some cats that are not fluffy’. This is opposed to the incorrect negation, ‘all cats are not fluffy’, meaning ‘no cats are fluffy’.
- The implication 'If P then Q' is a statement which may be true or false, but when read as an everyday sentence, may appear to draw a link between unrelated concepts (‘If $ABCD$ is a rectangle, then today is Sunday’), or appear to imply an incorrect fact (‘If $ABCD$ is a rectangle, then $ABCD$ is a square’). The truthfulness of both these implications is yet to be determined and relies on what is known about P and Q.

- The negation of the implication 'If P then Q' as 'P and not Q' can be understood by recognising 'If P then Q' does not hold if you have P, but do not have Q. The equivalence between 'If P then Q' and 'not P or Q' could be engaged with through discussion or other means.
- A statement can be disproved by a single counterexample. Students should consider whether a statement can be proved using examples, with the goal of recognising proof by exhaustion as a valid method of proof with an understanding of its limitations.
- The intuitive understanding of number properties such as why $n(n+1)$ is always even, and why $n(n+1)(n+2)$ is always divisible by 6 for $n \in \mathbb{Z}$, can be proven by considering the case when n is odd (letting $n = 2k \pm 1$) and the case when n is even (letting $n = 2k$) for some $k \in \mathbb{Z}$.
- To prove that $A \neq B$, for real numbers A and B is equivalent to proving either $A < B$ or $A > B$. Students can use this idea to prove $(5+2)^n \neq 5^n + 2^n$. Interestingly, many programming languages use '<>' to represent '≠'.
- The relationship between the arithmetic mean and geometric mean (the AM–GM inequality) can be extended to n non-negative terms for $n \geq 3$; the proof for simpler cases such as for $n = 3$ and $n = 4$ could be modelled.
- Students may benefit from being extended to analyse further logic statements using tools such as truth tables, though this is not essential content.
- Some students may benefit from being extended through exploration of the truthfulness of the implication 'If P then Q' when P is false and understand the necessity of these 'vacuous truths', but this is not essential content.
- To improve the clarity of a proof by induction, it is often helpful to include 'signpost' statements such as, 'show result is true for $n = 1$' or 'assume result is true for $n = k$' to help outline and remind oneself the step being worked on before carrying out the algebraic manipulations required for that step. While these statements are not necessary, they help establish the structure of a proof by induction. It is important to remember that a proof is written for other readers, and practices that improve the readability and the logical flow of a proof are encouraged.
- A convenient and common notation for the result to be proven is $P(n)$ (P for 'proposition'). A proof by induction would then consist of proving $P(a)$ is true (where the base case is when $n = a$, where a is a non-negative integer), assume $P(k)$ is true (for some k), then proving $P(k+1)$ is true using the assumption, followed with an appropriate conclusion. Be mindful not to mix the $P(n)$ notation with a similar notation used for polynomials. Some resources will use $S(n)$ (S for 'statement') as a workaround, particularly when proving results involving polynomials.
- Suitable conclusions such as '$\therefore$ result is true for $n = 1$' or '$\therefore P(k+1)$ is true if $P(k)$ is assumed true' should be included at each critical step of a proof by induction.

Diversity of learners

- Teachers should consider the accessibility of tools and resources. Considerations for tools such as spreadsheets, calculators and applications may include compatibility with screen readers and other assistive technologies including alternative input devices and the use of accessible calculators (see approved calculators). Considerations for resources may include the language demands, readability of and long descriptions for images and providing alternative text for graphs and diagrams.
- Students with disability may require adjustments including access to visual or digital aids to communicate and demonstrate knowledge, understanding and skills.
- Students for whom English is an additional language or dialect (EAL/D) may require additional support and targeted strategies in learning language specific to mathematics.

- The pace and complexity of learning may need to be adjusted for students who progress at different rates.

Teaching advice for Further work with vectors

Content

§ Further work with vectors

Teaching advice

Purpose

A knowledge of vectors enables the understanding of the behaviour of objects in two dimensions and ways in which this behaviour can be expressed, including the consideration of position, displacement and movement.

The study of vectors is important in developing students' understanding of an object's representation and behaviour in two dimensions using a variety of notations, and how to use these notations effectively to explore the geometry of a situation. Vectors are used in many fields of study, including engineering, structural analysis and navigation.

Key terms

- Cartesian equation
- Cauchy–Schwarz inequality
- collinear
- commutative law
- direction vector
- distributive law
- gradient
- gradient–intercept form
- parallel
- parameter
- position vector
- scalar
- scalar product
- skew lines
- vector
- vector equation

Related Stage 5 Path content

- Linear relationships C
- Non-linear relationships C
- Equations C
- Properties of geometrical figures C
- Circle geometry

Sequencing advice

The content in the focus area Further work with vectors builds on content in:

- Mathematics Extension 1 Year 12 Introduction to vectors.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
The midpoint and distance formulas in coordinate geometry are connected to vector equations of circles and spheres.	Mathematics Stage 5 Linear relationships C
The form $r = a + \lambda b$ is analogous to the point –gradient form of a line $y = mx + c$. Connections can be made with the direction vector b and the gradient m.	Mathematics Advanced Year 11 Working with functions
Determining if two lines in three dimensions intersect requires the skill of solving equations simultaneously.	Mathematics Stage 5 Equations C Mathematics Advanced Year 11 Working with functions
Defining curves parametrically using vectors requires skills of eliminating a parameter to find the Cartesian equation of the curve.	Mathematics Stage 5 Non-linear relationships C Mathematics Advanced Year 11 Working with functions Mathematics Extension 1 Year 11 Further work with functions
Proving geometrical results using vectors involves an understanding of geometrical proof and may involve properties of triangles and quadrilaterals and circle geometry theorems.	Mathematics Stage 5 ▪ Properties of geometrical figures C ▪ Circle geometry
Finite sequences of real numbers such as x_1, x_2, x_3 and y_1, y_2, y_3 can be represented as vectors by defining $a = x_1 i + x_2 j + x_3 k$ and $b = y_1 i + y_2 j + y_3 k$ and can therefore be operated on as vectors.	Mathematics Advanced Sequences and series
Vectors are used to represent complex numbers. An application of representing complex numbers as vectors is using them to describe rotations on a 2D plane.	Mathematics Extension 2 Introduction to complex numbers
The vector form of a circle (sphere) with centre at origin and at any point in the plane (space) is analogous to the form given in complex numbers and requires the translation of a curve.	Mathematics Advanced Year 11 ▪ Working with functions ▪ Graph transformations Mathematics Extension 2 Introduction to complex numbers
The Cauchy–Schwarz inequality for vectors	Mathematics Extension 2 The nature of proof

in two or three dimensions immediately gives the result for real numbers of two or three variables respectively.	
Vectors are useful in representing and modelling forces used to solve physical problems that are encountered in everyday life including in engineering and architecture.	Mathematics Extension 1 Year 12 Introduction to vectors Real-world applications
Vectors are used to define Bézier curves, which are used in computer graphics in architecture, game design, graphic design and product design. Information and data are treated as vectors for calculations in computer science, statistics and actuarial studies. Vectors are essential in navigation for aircraft in aviation.	Real-world applications
Vectors form the basis of vector calculus and linear algebra. Vectors can be used to explore the effects of changing the basis of a coordinate system, explore the idea of linear independence and explore the cross product between two three-dimensional vectors.	Pure and Applied Mathematics (Tertiary)

Considerations

- The vector equation of a line $r = a + \lambda b$ should be intuitively thought of as first finding a point on the line with position vector a, then traversing in the direction of vector b, where the parameter λ can be thought of as how many 'steps' of length $|b|$ have been traversed (and in which direction). This understanding should aid the drawing of lines given as vector equations and locating points on these lines. Equations of intervals naturally follow by limiting the number of 'steps' (i.e. restricting the value of λ). While not studied in this syllabus, equations of planes of the form $r = a + \lambda b + \mu c$ (for non-parallel vectors b and c, and scalar parameters λ and μ) can be understood in the same way.
- The circle with vector equation $|r| = R$ should be compared to the subset of the complex plane satisfying the condition $|z| = R$ for some $z \in \mathbb{C}$. An understanding that the equation $|r| = R$ represents the set of position vectors of fixed length R should assist students with understanding why this same formula in 3D represents a sphere of radius R. Discussions of how to describe circles in 3D using parametric equations, and vector equations, may follow.
- Proper notation and logical reasoning are key to a well-written proof. Taking note of how proofs are set out in reputable mathematical publications can be useful.
- Below is a list of results for vector proofs worth considering:
  - Pythagoras' theorem
  - the medians of a triangle are concurrent at its centroid
  - the centroid of ΔABC is represented by the vector $\frac{1}{3}(a+b+c)$, where a, b and c are the position vectors of A, B and C respectively

- the centroid of a triangle divides each median in a ratio of $1:2$
- the altitudes of a triangle are concurrent at its orthocentre
- the perpendicular bisectors of a triangle are concurrent at its circumcentre
- the interval joining the midpoints of 2 sides of a triangle is parallel to the third side but is half its length
- the diagonals of a parallelogram bisect each other
- the diagonals of a parallelogram are equal in length if and only if adjacent sides are perpendicular to each other
- the diagonals of a parallelogram meet at right angles if and only if it is a rhombus
- the diagonals of a rhombus bisect the vertex angles
- the midpoints of the sides of a quadrilateral join to form a parallelogram
- the midpoints of a pair of opposite sides and the midpoints of the diagonals of a quadrilateral join to form a parallelogram
- the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides
- the angle in a semicircle is a right angle
- the interval from the centre of a circle to the midpoint of a chord bisects the chord.

Diversity of learners

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Teaching advice for Introduction to complex numbers

Content

§ Introduction to complex numbers

Teaching advice

Purpose

The study of complex numbers involves investigating and extending understanding of the real number system to include complex numbers. The use of complex numbers is integral to many areas of life and modern-day technology such as electronics. Knowledge of complex numbers enables exploration of the ways different mathematical representations inform each other, and the development of understanding of the relationship between algebra, geometry and the extension of the real number system. The study of complex numbers prepares students for further study in mathematics and its applications.

Key terms

- argument
- Cartesian form of a complex number
- complex conjugate root theorem
- complex number
- complex plane
- conjugate
- de Moivre's theorem
- dilation
- discriminant
- integer
- modulus
- modulus–argument form
- polar form
- polynomial
- principal argument
- proof
- quadratic equation
- rational number
- real number
- roots of unity
- rotation
- triangle inequality
- vector

Related Stage 5 Path content

- Algebraic techniques C
- Indices B
- Indices C
- Equations C
- Linear relationships C
- Non-linear relationships C
- Polynomials
- Functions and other graphs

- Trigonometry D
- Circle geometry

Sequencing advice

The content in the focus area Introduction to complex numbers builds on content in:

- Mathematics Extension 1 Year 12 Proof by mathematical induction
- Mathematics Extension 1 Year 12 Introduction to vectors
- Mathematics Extension 2 The nature of proof.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Plotting complex numbers on the complex plane relates to plotting points on the Cartesian plane of given coordinates, including those with coordinates that are not whole numbers.	Mathematics Stage 4 Linear relationships
Working with complex numbers involves manipulating algebraic expressions, using the difference of 2 squares, factorising quadratic expressions, using index laws and surds, solving quadratic equations, using simultaneous equations and working with polynomials.	Mathematics Stage 5 ▪ Algebraic techniques C ▪ Indices B ▪ Indices C ▪ Equations C ▪ Polynomials Mathematics Advanced Year 11 Working with functions Mathematics Extension 1 Year 11 Further work with functions
Working with complex numbers involves using the distance between 2 points and coordinate geometry methods.	Mathematics Stage 5 Linear relationships C
The quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ can be applied to solve quadratic equations where the solutions are complex numbers.	Mathematics Stage 5 Equations C Mathematics Advanced Year 11 Working with functions
The discriminant can be used to determine whether the solution to a quadratic equation is real or imaginary.	Mathematics Advanced Year 11 Working with functions
Working with complex conjugates is similar to rationalising denominators involving surds.	Mathematics Stage 5 Indices C Mathematics Advanced Year 11 Working with functions
Graphing regions and curves in the complex plane is connected to graphs and equations of non-linear relationships.	Mathematics Stage 5 ▪ Non-linear relationships C ▪ Functions and other graphs ▪ Circle geometry Mathematics Advanced Year 11 Working with functions
Circle geometry theorems may be used in graphing regions and curves in the complex plane.	Mathematics Stage 5 Circle geometry
The modulus function is an extension or	Mathematics Advanced Year 11 Working

Finding the correct argument of a complex number and converting between forms of complex numbers involves using trigonometry.	Mathematics Stage 5 Trigonometry D Mathematics Advanced Year 11 Trigonometry and measure of angles
Polynomial techniques can be combined with the complex conjugate root theorem, de Moivre's theorem and roots of complex numbers to solve problems.	Mathematics Extension 1 Year 11 Polynomials
The triangle inequality for complex numbers is a generalisation of the real number case.	Mathematics Extension 2 The nature of proof
Complex numbers can be represented as vectors.	Mathematics Extension 1 Year 12 Introduction to vectors Mathematics Extension 2 Further work with vectors
The trigonometric identities proven by de Moivre's theorem and the binomial theorem can be used to find the integral of the power of a trigonometric function.	Mathematics Extension 1 Year 11 The binomial theorem Mathematics Extension 1 Year 12 Further calculus skills
Complex numbers can be used in proofs, proving de Moivre's theorem by mathematical induction.	Mathematics Extension 2 The nature of proof
Complex numbers relate to, and are used extensively in: Fourier transformations; abstract algebra (groups, rings and field theory); Galois theory); complex analysis; algebraic number theory; probability theory.	Pure Mathematics (Tertiary) Statistics (Tertiary)
Complex numbers are used in signal processing in engineering and music production. Complex numbers are used in modelling and/or solving problems related to electromagnetism, electrical circuit analysis, control theory, quantum mechanics, and solving differential equations. Complex numbers are used for rotations in digital graphic design and game development.	Real-world applications

Considerations

- The inclusion of 0 as an element of the natural number set ($\mathbb{N}$) is not universally agreed upon. To refer to the set of positive integers, i.e. $\{1,2,3,\ldots\}$, one could use the notation $\mathbb{Z}^+$, while the set of non-negative integers, i.e. $\{0,1,2,3,\ldots\}$, could then be notated as $\mathbb{Z}^+ \cup \{0\}$.

- The complex plane is also referred to as the Argand plane or Argand diagram.
- When finding the square roots of a complex number $a+ib$, it is advisable to use the language 'find the square roots of $a+ib$' or carefully notate it as $(x+iy)^2=a+ib$, instead of $x+iy=\sqrt{a+ib}$.
- For some students it may be worth noting that the argument of a complex number, $arg(z)$, is a many-to-many relation, while the principal argument of a complex number, $Arg(z)$, is a many-to-one function, noting that 0 is not in the domain of either the $arg(z)$ relation or the $\mathrm{Arg}(z)$ function. This is not essential content.
- The proof of $r_1(\cos\theta_1+i\sin\theta_1)\times r_2(\cos\theta_2+i\sin\theta_2)=r_1r_2(\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2))$ requires the identities $\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$ and $\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$. This important result could serve as a precursor to de Moivre's theorem.
- Students could explore polar grids and the plotting of complex numbers in modulus–argument form on these grids to aid visualisation, particularly when working with roots of unity.
- Students could use technology to examine the points of intersection of the graphs of $x^2-y^2=a$ and $2xy=b$ when finding the square roots of $a+ib$ to visualise the relationship between the two square roots of a complex number.
- Students may be extended by exploring the fundamental theorem of algebra and its algebraic and geometric implications but this is not essential content.
- Students should recognise the ability to model rotations on the complex plane as an advantage of using complex numbers over Euclidean vectors.

Diversity of learners

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Teaching advice for Further integration

Content

§ Further integration

Teaching advice

Purpose

Calculus involves the study of how things change and provides a framework for developing quantitative models of change and deducing their consequences. This focus area involves the development of a broader range of techniques and strategies to solve complex problems related to differential equations and integration.

The study of calculus is important in developing students' knowledge, understanding and capacity to operate with and model change in situations involving a variety of functions, use algebraic and graphical techniques to analyse and solve problems and to predict future outcomes with relevance to, for example, Chemistry, Physics and the construction industry.

