NSW Education Standards Authority

Multiple Syllabuses

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Generated Nov 2025

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Mathematics Advanced 11–12 (2024)

Implementation from 2026

The new Mathematics Advanced 11–12 Syllabus (2024) is to be implemented from 2026 and will replace the **Mathematics Advanced Stage 6 Syllabus (2017)**.

2025

- Plan and prepare to teach the new syllabus

2026, Term 1

- Start teaching the new syllabus for Year 11
- Start implementing the new Year 11 school-based assessment requirements
- Continue to teach the **Mathematics Advanced Stage 6 Syllabus (2017)** for Year 12

2026, Term 4

- Start teaching the new syllabus for Year 12
- Start implementing the new Year 12 school-based assessment requirements

2027

- First HSC examination for the new syllabus

Overview

Syllabus overview

Organisation of Mathematics Advanced 11–12

The organisation of outcomes and content for Mathematics Advanced 11–12 highlights the important role Working mathematically plays across all areas of mathematics and reflects the strengthened connections between concepts. Working mathematically has been embedded in the outcomes and content of the syllabus.

Mathematics Advanced outcomes and their related content are organised into 7 areas of study:

- Functions
- Trigonometric functions
- Sequences and series
- Calculus
- Exponential and logarithmic functions
- Statistical analysis
- Financial mathematics

Figure 1 shows the organisation of Mathematics Advanced 11–12, including the focus areas to be completed in each area of study.

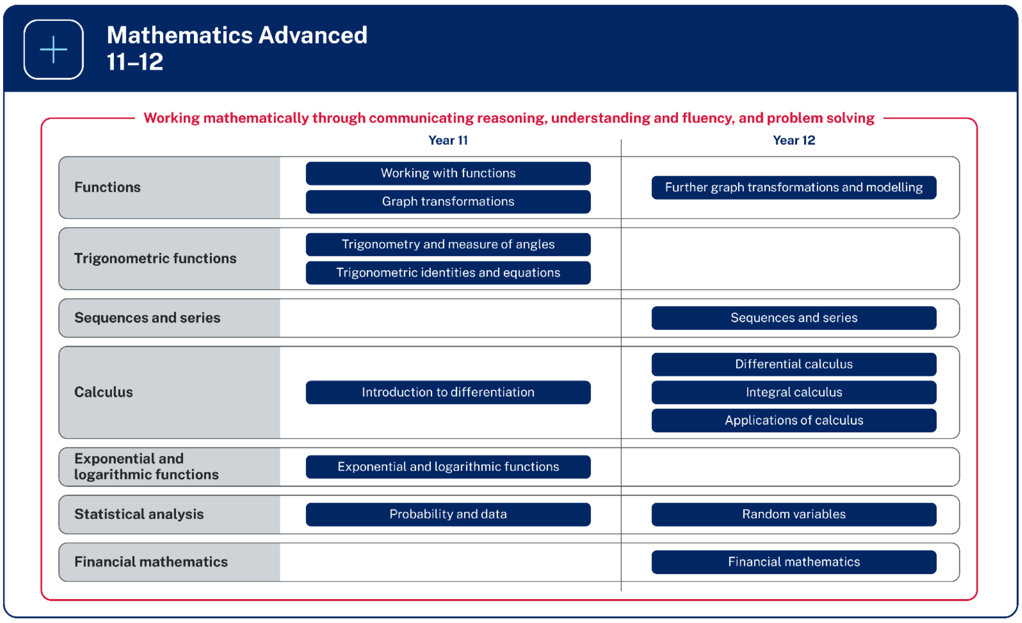


Figure 1: Organisation of Mathematics Advanced 11–12

Image long description: Seven rows represent the areas of study for Mathematics Advanced 11–12. Each row is divided in half, showing the focus areas for Year 11 and Year 12. The Functions area of study includes Year 11 focus areas Working with functions and Graph transformations and the Year 12 focus area Further graph transformations and modelling. The Trigonometric functions area of study includes the Year 11 focus areas Trigonometry and measure of angles and Trigonometric identities and equations. The Exponential and logarithmic function area of study includes the Year 11 focus area Exponential and logarithmic functions. The Sequences and series area of study includes the Year 12 focus area Sequences and series. The Calculus area of study includes the Year 11 focus area Introduction to differentiation, and the Year 12 focus areas Differential calculus, Integral calculus and Applications of calculus. Finally, the Statistical analysis area of study includes the Year 11 focus area Probability and data, and the Year 12 focus area Random variables. The Financial mathematics area of study includes the Year 12 focus area Financial mathematics. All content is surrounded by a box labelled with the phrase, ‘Working mathematically through communicating reasoning, understanding and fluency, and problem solving’.

Protocols for collaborating with Aboriginal and Torres Strait Islander Communities and engaging with Cultural works

NESA is committed to working in partnership with Aboriginal Communities and supporting teachers, schools and schooling sectors to improve educational outcomes for young people.

It is important to respect appropriate ways of interacting with Aboriginal Communities and Cultural material when teachers plan, program and implement learning experiences that focus on Aboriginal and Torres Strait Islander Priorities.

Indigenous Cultural and Intellectual Property (ICIP) protocols need to be followed. Aboriginal and Torres Strait Islander Peoples’ ICIP protocols include Cultural Knowledges, Cultural Expression and Cultural Property and documentation of Aboriginal and Torres Strait Islander Peoples’ Identities and lived experiences. It is important to recognise the diversity and complexity of different Cultural groups in NSW, as protocols may differ between local Aboriginal Communities.

Teachers should work in partnership with Elders, parents, Community members, Cultural Knowledge Holders, or a local, regional or state Aboriginal Education Consultative Group. It is important to respect Elders and the roles of men and women. Local Aboriginal Peoples should be invited to share their Cultural Knowledges with students and staff when engaging with Aboriginal histories and Cultural Practices.

Course description

Mathematics Advanced 11–12 focuses on mathematical ways of viewing the world to investigate concepts, such as order, relation, pattern, uncertainty and generality. The course provides students with the opportunity to explore mathematical problems through observation, reflection and reasoning.

What students learn

Through the study of Mathematics Advanced 11–12, students:

- develop knowledge, understanding and skills in Working mathematically and communicating concisely and precisely
- consider various applications of mathematics in a broad range of contemporary contexts through mathematical modelling
- gain an appropriate mathematical background for future pathways which involve mathematics and its applications at the tertiary level.

Course structure and requirements

Mathematics Advanced consists of the courses Mathematics Advanced Year 11 and Mathematics Advanced Year 12. Students must study both Mathematics Advanced Year 11 and Mathematics Extension 1 Year 11 courses before they can study Year 12 Mathematics Extension courses.

Alternatively, students can study both Mathematics Advanced Year 11 and Mathematics Advanced Year 12 before they begin either Mathematics Extension 1 Year 11 and Mathematics Extension 1 Year 12, or Mathematics Extension 1 Year 12 and Mathematics Extension 2 Year 12.