Key terms

- completing the square
- definite integral
- degree (polynomial)
- denominator
- distinct
- function
- linear
- identity
- indefinite integral
- integrand
- integration
- integration by parts
- partial fractions
- proof
- quadratic expression
- recurrence
- substitution
- t-formulas

Related Stage 5 Path content

- Algebraic techniques B
- Algebraic techniques C
- Indices B
- Indices C
- Trigonometry D

Sequencing advice

The content in the focus area Further integration builds on content in:

- Mathematics Advanced Year 12 Integral calculus
- Mathematics Advanced Year 12 Applications of calculus
- Mathematics Extension 1 Year 12 Further calculus skills

- Mathematics Extension 1 Year 12 Further applications of calculus.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Algebraic manipulation may be required to obtain an expression of a particular form prior to integrating.	Mathematics Stage 5 ▪ Algebraic techniques B ▪ Algebraic techniques C Mathematics Advanced Year 11 Working with functions
Use of index laws and manipulation of surds may be required to obtain an expression of a particular form prior to integrating.	Mathematics Stage 5 ▪ Indices B ▪ Indices C Mathematics Advanced Year 11 Working with functions
The product rule for differentiation is the basis for integration by parts.	Mathematics Advanced Year 11 Introduction to differentiation
Trigonometric identities and equations are used in integration techniques and applications of integration.	Mathematics Stage 5 Trigonometry D Mathematics Advanced Year 11 Trigonometric identities and equations Mathematics Extension 1 Year 11 Further trigonometry
Results involving integration are used in proofs and inequalities.	Mathematics Extension 2 The nature of proof
De Moivre's theorem can be used to prove trigonometric identities, which can then be used to integrate powers of trigonometric functions.	Mathematics Extension 2 Introduction to complex numbers
The complex number i can be used to decompose partial fractions with irreducible quadratic denominators that don't contain an x term.	Mathematics Extension 2 Introduction to complex numbers
Integration techniques can be used to solve mechanics problems.	Mathematics Extension 2 Applications of calculus to mechanics
Integration is used extensively in probability theory and statistics, for example, integration by parts can be used to find the mean and variance of a continuous probability density function (PDF).	Mathematics Advanced Year 12 Random variables Real-world applications
Integration is used in solving differential equations,	Mathematics Extension 1 Year 12

vector calculus and multi-variable calculus.	Further applications of calculus Mathematics Extension 1 Year 12 Introduction to vectors Tertiary mathematics
Integration is used in convolution of two signals in digital signal processing in electrical engineering. Integration is used in fluid dynamics in physics and engineering.	Real-world applications

Considerations

- Students may benefit from a review of all calculus skills prior to studying this focus area, including integration by a given substitution as well as the use of trigonometric identities, studied as part of the Mathematics Advanced and Mathematics Extension 1 courses.
- Students may find it helpful to be familiar with the following derivatives and their associated integrals to aid the evaluation of the integrals encountered in this focus area:
  - $\frac{d}{dx}\left[\sqrt{f(x)}\right]=\frac{f'(x)}{2\sqrt{f(x)}}$ and consequently $\int \frac{f'(x)}{2\sqrt{f(x)}}dx=\sqrt{f(x)}+c$
  - $\frac{d}{dx}\left[\frac{1}{f(x)}\right]=-\frac{f'(x)}{\left[f(x)\right]^2}$ and $\int \frac{f'(x)}{\left[f(x)\right]^2}dx=-\frac{1}{f(x)}+c$
  - $\frac{d}{dx}(cosec\,x)=-cosec\,x\cot x$ and $\int cosec\,x\cot x\,dx=-cosec\,x+c$
  - $\frac{d}{dx}(sec\,x)=sec\,x\tan x$ and $\int sec\,x\tan x\,dx=sec\,x+c$
  - $\frac{d}{dx}(\cot x)=-cosec^2 x$ and $\int cosec^2 x\,dx=-\cot x+c$
- While working with $\int \sin^2 x\,dx$ and $\int \cos^2 x\,dx$, it may be useful to also consider or review $\int sec^2 x\,dx$, $\int cosec^2 x\,dx$, $\int \tan^2 x\,dx$ and $\int \cot^2 x\,dx$ at the same time.
- Another useful technique when working with recurrence relations where I_n is related to $I_{n\pm1}$ is to group these terms on one side and consider the integral formed by $I_n \pm I_{n\pm1}$. A similar approach could be used for those recurrence relations where I_n is related to $I_{n\pm2}$, especially if trigonometric functions are involved, as the integral formed by considering $I_n \pm I_{n\pm2}$ often contains Pythagorean trigonometric identities that can be simplified to aid the evaluation of the integral.
- Students may benefit from, but are not expected to, recognise and use the results $\int \frac{dx}{\sqrt{x^2+a^2}}=\ln\left(x+\sqrt{x^2+a^2}\right)+c$ and $\int \frac{dx}{\sqrt{x^2-a^2}}=\ln\left(x+\sqrt{x^2-a^2}\right)+c$ (where $x>a$). These results can be shown using a trigonometric substitution, however, the result for $\int sec\,\theta\,d\theta$ is required.
- Students may benefit from, but are not expected to, recognise that t-formula substitutions can differ from the standard definition of $t=\tan\frac{\theta}{2}$ so long as the double/half-angle relationships are observed. However, when defined differently, e.g. 'Let $t=\tan\theta$', the definition of t must be clearly stated and the derivation of $\frac{d\theta}{dt}=\frac{1}{k(1+t^2)}$ from the definition $t=\tan(k\theta)$ should be shown.
- Alternative methods for partial fraction decomposition which may benefit students include the Heaviside cover-up method, and substituting simple complex numbers.

- The method of integration by parts can be derived from the product rule for differentiation.
- Students may explore the use of a tabular method when integrating by parts.

Diversity of learners

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- Students with disability may require adjustments including access to visual or digital aids to communicate and demonstrate knowledge, understanding and skills.
- Students for whom English is an additional language or dialect (EAL/D) may require additional support and targeted strategies in learning language specific to mathematics.
- The pace and complexity of learning may need to be adjusted for students who progress at different rates.

Teaching advice for Applications of calculus to mechanics

Content

§ Applications of calculus to mechanics

Teaching advice

Purpose

Mechanics involves the study of change in the motion of objects when acted upon by forces. A knowledge of mechanics enables understanding of the behaviour of objects according to mathematical law to model physical systems and predict the behaviour of objects that are under the influence of forces such as gravity and air resistance.

Key terms

- acceleration
- amplitude
- component
- constant
- displacement
- force
- gravity
- magnitude
- mass
- Newton's laws of motion
- oscillate
- period
- projectile
- projection (vectors)
- rectilinear
- resisted motion
- resistive force
- simple harmonic motion
- speed
- terminal velocity
- trajectory
- vector
- velocity

Related Stage 5 Path content

- Variation and rates of change A
- Variation and rates of change B
- Algebraic techniques B
- Algebraic techniques C
- Equations C
- Non-linear relationships C
- Logarithms
- Trigonometry D

Sequencing advice

The content in the focus area Applications of calculus to mechanics builds on content in:

- Mathematics Advanced Year 12 Further graph transformations and modelling
- Mathematics Advanced Year 12 Integral calculus
- Mathematics Advanced Year 12 Applications of calculus
- Mathematics Extension 1 Year 12 Further calculus skills
- Mathematics Extension 1 Year 12 Further applications of calculus
- Mathematics Extension 1 Year 12 Introduction to vectors.

Making connections

Connections	Course/Stage/Focus area/Post-school paths
Solving mechanics problems may involve algebraic manipulations, rearranging equations, using logarithms to solve equations, exact values of trigonometric ratios and trigonometric identities.	Mathematics Stage 5 ▪ Algebraic techniques A ▪ Algebraic techniques B ▪ Equations C ▪ Logarithms ▪ Trigonometry D Mathematics Advanced Year 11 ▪ Working with functions ▪ Exponential and logarithmic functions ▪ Trigonometry and measure of angles ▪ Trigonometric identities and equations
Mechanics involves considering proportionality and interpreting rates of change.	Mathematics Stage 5 ▪ Variation and rates of change A ▪ Variation and rates of change B Mathematics Advanced Year 11 Introduction to differentiation Mathematics Advanced Year 12 Applications of calculus Mathematics Extension 1 Year 12 Further applications of calculus
Simple harmonic motion involves transformations of trigonometric functions and trigonometric ratios of sums and differences of angles.	Mathematics Advanced Year 12 Further graph transformations and modelling Mathematics Extension 1 Year 11 Further trigonometric identities Mathematics Extension 1 Year 11 Further trigonometry
The projectiles moving subject to a resistance have a trajectory which is different to those moving only under the influence of gravity.	Mathematics Extension 1 Year 12 Introduction to vectors
Equations of motion encountered in problems are modelled using differential equations.	Mathematics Extension 1 Year 12 Further applications of calculus
Integration is foundational to solving the differential equations formed in mechanics problems.	Mathematics Extension 1 Year 12 Further applications of calculus Mathematics Extension 2 Further integration
Vectors are used to represent displacement, velocity and forces in mechanics problems.	Mathematics Extension 1 Year 12 Introduction to vectors Mathematics Extension 2 Further work with vectors
Mechanics problems involving forces are	Real-world applications

encountered in engineering. Biomechanics is the study of the effects of forces on the body and is used in physics, medicine, exercise science and engineering. The Laws of Motion and forces are used in fluid dynamics in physics, engineering and meteorology. Newton's laws are used to model planetary motion in astronomy. Quantum mechanics in physics and computer engineering builds on classical mechanics.	

Considerations

- In this focus area students explore problems involving statics (working with objects at rest or moving with constant velocity) and dynamics (working with objects with acceleration). Problems in dynamics can be divided into kinematics (working on motion of objects, which includes displacement, velocity and acceleration) and kinetics (study of relationships between motion and its causes, which include forces).
- Newton's laws of motion should be given to illustrate the application of calculus to the physical world.
  - Newton's First Law (Law of Inertia): an object remains in a state of rest or uniform motion in a straight line unless it is compelled to change that state by the action of an external force.
    - Only an unbalanced force can cause acceleration.
    - The resultant force is that which generates the equation of motion.
  - Newton's Second Law: the rate of change of momentum (mv) is proportional to the applied resultant force and occurs in the direction of the force.
    - From this second law, $F \propto \frac{d(mv)}{dt}$, hence $F = km\left(\frac{dv}{dt}\right) = km\ddot{x}$. Through the choice of a suitable unit of force k can be set as $k = 1$ and hence $F = m\ddot{x} = ma$.
    - The unit of force is a newton (N) which represents the force required to give a 1 kg mass an acceleration of 1 m/s^2. 1 kg weight is approximately 9.8 N.
  - Newton's Third Law: the forces of action and reaction between contacting bodies are equal in magnitude and opposite in direction.
    - This law confirms the existence of the reaction force.
- The results $\ddot{x} = v\frac{dv}{dx}$ and $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ can be derived using the chain rule.
- Students should be able to mathematically represent motion described in physical terms. They should be able to explain in physical terms, aspects of the motion from mathematical descriptions of the motion.
- Students should examine graphs which show the displacement, velocity or acceleration of a particle moving in a horizontal line and describe the motion in words. Beginning with a graph of the motion as a function of time, students should be able to identify key aspects of the motion and draw related graphs of motion. For example, given a velocity–time graph, draw a possible acceleration–time graph or displacement–time graph.
- Essential syllabus content is restricted to problems involving constant mass.
- Simulations, animations and videos can be used to provide visual illustrations of the physical situations. Useful applets may be available online, and relevant data may be obtained from reliable online sources to model mechanics problems.

- Force is commonly measured in newtons (N). By Newton's second law, where $F = ma$, the unit for forces can also be expressed as kgm/s^2 or $kgms^{-2}$. The use of SI units (international system of units) should be discussed to support students' understanding.
- The difference between resistance and retardation needs clarification. To avoid confusion, it is advised that the term 'deceleration' not be used. Discussions regarding the meanings and significance of negative displacements, velocities and accelerations would be beneficial as well.
- Whether an object is 'speeding up' or 'slowing down' depends on its velocity and acceleration. When describing motion, velocity and acceleration go hand-in-hand. It is possible for an object to experience zero velocity but a non-zero acceleration, and it is possible for an object to experience zero acceleration but a non-zero velocity. These scenarios should be explored and discussed.

Diversity of learners

- Teachers should consider the accessibility of tools and resources. Considerations for tools such as spreadsheets, calculators and applications may include compatibility with screen readers and other assistive technologies including alternative input devices and the use of accessible calculators (see approved calculators). Considerations for resources may include the language demands, readability of and long descriptions for images and providing alternative text for graphs and diagrams.
- Students with disability may require adjustments including access to visual or digital aids to communicate and demonstrate knowledge, understanding and skills.
- Students for whom English is an additional language or dialect (EAL/D) may require additional support and targeted strategies in learning language specific to mathematics.
- The pace and complexity of learning may need to be adjusted for students who progress at different rates.

Glossary

Mathematics Advanced 11–12 (2024) Terms

absolute value

The magnitude or size of a real number, i.e. the distance of the number from the origin on a number line.

Formally, it has the piecewise definition $|x| = \begin{pmatrix} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{pmatrix}$.

acceleration

The rate at which velocity changes over time.

acute angle

An angle smaller than a right angle, between 0° and 90°.

Extended in Mathematics Advanced and Extension 1: In circular measure, an angle lying in the interval $\left[0, \frac{\pi}{2}\right)$.

algebraic expression

A mathematical statement formed by combining numbers and algebraic symbols using arithmetic operations, e.g. $a^2 + 3ab - 2b^2$.

algebraic fraction

A fraction in which the numerator and/or the denominator are algebraic expressions.

ambiguous case (of the sine rule)

In trigonometry, refers to using the sine rule to calculate the size of an angle in a triangle where there are two possibilities for the angle, one obtuse and one acute, leading to two possible triangles. This occurs because, for an acute angle θ, $\sin(180° - \theta) = \sin\theta$.

amplitude

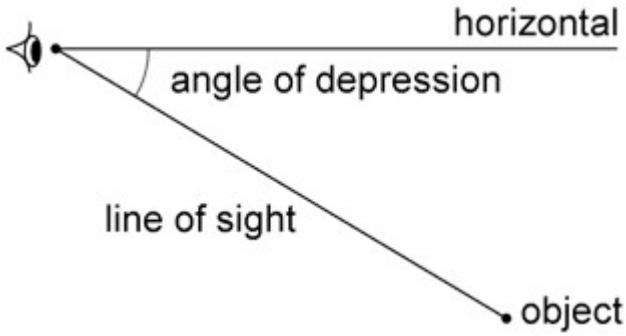
A function of the form $y = A\sin(nx + \alpha)$ or $y = A\cos(nx + \alpha)$ has amplitude A, that is half the distance between the maximum and minimum values.

angle

Formed by 2 straight lines meeting at a common endpoint, called the vertex. An angle can describe the amount of turn between its 2 arms (lines).

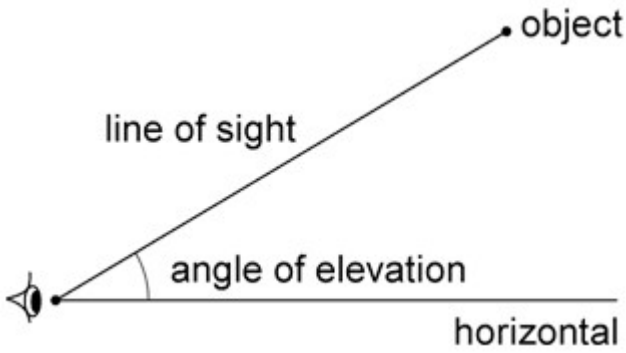
angle of depression

The angle between horizontal and the line of sight from an observer to an object that is lower than the observer.



angle of elevation

The angle between horizontal and the line of sight from an observer to an object that is higher than the observer.



angle of inclination

The angle a straight line makes with the positive x-axis.

annuity

A compound interest investment to which payments are made, or a single sum invested from which payments are received, on a regular basis for a fixed period of time.

arc

A part of a circle's circumference.

Extended in Mathematics Extension 1: A part of a curve.

arc length

The distance between two points on a curve. In a circle the length of an arc is given by $l = r\theta$, where l is the arc length, r is the radius and θ is the angle subtended at the centre, measured in radians.

area of a triangle

Calculated using $A = \frac{1}{2}bh$, where b is the base length and h is the perpendicular height. For a triangle ABC with sides a and b about the angle C, the area of the triangle is $A = \frac{1}{2}ab\sin C$.

arithmetic progression (AP)

A sequence of numbers in which the difference between any two successive terms of the sequence is constant. Also known as arithmetic sequence.

arithmetic sequence

A sequence of numbers in which the difference between any two successive terms of the sequence is constant. Also known as arithmetic progression.

array

One of several different arrangements that can be used to model multiplicative situations. It is made by arranging a set of objects, such as counters, into columns and rows. Each column must contain the same number of objects as the other columns, and each row must contain the same number of objects as the other rows.

association

Relationship between pairs of numerical variables e.g. in terms of strength, form and direction.

asymptote

A straight line (or another curve) that a curve approaches as x tends to $\pm\infty$, or to some particular value.

For example, the curve $y=2^x$ has an asymptote $y=0$ as x tends to -∞, and the curve $y=\frac{1}{x}$ has asymptotes $x=0$ and $y=0$ as x tends to 0 and $\pm\infty$ respectively.

average

A number expressing a central or typical value in a set of data. While it usually refers to the arithmetic mean, that is, the sum of a set of numbers divided by the number of numbers in the set, it may also refer to other measures of centre.

average rate of change

The change in one quantity divided by the corresponding change in another quantity. For a function $y=f(x)$ over the domain $[a,b]$, the average rate of change of y with respect to x is, $\frac{\text{change in } y}{\text{change in } x}=\frac{\Delta y}{\Delta x}=\frac{f(b)-f(a)}{b-a}$.

axis of symmetry

A straight line that divides a shape or curve into two so that one side is a reflection of the other in the given line.

bearing

A direction from one point on the Earth's surface to another. Two types of bearings may be used: compass bearing and true bearing.

See compass bearing and true bearing

break-even point

The point at which income from production and cost of production are equal.

Cartesian coordinate system

A system that specifies the position of each point in a plane, formed by the intersection of 2 perpendicular number lines called axes. Each point is represented by an ordered pair $(x,y)$of real numbers called the coordinates of the point. The point where the axes meet is called the origin and has coordinates $(0,0)$.

chain rule

A formula for the derivative of the composite of two differentiable functions. If y is a function of u, and u is a function of x, then the chain rule states that $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$. If $h(x)$ is the composite function $f(g(x))$, then $h'(x)=f'(g(x))g'(x)$.

change of base rule

A rule for writing a logarithm with a particular base as the ratio of two logarithms with a different base: $log_a x=\frac{log_b x}{log_b a}$.

circle

All points equal to a given distance from a fixed point, the centre. When used to describe a shape, a circle includes all points inside the boundary.

circumference

The boundary of a circle. The length of the circumference C is given by $C=\pi d$, where d is the diameter. Alternatively, it is given by $C=2\pi r$, where r is the radius.

coefficient

A numerical quantity that multiplies a variable in an algebraic expression, e.g. 5 is the coefficient of $5x$. Variables with no specified coefficient have a coefficient of 1.

common difference

The constant difference between successive terms of an arithmetic sequence.

common ratio

The constant ratio between successive terms of a geometric sequence.

compass bearing

Angles either side of north or south. For example, a compass bearing of N50°E is found by facing north and moving through an angle of 50° to the east.

complement

All of the outcomes that are not in a given event. All the elements of a set that are not in a given subset.

complementary angles

Two adjacent angles that form a right angle, i.e. the sum of the angles measured in degrees is 90°.

complementary events

Two mutually exclusive outcomes in a probability experiment, such that $P(A)+P(B)=1$ where $P(A)$ is the probability of event A and $P(B)$ the probability of event B.

Extended in Mathematics Standard and Advanced: Two mutually exclusive events that make up the sample space.

completing the square

Rearranging a quadratic expression $a x^2 + b x + c$ into the form $(p x + q)^2 + r$.

composite functions

When the output of one function becomes the input of a second function.

For example, $f(g(x))$(read as'f of g of x') is a composite function where the outputs of function g are taken as the inputs of function f.

compound interest

The interest earned by investing a sum of money (the principal) when each successive interest payment is added to the principal for calculating the next interest payment.

concave down

A function in which all points on a graph between any two given points on the graph lie on or above the chord joining the given points.

concave up

A function for which all points on a graph between any two given points on the graph lie on or below the chord joining the given points.

conditional probability

Consideration of whether the knowledge of the occurrence of one event, A, affects the probability of occurrence of another event, B.

Extended in Mathematics Advanced: The probability of an event A occurring, given that another event B has occurred, notated as $P(A|B)$ (read as 'probability of A given B').

conjugate

For a surd $a\sqrt[n]{b} + x\sqrt[n]{y}$, where a, b, x and y are real numbers, the conjugate is $a\sqrt[n]{b} - x\sqrt[n]{y}$.

In complex numbers, the conjugate of $z = x + i\,y$ is $\overline{z} = x - i\,y$. z and $\overline{z}$ are complex conjugates.

constant

A fixed numerical value. For example, in the algebraic expression $x + 11$, the number 11 is a constant.

constant function

A function that has only one value.

continuous function

The graph of a continuous function is an unbroken curve. It can be drawn without lifting the pen off the paper.

continuous random variable

A continuous random variable is a numerical variable that can take any value along a continuum.

contribution

An amount of money that is paid into an annuity. The contribution can be made as a lump sum or as a series of payments.

converse

If one relationship is true, then the opposite is also true.