The course numbers and units for each year of study are set out below.

Course numbers:

- Mathematics Advanced (Year 11, 2 units): TBA
- Mathematics Advanced (Year 12, 2 units): TBA

Year 11 course structure

Area of study	Focus areas
Functions	Working with functions Graph transformations
Trigonometric functions	Trigonometry and measure of angles Trigonometric identities and equations
Calculus	Introduction to differentiation
Exponential and logarithmic functions	Exponential and logarithmic functions
Statistical analysis	Probability and data

Year 12 course structure

Area of study	Focus area
Functions	Further graph transformations and modelling
Calculus	Differential calculus Integral calculus Applications of calculus
Sequences and series	Sequences and series
Statistical analysis	Random variables
Financial mathematics	Financial mathematics

Further information for Mathematics Advanced Year 11

- Course number: TBA
- Course hours: 120
- Course units: 2
- Enrolment type: Elective
- Endorsement type: Board developed
- Study via self-tuition: Yes

Exclusions

- Mathematics Standard (Year 11, 2 units): TBA
- Mathematics Life Skills (Year 11, 2 units): TBA

Further information for Mathematics Advanced Year 12

- Course number: TBA

- Course hours: 120
- Course units: 2
- Enrolment type: Elective
- Endorsement type: Board developed
- Study via self-tuition: Yes

Exclusions

- Mathematics Standard 1 (Year 12, 2 units): TBA
- Mathematics Standard 2 (Year 12, 2 units): TBA
- Mathematics Life Skills (Year 12, 2 units): TBA

Rationale

Mathematics in the Stage 6 Curriculum

Mathematical ideas have evolved and continue to develop across cultures and have been practised in Australia by Aboriginal and Torres Strait Islander Peoples for thousands of years.

The utility of mathematics extends from counting and measuring to explaining real-world phenomena and its inherent beauty. It has evolved in sophisticated ways to become the language used to describe and understand much of the modern world.

The creative application of mathematical concepts is integral to scientific and technological advancement. The symbolic nature of mathematics provides a precise means of communication.

The Mathematics Stage 6 syllabuses are designed to offer students opportunities to think mathematically. Mathematical thinking is supported through questioning, communicating, reasoning, reflecting and the use of appropriate technology, and is encouraged through providing students with opportunities to generalise, challenge, find connections and to think critically and creatively. Using mathematical thinking, students apply their knowledge and skills to deepen their understanding of the world.

Mathematics Advanced

Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 form a continuum to provide students with opportunities to acquire knowledge, understanding and skills in mathematical concepts at progressively higher levels and with applications in an increasing number of contexts. Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 provide students with opportunities to learn about the interconnected nature of mathematics, its beauty and its functionality. The concepts and techniques of differential and integral calculus are the basis of the courses.

The *Mathematics Advanced 11–12 Syllabus* (2024) is designed to encourage students to appreciate mathematical ways of viewing the world to investigate concepts, such as order, relation, pattern, uncertainty and generality. The course provides students with the opportunity to explore mathematical problems through observation, reflection and reasoning.

The *Mathematics Advanced 11–12 Syllabus* (2024) enables students to use mathematical models and serves as a basis for further studies at the tertiary level in science and commerce that require mathematics and its applications.

Aim

The aim of Mathematics Advanced in Years 11 and 12 is to enable students to enhance their knowledge and understanding from Stage 5 of how to work mathematically, make mathematical connections, develop their understanding of the relationship between real-world problems and mathematical models, and extend their skills to apply the language of mathematics to communicate in a concise and systematic manner.

Table of outcomes

MAO-WM-01 Working mathematically

develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly

This outcome is aligned to all content in each Stage.

Year 11	Year 12
MAV-11-01 applies algebraic techniques and the laws of indices and surds to manipulate expressions and solve problems	**MAV-12-01** uses algebraic and graphical techniques to analyse trigonometric functions
MAV-11-02 uses functions and relations to model, analyse and solve problems	**MAV-12-02** models, analyses and solves practical problems involving functions and their transformations
MAV-11-03 analyses and solves algebraic and graphical problems involving transformations of functions and relations	**MAV-12-03** uses arithmetic and geometric sequences and series to model and solve problems
MAV-11-04 applies trigonometry to solve problems involving geometric shapes	**MAV-12-04** selects and applies differentiation methods to solve problems
MAV-11-05 uses periodic functions to solve trigonometric equations and prove trigonometric identities	**MAV-12-05** solves problems involving indefinite and definite integrals
MAV-11-06 interprets the meaning of the derivative and determines the derivative of functions to solve problems	**MAV-12-06** applies calculus to graph functions and model and solve problems involving optimisation, rates of change and motion in a line
MAV-11-07 applies exponential and logarithmic laws to simplify expressions, solve equations and prove results	**MAV-12-07** solves problems involving discrete probability distributions, continuous random variables and the normal distribution
MAV-11-08 analyses graphs of exponential and logarithmic functions	**MAV-12-08** models and solves problems to make informed decisions about financial situations
MAV-11-09 solves problems involving probability in a variety of contexts	
MAV-11-10 displays and analyses datasets using summary statistics and graphical representations	

Outcomes and content for Year 11

Working with functions

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- applies algebraic techniques and the laws of indices and surds to manipulate expressions and solve problems **MAV-11-01**
- uses functions and relations to model, analyse and solve problems **MAV-11-02**

Content

Algebraic techniques

- Use index laws to simplify expressions and solve problems involving positive, negative, zero or fractional indices
- Expand, factorise and simplify algebraic expressions
- Simplify expressions involving algebraic fractions

Example(s):

Simplify $\frac{a^3b-ab^3}{a^2+2ab+b^2}$, $\frac{m^3+m^2}{x^2-x} \div \frac{m+1}{x-x^3}$ and $\frac{x^2}{x^2+3x+2}-\frac{x}{x+1}$.

- Expand and simplify expressions involving surds
- Identify the conjugate of $\sqrt{a} \pm \sqrt{b}$, and rationalise the denominators of expressions of the form $\frac{a}{\sqrt{b} \pm \sqrt{c}}$ and $\frac{\sqrt{a} \pm \sqrt{d}}{\sqrt{b} \pm \sqrt{c}}$, where a, b, c and d are positive rational numbers
- Solve quadratic equations $ax^2+bx+c=0$ by factorisation, completing the square and using the quadratic formula $x=\frac{-b \pm \sqrt{b^2-4ac}}{2a}$ where a, b and c are real numbers and $a \neq 0$
- Define the discriminant as $\Delta=b^2-4ac$ and use it to solve problems involving the number of real roots of a quadratic equation and determine the conditions for roots to be equal, distinct, real or rational

Example(s):

Find k if the equation $(2k+3)x^2-4kx+4=0$ has equal roots.