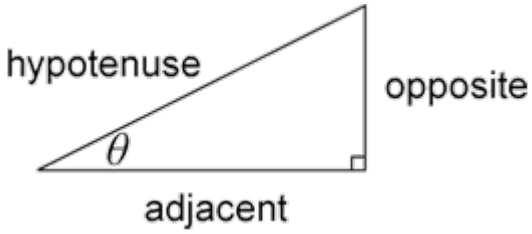
The converse of a statement 'If P then Q' is 'If Q then P'. For example, the converse of Pythagoras' theorem is that if the sum of the squares of 2 sides of a triangle is equal to the square of the third side, then the triangle is right-angled.

Extended in Mathematics Extension 2: The converse of $P \rightarrow Q$ is $Q \rightarrow P$ or $P \leftarrow Q$.

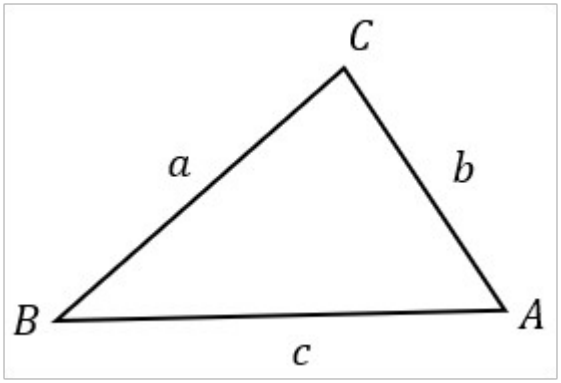
cosecant ratio

For an angle, its cosecant is the reciprocal of its sine, $cosec\,\theta = \frac{1}{\sin\theta}$.

In any right-angled triangle, $cosec\,\theta = \frac{hypotenuse}{opposite}$ where $0 < \theta < 90°$.



cosine rule



A formula $c^2 = a^2 + b^2 - 2ab\cos C$ that relates the lengths of the sides of a triangle to the cosine of one of its angles.

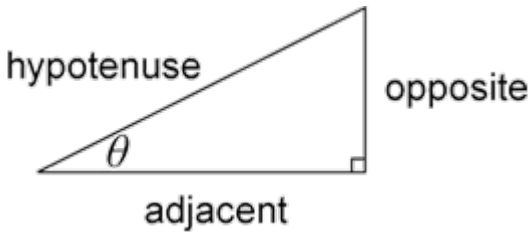
cost

The price paid to acquire, produce, accomplish or maintain anything.

cotangent ratio

For an angle, its cotangent is the reciprocal of its tangent, $\cot\theta = \frac{1}{\tan\theta}$.

In any right-angled triangle, $\cot\theta = \frac{adjacent}{opposite}$ where $0 < \theta < 90°$.



cubic function

A polynomial function of degree 3, that is, a function of the form $f(x) = a x^3 + b x^2 + c x + d$.

cumulative distribution function (CDF)

For a continuous random variable X, the cumulative distribution function (CDF), given by the function $F(x)$, is $F(x) = P(X \leq x)$.

cumulative frequency

The accumulating total of frequencies within an ordered dataset.

cumulative frequency histogram

A visual representation of data using bars to represent the class boundaries and the cumulative frequencies.

The area of each bar is proportional to the cumulative frequency of the observations up to the end of that class.

cumulative frequency polygon

A series of straight lines representing the cumulative frequency for a given dataset. Sometimes called the ‘ogive’.

degree (networks)

The degree of a vertex is the number of edge ends at that vertex.

degrees (geometry)

A unit for measuring an angle. Angles are measured as a proportion of a full turn, which is equivalent to 360 degrees, so that one degree, written as $1°$ is equal to $\frac{1}{360}$ of a full turn.

denominator

Comes from Latin, meaning ‘that which gives a name’ [de- “completely”; nomen “name”]. The denominator of a fraction identifies the name of the fractional parts (e.g. thirds, quarters or fifths).

Extended in 7–10: In the fraction $\frac{a}{b}$, *b* is the denominator.

dependent variable

The variable used to represent the output values of a function. A dependent variable is generally represented on the vertical axis of a graph.

derivative

The result obtained after differentiation. For the function $f(x)$, the derivative is the gradient function of $f(x)$, and is denoted $f'(x)$. The derivative of $f(x)$ from first principles is given by $f'(x)=lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. The derivative of y with respect to x is denoted $\frac{dy}{dx}$ or y'.

differentiable

A function which can be differentiated at each point in its domain.

A function $f(x)$ is differentiable at $x=a$ if $lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ exists, that is if there is a non-vertical tangent to the graph of $f(x)$ at the point $P(a,f(a))$.

differentiation

The process used to find the derivative of a function.

dilation

A process of stretching or compressing the graph of a function. This could happen either in the x or y direction or both.

direct variation

A proportional relationship where one quantity directly varies with respect to a change in another quantity. This implies that if there is an increase (or decrease) in one quantity then the other quantity will experience a proportionate increase (or decrease).

discontinuity

A point at which the graph of a function is broken (not continuous) and the function is undefined.

discrete probability distribution

Specifies the probabilities with which a discrete random variable takes each of its values.

discrete random variable

A numerical variable whose values can be listed.

discrete variable

Individual and countable items that can be listed.

Extended in Mathematics Advanced: Also known as a discrete random variable.

discriminant

In the quadratic expression ax^2+bx+c, the discriminant $\Delta=b^2-4ac$

disjoint (sets)

Two sets which do not have any common elements.

displacement

The change in position of an object, after a period of time, from its original position. Displacement is a vector quantity. The displacement may be positive, negative or zero.

distance

The length between two points. Distance is a positive scalar quantity.

distance–time graph

A line graph that relates distance and time, with time on the horizontal axis and distance on the vertical axis.

distinct

Different, not equal.

domain

The set of allowable values of x in a function or relation.

Extended in Mathematics Advanced, Extension 1 and Extension 2: For a function or relation, it is the set of real numbers on which the function or relation is defined.

element(s)

A member of a set. For example, 3 is a member of the set of natural numbers $N=\{0,1,2,3,4,\ldots\}$. This relation can be written more concisely as $3\in N$ ('3 is an element of the set N').

empirical rule (for a normally distributed random variable)

The empirical rule for normally distributed random variables is:

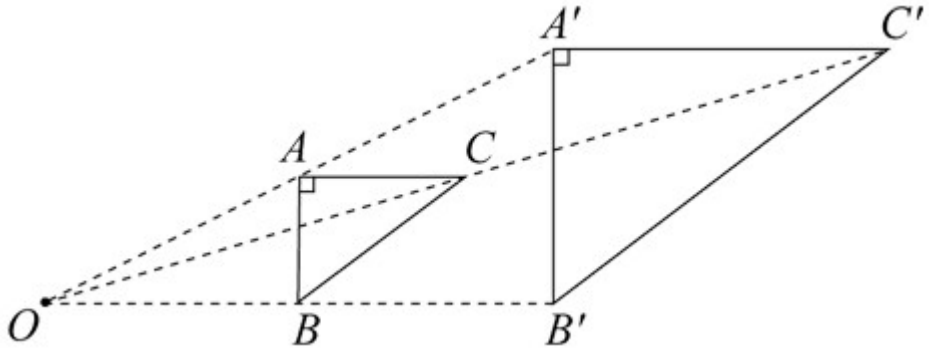
- approximately 68% of data will have z-scores between −1 and 1
- approximately 95% of data will have z-scores between −2 and 2
- approximately 99.7% of data will have z-scores between −3 and 3.

empty set

The unique set characterised by the property that it has no elements at all.

enlargement

A type of transformation that increases or decreases the size of an image without changing its shape, depending on the size of the enlargement factor.



equal

If two quantities or mathematical expressions are equal, then the quantities have the same value, or the expressions represent the same mathematical object.

If $a=b$ then either may be substituted for the other.

For $\sin\theta = \frac{opposite}{hypotenuse}$ and $f(x) = 3x - 1$, the '=' sign indicates assignment by definition.

equally likely outcomes

An event for which all outcomes have the same probability of occurring. For example, in tossing a fair coin, the outcome 'head' and the outcome 'tail' are equally likely. In this situation, P(head)=P(tail)=0.5.

equation

An expression showing the equality of 2 quantities, using the = sign between them. A mathematical formula asking for a solution so that the 2 expressions in that variable are equal, e.g. $x^2 - 1 = x$.

equivalent

Two things are equivalent if they have the same value.

See equal, equals sign.

Extended in Mathematics Advanced, Extension 1 and Extension 2: Two things are equivalent if they have the same value, function or meaning. For example, $y = a^x$ and $x = \log_a y$ are equivalent. Statements P and Q are equivalent if P implies Q and Q implies P.

Euler's number

An irrational number, $e \approx 2.71828182845\ldots$, which can be defined in various ways including: $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$ and $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$. It has the property $\frac{d}{dx} e^x = e^x$.

even function

A function is even if its graph is unchanged under reflection in the y-axis.

An even function $f(x)$ has the property $f(-x) = f(x)$, for all values of x in the domain.

event

Situations that occur in the everyday context, e.g. having lunch. In the context of probability, the set of possible outcomes.

Extended in 7–10: A subset of the sample space for a random experiment. For example, the set of possible outcomes from tossing 2 coins is $\{HH, HT, TH, TT\}$, where H represents a 'head' and T is a 'tail'. For example, if A is the event 'at least one head is obtained', then $A = \{HH, HT, TH\}$.

exact value

A precise numerical value e.g. $\sqrt{3}$ as opposed to the approximation 1.732.

expected value

In statistics, the expected value $E(X)$ of a random variable X is a measure of the central tendency of its distribution. Also known as the expectation or mean. $E(X)$ is calculated differently depending on whether the random variable is discrete or continuous.

experiment (random)

A process with an observable result. The results of the experiment are the set of possible outcomes, called the sample space.

exponential decay

Occurs when the rate of change of a mathematical function is negative and proportional to the function's current value.

exponential function

A function of the form $y = a^x$, where x is the independent variable and the base $a > 0$. The graph of an exponential function is an exponential curve.

exponential growth

Occurs when the rate of change of a mathematical function is positive and proportional to the function's current value.

expression

Two or more numbers or variables connected by operations. For example, $17 - 9$, $8 \times (2 + 3)$, $2a + 3b$ are all expressions. Expressions do not include an equals sign.

Extended in Mathematics Advanced, Extension 1 and Extension 2: The result of combining numbers, variables and operations in a meaningful way. For example, $\frac{x^3 - 1}{\sin y}$ is an expression, but $(x^3 -) \int / \sin y$ is not a validly formed expression.

factorise

To express a number or algebraic expression as a product. For example, 15 is factorised when expressed as a product: $15 = 3 \times 5$ and $x^2 - 3x + 2$ is factorised when written as a product: $x^2 - 3x + 2 = (x + 1)(x - 2)$.

finite

A value between $-\infty$ and ∞.

frequency

The number of times that a particular value occurs in a dataset. For grouped data, it is the number of observations that lie in that group or class interval. For example, when rolling a dice 20 times, 'the frequency of a 6' means how many times the number 6 comes up.

function

A function f assigns to each element of one set S precisely one element of a second set T.

The functions most commonly encountered in elementary mathematics are real functions of real variables. For such functions, the domain are sets of real numbers.

Functions are usually defined by a formula for $f(x)$ in terms of x. For example, the formula $f(x) = x^2$, defines the 'squaring function' that maps each real number x to its square x^2.

Fundamental Theorem of Calculus

Relates differentiation and integration.

For a continuous function f on the interval $[a, b)$ it has two equivalent forms:

for $x \leq b$,

$$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$$

and $\int_a^b f(x)\,dx = F(b) - F(a)$,

where F is any antiderivative of f.

future value

The total value of an investment or annuity at the end of a specified term, including all contributions and interest earned.

general form (of a straight line)

A straight line in general form is given as $ax + by + c = 0$, where a, b and c are constants.

geometric progression (GP)

See geometric sequence

geometric sequence

A sequence of numbers where each term after the first is found by multiplying the previous term by a fixed number called the common ratio. Sometimes called a geometric progression (GP).

geometric series

A geometric series is a sum whose terms form a geometric sequence.

global maximum

The largest value of the function on its domain.

global minimum

The smallest value of the function on its domain.

gradient

The slope of a line. It is calculated as the gradient of a line segment it contains. If $A(x_1, y_1)$ and $B(x_2, y_2)$ are 2 distinct points on a line, the gradient of the line (or line segment AB) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

gradient–intercept form

$y = mx + c$ is the gradient–intercept form of a straight line, where m is the gradient of the line and $(0, c)$ is the point at which the line intersects the y-axis. c is called the y-intercept.

graph

A visual representation of statistical data or of a relationship between variables. Ordered pairs of values (x, y) that represent the function or relation are plotted to form a graph. Graphs of statistical data include dot plots, box plots, column graphs, divided bar graphs and histograms.

Extended in Mathematics Advanced, Extension 1, Extension 2: For a function f, its graph consists of all ordered pairs $(x, f(x))$ for x in the domain of f.

identity

An identity is a statement involving a variable(s) that is true for all possible values of the variable(s).

indefinite integral

The set of all primitives of a continuous function f notated $\int f(x)\,dx$. So, $\int f(x)\,dx = F(x) + C$ where C is an arbitrary constant of integration, and $F'(x) = f(x)$.

independent events

Two events are independent if knowing the outcome of one event tells us nothing about the outcome of the other event.

independent variable

A variable used to represent values in the domain (input values) of a function. Generally represented on the horizontal axis of a graph.

index

Also called 'exponent'. The power to which a number or algebraic expression is to be raised. The index or exponent is written as a superscript. Positive integral exponents indicate the number of times a term is to be multiplied by itself.

indices

The plural of the term index.

inequality

A statement that one number or algebraic expression is less than (or greater than) another. There are 4 types of inequalities:

- a is less than b is written $a < b$
- a is greater than b is written $a > b$
- a is less than or equal to b is written $a \leq b$
- a is greater than or equal to b is written $a \geq b$.

Extended in Advanced, Extension 1, Extension 2: An order relation between one number or algebraic expression and another.

infinite

Limitless, exceeding any finite amount.

See finite

instantaneous rate of change

The rate of change at a particular moment. For a differentiable function, the instantaneous rate of change at a point is its derivative at that point, so it equals the gradient of the tangent to its graph at the point.

integer

A whole number, positive, negative or zero, e.g. −3, −2, −1, 0, 1, 2 …

Extended in Advanced, Extension 1, Extension 2: The set of integers is usually denoted by $\mathbb{Z}$.

integration

The process of finding the integral of a function. The inverse of differentiation.

intercept

The point at which a curve or function crosses an axis or other curve in a plane. The point at which a curve crosses the x-axis $(y=0)$ is called the x-intercept and the point at which a curve crosses the y-axis $(x=0)$ is called the y-intercept.

Extended in Advanced and Extension 1: The x- and y-intercepts are sometimes taken to mean the signed distance from the point at which the curve crosses the axis to the origin, e.g. for the line $y=mx+c$, the y-intercept is c.

interest

Money paid or received in return for using or lending money.

interest rate

A percentage at which interest is charged or paid.

intersection (sets)

For sets A and B, it is the set of elements that are in both A and in B. The intersection of A and B is denoted $A \cap B$.

interval notation

Notation for representing an interval on the real number line by its endpoints. Parentheses and/or square brackets are used respectively to show whether the endpoints are excluded or included.

For $a<b$, (a,b) is an open interval, and $[a,b]$ is a closed interval.

inverse variation

When one variable increases as the other variable decreases. For example, if y is said to be 'inversely proportional' to x, the equation is of the form $y=\frac{k}{x}$, where k is a constant of variation (or proportion). Also known as inverse proportion.

investment

An allocation of money or capital into an asset or venture which is expected to generate a profit or return.

irrational number

A number that cannot be expressed as a fraction in the form $\frac{m}{n}$, where m and n are integers and n is non-zero. The decimal form of irrational numbers does not terminate and is non-recurring.

linear

Something that can be represented or modelled by a straight line.

linear equation

An equation involving linear expressions. The general form of a linear equation in one variable is $ax+b=c$ where a, b and c are constants.

linear function

y is a linear function of x if $y=mx+c$ where m and c are constants.

The graph of a linear function is a straight line, where m is the gradient and c is the y-intercept.

See linear relationship

loan

An amount of money borrowed from a bank or other financial institution.

local maximum

A point on a function is a local maximum if its function value is larger than the function values near it.

local minimum

A point on a function is a local minimum if its function value is smaller than the function values near it.

logarithm

The logarithm of a positive number x is the power to which a given number b, called the base, must be raised in order to produce the number x. The logarithm of x, to the base b is denoted by $\log_b x$.

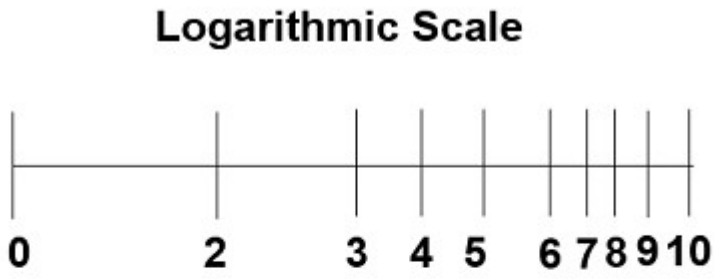
Algebraically: $\log_b x = y$ is equivalent to $b^y = x$.

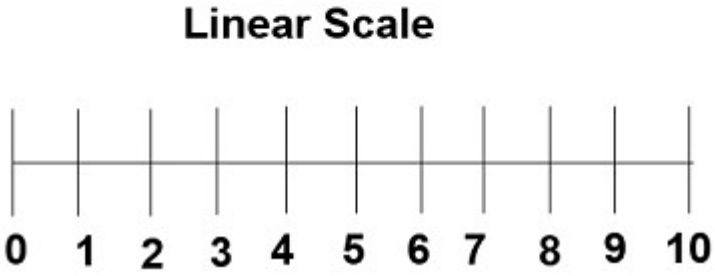
logarithmic function

A function, such as $y=\log_a x$, which is the inverse of the exponential function, $y=a^x$. Here x is the independent variable and $a>0$ is the base of the logarithm.

logarithmic scale

A scale where successive endpoint values of intervals increase by a constant factor (multiplicatively). Contrast with linear scales in which the increase is a constant amount.





magnitude

The size or absolute value of a number. For example, +4 and -4 have a magnitude of 4.

Extended in Mathematics Extension 1 and Mathematics Extension 2: For a vector, the magnitude is its length, given by $|x\,i + y\,j) = \sqrt{x^2 + y^2}$ for 2D vectors and $|x\,i + y\,j + z\,k) = \sqrt{x^2 + y^2 + z^2}$ for 3D vectors.

mean

The sum of values in a dataset divided by the total number of values in the dataset. Also called the average.

measure of centre

A statistic that is used to summarise a dataset. There are 3 common measures of centre for a dataset: mode, median and mean.

median

The value in a set of ordered data that divides the data into 2 parts. It is frequently called the 'middle value'.