Introduction to functions and relations

- Describe a relation between two sets as an association between the elements of one set and the elements of the other set

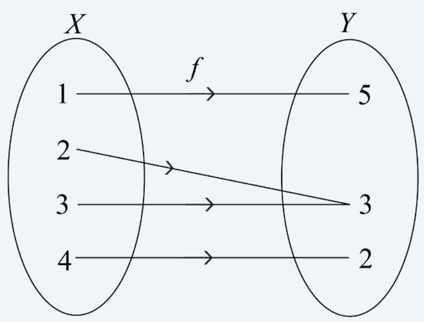
Example(s):

A relation between the elements of a set A and a set B can be described as the set of ordered pairs (x, y) where x is a member of A and y is a member of B related to x. Consider the relation between the set of 'names of students in the class' and 'month of birth'.

- Define a real function f of a real variable x as a relation where each element x of a given set is associated with exactly one element y of the second set

Example(s):

f is a function in which every element in X is associated with exactly one element of Y. g is not a function but a relation, as element 1 in set X is assigned to the two elements 5 and 6 in set Y.



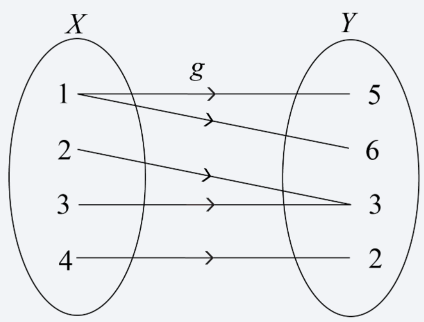


Image long description: The first image shows a function f associating the elements 1, 2, 3 and 4 in a set X to the elements 5, 3 and 2 in a set Y, so that 1 is mapped to 5, 2 and 3 are mapped to 3, and 4 is mapped to 2. The second image shows relation g associating the elements 1, 2, 3, and 4 in a set X with the elements 5, 6, 3, and 2 in a set Y, so that 1 is mapped to 5 and 6, 2 and 3 are mapped to 3, and 4 is mapped to 2.

- Recognise that a relation is a rule which can be represented by an algebraic formula, a table of values, a set of ordered pairs (x, y) or a graph

Example(s):

$G = \{(1,5),(4,2),(2,3),(3,3),(1,6)\}$ is a relation.

- Use the notation $f(x)$ to identify the unique value of y associated with x when working with functions, and refer to it as the value of f at x
- Refer to x in a function $y = f(x)$ as the independent variable of the function, and refer to y as the dependent variable
- Substitute numeric and algebraic expressions into the formulas of functions
- Apply the vertical line test on the graph of a relation to determine whether it represents a function
- Define the domain of a function f as the set of real numbers on which f is defined

- Define the range of a function f as the set of values of $f(x)$ obtained as x varies over the domain of f
- Refer to a value in the domain of a function at which the function is 0 as a zero of the function
- Recognise that the x-intercepts of the graph of $y=f(x)$ are its zeroes, the solutions of $f(x)=0$, and that the y-intercept is $f(0)$

Linear functions

- Determine the equations of straight lines in gradient–intercept form $y=mx+c$ with gradient m and y-intercept c
- Determine the equations of straight lines in general form $ax+by+c=0$, where a, b and c are constants
- Determine the equation of a straight line passing through a point (x_1, y_1) with gradient m using the point–gradient formula $y-y_1=m(x-x_1)$
- Determine the equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) by calculating its gradient m using the formula $m=\frac{y_2-y_1}{x_2-x_1}$

Example(s):

Find the equation of the line that has a gradient of 3 and passes through the point $(2,-5)$ and hence determine whether the point $(5,8)$ lies on that line.

- Determine the x-intercept and y-intercept of a straight line given its equation
- Choose and apply appropriate techniques to graph a straight line given its equation
- Find the equation of a line that is parallel or perpendicular to a given line
- Solve linear inequalities and graph the solution on a number line

Quadratic and cubic functions

- Identify the x-intercepts of a parabola whose quadratic function is expressed in factored form
- Use the discriminant to determine the number of x-intercepts on a parabola and justify its position in relation to the x-axis

Example(s):

Show that the discriminant of $y=-x^2+2x-2$ is negative and justify why its graph lies entirely below the x-axis.

- Show by completing the square on the general quadratic $y=ax^2+bx+c$ that the axis of symmetry is $x=-\frac{b}{2a}$ where a, b and c are constants
- Identify the axis of symmetry and vertex of a parabola by completing the square on its quadratic function

- Choose and apply appropriate techniques to graph a parabola of the form $y=ax^2+bx+c$ by identifying its x-intercepts if they exist, its y-intercept, its axis of symmetry using $x=-\frac{b}{2a}$ and its vertex

Example(s):

Express $y=x^2+2x+3$ in the form $y=(x-a)^2+c$ and determine the coordinates of the vertex. Graph the parabola showing its y-intercept.

- Find the equation of a parabola given sufficient graphical features
- Use the fact that two quadratic functions are equal for all values of x if and only if the corresponding coefficients are equal to solve related problems

Example(s):

Find the values of a, b and c if the quadratic functions $f(x)=x^2-x$ and $f(x)=a(x+3)^2+bx+c-1$ have the same graph.

- Recognise that solving $f(x)=k$ for some constant k corresponds to finding the x-coordinate(s) of the intersection of the graphs $y=f(x)$ and $y=k$
- Solve problems by finding the solution to simultaneous equations involving a linear and a quadratic function, or two quadratic functions, both algebraically and graphically
- Solve quadratic inequalities
- Recognise and graph cubic functions of the form $f(x)=kx^3$ and $f(x)=k(x-a)(x-b)(x-c)$, where a, b, c and k are constants and $k\neq 0$

Reciprocal functions

- Graph functions of the form $f(x)=\frac{k}{x}$, where k is a constant and $k\neq 0$, and identify their hyperbolic shape and their asymptotes
- Describe the behaviour of $f(x)=\frac{k}{x}$ as $x\to\infty$ and $x\to-\infty$

Example(s):

For $f(x)=\frac{-2}{x}$ as $x\to\infty, f(x)\to 0$ and as $x\to-\infty, f(x)\to 0$.

Constructing and using functions

- Construct and use linear functions to model and solve problems in real-world situations, identifying the independent and dependent variables and any restrictions on these variables, and justify conclusions in the context of the problem
- Use linear inequalities to model and solve problems in real-world situations, and justify conclusions in the context of the problem

- Solve practical problems involving a pair of simultaneous linear equations both algebraically and graphically, with and without graphing applications, and justify conclusions in the context of the problem
- Construct and use simultaneous equations to model and solve a problem where cost and revenue are represented by linear equations, identify and analyse the break-even point, and justify conclusions in the context of the problem
- Model and solve practical problems involving quadratic functions and justify conclusions in the context of the problem

Direct and inverse variation

- Develop models of the form $y=kx^n$, where k is a non-zero constant, from descriptions of situations in which one quantity varies directly with another
- Develop the model $y=\frac{k}{x^n}$, where k is a non-zero constant, from descriptions of situations in which one quantity varies inversely with another
- Evaluate k in the equations $y=kx^n$ and $y=\frac{k}{x^n}$, given one pair of values for the variables, and use the resulting formula to find other values of the variables
- Analyse and solve problems involving direct and inverse variation

Circles and semicircles

- Derive the equation of a circle of radius r with centre at the origin by considering Pythagoras' theorem
- Graph circles of the form $x^2+y^2=r^2$ from their equations
- Determine the equation of a circle of the form $x^2+y^2=r^2$ given its graph
- Identify and graph the semicircles $y=\sqrt{r^2-x^2}$, $y=-\sqrt{r^2-x^2}$, $x=\sqrt{r^2-y^2}$ and $x=-\sqrt{r^2-y^2}$

Properties of functions, relations and graphs

- Extend the definitions of domain and range to relations
- Recognise domains and ranges of functions and relations given in interval notation, as inequalities and as worded descriptions
- Determine and describe the domain and range of functions and relations, using interval notation, inequalities or worded descriptions

> **Example(s):**
>
> The domain and range of $y=\sqrt{x-4}$ can be expressed in interval notation as $[4,\infty)$ and $[0,\infty)$ respectively. The range of $y=2^x$ is the set of y satisfying $y>0$. The domain of $2x-3y+2=0$ is all real x.

- Define a function to be even if its graph is unchanged under reflection in the y-axis, and odd if its graph is unchanged under rotation of $180°$ about the origin
- Develop and use the tests that a function $f(x)$ is odd if $f(-x)=-f(x)$ and a function $f(x)$ is even if $f(-x)=f(x)$
- Solve problems involving even and odd functions
- Use the composite function $f(g(x))$, where the output of $g(x)$ becomes the input of $f(x)$

- Determine the equations of composite functions

> **Example(s):**
>
> Given the functions $f(x) = x - 2$ and $g(x) = x^2$, determine the equation of $f(g(x))$ and evaluate $g(f(2))$.

Piecewise-defined functions

- Interpret piecewise-defined functions, where the function is defined differently in different parts of the domain
- Graph piecewise-defined functions involving functions covered in the scope of the Mathematics Advanced course, test if they are even or odd, and determine the domain and range

> **Example(s):**
>
> Graph the function $f(x) = \begin{cases} x^2, & \text{for } x \le -1, \\ x - 1, & \text{for } -1 < x \le 1, \\ -x^3, & \text{for } x > 1. \end{cases}$. Mark endpoints of the graph with a closed circle or an open circle depending on whether the endpoint is included or not included.

- Define informally that a function is continuous at a point if the curve can be drawn through the point without lifting the pen off the paper
- Identify points where piecewise-defined functions and other functions are not continuous

> **Example(s):**
>
> Identify any places where the graphs of $f(x) = \frac{x^2 - 1}{x - 1}$ and $f(x) = \frac{|x|}{x}$ are not continuous and graph the functions.

- Define a discontinuity of a function informally as a point where the function is not continuous

Absolute value functions

- Define the absolute value $|x|$ of a number x, also known as the magnitude of x, to be the distance from the origin to x on the number line
- Establish and use the piecewise definition $|x| = \begin{cases} x, \text{for } x \ge 0, \\ -x, \text{for } x < 0 \end{cases}$
- Show using numerical substitutions that $\sqrt{x^2} = |x|$ and use the result
- Graph the function $y = |x|$, describe its symmetry, and identify its domain and range
- Graph $y = |ax + b|$ with and without graphing applications, and identify its symmetry, domain and range

- Solve absolute value equations of the form $|ax+b|=k$ algebraically and graphically

Example(s):

Graph $y=4$ and $y=|2x-2|$ on the same number plane and hence solve $|2x-2|=4$.

Trigonometry and measure of angles

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- applies trigonometry to solve problems involving geometric shapes **MAV-11-04**

Content

Trigonometry with acute angles

- Use Pythagoras' theorem to find the exact sine, cosine and tangent ratios for angles of $30°$, $45°$ and $60°$
- Apply trigonometry to solve problems involving right-angled triangles in two dimensions, true and compass bearings, and angles of elevation and depression

Trigonometry with angles of any magnitude

- Represent angles of any magnitude using rays from the origin in the Cartesian plane and describe how one ray represents infinitely many angles
- Develop the definitions $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$ and $\tan\theta = \frac{y}{x}$, where (x, y) is a point on the circle of radius r, centred at the origin, and θ is the angle between the positive x-axis and the radius drawn to this point
- Use the definitions $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$ and $\tan\theta = \frac{y}{x}$ to evaluate $\sin\theta$, $\cos\theta$ and $\tan\theta$ where θ is a multiple of $90°$, and identify the values of θ for which these ratios are undefined
- Extend and apply the definitions $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$ and $\tan\theta = \frac{y}{x}$ for angles of any magnitude
- Identify the related angle of an angle of any magnitude, excluding multiples of $90°$, as the acute angle between the ray and the x-axis and obtain the values of the trigonometric functions of an angle of any magnitude from the trigonometric functions of the related angle
- Develop and use the trigonometric ratios for angles that can be written in the form $\theta = 180° \pm A$, and $\theta = 360° - A$, where $0° < A < 90°$

Example(s):

Find the exact value of $\sin(-120°)$ and $\tan 585°$.

- Establish and use the results $\cos(-\theta) = \cos\theta$, $\sin(-\theta) = -\sin\theta$ and $\tan(-\theta) = -\tan\theta$
- Examine the proof of the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, cosine rule $c^2 = a^2 + b^2 - 2ab\cos C$ and the area of a triangle formula $A = \frac{1}{2}ab\sin C$ for a given triangle ABC

- Use graphing applications or geometric construction to examine the ambiguous case of the sine rule, in which there are two possible solutions for an angle, and the condition for it to arise

Example(s):

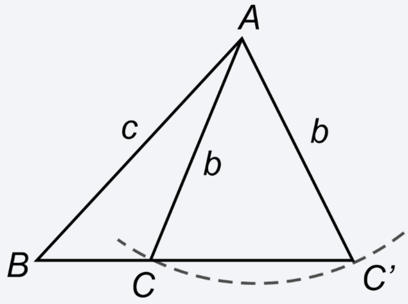


Image long description: A triangle ABC' with sides b and c contains a smaller triangle with 2 sides b. The internal side b intersects line BC' at point C. An arc connects points C and C', demonstrating that the movement of point C can adjust the angle ACB so it is either obtuse or acute.