Extended in Mathematics Standard and Advanced: If the number of values in the ordered dataset is even, it is the average of the two middle values. If the number of values in the dataset is odd, the median is the middle value.

minute (geometry)

One sixtieth of a degree, also equivalent to sixty seconds. Often represented by the sign ($'$). For example, $14\,°\,25\,'$ (fourteen degrees and twenty five minutes).

mode

The most frequently occurring value in a set of data.

Extended in 7–10: There can be more than one mode. When there are 2 modes, the dataset is said to be bimodal.

model

A mathematical, conceptual or physical representation that describes, simplifies, clarifies or provides an explanation of the structure, workings or relationships within an object, system or idea. Models can provide a means of testing and predicting behaviour within limited conditions. Models may be physical or exist in digital form.

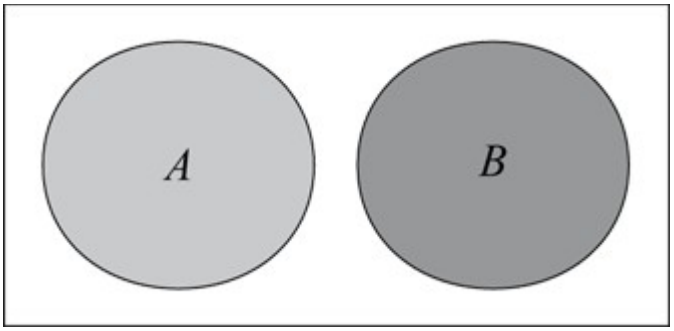
multistage events

An event that consists of two or more simple experiments. For example, tossing a coin three times (repeated trials), or both tossing a coin and rolling a dice (possible outcomes would be H and 5, T and 2).

mutually exclusive events

Two events that cannot have simultaneous outcomes in the same chance experiment. For example, when a fair coin is tossed twice, the events '*HH*' and '*TT*' cannot occur at the same time and are, therefore, mutually exclusive.

In a Venn diagram, as shown below, mutually exclusive events do not overlap.



Extended in Mathematics Advanced: For a random experiment two events A and B are mutually exclusive if $A \cap B = \varnothing$, that is, they cannot occur simultaneously.

natural logarithm

A logarithm to the base e. The logarithm of x to the base e is denoted as $\log_e x$ or $\ln x$.

normal (to a curve)

In calculus, the normal to a curve at a given point P is the straight line that is perpendicular to the tangent to the curve at a given point P.

normal distribution

A type of continuous distribution whose graph looks like this:

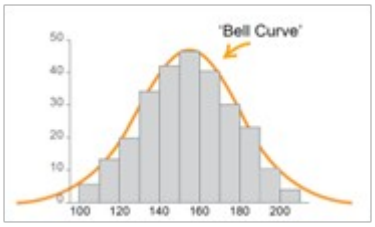


Image long description: The horizontal x-axis is labelled from 100 to 200 in increments of 20 from left to right. The y-axis is labelled from 0 to 50 in increments of 10 from the bottom to the top. Along the x-axis is a series of columns that rise to a peak halfway along the axis and fall back towards 0 on the far right of the x-axis. An orange 'bell curve' is laid over the peaks of the columns.

The mean, median and mode are equal and the scores are symmetrically arranged either side of the mean.

The graph of a normal distribution is often called a 'bell curve' due to its shape.

normal random variable

A variable which varies according to the normal distribution.

odd function

A function whose graph is unchanged under rotation of $180°$ about the origin.

An odd function $f(x)$ has the property $f(-x)=-f(x)$, for all values of x in the domain.

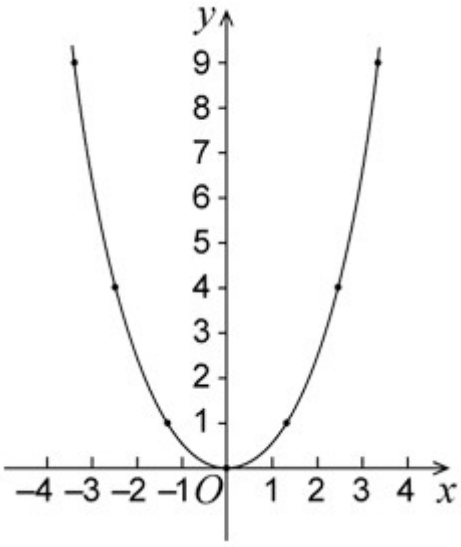
outcome

Possible result from an experiment or trial.

parabola

The graph of $y=x^2$. The point $(0,0)$ is called the vertex of the parabola and the y-axis is the axis of symmetry of the parabola.

Some other parabolas are the graphs of $y=ax^2+bx+c$ where $a\neq 0$.



parallel

Two distinct lines, rays or line segments in the same plane that have no points of intersection and so necessarily have the same gradient (slope).

The symbol $\parallel$ is used to express that one line, ray, or line segment is parallel to another.

partial sum of a sequence

The nth partial sum S_n of a sequence $a_1, a_2, \ldots$ is the sum of the first n terms of the sequence: $S_n = a_1 + a_2 + \ldots + a_n$, for all whole numbers $n \geq 1$.

period

A function f is periodic with period p if $f(x+p) = f(x)$, for all x, that is, the function repeats itself after each interval of length p.

For example, $\sin x$ and $\cos x$ have period 2π.

perpendicular

Two lines, rays, line segments, vectors, planes or other objects that intersect at a 90° angle (a right angle).

piecewise-defined function

A function which is defined differently for different parts of the domain. For example,

$$f(x) = \begin{cases} x^2, & \text{for } x \leq -1, \\ x - 1, & \text{for } -1 < x \leq 1, \\ -x^3, & \text{for } x > 1. \end{cases}$$

point of inflection

A point on a curve where the concavity changes. At a point of inflection, the tangent exists and crosses the curve.

primitive

For a function $f(x)$, a primitive is a function $F(x)$ whose derivative is $F'(x) = f(x)$. Primitive and antiderivative are synonyms.

probability

The chance of something happening shown on a scale from 0 and 1 (inclusive). For example, the probability that a fair coin toss will come up 'heads' is 0.5.

probability density function

For a random variable X, the probability density function (PDF), $f(x)$, is a function that describes the likelihood that the random variable takes a particular value, that is $P(a < X < b) = \int_a^b f(x)\,dx$.

probability distribution

For a random variable X, the probability distribution is the set of values x taken by the random variable, together with the probabilities $P(x)$ that $X = x$.

product rule

A rule for the derivative of the product of two differentiable functions: If $y = u\,v$ where u and v are both functions of x , then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$, or if $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$.

proof

A rigorous mathematical argument that demonstrates the truth of a given statement or proposition. A mathematical statement that has been established by means of a proof is called a theorem.

Pythagoras' theorem

The square of the length of the hypotenuse, c, of a right-angled triangle equals the sum of the squares of the lengths of the other 2 sides, a and b, such that $c^2 = a^2 + b^2$.

quadrant

A quarter of a circle.

Any one of the four parts into which the Cartesian plane is divided by the x- and y-axes. Thus, the first or positive quadrant consists of those points (x, y) with $x \geq 0$ and $y \geq 0$.

quadratic equation

An equation of the form $a x^2 + b x + c = 0$, where $a \neq 0$, b and c are constants.

quadratic expression

An expression of the form $a x^2 + b x + c$, where $a \neq 0$, b and c are constants.

quadratic formula

The roots of a quadratic equation $a x^2 + b x + c = 0$ where $a \neq 0$ are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

quadratic function

A function of the form $f(x) = a x^2 + b x + c$, where $a \neq 0$, b and c are constants.

quadratic inequality

An inequality reducible to inequalities of the form $a x^2 + b x + c < (\leq, >, \geq) 0$, where $a \neq 0$, b and c are constants.

quartiles

The values that divide an ordered dataset into 4 (approximately) equal parts. There are 3 quartiles. The first, the lower quartile (Q_1), divides off (approximately) the lower 25% of data values. The second quartile (Q_2) is the median. The third quartile, the upper quartile (Q_3), divides off (approximately) the upper 25% of data values.

quotient rule

A formula for the derivative of the ratio of two differentiable functions. If $y = \frac{u}{v}$, where u and v are both functions of x, then the quotient rule states that $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$, or if $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$.

radian

A unit of angular measure frequently used in mathematics.

1 radian is the angle between two radii of a circle which cut off on the circumference an arc equal to the radius.

radian measure

The size of an angle subtended by an arc of a circle in radian (or circular) measure is given by the ratio $\frac{\text{arc length}}{\text{radius of a circle}}$.

Radian measure is frequently employed in mathematics. For example, in the formulas of differentiation for trigonometric function; $\frac{d}{dx}\sin x = \cos x$ the values of x relate to radian measure.

random variable

A variable whose possible values are outcomes of a statistical experiment or a random phenomenon.

range (function)

The set of values of the dependent variable for which a function is defined.

rate of change

For a function, the relative change in a function's value $f(x)$ to a change in x. Instantaneous rate of change is measured by its derivative.

For example, the velocity of a moving object is the rate of change of its position over time.

rational number

An element of the infinite set of numbers $Q = \{\frac{m}{n}$, where m and n are integers and $n \neq 0\}$. It may be expressed in decimal form, e.g. $\frac{1}{8} = 0.125$ and $\frac{4}{9} = 0.444\ldots$

real number

A number which can be represented by a point on a number line. The set of real numbers is the set of all rational and irrational numbers.

reciprocal

If x is any real number, then the reciprocal of that number will be $\frac{1}{x}$. For example, the reciprocal of 4 is $\frac{1}{4}$.

reducing balance loan

A reducing balance loan is a compound interest loan where the loan is repaid by making regular payments and the interest paid is calculated on the amount still owing (the reducing balance of the loan) after each payment is made.

reduction

A type of transformation that decreases the size of an image by a factor without changing its shape.

reflection

A transformation of a shape formed by creating a mirror image on the other side of a given line.

relation

An association between the elements of one set, A, and the elements of another set, B. It may be represented as a set of ordered pairs (a, b), where a is in A, b is in B and b is related to a.

relative frequency

Given by the ratio $\frac{f}{n}$, where f is the frequency of occurrence of a particular data value or group of data values in a dataset, and n is the number of data values in the dataset.

repayment

A set amount, paid at regular intervals over the period of a loan, to pay off a loan.

revenue

The total income generated from normal business operations, such as from the sale of items produced.

root

The solution to an equation. For example, $(x-1)(x+3)=0$ has roots $x=1$ and $x=-3$.

rotation

A turn about a point through a specified angle. For example, a rotation about the origin through an anticlockwise angle of 45° transforms the line $y=0$ into the line $y=x$.

sample

A subset of a population used to estimate characteristics of the population. For example, a randomly selected group of 8-year-olds (sample) selected to estimate the height of 8-year-olds in Australia (population).

secant

The straight line passing through 2 points on the graph of a function.

secant (ratio)

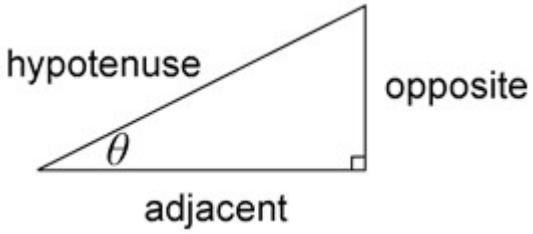
For an angle, the secant is the reciprocal of its cosine, $sec\,\theta = \frac{1}{\cos\theta}$.

In any right-angled triangle,

$sec\,\theta = \frac{hypotenuse}{adjacent}$

where

$0 < \theta < 90°$.



second derivative

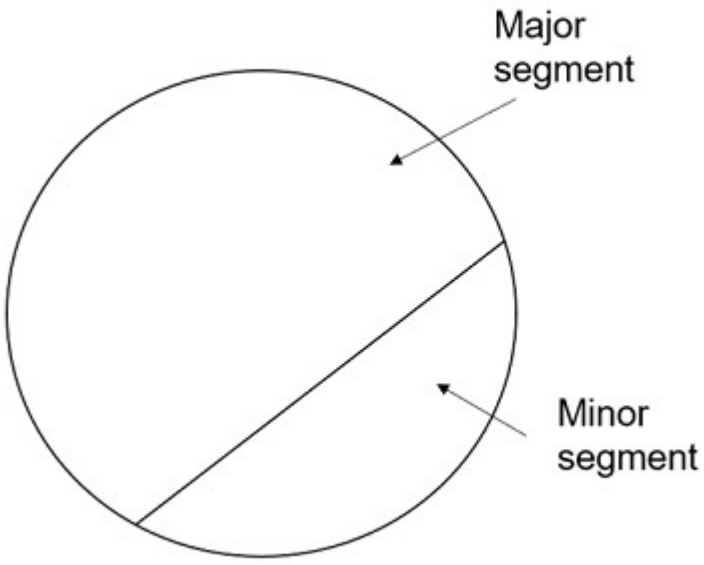
The derivative of the first derivative. It is denoted by: $f''(x)$, $\frac{d^2y}{dx^2}$ or y''.

sector

The plane figure enclosed by 2 radii or a circle and the arc between them.

segment

A plane figure enclosed by a chord and the arc joining the endpoints of a chord.



semicircle

Half of a circle.

sequence

An enumerated list of elements: $a_1, a_2, \ldots, a_n, \ldots$where a_n is referred to as the nth term.

In a sequence, repetitions are allowed and order matters. The length (possibly infinite) of the sequence is the number of terms in it.

series

The sum of the terms of an infinite sequence. The notation $a_1+a_2+a_3+\ldots$ is used for the series corresponding to the sequence $a_1, a_2, a_3 \ldots$

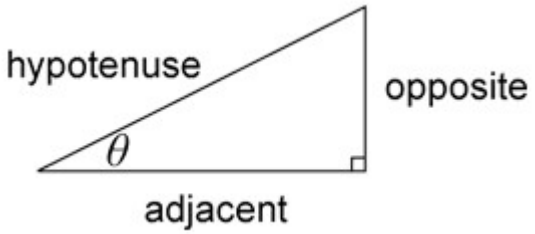
set

A collection of objects or elements, usually specified by listing its elements, e.g. $\{1,2,3,4\}$; by describing it in words, e.g. 'the set of primes'; or by using a rule such as $\{(x, y): y=2x+1, x \in N\}$.

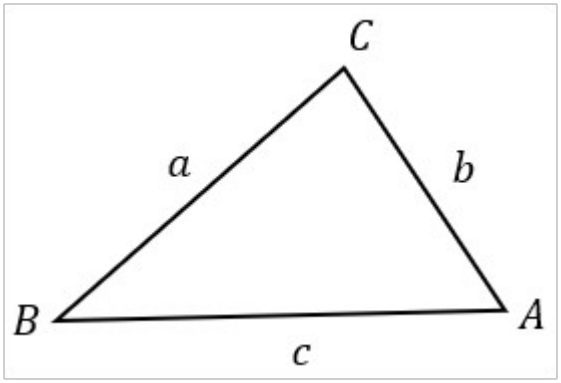
Extended in Mathematics Advanced, Extension 1 and Extension 2: A collection of distinct, unordered objects, referred to as members or elements of the set.

sine ratio

In any right-angled triangle, $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ where $0^\circ < \theta < 90^\circ$.



sine rule



Relates the lengths of the sides of a triangle to the sines of its angles: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

speed

The absolute value of an object's velocity. It represents how fast the object is moving. For an object moving along the x-axis, average speed is calculated as $\text{Average speed} = \dfrac{\text{distance travelled}}{\text{time taken}}$.

If its position at time t is $x(t)$ the instantaneous speed is $\left|\dfrac{dx}{dt}\right|$.

standard deviation

A measure of the variability or spread of a dataset. It gives an indication of the degree to which the individual data values are spread around their mean.

Extended in Mathematics Standard and Advanced: For a random variable, the standard deviation is the square root of its variance.

It provides a measure of the spread of the probability density function.

stationary point

A stationary point on the graph $y=f(x)$ of a differentiable function is a point where $f'(x)=0$.

A stationary point could be classified as a local or global maximum or minimum or a horizontal point of inflection.

subset

B is a subset of set A if every member of set B is a member of set A and is written as $B \subseteq A$, i.e. 'B is a subset of A'. If there are elements of A not in B, then B is a proper subset of A, written $B \subset A$.

substitution

The process of replacing a variable in an algebraic expression, formula, equation or function consistently by a particular value, another variable, expression or function.

Simultaneous equations can be solved using substitution by isolating one variable in one equation and using its value to replace that variable in another equation.

See simultaneous equations

Extended in Mathematics Extension 1 and Extension 2: Integration by substitution is a method used for evaluating integrals. The substitution could be expressed as a function of the variable of integration, or with the variable of integration as the subject of the substitution.

subtend

In geometry, an angle subtended by an arc or interval is the angle whose 2 rays pass through the endpoints of the arc or interval. A possible synonym for 'subtends' is 'makes'.

summation notation

Notation used to express a sum. For example, the sum $1+3+5+7+9$ can be represented using summation notation as $\sum_{k=1}^{5}(2k-1)$. Also known as sigma notation.

surd

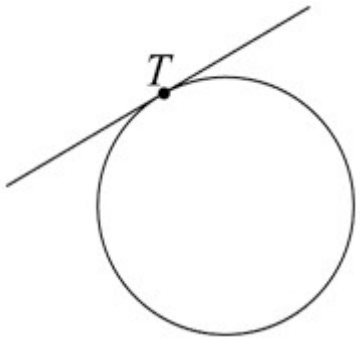
A numerical expression involving one or more irrational roots of numbers.

Extended in Mathematics Advanced: An irrational root of a number, e.g. $\sqrt{5}$ and $\sqrt[3]{\square}$.

More generally sums involving rational multiples of surds are sometimes also referred to as surds, e.g. $2+\frac{3\sqrt{7}}{4}$.

tangent

A line that intersects a circle at just one point. It touches the circle at that point of contact but does not pass inside it.

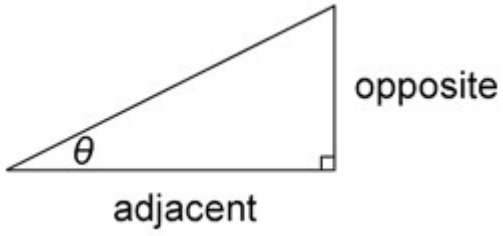


Extended in Mathematics Advanced, Extension 1 and Extension 2: For a curve at a given point P the tangent can be described intuitively as the straight line that 'just touches' the curve at that point. At P the curve has 'the same direction' as the tangent. In this sense, it is the best straight-line approximation to the curve at point P. For a differentiable function $f(x)$ the tangent to its graph at the point $P(a, f(a))$ has slope $f'(a)$.

tangent ratio

The ratio of the length of the altitude, a, of a right-angled triangle to the length of its base, b, for a given base angle, theta, θ.

In any right-angled triangle, $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$, where $0° < \theta < 90°$.



transformation

A procedure or set of procedures that changes the size and/or shape of an image. A transformation operates on points in the plane to change aspects, such as the position, size or shape of curves and other figures.