- Apply the sine rule, cosine rule and formula for the area of a triangle to solve problems where angles are measured in degrees, or degrees and minutes

Radians

- Recognise that both ratios $\frac{\text{arc length}}{\text{circumference}}$ and $\frac{\text{area of sector}}{\text{area of circle}}$ are equal to $\frac{\theta}{\text{one revolution}}$ where θ is the angle at the centre of a circle subtended by the arc
- Define the angle size of θ in radian measure as the ratio $\frac{\text{arc length}}{\text{radius of a circle}}$
- Explain why $360° = 2\pi$ radians and $1 \text{ radian} = \frac{180}{\pi}$ degrees
- Convert between degrees and radians and find the exact sine, cosine and tangent ratios for integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$
- Graph $y = \sin x$, $y = \cos x$ and $y = \tan x$ over domains given in degrees or radians, showing intercepts with the x-axis and y-axis and any asymptotes, determine their domains and ranges, and period and amplitude where appropriate, and whether each is even or odd or neither
- Establish and use the formula $l = r\theta$ for the length l of arc subtending an angle θ in radians at the centre of a circle of radius r
- Prove and use the formula $A = \frac{1}{2}r^2\theta$ for the area of a sector with angle θ in radians at the centre of a circle of radius r
- Solve problems involving arc lengths and areas of major and minor sectors and segments

Trigonometric identities and equations

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- uses periodic functions to solve trigonometric equations and prove trigonometric identities **MAV-11-05**

Content

- Define the trigonometric functions $sec\,\theta$, $cosec\,\theta$ and $\cot\theta$ for acute angles θ using ratios of sides in a right-angled triangle
- Find the exact secant, cosecant and cotangent ratios for angles of $30°$, $45°$ and $60°$
- Justify the values of $sec\,\theta$, $cosec\,\theta$ and $\cot\theta$ at $0°$ and $90°$ by examining the ratios of sides in a right-angled triangle as θ tends to $0°$ and $90°$
- Develop the definitions $sec\,\theta=\frac{r}{x}$, $cosec\,\theta=\frac{r}{y}$ and $\cot\theta=\frac{x}{y}$ using a circle of radius r centred at the origin
- Establish the reciprocal ratios $sec\,A=\frac{1}{\cos A}$, $cosec\,A=\frac{1}{\sin A}$, and $\cot A=\frac{1}{\tan A}$, identifying the angles to be excluded in each identity
- Determine the exact value of the secant, cosecant and cotangent ratios for angles that are integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$, if they exist
- Establish the identities $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and $\cot\theta=\frac{\cos\theta}{\sin\theta}$, identifying the angles to be excluded in each identity
- Prove the complementary angle identities $\sin(90°-\theta)=\cos\theta$, $\cos(90°-\theta)=\sin\theta$, $\tan(90°-\theta)=\cot\theta$, $\cot(90°-\theta)=\tan\theta$, $sec(90°-\theta)=cosec\,\theta$ and $cosec(90°-\theta)=sec\,\theta$ identifying the angles to be excluded in each identity
- Evaluate trigonometric expressions using angles of any magnitude and complementary angle identities

Example(s):

Given $\cot\alpha=-\frac{15}{8}$, and that $\frac{\pi}{2}<\alpha<\pi$, find the values of $cosec\,\alpha$ and $\cos\alpha$.

- Solve equations involving trigonometric ratios of angles, specified in degrees or radians, on a restricted domain

Example(s):

Solve $\sqrt{3}\sin x=\cos x$ for $0°\le x\le 360°$ and $2\cos 2x=0$ for $-\pi\le x\le\pi$.

- Prove the Pythagorean identity $\cos^2 x + \sin^2 x = 1$, and the identities $1 + \tan^2 x = sec^2 x$ and $1 + \cot^2 x = cosec^2 x$
- Apply trigonometric identities to solve problems, simplify expressions and prove further trigonometric identities using substitution and/or reduction to $\sin x$ and $\cos x$

Example(s):

Prove that $sec^2 x + sec\, x \tan x = \dfrac{1 + \sin x}{\cos^2 x}$ and hence prove that $sec^2 x + sec\, x \tan x = \dfrac{1}{1 - \sin x}$.

- Solve problems involving trigonometric equations, including those that reduce to quadratic equations, on a restricted domain

Example(s):

Express $5\cot^2 x - 2\,cosec\, x + 2$ in terms of $cosec\, x$ and hence solve the equation $5\cot^2 x - 2\,cosec\, x + 2 = 0$ for $0 \le x < 2\pi$.

Introduction to differentiation

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- interprets the meaning of the derivative and determines the derivative of functions to solve problems **MAV-11-06**

Content

Estimating change

- Define the average rate of change of y with respect to x for a function $y=f(x)$ over the domain $[a,b]$ as $\frac{\Delta y}{\Delta x}=\frac{\text{change in } y}{\text{change in } x}$, that is $\frac{\Delta y}{\Delta x}=\frac{f(b)-f(a)}{b-a}$, and recognise $\frac{f(b)-f(a)}{b-a}$ as the gradient of the secant through $(a,f(a))$ and $(b,f(b))$ on the graph of $y=f(x)$
- Recognise speed as a rate of change of distance with respect to time
- Use the definition for average rate of change to determine the average speed of an object from a given distance–time graph
- Describe the difference between the average speed of an object and its instantaneous speed
- Determine that the instantaneous speed of an object at time t can be approximated by the average speed between its position at time t and its position some time later, and explain how this approximation can be improved
- Relate the instantaneous speed of an object to the gradient of the tangent at that point on its distance–time graph
- Estimate the instantaneous speed of an object from its distance–time graph
- Recognise when modelling with a linear function that its gradient is the rate of change and determine the rate of change for linear functions in practical situations
- Recognise when modelling with a non-linear function that the rate of change is not constant and is represented by the gradient of the tangent to the curve at each point on the curve

- Estimate the instantaneous rate of change of a non-linear function at a given point from a given graph of a practical situation

Example(s):

The graph shows the displacement of a particle that is moving in a horizontal line. Estimate the velocity of the particle as it passes through the origin.

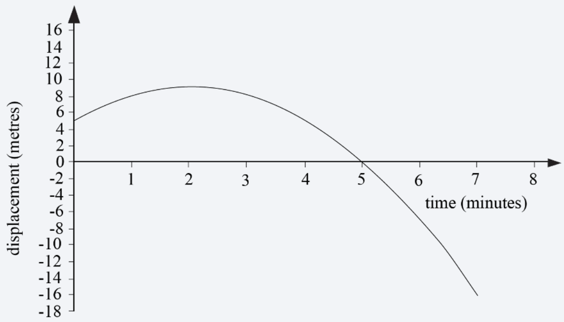


Image long description: A graph of a distance–time function with displacement in metres on the y-axis and time in minutes on the x-axis. It depicts a distance-time function that is in the shape of a concave down parabola with a distance intercept of 5 metres and time intercept of 5 minutes. The vertex of the parabola is at the point $(2,9)$.

The derivative

- Examine the gradient of a curve at a point on the curve using graphing applications

Example(s):

Use a graphing application to magnify the graph of $y=x^2$ at $x=1$. Discuss the 'straightness' of the graph when magnified.