Translations, reflections, rotations, dilations, enlargements are all examples of transformations.

translation

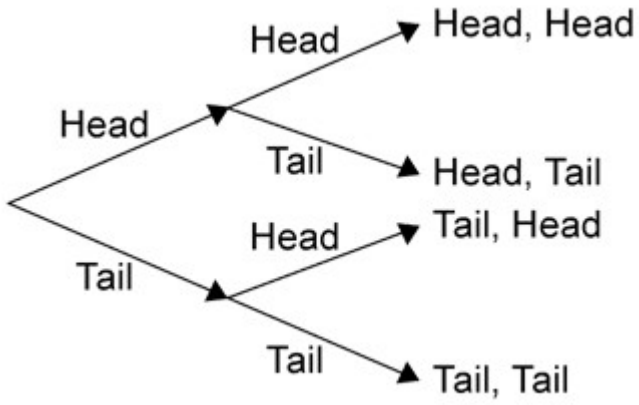
A type of transformation that moves a shape (or all the points in a plane) by the same amount to the left or right, or up or down.

trapezoidal rule

A method of approximating the area of an irregular shape, or a region bounded by a curve and an axis, by slicing the area up into trapeziums (trapezia) of equal height.

tree diagram

A diagram consisting of line segments (edges) connected to points (vertices) like the branches of a tree. It shows the relationship between sets, events or the set of outcomes of a multi-step random experiment.



See probability tree

trial

A single performance of a random experiment. Successive trials refers to repeated performances of the same experiment each of which will therefore have the same set of possible outcomes (sample space).

When there are only two possible outcomes they are known as Bernoulli trials.

trigonometric ratios

The relationship between the angles and sides of right-angled triangles. The 3 basic trigonometric ratios are sine, cosine and tangent.

See sine ratio, cosine ratio and tangent ratio

true bearing

Measured in degrees clockwise from true north and written with 3 digits to specify the direction. For example, the direction of north is specified as $000°$T, east is specified as $090°$T, south is specified as $180°$T and north-west is specified as $315°$T.

turning points

Points at which the gradient of the graph of a function changes direction, so the function has either a local minimum or local maximum at those points.

two-way table

A common way of displaying the two-way frequency distribution that arises when a group is categorised according to 2 criteria.

	Curly hair	Straight hair	Total
Red hair	1	1	2
Brown hair	8	4	12
Blonde hair	1	3	4
Black hair	7	2	9
Total	17	10	27

uniform distribution

A probability distribution in which all values of the random variable are equally likely.

union (of sets)

For sets A and B, the union is the set of elements which are in A or B or both and is written as $A \cup B$, i.e. 'A union B'.

universal set

The universal set for a particular problem or context contains all elements involved in the problem.

variable (algebra)

Things that are measurable or observable that are expected to either change over time or between individual observations. They are often designated by symbols, such as n, x and y, to represent members of a set. For example, the variable x could represent an unspecified real number.

variable (statistics)

Something measurable or observable that is expected to change either over time or between individual observations. For example, the age of students, their hair colour or a playing field's length or its shape.

variance

In statistics, the variance $Var(X)$ of a random variable X is a measure of the spread of its distribution. $Var(X)$ is calculated differently depending on whether the random variable is discrete or continuous.

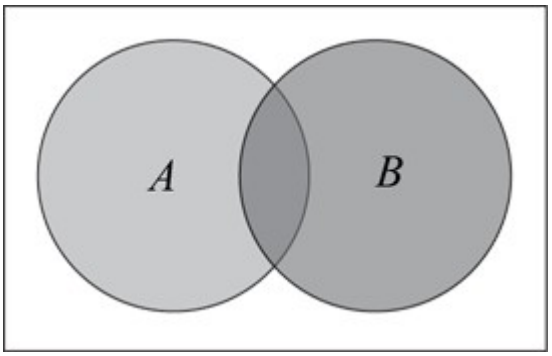
velocity

The rate of change of an object's position with respect to time. It is a vector quantity, meaning it has both magnitude and direction. The SI units for velocity are metres per second (ms^{-1}).

Extended in Mathematics Advanced, Extension 1, Extension 2: A particle moving along the x-axis whose position at time t is $x(t)$ has velocity $\frac{dx}{dt}$.

Venn diagram

Graphical representations, using several typically overlapping circles, showing elements of sets in relation to properties or attributes. They are drawn for some specified universal set.



vertex

Where 2 straight sides of a two-dimensional shape meet.

Extended in 7–12: A vertex is a point in the plane where lines meet and do not extend beyond, or a point in space where several edges meet. A vertex can also refer to a node in a network. It is also the turning point of a parabola.

vertical line test

Determines whether a relation or graph is also a function. If a vertical line intersects or touches a graph at more than one point, then the graph is not a function.

volume

The amount of space occupied by an object.

whole number

Any of the positive integers or 0.

z-score

A z-score is a statistical measurement of how many standard deviations a raw score is above or below the mean. A z-score can be positive or negative, indicating whether it is above or below the mean, or zero. Also known as a standardised score.

zero (of a function)

A point in the domain of a function where the value of the function is zero. A solution of the equation $f(x)=0$.

Mathematics Extension 1 11–12 (2024) Terms

absolute value

The magnitude or size of a real number, i.e. the distance of the number from the origin on a number line.

Formally, it has the piecewise definition $|x| = \begin{pmatrix} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{pmatrix}$.

acceleration

The rate at which velocity changes over time.

acceleration vector

$\ddot{x}(t)$ and $\ddot{y}(t)$ are functions of time that describe the acceleration of an object in the x-direction and y-direction respectively. $a = \ddot{x}(t)i + \ddot{y}(t)j$ is an acceleration vector in two dimensions.

acute angle

An angle smaller than a right angle, between 0° and 90°.

Extended in Mathematics Advanced and Extension 1: In circular measure, an angle lying in the interval $\left[0, \frac{\pi}{2}\right)$.

algebraic expression

A mathematical statement formed by combining numbers and algebraic symbols using arithmetic operations, e.g. $a^2 + 3ab - 2b^2$.

amplitude

A function of the form $y = A\sin(nx + \alpha)$ or $y = A\cos(nx + \alpha)$ has amplitude A, that is half the distance between the maximum and minimum values.

angle

Formed by 2 straight lines meeting at a common endpoint, called the vertex. An angle can describe the amount of turn between its 2 arms (lines).

arc

A part of a circle's circumference.

Extended in Mathematics Extension 1: A part of a curve.

asymptote

A straight line (or another curve) that a curve approaches as x tends to $\pm\infty$, or to some particular value.

For example, the curve $y = 2^x$ has an asymptote $y = 0$ as x tends to $-\infty$, and the curve $y = \frac{1}{x}$ has asymptotes $x = 0$ and $y = 0$ as x tends to 0 and $\pm\infty$ respectively.

Bernoulli distribution

A discrete probability distribution of a Bernoulli random variable.

Bernoulli random variable

Represents the result of a Bernoulli trial. It takes two possible values, a value of 1 representing 'success' (with probability p) and a value of 0 representing 'failure' (with probability $1-p$).

Bernoulli trial

An experiment with only two possible outcomes, often labelled 'success' (with probability p) and 'failure' (with probability $1-p$).

binomial coefficient

The coefficient of the term $x^{n-r}y^r$ in the expansion of $(x+y)^r$. It is written as ${}^nC_r$ or $\binom{n}{r}$ where $r=0,1,2,\ldots,n$ and is given by $\frac{n!}{r!(n-r)!}$.

In Pascal's triangle, the uppermost row is row 0. The entries in row n are the binomial coefficients ${}^nC_r$ or $\binom{n}{r}$ where $r=0,1,2,\ldots,n$ of the expansion of $(x+y)^n$.

binomial distribution

The discrete probability distribution of the number of successes in a sequence of n independent Bernoulli trials, each of which yields success with probability p.

binomial expansion

The algebraic expansion of a power of a binomial expression. For example, $16x^4+32x^3+24x^2+8x+1$ is the binomial expansion of $(2x+1)^4$.

binomial experiment

Performance of a fixed number of Bernoulli trials.

binomial expression

An algebraic expression with 2 terms. For example, $x+y$ is a binomial expression in the 2 terms x and y.

binomial random variable

Represents the number of successes in n independent Bernoulli trials.

binomial theorem

The formula for the expansion of a power of a binomial expression:

$(x+y)^n={}^nC_0x^n+{}^nC_1x^{n-1}y+{}^nC_2x^{n-2}y^2+\ldots+{}^nC_{n-1}xy^{n-1}+{}^nC_ny^n$, when n is a positive integer and ${}^nC_0, {}^nC_1, \ldots, {}^nC_n$ are binomial coefficients.

Cartesian coordinate system

A system that specifies the position of each point in a plane, formed by the intersection of 2 perpendicular number lines called axes. Each point is represented by an ordered pair (x, y) of real numbers called the coordinates of the point. The point where the axes meet is called the origin and has coordinates $(0,0)$.

Cartesian equation

The equation of a relation or a function expressed in terms of the Cartesian coordinates x and y. May sometimes be formed from two parametric equations by eliminating the parameter.

chain rule

A formula for the derivative of the composite of two differentiable functions. If y is a function of u, and u is a function of x, then the chain rule states that $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$. If $h(x)$ is the composite function $f(g(x))$, then $h'(x)=f'(g(x))g'(x)$.

circle

All points equal to a given distance from a fixed point, the centre. When used to describe a shape, a circle includes all points inside the boundary.

coefficient

A numerical quantity that multiplies a variable in an algebraic expression, e.g. 5 is the coefficient of $5x$. Variables with no specified coefficient have a coefficient of 1.

column vector notation

A notation used to represent a vector in two or three dimensions. For example the vectors $a=4i+5j$ and $b=i-2j+3k$ can be represented in column vector notation as: $a=\begin{pmatrix}4\\5\end{pmatrix}$ and $b=\begin{pmatrix}1\\-2\\3\end{pmatrix}$ respectively.

combination

A selection of r distinct objects from n distinct objects, where order is not important. The number of such combinations is given by $\frac{n!}{r!(n-r)!}$, the binomial coefficient ${}^{n}C_{r}$ or $\binom{n}{r}$.

component

One of the parts of a vector which is parallel to a particular axis or lying in a specified direction.

See component form (vectors)

component form (vectors)

Notation used to express a vector in terms of unit vectors. For example, vectors $a=\begin{pmatrix}1\\-2\end{pmatrix}$ and $b=\begin{pmatrix}4\\5\\6\end{pmatrix}$ can be expressed in component form as $a=i-2j$ and as $b=4i+5j+6k$ respectively, where i is a unit vector in the x-direction, j is a unit vector in the y-direction, and k is a unit vector in the z-direction.

composite functions

When the output of one function becomes the input of a second function.

For example, $f(g(x))$(read as'f of g of x') is a composite function where the outputs of function g are taken as the inputs of function f.

constant

A fixed numerical value. For example, in the algebraic expression $x+11$, the number 11 is a constant.

coordinate plane

A two-dimensional plane formed by two perpendicular number lines, intersecting at the origin. The xy-plane, the xz-plane and the yz-plane are the three coordinate planes.

cubic function

A polynomial function of degree 3, that is, a function of the form $f(x)=ax^3+bx^2+cx+d$.

degree (of a polynomial)

The highest power of x that appears in a polynomial $P(x)$.

denominator

Comes from Latin, meaning 'that which gives a name' [de- "completely"; nomen "name"]. The denominator of a fraction identifies the name of the fractional parts (e.g. thirds, quarters or fifths).

Extended in 7–10: In the fraction $\frac{a}{b}$, *b* is the denominator.

derivative

The result obtained after differentiation. For the function $f(x)$, the derivative is the gradient function of $f(x)$, and is denoted $f'(x)$. The derivative of $f(x)$ from first principles is given by $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. The derivative of y with respect to x is denoted $\frac{dy}{dx}$ or y'.

differential equation

Any equation containing the derivative of an unknown function, for example $y'=y(x-y)$ and $x+y\frac{dy}{dx}=0$.

dilation

A process of stretching or compressing the graph of a function. This could happen either in the x or y direction or both.

directed line segment

A part of a line between two endpoints, with fixed length and a direction.

discrete probability distribution

Specifies the probabilities with which a discrete random variable takes each of its values.

displacement

The change in position of an object, after a period of time, from its original position. Displacement is a vector quantity. The displacement may be positive, negative or zero.

displacement vector

Represents the displacement from one point to another.

distance

The length between two points. Distance is a positive scalar quantity.

distinct

Different, not equal.

dividend

The number that is divided or distributed in a division problem. For example, in $12 \div 3 = 4$, 12 is the dividend.

divisor

A number which is divided into another number.

In polynomials, $P(x) = A(x)Q(x) + R(x)$ is the result of dividing a polynomial $P(x)$ by a divisor $A(x)$, that is not the zero polynomial, with quotient $Q(x)$ and remainder $R(x)$.

domain

The set of allowable values of x in a function or relation.

Extended in Mathematics Advanced, Extension 1 and Extension 2: For a function or relation, it is the set of real numbers on which the function or relation is defined.

dot product

See scalar (dot) product

element(s)

A member of a set. For example, 3 is a member of the set of natural numbers $N = \{0, 1, 2, 3, 4, \ldots\}$. This relation can be written more concisely as $3 \in N$ ('3 is an element of the set N').

equal

If two quantities or mathematical expressions are equal, then the quantities have the same value, or the expressions represent the same mathematical object.

If $a = b$ then either may be substituted for the other.

For $\sin\theta = \frac{opposite}{hypotenuse}$ and $f(x) = 3x - 1$, the '=' sign indicates assignment by definition.

equation

An expression showing the equality of 2 quantities, using the = sign between them. A mathematical formula asking for a solution so that the 2 expressions in that variable are equal, e.g. $x^2 - 1 = x$.

equivalent

Two things are equivalent if they have the same value.

See equal, equals sign.

Extended in Mathematics Advanced, Extension 1 and Extension 2: Two things are equivalent if they have the same value, function or meaning. For example, $y = a^x$ and $x = \log_a y$ are equivalent. Statements P and Q are equivalent if P implies Q and Q implies P.

even function

A function is even if its graph is unchanged under reflection in the y-axis.

An even function $f(x)$ has the property $f(-x) = f(x)$, for all values of x in the domain.

exact value

A precise numerical value e.g. $\sqrt{3}$ as opposed to the approximation 1.732.

expected frequency

The number of times that a particular event or outcome is predicted to occur using theoretical probability.

expected value

In statistics, the expected value $E(X)$ of a random variable X is a measure of the central tendency of its distribution. Also known as the expectation or mean. $E(X)$ is calculated differently depending on whether the random variable is discrete or continuous.

experiment (random)

A process with an observable result. The results of the experiment are the set of possible outcomes, called the sample space.

expression

Two or more numbers or variables connected by operations. For example, $17 - 9$, $8 \times (2 + 3)$, $2a + 3b$ are all expressions. Expressions do not include an equals sign.

Extended in Mathematics Advanced, Extension 1 and Extension 2: The result of combining numbers, variables and operations in a meaningful way. For example, $\dfrac{x^3 - 1}{\sin y}$ is an expression, but $(x^3 -) \int / \sin y$ is not a validly formed expression.

factor theorem

If $p(x)$ is a polynomial and $p(a) = 0$ for some number a, then $p(x)$ is divisible by $x - a$. The factor theorem can be used to obtain factors of a polynomial.

factorial

The product of the first n positive integers is denoted by $n!$, read as 'n factorial', that is, $n! = n(n-1)(n-2) \times \cdots \times 3 \times 2 \times 1$, with the definition that $0! = 1$.

finite

A value between $-\infty$ and ∞.

force

A push or pull between objects, which may cause one or both objects to change speed, and/or direction of their motion, and/or their shape.

function

A function f assigns to each element of one set S precisely one element of a second set T.

The functions most commonly encountered in elementary mathematics are real functions of real variables. For such functions, the domain are sets of real numbers.

Functions are usually defined by a formula for $f(x)$ in terms of x. For example, the formula $f(x)=x^2$, defines the 'squaring function' that maps each real number x to its square x^2.

gradient

The slope of a line. It is calculated as the gradient of a line segment it contains. If $A(x_1, y_1)$ and $B(x_2, y_2)$ are 2 distinct points on a line, the gradient of the line (or line segment AB) is given by $m=\frac{y_2 - y_1}{x_2 - x_1}$.

graph

A visual representation of statistical data or of a relationship between variables. Ordered pairs of values (x, y) that represent the function or relation are plotted to form a graph. Graphs of statistical data include dot plots, box plots, column graphs, divided bar graphs and histograms.

Extended in Mathematics Advanced, Extension 1, Extension 2: For a function f, its graph consists of all ordered pairs $(x, f(x))$ for x in the domain of f.

gravity/gravitational

The force of attraction that objects with mass exert on each other.

identity

An identity is a statement involving a variable(s) that is true for all possible values of the variable(s).

indefinite integral

The set of all primitives of a continuous function f notated $\int f(x)\,dx$. So, $\int f(x)\,dx = F(x)+C$ where C is an arbitrary constant of integration, and $F'(x)=f(x)$.

inequality

A statement that one number or algebraic expression is less than (or greater than) another. There are 4 types of inequalities:

- a is less than b is written $a<b$
- a is greater than b is written $a>b$
- a is less than or equal to b is written $a\leq b$
- a is greater than or equal to b is written $a\geq b$.

Extended in Advanced, Extension 1, Extension 2: An order relation between one number or algebraic expression and another.

infinite

Limitless, exceeding any finite amount.

See finite

integer

A whole number, positive, negative or zero, e.g. −3, −2, −1, 0, 1, 2 …

Extended in Advanced, Extension 1, Extension 2: The set of integers is usually denoted by $\mathbb{Z}$.

integration

The process of finding the integral of a function. The inverse of differentiation.

intercept

The point at which a curve or function crosses an axis or other curve in a plane. The point at which a curve crosses the x-axis $(y=0)$ is called the x-intercept and the point at which a curve crosses the y-axis $(x=0)$ is called the y-intercept.

Extended in Advanced and Extension 1: The x- and y-intercepts are sometimes taken to mean the signed distance from the point at which the curve crosses the axis to the origin, e.g. for the line $y=mx+c$, the y-intercept is c.

inverse function

$f^{-1}(x)$ is the inverse function of $f(x)$ if the relationships $f\left(f^{-1}(x)\right)=x$ and $f^{-1}\left(f(x)\right)=x$ hold.

f^{-1} reverses or undoes the effect of the function f.

An inverse function exists for any one-to-one function and has domain the range of the function.

leading coefficient

The coefficient of the term involving the highest power in a polynomial.

leading term

The term involving the highest power in a polynomial.

linear

Something that can be represented or modelled by a straight line.

linear function

y is a linear function of x if $y=mx+c$ where m and c are constants.

The graph of a linear function is a straight line, where m is the gradient and c is the y-intercept.