- Approximate the gradient of a curve $f(x)=x^n$ at a point $P(c,f(c))$ by considering the gradient of the secant through P and $Q(c+h,f(c+h))$ as the magnitude of h approaches zero, using graphing applications or a spreadsheet

Example(s):

Use a spreadsheet or graphing application to examine the gradient of $f(x)=x^n$, $n=2$ and 3 at $P(c,f(c))$ for various values of c by considering the value of $\dfrac{f(c+h)-f(c)}{h}$ as h steadily decreases.

- Infer that $n x^{n-1}$ is the gradient of $f(x)=x^n$ and verify the result using graphing applications

Example(s):

Use a graphing application to find an approximation for the gradient of $f(x)=x^n$ by considering the graph of $y=\frac{f(x+0.001)-f(x)}{h}$ for $n=2$ and 3. Summarise the results for the gradients of $y=1$, $y=x$, $y=x^2$ and $y=x^3$ and generalise the result for $y=x^n$.

Use a graphing application to verify the result for positive, negative and fractional values of n.

- Define $f'(x)$, for any function $f(x)$ and any value x, to be the gradient of the tangent to the curve $y=f(x)$ at the point $P(x, f(x))$ if the tangent exists and is not vertical
- Refer to $f'(x)$ as the derivative of $f(x)$ or the gradient, or derived, function of $f(x)$
- Define differentiation as the process of finding the derivative of a function
- Find derivatives of constant and linear functions
- Define the derivative of the function $f(x)$ from first principles, as the limiting value of the gradient of the secant $\frac{f(x+h)-f(x)}{h}$ as h approaches zero, when this limiting value exists, and use the notation $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
- Use first principles to find the derivative of quadratic functions

Example(s):

Use differentiation by first principles to verify that if $y=3x^2-x$, then $y'=6x-1$.

Calculations with the derivative

- Use the notation $\frac{dy}{dx}$ and y' for the derivative of y when y is a function of x
- Use the notation $\frac{d}{dx}(f(x))$ and $f'(x)$ for the derivative of a function $f(x)$
- Use the formula $\frac{d}{dx}(x^n)=n x^{n-1}$ for all real values of n

Example(s):

Find the derivative of $f(x)=x^7$, $f(t)=\sqrt[\square]{t}$ and $g(x)=\frac{1}{x^3}$.

- Apply the fact that the derivative of a sum is the sum of the derivatives: $\frac{d}{dx}(f(x)+g(x))=f'(x)+g'(x)$, and the derivative of a multiple of a function is the multiple of its derivative: $\frac{d}{dx}(k\cdot f(x))=k\cdot f'(x)$

- Use the rules for differentiation to find equations of tangents and normals to a curve at points on the curve
- Find points on a curve where the tangent or normal has a given gradient
- Examine and use the relationship between the angle of inclination of a line or tangent to a curve, θ, with the positive x-axis, and the gradient, m, of that line or tangent, and establish that $\tan\theta = m$
- Apply the product rule: if $y = uv$, where u and v are both differentiable functions of x, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ or if $h(x) = f(x)g(x)$ for differentiable functions $f(x)$ and $g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$
- Apply the quotient rule: if $y = \frac{u}{v}$, where u and v are both functions of x, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ or if $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
- Apply the chain rule: if y is a differentiable function of u, and u is a differentiable function of x, then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ or if $h(x) = f(g(x))$ for differentiable functions $f(x)$ and $g(x)$ then $h'(x) = f'(g(x))g'(x)$
- Identify and apply the product, quotient or chain rule, or a combination of the rules, as appropriate to differentiate a given function

Graphical applications of the derivative

- Interpret $f(x)$ as increasing at $x = c$ when $f'(c) > 0$ and decreasing at $x = c$ when $f'(c) < 0$
- Describe the behaviour of a function at a point as stationary when the tangent at the point is parallel to the x-axis, and recognise that $f(x)$ is stationary at $x = c$ when $f'(c) = 0$
- Graph $y = f'(x)$ for a given graph of a function $y = f(x)$
- Numerically estimate the value of the derivative at a point on the graph of a power of x, with and without the use of digital tools
- Identify stationary points on the graph of a cubic function, and the values of x for which the function is increasing and/or decreasing, by first calculating the derivative, and justify conclusions

The derivative as a rate of change

- Interpret $f'(c)$ as the instantaneous rate of change of the function $f(x)$ at $x = c$
- Define and distinguish between displacement and distance and between velocity and speed
- Use graphs of functions and their derivatives, without the use of algebraic techniques, to describe and interpret physical phenomena
- Use the notation $\frac{dx}{dt}$ or $\dot{x}$ to represent the velocity of a particle with displacement x from a point as a function of time t
- Solve problems by determining the velocity of a particle moving in a straight line, given its displacement from a point as a function of time

Exponential and logarithmic functions

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- applies exponential and logarithmic laws to simplify expressions, solve equations and prove results **MAV-11-07**
- analyses graphs of exponential and logarithmic functions **MAV-11-08**

Content

Exponential functions

- Graph the exponential functions $y=k\left(a^x\right)$ and $y=k\left(a^{-x}\right)$ for constants a and k where $a>0$, $a\neq 1$ and $k\neq 0$, and identify its asymptote, y-intercept, domain and range
- Describe the behaviour of $y=k\left(a^x\right)$ and $y=k\left(a^{-x}\right)$ as $x\to\infty$ and $x\to-\infty$

Example(s):

For $y=2^x$, as $x\to-\infty$, $y\to 0$ and as $x\to\infty$, $y\to\infty$.

- Examine the gradient of the tangent to the curve $y=a^x$ at its y-intercept for varying values of a, and verify using graphing applications that there is a unique number $e\approx 2.71828182845...$, such that the gradient of the tangent to $y=e^x$ at $x=0$ is 1, and call this number e Euler's number
- Examine the gradient function of $y=a^x$ with graphing applications and identify that the gradient function is $\frac{d}{dx}\left(a^x\right)=k\,a^x$, for some constant k depending only on a
- Conclude that Euler's number, e, is a unique number such that $\frac{d}{dx}\left(e^x\right)=e^x$, that is, e^x is its own derivative

Logarithmic functions

- Define the logarithm of a number y, where $y>0$, to any positive base a as the index to which a is raised to give y
- Use the notation $\log_a y$ for the logarithm of y to the base a
- Define the natural logarithm $\ln a=\log_e a$
- Use digital tools to determine rational and irrational values of exponential and logarithmic expressions

Example(s):

Evaluate $4^{\frac{1}{3}}$, e^3 and $\log_{10}5$ to two significant figures.

- Explain that $y=a^x$ is equivalent to $x=\log_a y$ for $a>0$and $a\neq 1$, and use the equivalence to solve equations of the form $a^x=b$, for $a>0$

Example(s):

Solve $e^{x-2}=12$.