See linear relationship

logistic function

A common S-shaped curve with the equation $y=\dfrac{c}{1+a\cdot b^{x}}$ where a, b and c are constants, and c is the carrying capacity.

magnitude

The size or absolute value of a number. For example, +4 and -4 have a magnitude of 4.

Extended in Mathematics Extension 1 and Mathematics Extension 2: For a vector, the magnitude is its length, given by $|xi+yj|=\sqrt{x^2+y^2}$ for 2D vectors and $|xi+yj+zk|=\sqrt{x^2+y^2+z^2}$ for 3D vectors.

mass

The amount of matter in an object.

mathematical induction

A method used to prove a statement involving a natural number n is true for all natural numbers larger than some given one.

mean

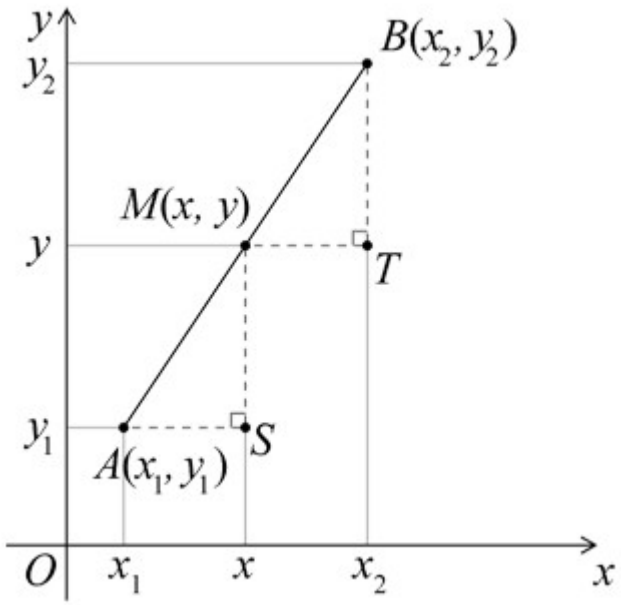
The sum of values in a dataset divided by the total number of values in the dataset. Also called the average.

midpoint

A point on a line segment or interval that divides the segment into 2 equal parts.

Let (x_1, y_1) and (x_2, y_2) be points in the Cartesian plane. Then the midpoint M of line segment AB has coordinates $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

This can be seen from the congruent triangles below.



model

A mathematical, conceptual or physical representation that describes, simplifies, clarifies or provides an explanation of the structure, workings or relationships within an object, system or idea. Models can provide a means of testing and predicting behaviour within limited conditions. Models may be physical or exist in digital form.

monic

A polynomial in which the coefficient of the leading term is 1.

multiplication principle

If one event has m possible outcomes and a second independent event has n possible outcomes, then there is a total of $m \times n$ possible outcomes for the two combined events.

multiplicity (of a root/zero)

The number of times a particular number is a zero for a given polynomial or a root for a polynomial equation.

obtuse angle

An angle bigger than a right angle (90°) but smaller than a straight angle (2 right angles, or 180°).

odd function

A function whose graph is unchanged under rotation of $180°$ about the origin.

An odd function $f(x)$ has the property $f(-x)=-f(x)$, for all values of x in the domain.

one-to-one

A correspondence between two sets where each element of one set is paired with one and only one element of another set.

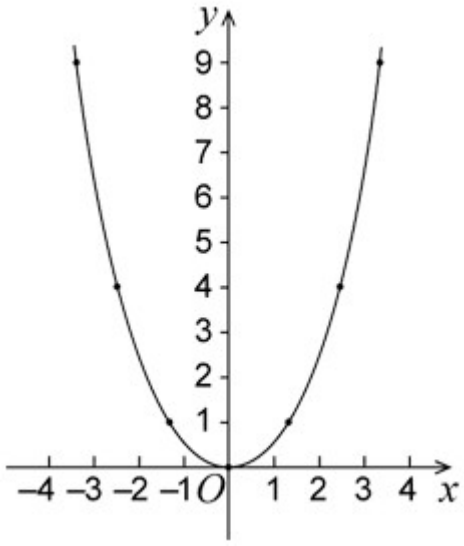
outcome

Possible result from an experiment or trial.

parabola

The graph of $y=x^2$. The point $(0,0)$ is called the vertex of the parabola and the y-axis is the axis of symmetry of the parabola.

Some other parabolas are the graphs of $y=ax^2+bx+c$ where $a\neq 0$.



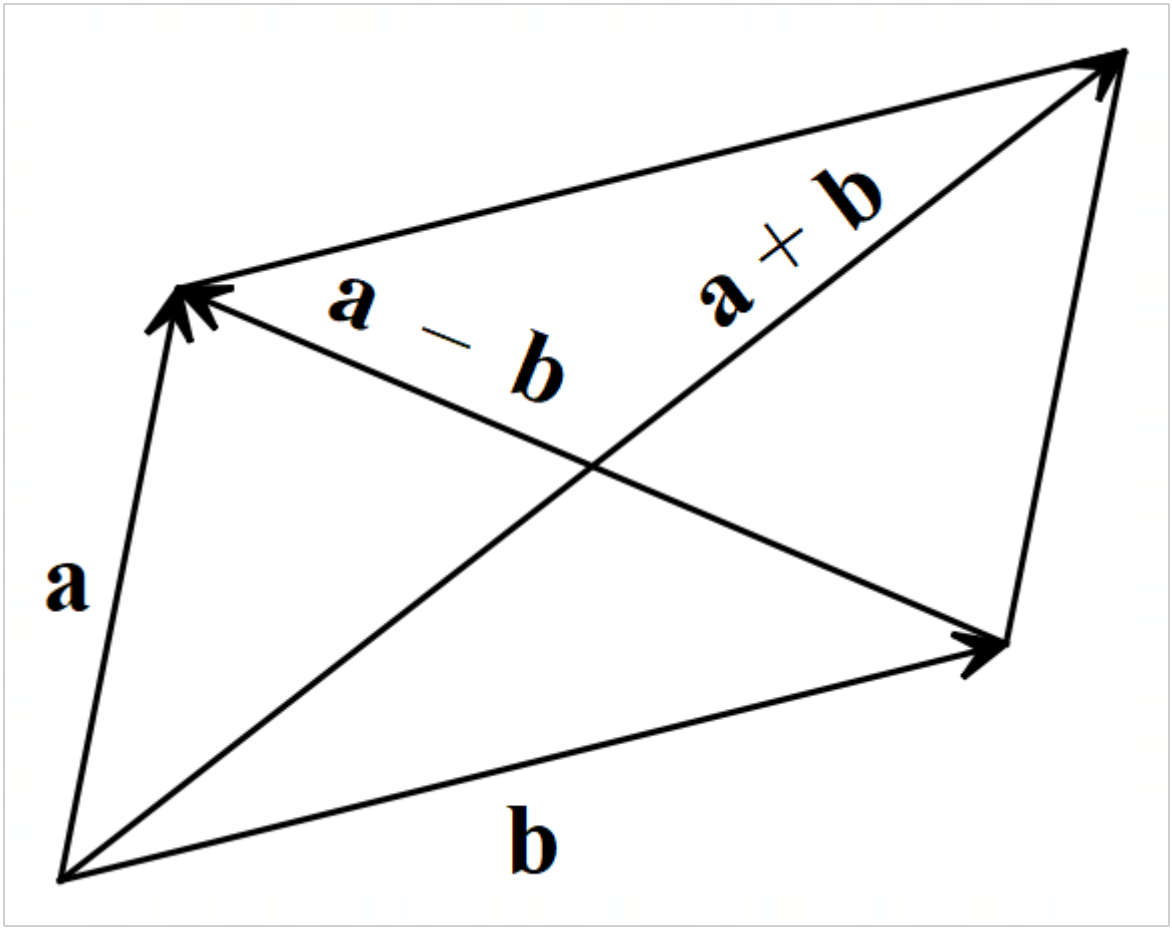
parallel

Two distinct lines, rays or line segments in the same plane that have no points of intersection and so necessarily have the same gradient (slope).

The symbol $/\!/$ is used to express that one line, ray, or line segment is parallel to another.

parallelogram law

A visual representation of the geometric constructions of vector addition and subtraction. For vectors a and b represented by directed line segments with the same initial point, the sum $a + b$ and difference $a - b$ are the diagonals of a parallelogram.



parameter

A quantity that is characteristic of a system.

In statistics, the population mean and the population standard deviation, μ and σ are parameters of the population.

See parametric equations

parametric equations

A type of equation that uses a parameter as the independent variable.

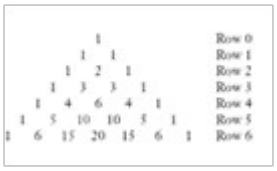
For example, $x = 3\sin\theta$ and $y = 3\cos\theta$, are a pair of parametric equations, where different values of the parameter θ will give different points on the number plane.

partial fraction

A way of splitting a rational expression as a sum of two or more simpler rational expressions, for example: $\frac{1}{x^2-1}=\frac{½}{x-1}+\frac{½}{x+1}$

Pascal's triangle

A triangular figure with rows of numbers starting and ending with 1, where each interior number is equal to the sum of the two numbers immediately above it.



See binomial coefficient

period

A function f is periodic with period p if $f(x+p)=f(x)$, for all x, that is, the function repeats itself after each interval of length p.

For example, $\sin x$ and $\cos x$ have period 2π.

permutation

An arrangement of r distinct objects taken from n distinct objects where order is important, where $r=0,1,2,\ldots,n$. The number of such permutations is denoted by ${}^nP_r$, and is given by $\frac{n!}{(n-r)!}$.

perpendicular

Two lines, rays, line segments, vectors, planes or other objects that intersect at a 90° angle (a right angle).

point of inflection

A point on a curve where the concavity changes. At a point of inflection, the tangent exists and crosses the curve.

polynomial

An expression made up of non-negative integer powers of the same variable and coefficients combined using addition, subtraction and multiplication.

population

The complete set of individuals, objects, places, etc., that we want information about.

Extended in Mathematics Standard and Mathematics Extension 1: In statistics it is the entire dataset from which a statistical sample may be drawn.

position vector

The vector running from the origin to a fixed point P in the plane. A position vector can also be expressed as a function of time, for example $r=x(t)i+y(t)j$, to describe the changing positions of an object, where $x(t)$ and $y(t)$ are the x- and y-coordinates of its position at time t.

probability

The chance of something happening shown on a scale from 0 and 1 (inclusive). For example, the probability that a fair coin toss will come up ‘heads’ is 0.5.

probability distribution

For a random variable X, the probability distribution is the set of values x taken by the random variable, together with the probabilities $P(x)$ that $X = x$.

product rule

A rule for the derivative of the product of two differentiable functions: If $y = uv$ where u and v are both functions of x , then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$, or if $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$.

projectile

An object initially propelled by an external force, after which the only forces acting on it are gravity and possibly a resistive force due to its motion through a surrounding medium.

projection (vectors)

The vector component of a in the direction of vector b is the projection of a vector a onto a vector b .

$proj_b a = \left(\frac{a \cdot b}{|b|^2}\right) b.$

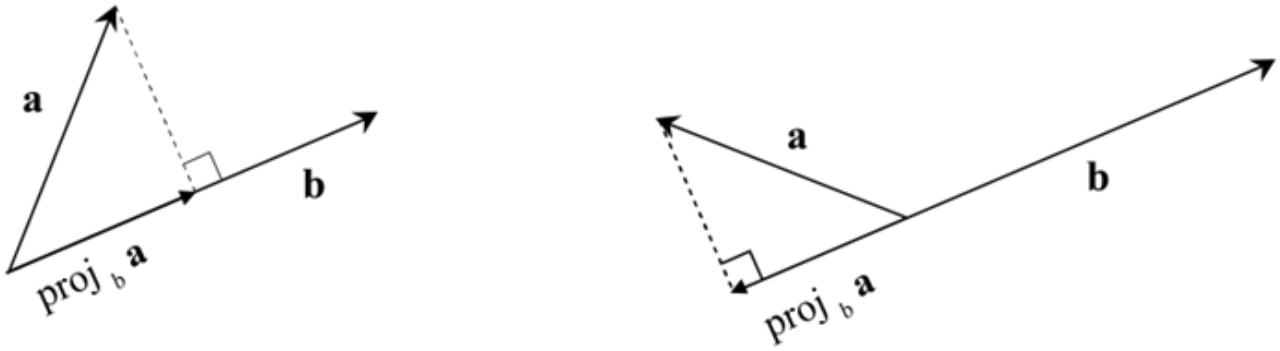


Image long description: Two vector diagrams both representing vectors, each showing two arrows connected at a point. In the first diagram, the angle between the arrows is acute. One arrow is labelled with boldface a, the other with boldface b, with the b arrow as the longer vector, pointing up and to the right. A dotted line perpendicular to arrow b joins the tip of arrow a. There is an arrow drawn on top of arrow b from the vertex, which is labelled proj b a, with a subscripted b and boldface a. In the second diagram the angle between the arrows is obtuse. One arrow is labelled with boldface a and the other with boldface b, with b as the longer vector, pointing up and to the right. A dotted line perpendicular to arrow b joins the tip of arrow a, pointing up and to the left, to an extension of arrow b, pointing down and to the left. The arrow drawn on top of arrow b from the vertex is labelled proj b a with a subscripted b and boldface a.

proof

A rigorous mathematical argument that demonstrates the truth of a given statement or proposition. A mathematical statement that has been established by means of a proof is called a theorem.

quadratic expression

An expression of the form ax^2+bx+c, where $a\neq 0$, b and c are constants.

quadratic function

A function of the form $f(x)=ax^2+bx+c$, where $a\neq 0$, b and c are constants.

quotient

The result of dividing one number or algebraic expression by another.

quotient rule

A formula for the derivative of the ratio of two differentiable functions. If $y=\frac{u}{v}$, where u and v are both functions of x, then the quotient rule states that $\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$, or if $h(x)=\frac{f(x)}{g(x)}$ then $h'(x)=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$.

radian

A unit of angular measure frequently used in mathematics.

1 radian is the angle between two radii of a circle which cut off on the circumference an arc equal to the radius.

random variable

A variable whose possible values are outcomes of a statistical experiment or a random phenomenon.

range (function)

The set of values of the dependent variable for which a function is defined.

rate of change

For a function, the relative change in a function's value $f(x)$ to a change in x. Instantaneous rate of change is measured by its derivative.

For example, the velocity of a moving object is the rate of change of its position over time.

real number

A number which can be represented by a point on a number line. The set of real numbers is the set of all rational and irrational numbers.

reciprocal

If x is any real number, then the reciprocal of that number will be $\frac{1}{x}$. For example, the reciprocal of 4 is $\frac{1}{4}$.

reflection

A transformation of a shape formed by creating a mirror image on the other side of a given line.

relation

An association between the elements of one set, A, and the elements of another set, B. It may be represented as a set of ordered pairs (a, b), where a is in A, b is in B and b is related to a.

remainder

The part 'left over' when dividing a number into equal groups. The remainder forms part of the next group.

Extended in 7–10: The amount left over when one number or algebraic quantity a is divided by another b. If a is divisible by b, then the remainder is 0. For example, when 68 is divided by 11, the remainder is 2, because 68 can be expressed as $68 = 6 \times 11 + 2$.

Extended in Mathematics Extension 1: What is left over after a division. When division of a polynomial $P(x)$ by a non-zero polynomial $Q(x)$ gives $P(x) = A(x)Q(x) + R(x)$, where the degree of $R(x)$ is less than the degree of $Q(x)$, then $R(x)$ is the remainder.

remainder theorem

A rule to determine the remainder when one polynomial $P(x)$ is divided by a linear polynomial $(x - \alpha)$. The remainder is equal to $P(\alpha)$.

root

The solution to an equation. For example, $(x - 1)(x + 3) = 0$ has roots $x = 1$ and $x = -3$.

sample

A subset of a population used to estimate characteristics of the population. For example, a randomly selected group of 8-year-olds (sample) selected to estimate the height of 8-year-olds in Australia (population).

sample mean

The sum of the values in a sample of data, divided by the total number of values in the sample, denoted $\bar{x}$.

sampling distribution

The distribution of a sample statistic over all samples of the same size.

scalar

The term used to describe real numbers when they are used to multiply (scale) vectors. For example, -3 in $-3(2i - j) = -6i + 3j$, is acting as a scalar.

scalar (dot) product

Also called the dot product. An operation ($\cdot$) on two vectors a and b that outputs a scalar $a \cdot b$.

The scalar product in two dimensions, for vectors $a = x_1 i + y_1 j$ and $b = x_2 i + y_2 j$ is $a \cdot b = x_1 x_2 + y_1 y_2$. Similarly in three dimensions.

Also, in either two or three dimensions $a \cdot b = |a||b| \cos\theta$, where θ is the included angle between a and b.

set

A collection of objects or elements, usually specified by listing its elements, e.g. $\{1,2,3,4\}$; by describing it in words, e.g. 'the set of primes'; or by using a rule such as $\{(x,y): y=2x+1, x \in N\}$.

Extended in Mathematics Advanced, Extension 1 and Extension 2: A collection of distinct, unordered objects, referred to as members or elements of the set.

slope field

A graphical representation of the tangent lines to the solutions of a first-order differential equation.

statement

A statement is an assertion that is either true or false but not both.

Also referred to as a proposition.

subset

B is a subset of set A if every member of set B is a member of set A and is written as $B \subseteq A$, i.e. 'B is a subset of A'. If there are elements of A not in B, then B is a proper subset of A, written $B \subset A$.

substitution

The process of replacing a variable in an algebraic expression, formula, equation or function consistently by a particular value, another variable, expression or function.

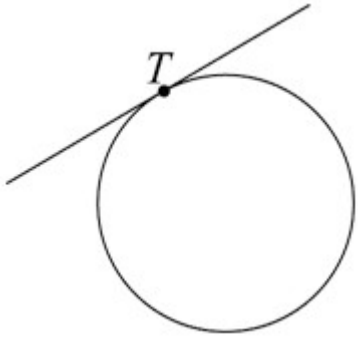
Simultaneous equations can be solved using substitution by isolating one variable in one equation and using its value to replace that variable in another equation.

See simultaneous equations

Extended in Mathematics Extension 1 and Extension 2: Integration by substitution is a method used for evaluating integrals. The substitution could be expressed as a function of the variable of integration, or with the variable of integration as the subject of the substitution.

tangent

A line that intersects a circle at just one point. It touches the circle at that point of contact but does not pass inside it.



Extended in Mathematics Advanced, Extension 1 and Extension 2: For a curve at a given point P the tangent can be described intuitively as the straight line that 'just touches' the curve at that point. At P the curve has 'the same direction' as the tangent. In this sense, it is the best straight-line approximation to the curve at point P. For a differentiable function $f(x)$ the tangent to its graph at the point $P(a, f(a))$ has slope $f'(a)$.

transformation

A procedure or set of procedures that changes the size and/or shape of an image. A transformation operates on points in the plane to change aspects, such as the position, size or shape of curves and other figures.

Translations, reflections, rotations, dilations, enlargements are all examples of transformations.

translation

A type of transformation that moves a shape (or all the points in a plane) by the same amount to the left or right, or up or down.

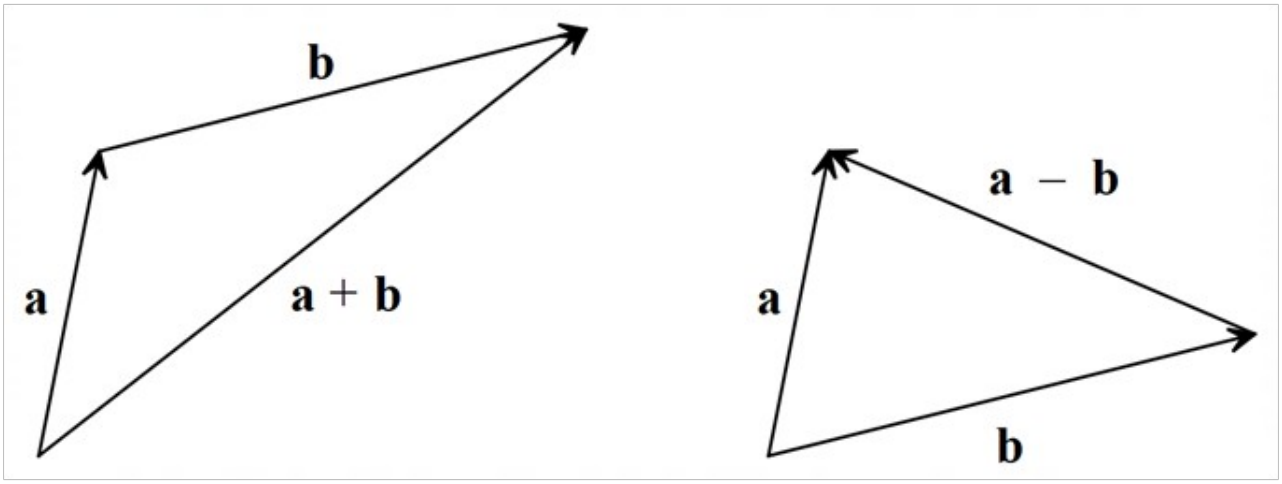
trial

A single performance of a random experiment. Successive trials refers to repeated performances of the same experiment each of which will therefore have the same set of possible outcomes (sample space).

When there are only two possible outcomes they are known as Bernoulli trials.

triangle law

For vector addition, when two vectors are represented as two sides of a triangle, the third side represents the resultant vector:



turning points

Points at which the gradient of the graph of a function changes direction, so the function has either a local minimum or local maximum at those points.

unit vector

A vector with a magnitude of 1.

variable (algebra)

Things that are measurable or observable that are expected to either change over time or between individual observations. They are often designated by symbols, such as n, x and y, to represent members of a set. For example, the variable x could represent an unspecified real number.

variable (statistics)

Something measurable or observable that is expected to change either over time or between individual observations. For example, the age of students, their hair colour or a playing field's length or its shape.

variance

In statistics, the variance $Var(X)$ of a random variable X is a measure of the spread of its distribution. $Var(X)$ is calculated differently depending on whether the random variable is discrete or continuous.

vector

A quantity having both magnitude and direction.

velocity

The rate of change of an object's position with respect to time. It is a vector quantity, meaning it has both magnitude and direction. The SI units for velocity are metres per second (ms^{-1}).

Extended in Mathematics Advanced, Extension 1, Extension 2: A particle moving along the x-axis whose position at time t is $x(t)$ has velocity $\frac{dx}{dt}$.

volume

The amount of space occupied by an object.

whole number

Any of the positive integers or 0.

zero polynomial

A polynomial which has all coefficients equal to zero.

zero vector

The vector of magnitude 0, namely $0i+0j$ in two dimensions and $0i+0j+0k$ in three dimensions. Denoted as 0 When handwritten, it is expressed as $\underset{\sim}{0}$, to be distinguished from the number 0.

Mathematics Extension 2 11–12 (2024) Terms

acceleration

The rate at which velocity changes over time.

amplitude

A function of the form $y = A\sin(nx+\alpha)$ or $y = A\cos(nx+\alpha)$ has amplitude A, that is half the distance between the maximum and minimum values.

angle

Formed by 2 straight lines meeting at a common endpoint, called the vertex. An angle can describe the amount of turn between its 2 arms (lines).

argument

When a non-zero complex number z is represented by a point P in the complex plane an argument of z, denoted $arg\,z$, is an angle θ that OP (where O is the origin) makes with the positive real axis, with successive pairs differing by 2π.

arithmetic mean

For the numbers $x_1, x_{2,} x_3, \ldots, x_n$ the arithmetic mean is: $\frac{x_1+x_2+x_3+\ldots+x_n}{n}$.

binomial theorem

The formula for the expansion of a power of a binomial expression:

$(x+y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} y + {}^nC_2 x^{n-2} y^2 + \ldots + {}^nC_{n-1} x\, y^{n-1} + {}^nC_n y^n$, when n is a positive integer and ${}^nC_0, {}^nC_1, \ldots, {}^nC_n$ are binomial coefficients.

Cartesian coordinate system

A system that specifies the position of each point in a plane, formed by the intersection of 2 perpendicular number lines called axes. Each point is represented by an ordered pair $(x, y)$of real numbers called the coordinates of the point. The point where the axes meet is called the origin and has coordinates $(0,0)$.

Cartesian equation

The equation of a relation or a function expressed in terms of the Cartesian coordinates x and y. May sometimes be formed from two parametric equations by eliminating the parameter.

Cartesian form (of a complex number)

A form that expresses a complex number z in terms of its real and imaginary parts, $z = a + ib$, where a and b are real numbers.

a is referred to as the real part of the complex number and b is referred to as the imaginary part of the complex number.

Cauchy–Schwarz inequality

The Cauchy–Schwarz inequality for vectors is given by: $|a \cdot b) \leq |a)|b)$.

circle

All points equal to a given distance from a fixed point, the centre. When used to describe a shape, a circle includes all points inside the boundary.

coefficient

A numerical quantity that multiplies a variable in an algebraic expression, e.g. 5 is the coefficient of $5x$. Variables with no specified coefficient have a coefficient of 1.

collinear

Describes points that lie on the same straight line.

column vector notation

A notation used to represent a vector in two or three dimensions. For example the vectors $a = 4i + 5j$ and $b = i - 2j + 3k$ can be represented in column vector notation as: $a = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ respectively.

commutativity (commutative property)

Commutativity (commutative property) of addition or multiplication means that 2 numbers can be added or multiplied in any order and the solution will be the same.

Commutative law, commutativity and turn-around facts are interchangeable terms.

complex conjugate root theorem

A theorem regarding the complex roots of a polynomial with real coefficients: If $a + ib$ is a root of a polynomial equation in one variable with real coefficients, then its complex conjugate $a - ib$ is also a root of the polynomial.

complex number

A number of the form $x + iy$, where x and y are real numbers and $i^2 = -1$.

complex plane

A Cartesian plane in which the real part of a complex number is plotted on the horizontal axis, referred to as the real axis, and the imaginary part on the vertical axis, referred to as the imaginary axis. Sometimes called the Argand plane.

component

One of the parts of a vector which is parallel to a particular axis or lying in a specified direction.

See component form (vectors)

conjugate

For a surd $a\sqrt{b} + x\sqrt{y}$, where a, b, x and y are real numbers, the conjugate is $a\sqrt{b} - x\sqrt{y}$.

In complex numbers, the conjugate of $z = x + iy$ is $\overline{z} = x - iy$. z and $\overline{z}$ are complex conjugates.

constant

A fixed numerical value. For example, in the algebraic expression $x+11$, the number 11 is a constant.

contrapositive

A proposition formed by exchanging the hypothesis and conclusion of a conditional statement and negating both. The contrapositive of the statement 'if P then Q' is 'if not Q then not P'. The contrapositive of a true statement is also true.

converse

If one relationship is true, then the opposite is also true.

The converse of a statement 'If P then Q' is 'If Q then P'. For example, the converse of Pythagoras' theorem is that if the sum of the squares of 2 sides of a triangle is equal to the square of the third side, then the triangle is right-angled.

Extended in Mathematics Extension 2: The converse of $P \rightarrow Q$ is $Q \rightarrow P$ or $P \leftarrow Q$.

counterexample

An instance/example where the proposition is not true, thereby showing the proposition to be false.

differential equation

Any equation containing the derivative of an unknown function, for example $y' = y(x - y)$ and $x + y\frac{dy}{dx} = 0$.

dilation

A process of stretching or compressing the graph of a function. This could happen either in the x or y direction or both.

direction vector

For a straight line or line segment, a direction vector is a vector whose direction is that of the line or line segment.

discriminant

In the quadratic expression $ax^2 + bx + c$, the discriminant $\Delta = b^2 - 4ac$

distinct

Different, not equal.

distributivity (Distributive law)

Multiplication of numbers is distributive over addition because the product of one number with the sum of 2 others equals the sum of the products of the first number with each of the others. For example, the product of 3 with $(4+5)$ gives the same result as the sum of 3×4 and 3×5:

$3 \times (4+5) = 3 \times 9 = 27$ and $3 \times 4 + 3 \times 5 = 12 + 15 = 27$

The distributive law is expressed algebraically as follows:

$a(b+c) = ab + ac$, for all numbers a, b and c.

domain

The set of allowable values of x in a function or relation.

Extended in Mathematics Advanced, Extension 1 and Extension 2: For a function or relation, it is the set of real numbers on which the function or relation is defined.

dot product

See scalar (dot) product

equation

An expression showing the equality of 2 quantities, using the = sign between them. A mathematical formula asking for a solution so that the 2 expressions in that variable are equal, e.g. $x^2 - 1 = x$.

equivalent

Two things are equivalent if they have the same value.

See equal, equals sign.

Extended in Mathematics Advanced, Extension 1 and Extension 2: Two things are equivalent if they have the same value, function or meaning. For example, $y = a^x$ and $x = \log_a y$ are equivalent. Statements P and Q are equivalent if P implies Q and Q implies P.

expression

Two or more numbers or variables connected by operations. For example, $17 - 9$, $8 \times (2 + 3)$, $2a + 3b$ are all expressions. Expressions do not include an equals sign.

Extended in Mathematics Advanced, Extension 1 and Extension 2: The result of combining numbers, variables and operations in a meaningful way. For example, $\frac{x^3 - 1}{\sin y}$ is an expression, but $(x^3 -) \int / \sin y$ is not a validly formed expression.

force

A push or pull between objects, which may cause one or both objects to change speed, and/or direction of their motion, and/or their shape.

function

A function f assigns to each element of one set S precisely one element of a second set T.

The functions most commonly encountered in elementary mathematics are real functions of real variables. For such functions, the domain are sets of real numbers.

Functions are usually defined by a formula for $f(x)$ in terms of x. For example, the formula $f(x) = x^2$, defines the 'squaring function' that maps each real number x to its square x^2.

geometric mean

The nth root of the product of n numbers. For a set of numbers $a_1, a_2, \ldots, a_n$, the geometric mean is defined as $\sqrt[n]{\square}$.

gradient

The slope of a line. It is calculated as the gradient of a line segment it contains. If $A(x_1, y_1)$ and $B(x_2, y_2)$ are 2 distinct points on a line, the gradient of the line (or line segment AB) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

gradient–intercept form

$y = mx + c$ is the gradient–intercept form of a straight line, where m is the gradient of the line and $(0, c)$ is the point at which the line intersects the y-axis. c is called the y-intercept.

graph

A visual representation of statistical data or of a relationship between variables. Ordered pairs of values (x, y) that represent the function or relation are plotted to form a graph. Graphs of statistical data include dot plots, box plots, column graphs, divided bar graphs and histograms.

Extended in Mathematics Advanced, Extension 1, Extension 2: For a function f, its graph consists of all ordered pairs $(x, f(x))$ for x in the domain of f.

gravity/gravitational

The force of attraction that objects with mass exert on each other.

identity

An identity is a statement involving a variable(s) that is true for all possible values of the variable(s).

implication

A relationship between two propositions where the second is a direct consequence of the first. A statement in the form 'If P then Q', or equivalently 'P is necessary for Q' or 'P implies Q', represented as $P \Rightarrow Q$ or $P \rightarrow Q$.

indefinite integral

The set of all primitives of a continuous function f notated $\int f(x)\,dx$. So, $\int f(x)\,dx = F(x) + C$ where C is an arbitrary constant of integration, and $F'(x) = f(x)$.

inequality

A statement that one number or algebraic expression is less than (or greater than) another. There are 4 types of inequalities:

- a is less than b is written $a < b$
- a is greater than b is written $a > b$
- a is less than or equal to b is written $a \leq b$
- a is greater than or equal to b is written $a \geq b$.

Extended in Advanced, Extension 1, Extension 2: An order relation between one number or algebraic expression and another.

integer

A whole number, positive, negative or zero, e.g. −3, −2, −1, 0, 1, 2 …

Extended in Advanced, Extension 1, Extension 2: The set of integers is usually denoted by $\mathbb{Z}$.

integrand

A function that is to be integrated.

integration

The process of finding the integral of a function. The inverse of differentiation.

integration by parts

A process of transforming the integral of a product of functions into an integral which can be more easily found. It is derived using the product rule for differentiation.

See product rule

linear

Something that can be represented or modelled by a straight line.

logarithm

The logarithm of a positive number x is the power to which a given number b, called the base, must be raised in order to produce the number x. The logarithm of x, to the base b is denoted by $\log_b x$.

Algebraically: $\log_b x = y$ is equivalent to $b^y = x$.

magnitude

The size or absolute value of a number. For example, +4 and -4 have a magnitude of 4.

Extended in Mathematics Extension 1 and Mathematics Extension 2: For a vector, the magnitude is its length, given by $|x\,i + y\,j| = \sqrt{x^2 + y^2}$ for 2D vectors and $|x\,i + y\,j + z\,k| = \sqrt{x^2 + y^2 + z^2}$ for 3D vectors.

mass

The amount of matter in an object.

mathematical induction

A method used to prove a statement involving a natural number n is true for all natural numbers larger than some given one.

model

A mathematical, conceptual or physical representation that describes, simplifies, clarifies or provides an explanation of the structure, workings or relationships within an object, system or idea. Models can provide a means of testing and predicting behaviour within limited conditions. Models may be physical or exist in digital form.

modulus

For a complex number $z = a + i\,b$, the modulus, denoted as $|z|$, is given by $|z| = \sqrt{a^2 + b^2}$.

modulus–argument form (of a complex number)

A complex number z expressed in terms of its modulus r and argument θ, i.e. $z = r(\cos\theta + i\sin\theta)$.

negation

If P is a statement then the statement 'not P' is the negation of P. The negation of P is denoted by $\neg P$ or $\sim P$.

Newton's laws of motion

Law 1: an object will remain at rest, or continue moving with the same speed and in the same direction, unless acted upon by a net external force.

Law 2: the acceleration of an object is proportional to the forces applied, that is, $\text{force} = \text{mass} \times \text{acceleration}$.

Law 3: for every action, there is an equal and opposite reaction.

numerator

In the fraction $\frac{a}{b}$, a is the numerator. If an object is divided into b equal parts, then the fraction $\frac{a}{b}$ represents a of these parts all together. For example, if a line segment is divided into 5 equal parts, each of these parts is one-fifth ($\frac{1}{5}$) of the whole and 3 of these parts taken together corresponds to the fraction $\frac{3}{5}$.

one-to-one

A correspondence between two sets where each element of one set is paired with one and only one element of another set.

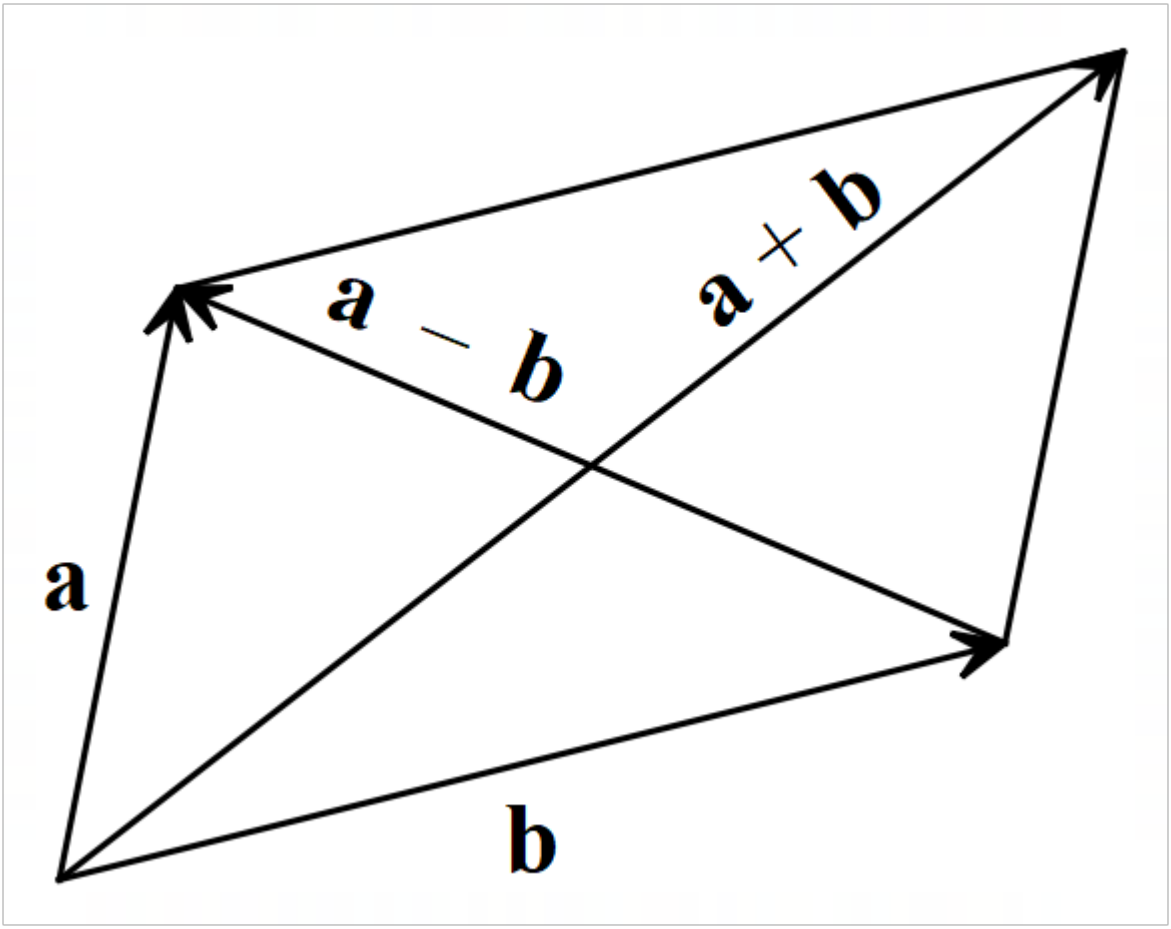
parallel

Two distinct lines, rays or line segments in the same plane that have no points of intersection and so necessarily have the same gradient (slope).