- Recognise and use the logarithmic properties: $\log_a a^x=x$ for all real x, $a^{\log_a x}=x$ where $x>0$

Example(s):

Solve $e^{2\ln x}-\ln\left(e^x\right)-4=0$.

- Derive the laws of logarithms from the laws of indices $\log_a m+\log_a n=\log_a(mn)$, $\log_a m-\log_a n=\log_a\left(\dfrac{m}{n}\right)$ and $\log_a\left(m^n\right)=n\log_a m$
- Justify the logarithmic results $\log_a a=1$, $\log_a 1=0$ and $\log_a\left(\dfrac{1}{x}\right)=-\log_a x$
- Apply the logarithmic laws and results to simplify expressions, solve equations and prove results, using digital tools where necessary

Example(s):

Solve $\log_2 x+\log_2(x-3)-\log_2(x+2)=2$.

- Prove the change of base rule $\log_a x=\dfrac{\log_b x}{\log_b a}$ and use it to solve problems
- Graph the logarithmic function $y=\log_a x$ for $a>0$ and $a\neq 1$
- Use graphing applications to identify the graphs of $y=a^x$ and $y=\log_a x$ as reflections of each other in the line $y=x$, in cases where $a>1$ and $0<a<1$

Graph transformations

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- analyses and solves algebraic and graphical problems involving transformations of functions and relations **MAV-11-03**

Content

- Establish using graphing applications that replacing x by $-x$ in the equation of a graph corresponds to the reflection of the graph in the y-axis, and that replacing y by $-y$ corresponds to a reflection in the x-axis
- Establish using graphing applications that replacing x by $x-a$ in the equation of a graph corresponds to a horizontal translation of a graph by a, that replacing y by $y-b$ corresponds to a vertical translation by b, and recognise that the sign of a and b determines the direction of the translation
- Describe a horizontal dilation as a stretch from the y-axis and a vertical dilation as a stretch from the x-axis
- Establish using graphing applications that replacing x by $\frac{x}{k}$ in the equation of a graph corresponds to a horizontal dilation of a graph by a factor of k, that replacing y by $\frac{y}{l}$ corresponds to a vertical dilation by a factor of l, and determine the effects on the graph when the magnitude of the dilation factor lies between 0 and 1

Example(s):

A dilation stretches up and down from the x-axis, or right and left from the y-axis.

- Establish, using graphing applications, replacing x by $\frac{x}{k}$ and replacing y by $\frac{y}{k}$ in the equation of a graph corresponds to a dilation by a factor of k, that is, an enlargement of the graph when $k>1$ and a reduction of the graph when $0<k<1$

Example(s):

An enlargement enlarges a graph in all directions from the origin.

- Apply reflections, translations and dilations to functions within the scope of the Mathematics Advanced course, excluding trigonometric functions, determine the function rule and the graph of the transformed function, the domain and range of the transformed graph, any intercepts, asymptotes and discontinuities where appropriate, and describe which transformations have been applied

Example(s):

Show that $y=\frac{x-2}{x-3}$ can be expressed as $y=1+\frac{1}{x-3}$, hence describe $y=\frac{x-2}{x-3}$ in terms of transformations from the graph of $y=\frac{1}{x}$ and graph $y=\frac{x-2}{x-3}$, identifying any asymptotes and any intercepts on the axes.

- Use the principles of translations to identify the centre, radius, domain and range of graphs of circles in the form $(x-a)^2+(y-b)^2=r^2$, where a, b and r are constants
- Find the centre and radius of circles of the form $x^2+y^2+ax+by+c=0$, where a, b and c are constants, by completing the squares
- Graph circles given their equations and find the equation of a circle from its graph
- Recognise that the order in which individual transformations are applied to functions is important

Example(s):

Recognise that $y=3f(2x+8)$ represents a horizontal stretch with a factor $\frac{1}{2}$; a shift left of 4 units; a vertical stretch with factor 3. Alternatively: a vertical stretch with a factor 3; horizontal stretch with factor $\frac{1}{2}$; a shift left of 4 units.

Probability and data

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- solves problems involving probability in a variety of contexts **MAV-11-09**
- displays and analyses datasets using summary statistics and graphical representations **MAV-11-10**

Content

Sets and set notation

- Define a set as a collection of objects, called the elements of the set, and represent a set using notation such as $\{2,3,5,7\}$ and $\{0,2,4,6,\ldots\}$
- Use the notation $n(A)$ or $|A|$ to represent the number of elements in a finite set A

Example(s):

The number of elements in set $A=\{1,3,5\}$ is written as $n(A)=3$, or $|A|=3$.

- Define the empty set as the set with no elements, denoted in set notation as $\varnothing$
- Use the notation $\bar{A}$, A' or A^c to represent the complement of a set A with respect to some universal set U
- Define A to be a subset of B if all the elements of A are elements of B
- Define the intersection $A \cap B$ of sets A and B to be the set of elements that are in A and in B
- Define the union $A \cup B$ of sets A and B to be the set of elements that are in A or in B
- Define sets A and B to be disjoint if $A \cap B = \varnothing$, that is, they have no elements in common
- Use Venn diagrams in practical situations to represent and interpret sets that may intersect in various ways within a universal set
- Establish and use the rule $|A \cup B| = |A| + |B| - |A \cap B|$

Probability

- Define an experiment or a trial to be any procedure that can be infinitely repeated and has a well-defined set of possible outcomes known as the sample space, denoted S
- Identify an event A as a subset A of the sample space S
- Define the probability of each outcome to be $\frac{1}{|S|}$ when all the outcomes are equally likely and the probability of the event A to be $P(A)=\frac{|A|}{|S|}$
- Interpret the notation $\bar{A}$ to be the event 'A does not occur', interpret $A \cap B$ to be the event 'A and B both occur', and interpret $A \cup B$ to be the event 'A or B occurs'
- Use Venn diagrams to represent the relationship between events within the same sample space, including mutually exclusive events, that is, events that as subsets of the sample space are disjoint
- Establish and use the rules $P(\bar{A}) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- Use arrays and tree diagrams to determine the outcomes and probabilities for multistage events

Conditional probability

- Define conditional probability as the probability that an event A occurs given that another event B has already occurred, and use the notation $P(A|B)$
- Examine conditional probability by restricting the sample space and event spaces in a Venn diagram, using a two-way table, a tree diagram and other arrays

Example(s):

The 2 diagrams below represent the same scenario. When calculating the probability of A given B, the sample space is restricted to event B, which has 63 elements. Event A shares 23 elements with event B. $P(A|B)=\dfrac{n(A \cap B)}{n(B)}=\dfrac{23}{63}$.