The symbol $/\!/$ is used to express that one line, ray, or line segment is parallel to another.

parallelogram law

A visual representation of the geometric constructions of vector addition and subtraction. For vectors a and b represented by directed line segments with the same initial point, the sum $a + b$ and difference $a - b$ are the diagonals of a parallelogram.



parameter

A quantity that is characteristic of a system.

In statistics, the population mean and the population standard deviation, μ and σ are parameters of the population.

See parametric equations

parametric equations

A type of equation that uses a parameter as the independent variable.

For example, $x = 3\sin\theta$ and $y = 3\cos\theta$, are a pair of parametric equations, where different values of the parameter θ will give different points on the number plane.

partial fraction

A way of splitting a rational expression as a sum of two or more simpler rational expressions, for example: $\frac{1}{x^2 - 1} = \frac{½}{x - 1} + \frac{½}{x + 1}$

period

A function f is periodic with period p if $f(x + p) = f(x)$, for all x, that is, the function repeats itself after each interval of length p.

For example, $\sin x$ and $\cos x$ have period 2π.

perpendicular

Two lines, rays, line segments, vectors, planes or other objects that intersect at a 90° angle (a right angle).

plane

A flat surface that extends indefinitely. In 3D Cartesian space, the $x\,y$, $y\,z$ and $x\,z$ planes are referred to as the coordinate planes.

polar form (of a complex number)

The complex number $z = a + i\,b$ can be expressed in polar form as: $z = r\cos\theta + r\,i\sin\theta = r\left(\cos\theta + i\sin\theta\right)$, where r is the modulus of the complex number and θ is its argument expressed in radians. This is also known as modulus-argument form.

polynomial

An expression made up of non-negative integer powers of the same variable and coefficients combined using addition, subtraction and multiplication.

position vector

The vector running from the origin to a fixed point P in the plane. A position vector can also be expressed as a function of time, for example $r = x(t)\,i + y(t)\,j$, to describe the changing positions of an object, where $x(t)$ and $y(t)$ are the x- and y-coordinates of its position at time t.

principal argument of a complex number

The argument of a non-zero complex number z in the interval $(-\pi, \pi]$, denoted as $Arg\,z$.

See argument

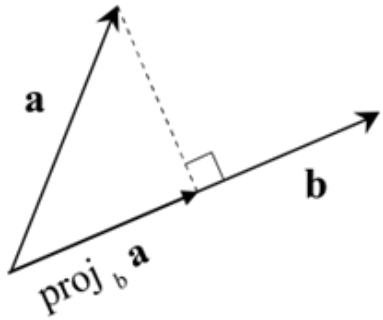
projectile

An object initially propelled by an external force, after which the only forces acting on it are gravity and possibly a resistive force due to its motion through a surrounding medium.

projection (vectors)

The vector component of a in the direction of vector b is the projection of a vector a onto a vector b .

$$proj_b\,a = \left(\frac{a \cdot b}{|b|^2}\right) b.$$



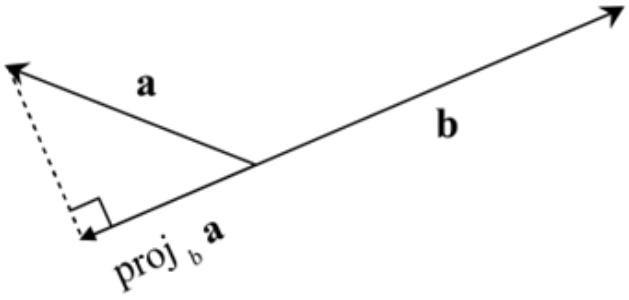


Image long description: Two vector diagrams both representing vectors, each showing two arrows connected at a point. In the first diagram, the angle between the arrows is acute. One arrow is

labelled with boldface a, the other with boldface b, with the b arrow as the longer vector, pointing up and to the right. A dotted line perpendicular to arrow b joins the tip of arrow a. There is an arrow drawn on top of arrow b from the vertex, which is labelled proj b a, with a subscripted b and boldface a. In the second diagram the angle between the arrows is obtuse. One arrow is labelled with boldface a and the other with boldface b, with b as the longer vector, pointing up and to the right. A dotted line perpendicular to arrow b joins the tip of arrow a, pointing up and to the left, to an extension of arrow b, pointing down and to the left. The arrow drawn on top of arrow b from the vertex is labelled proj b a with a subscripted b and boldface a.

proof

A rigorous mathematical argument that demonstrates the truth of a given statement or proposition. A mathematical statement that has been established by means of a proof is called a theorem.

proof by contradiction

The negation of an original statement is assumed but is proven false, usually by deriving a contradiction, so the negation of the original statement cannot be true, that is, the original statement is true.

proposition

See statement

quadratic equation

An equation of the form $a x^2 + b x + c = 0$, where $a \neq 0$, b and c are constants.

quantifiers

Symbols used to specify how many instances of the variable satisfy the statement. $\forall$ and $\exists$ are quantifiers. $\forall$ stands for the phrase ‘for all’, ‘for every’ or ‘for each’. $\exists$ stands for the phrase ‘there exists a’, ‘there is a’ or ‘for some’.

rational number

An element of the infinite set of numbers $Q = \{\frac{m}{n}$, where m and n are integers and $n \neq 0\}$. It may be expressed in decimal form, e.g. $\frac{1}{8} = 0.125$ and $\frac{4}{9} = 0.444\ldots$

real number

A number which can be represented by a point on a number line. The set of real numbers is the set of all rational and irrational numbers.

recurrence relation

A formula that defines each term of a sequence using a preceding term or preceding terms, for example $a_{n+1} = r\, a_n$ and $a_1 = a$ defines a geometric sequence with first term a and common ratio r.

recursive formula

A formula that defines each term of a sequence using a preceding term or preceding terms, for example $a_{n+1} = r\, a_n$ and $a_1 = a$ defines a geometric sequence with first term a and common ratio r.

resisted motion

Motion that encounters resisting forces such as air resistance.

root

The solution to an equation. For example, $(x-1)(x+3)=0$ has roots $x=1$ and $x=-3$.

rotation

A turn about a point through a specified angle. For example, a rotation about the origin through an anticlockwise angle of 45° transforms the line $y=0$ into the line $y=x$.

scalar

The term used to describe real numbers when they are used to multiply (scale) vectors. For example, -3 in $-3(2i-j)=-6i+3j$, is acting as a scalar.

scalar (dot) product

Also called the dot product. An operation ($\cdot$) on two vectors a and b that outputs a scalar $a \cdot b$.

The scalar product in two dimensions, for vectors $a=x_1 i+y_1 j$ and $b=x_2 i+y_2 j$ is $a \cdot b=x_1 x_2+y_1 y_2$. Similarly in three dimensions.

Also, in either two or three dimensions $a \cdot b=|a||b|\cos\theta$, where θ is the included angle between a and b.

sequence

An enumerated list of elements: $a_1, a_2, \ldots, a_n, \ldots$where a_n is referred to as the nth term.

In a sequence, repetitions are allowed and order matters. The length (possibly infinite) of the sequence is the number of terms in it.

set

A collection of objects or elements, usually specified by listing its elements, e.g. $\{1,2,3,4\}$; by describing it in words, e.g. 'the set of primes'; or by using a rule such as $\{(x,y): y=2x+1, x \in N\}$.

Extended in Mathematics Advanced, Extension 1 and Extension 2: A collection of distinct, unordered objects, referred to as members or elements of the set.

simple harmonic motion

Motion in which acceleration is proportional to, and in the opposite direction to, displacement. It is a repetitive movement back and forth through an equilibrium, or central position, so that the maximum displacement on one side of this position is equal to the maximum displacement on the other side.

skew lines

In three dimensions, lines that do not intersect and are not parallel to each other.

speed

The absolute value of an object's velocity. It represents how fast the object is moving. For an object moving along the x-axis, average speed is calculated as $\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$. If its position at time t is $x(t)$ the instantaneous speed is $\left|\frac{dx}{dt}\right|$.

squeeze theorem

A theorem to find the limit of a function that is bounded between two other functions: if $f(x) \leq g(x) \leq h(x)$ for all x that are near k, but not necessarily at k, and $\lim_{x \to k} f(x) = \lim_{x \to k} h(x) = L$, then $\lim_{x \to k} g(x) = L$.

statement

A statement is an assertion that is either true or false but not both.

Also referred to as a proposition.

substitution

The process of replacing a variable in an algebraic expression, formula, equation or function consistently by a particular value, another variable, expression or function.

Simultaneous equations can be solved using substitution by isolating one variable in one equation and using its value to replace that variable in another equation.

See simultaneous equations

Extended in Mathematics Extension 1 and Extension 2: Integration by substitution is a method used for evaluating integrals. The substitution could be expressed as a function of the variable of integration, or with the variable of integration as the subject of the substitution.

t-formulas

In trigonometry, relationships between $\sin A$, $\cos A$ and $\tan A$, to the tangent of half the angle. The t-formulas are $\sin A = \frac{2t}{1+t^2}$, $\cos A = \frac{1-t^2}{1+t^2}$ and $\tan A = \frac{2t}{1-t^2}$ where $t = \tan\frac{A}{2}$

terminal velocity

The constant velocity that a free-falling object approaches as the resistance of the medium through which the object is falling more closely cancels the downward gravitational force.

triangle inequality

A theorem describing the relationship between the absolute value of the sum of numbers to the absolute values of those numbers: $|a+b| \leq |a| + |b|$ for real or complex numbers a and b, or when a and b are vectors of equal dimension. Geometrically, that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

unit circle

A circle with a radius of 1 unit.

vector

A quantity having both magnitude and direction.

velocity

The rate of change of an object's position with respect to time. It is a vector quantity, meaning it has both magnitude and direction. The SI units for velocity are metres per second (ms^{-1}).

Extended in Mathematics Advanced, Extension 1, Extension 2: A particle moving along the x-axis whose position at time t is $x(t)$ has velocity $\frac{dx}{dt}$.

Global Terms

Aboriginal and Torres Strait Islander Peoples

Aboriginal Peoples are the first peoples of Australia and are represented by more than 250 language groups, each associated with a particular Country or territory. Torres Strait Islander Peoples are represented by 5 major island groups, and are associated with island territories to the north of Australia's Cape York which were annexed by Queensland in 1879.

An Aboriginal and/or Torres Strait Islander person is someone who:

- is of Aboriginal and/or Torres Strait Islander descent
- identifies as an Aboriginal person and/or Torres Strait Islander person, and
- is accepted as such by the Aboriginal and/or Torres Strait Islander community(ies) in which they live.

Aboriginal English

A recognised dialect of English which is the first, or home language, of many Aboriginal people. It differs from other dialects of English, such as Standard Australian English, in systematic ways including sounds, grammar, words and their meanings, and language use. Aboriginal English is a powerful vehicle for the expression of Aboriginal identity. Aboriginal English is not a target language study option for NSW Aboriginal Languages syllabuses.

Aboriginal Songline(s)

Texts that describe landscapes and directions of the tracks forged in lands, waters and skies by Creator Spirits during the Dreaming.

accessibility

The extent to which a system, environment or object may be used irrespective of a user's capabilities or abilities. For example, the use of assistive technologies (AT) to allow people with disability to use computer systems, or the use of icons in place of words to allow young children to use a system.

assistive technology (AT)

A device or system whose primary purpose is to maintain or improve an individual's functioning and independence to facilitate participation and enhance overall wellbeing. This includes technologies specifically designed to meet an individual's needs, eg eye gaze technology, as well as more general technologies that can be used by anyone, eg speech-to-text applications. Assistive technology can also be referred to as inclusive technology.

augmentative and alternative communication (AAC)

An umbrella term that encompasses the communication methods used to supplement or replace speech or writing. AAC can be unaided, such as gestures, body language and sign language, or aided such as pictures, symbols, objects or speech generating devices.

See communication systems

communication forms

The ways people communicate and the communicative behaviours they use. Communication forms can be non-symbolic and/or symbolic. Non-symbolic forms include sounds, gestures, facial expressions and eye movements. Symbolic forms can be aided or non-aided. Aided forms of symbolic communication include objects, symbols, photographs and drawings. Aided forms can be

digital. Non-aided forms of symbolic communication include formal gestures; speech; and signs, such as Key Word Sign.

copyright

The protection provided to the creators of original works and makers of sound recordings and films, that offers a legal framework for the control and reproduction or transmission of their literary, dramatic, artistic or musical works.

Country/Place

Country is used to describe a specific area of a nation or clan including physical, linguistic and spiritual features. Aboriginal communities' cultural associations with their Country may include or relate to languages, cultural practices, knowledge, songs, stories, art, paths, landforms, flora, fauna and minerals. These cultural associations may include custodial relationships with particular landscapes such as land, sea, sky, rivers as well as the intangible places associated with the Dreaming(s). Custodial relationships are extremely important in determining who may have the capacity to authentically speak for their Country.

Place is a space mapped out by physical or intangible boundaries that individuals or groups of Torres Strait Islander Peoples occupy and regard as their own. It is a space with varying degrees of spirituality.

culture

The customs, habits, beliefs/spirituality, social organisation and ways of life that characterise different groups and communities. Cultural characteristics give a group or individual a sense of who they are and help them make sense of the world in which they live. Culture is a shared system but inherently diverse – there can be individual and group differences within cultures. Everyone has culture – it is a lens through which we see the world.

custodians

In Aboriginal communities, an individual charged with maintaining and passing on particular elements of cultural significance, eg language, stories, songs, rituals and imagery.

See owners

d/Deaf

When referring to deaf people who belong to a linguistic and cultural minority known as the Deaf community, the 'D' may be capitalised in reference to the individual, the group, or the culture in order to accord respect and deference, for example, the Deaf community. When referring simply to audiological status or when cultural affiliation is not known, as in the case of a person with a hearing loss in general, the lowercase 'd', as in 'deaf' is the more common usage.

Deaf

A cultural identity for people with hearing loss who share a common culture and who usually have a shared sign language.

disability

An umbrella term for any or all of the following components:

- impairments: challenges in body function or structure
- activity limitations: difficulties in executing activities

- participation restrictions: challenges an individual may experience in involvement in life situations. (World Health Organization)

diversity

Differences that exist within a group, for example, age, sex, gender, gender expression, sexuality, ethnicity, ability/disability, body shape and composition, culture, religion/spirituality, learning differences, socioeconomic background, values and experiences.

Dreaming

The Dreaming has different meanings for different Aboriginal groups. The Dreaming can be seen as the embodiment of Aboriginal creation which gives meaning to everything; the essence of Aboriginal beliefs about creation and spiritual and physical existence. It establishes the rules governing relationships between the people, the land and all things for Aboriginal Peoples. The Dreaming is linked to the past, the present and the future. Where appropriate, refer to Aboriginal names for the Dreaming.

Elders

The custodians of knowledge and lore. They are chosen and accepted by their own communities as people who have the permission to disclose cultural knowledge and beliefs. Recognised Elders are highly respected people within Aboriginal communities. Proper consultation with local Aboriginal communities will often direct schools to recognised Elders.

first language(s)

The first language(s) that a person learns to speak.

Indigenous

An internationally recognised term for the first peoples of a land. In NSW the term Aboriginal person/Peoples is preferred.

See Aboriginal and Torres Strait Islander Peoples

Indigenous cultural and intellectual property (ICIP)

Includes, but is not limited to, objects, sites, cultural knowledge, cultural expression and the arts, that have been transmitted or continue to be transmitted through generations as belonging to a particular Indigenous group or Indigenous people as a whole or their territory.

See intellectual property

intellectual property

Non-material assets such as forms of cultural expression that belong to a particular individual or community. Intellectual property rights refer to the rights that the law grants to individuals for the protection of creative, intellectual, scientific and industrial activity, such as inventions.

See Indigenous cultural and intellectual property (ICIP), and copyright

Key Word Sign

A communication strategy that incorporates signing with speech. It is used to support language development for people with communication difficulties. Although Key Word Sign uses a simplified form of manual signing, it is different to Auslan, as it is not a signed language.

kinship

A key aspect of Aboriginal cultures and values. It includes the importance of all relationships and of being related to and belonging to the land.

language group

An Aboriginal community identified with a common language, both verbal and nonverbal, and with a particular territory. Used in preference to the term ‘tribe’.

language revival

The process and range of strategies for increasing knowledge and use of a language that is no longer spoken fluently across all generations in the context of language loss or language dispossession caused by colonisation. Aboriginal Languages and Torres Strait Islander Languages are being revived through community initiatives, linguistic research and school programs. ‘Language revival’ may be used as an overarching term that could also include ‘reclamation’, ‘revitalisation’, ‘renewal’ and ‘reawakening’.

local Aboriginal community

A local Aboriginal community is constituted by those people who are Aboriginal and who reside in the near locality. Aboriginal communities will have a rich and diverse history that has been seriously affected by dispossession and relations, which sees families with spiritual connection to Country residing beside those who have been forced to move from other locations. The notion of locality is complex and multilayered: schools should seek advice from a range of people and/or organisations representing local interests.

See community(ies)

off-Country

Taking place away from Aboriginal land or Country of origin.

See Country/Place

on-Country

Taking place on Aboriginal land or Country of origin.

See Country/Place

owners

Each Aboriginal Language is recognised as belonging to a particular geographical area and thus to the people who can claim a connection to that area. Aboriginal community members acquire ownership of their language(s) at birth. Language proficiency is not essential for ownership.

See custodians

protocols

The appropriate ways of behaving, communicating and showing respect for diversity of history and culture. This involves appreciation of the knowledge, standing and status of people within the local Aboriginal community and the school community. Protocols inevitably vary between communities, and between people within a community. In establishing a partnership between schools and Aboriginal communities, it is especially important that protocols are acknowledged and respected.

respect

A term used commonly in NSW Aboriginal communities to refer to the way an individual treats others. Showing respect occurs in many ways, such as waiting to speak, listening and demonstrating understanding, not asking too many direct questions, ensuring that people are not made to feel uncomfortable or uneasy, and generally showing regard for others' ideas, beliefs and culture.

sign

Hand signs (or hand talk) used to supplement or replace oral language. Signs form part of nonverbal communication for Aboriginal and Torres Strait Islander Peoples and may be used by people who are hearing, or d/Deaf or hard of hearing. Aboriginal and Torres Strait Islander Sign Languages may be used in some areas. Some Sign Languages may be associated with sacred ceremonial practices.

signage

The use of words, graphic designs and/or symbols used to communicate a message, eg information signs, plaques, warning signs, road signs, signs that show direction.

switch

A type of assistive technology that enables people with cognitive and/or physical disability to access a range of devices, including computers and communication devices. Switches can be activated by touch, or triggered without contact, such as through eye gaze, sound or blowing.

See assistive technology (AT)

yarning circle

Yarning circles are an important cultural practice for Aboriginal and Torres Strait Islander Peoples to learn within the collective group. Knowledge and information are shared in harmony and respect with all individuals.