	A	$\bar{A}$	Total
B	23	40	63
$\bar{B}$	30	7	37
Total	53	47	100

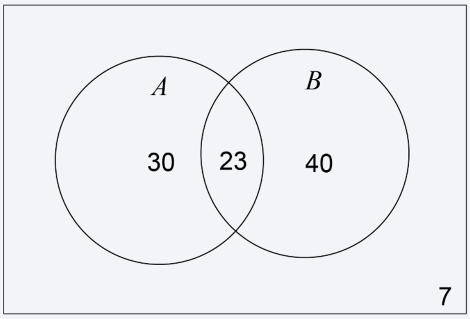


Image long description: In the table, row one includes column headers A, $\bar{A}$ and Total; row 2 includes row header B, 23, 40 and 63; row 3 includes row header B, 30, 7 and 37; and row 4 includes row header Total, 53, 47 and 100. In the Venn diagram, the 2 circles are labelled A and B. Circle A contains the number 30, circle B contains the number 40 and the overlapping area contains the number 23. The number 7 sits outside the circles.

- Establish that $P(A|B)=\dfrac{|A \cap B|}{|B|}$ when all outcomes are equally likely by restricting the sample space and event space, and hence $P(A|B)=\dfrac{P(A \cap B)}{P(B)}$, provided $|B| \neq 0$
- Use the formulas for $P(A|B)$ to solve practical problems involving conditional probability
- Define two events to be independent if the occurrence of one event does not affect the probability that the other event occurs
- Explain that two events A and B are independent means $P(A|B)=P(A)$ and $P(B|A)=P(B)$, and show algebraically that if one of these formulas is true, then the other is also true
- Use the formula $P(A|B)=\dfrac{P(A \cap B)}{P(B)}$, and the test $P(A|B)=P(A)$ for independence to prove that if two events are independent, then $P(A \cap B)=P(A) \times P(B)$, and to prove conversely that if $P(A \cap B)=P(A) \times P(B)$, then A and B are independent
- Solve practical problems involving independent events

Data

- Define a random variable as a variable whose value is the outcome of a random experiment

- Compare discrete random variables with continuous random variables, describe their differences, and give practical examples of each

Example(s):

The number of customers who arrive at the bank between two certain times is a discrete random variable. The time it takes for a bulb to burn out is a continuous random variable.

- Organise finite datasets using a table or a spreadsheet, listing the values, frequency, relative frequency, cumulative frequency, and cumulative relative frequency
- Graph the frequency, relative frequency, and cumulative frequency histograms and polygons of datasets, using spreadsheets or graphing applications, and identify the mode and median from the graphs, and from tables
- Use the relative frequency to estimate the probability of results in experiments

Outcomes and content for Year 12

Further graph transformations and modelling

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- uses algebraic and graphical techniques to analyse trigonometric functions **MAV-12-01**
- models, analyses and solves practical problems involving functions and their transformations **MAV-12-02**

Content

Transformations of trigonometric functions

- Use graphing applications to explore reflections, translations and dilations of, and confirm that the principles of transformations hold for, the trigonometric functions $f(x)=\sin x$, $f(x)=\cos x$ and $f(x)=\tan x$
- Apply the principles of transformations to graph transformations of the trigonometric functions $f(x)=\sin x$, $f(x)=\cos x$ and $f(x)=\tan x$, describe which transformations have been applied to $y=f(x)$ and determine the domain, range, horizontal translation, vertical translation, dilation factor, period and amplitude when applicable
- Solve equations and graphical problems involving transformations of the trigonometric functions $f(x)=\sin x$, $f(x)=\cos x$ or $f(x)=\tan x$, using graphing applications or otherwise, within a specified domain

Modelling with functions

- Model and solve problems involving periodic phenomena using transformations of the trigonometric functions $f(x)=\sin x$, $f(x)=\cos x$ or $f(x)=\tan x$, without the use of calculus
- Model and solve practical problems involving functions and their transformations, within the scope of the Mathematics Advanced course, using graphical and algebraic techniques
- Recognise that logarithmic scales are useful to represent a variable that ranges over a large set of values and explain when a logarithmic scale is suitable
- Use logarithmic scales to model and solve problems involving decibels (dB) in acoustics, seismic scales for earthquakes, magnitudes of stars and the pH scale in chemistry

Sequences and series

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- uses arithmetic and geometric sequences and series to model and solve problems **MAV-12-03**

Content

Sequences and series

- Define a sequence as an ordered list of objects
- Use the notation a_n, where n is a positive integer, to represent the nth term of a sequence
- Distinguish between a finite sequence $a_1, a_2, \ldots, a_m$ that terminates at its mth term, for some whole number $m \geq 1$, and an infinite sequence $a_1, a_2, \ldots$ that never terminates
- Define the nth partial sum S_n of a sequence $a_1, a_2, \ldots$ to be the sum of the first $n$terms of the sequence: $S_n = a_1 + a_2 + \ldots + a_n$, for all whole numbers $n \geq 1$
- Define a series formally as the sum of the terms of an infinite sequence and use the notation $a_1 + a_2 + a_3 + \ldots$ for the series corresponding to the sequence $a_1, a_2, a_3 \ldots$
- Use summation notation to represent the sum of terms a_i to a_j of a sequence where $j > i$, $\sum_{k=i}^{j} a_k = a_i + a_{i+1} + a_{i+2} + \ldots + a_{j-1} + a_j$

Example(s):

$$\sum_{k=1}^{5} (2k+1) = 3 + 5 + 7 + 9 + 11.$$

Arithmetic sequences and series

- Define a sequence a_n to be an arithmetic sequence, or arithmetic progression (AP), if every difference $a_n - a_{n-1}$ of successive terms is the same where $n \geq 2$, that is $a_n - a_{n-1} = d$ for some constant d called the common difference
- Develop the formula $a_n = a + (n-1)d$ for the nth term of an AP, where a is the first term and $n \geq 1$, and use it to solve problems
- Recognise that a_n is a linear function of n in an AP
- Develop the formula $S_n = \frac{n}{2}(a + a_n)$ for the nth partial sum of an AP, and use the formula to solve problems
- Develop the formula $S_n = \frac{n}{2}[2a + (n-1)d]$ for the nth partial sum of an AP, and use the formula to solve problems
- Apply the formulas for arithmetic sequences and their partial sums to model and solve growth and decay problems involving a quantity that is a linear function of time

Geometric sequences and series

- Define a sequence a_n to be a geometric sequence, or geometric progression (GP), if every ratio $\frac{a_n}{a_{n-1}}$ of successive terms is the same where $n \geq 2$, that is $a_n = r\, a_{n-1}$ for some non-zero real number r called the common ratio
- Develop the formula $a_n = a r^{n-1}$ for the nth term of a GP, where a is the first term and $n \geq 1$, and use it to solve problems
- Recognise that a_n is an exponential function of n in a GP
- Prove by expansion $(x-1)(x^{n-1} + x^{n-2} + \ldots + x^2 + x + 1) = x^n - 1$for whole numbers $n \geq 1$
- Develop the formula $S_n = \frac{a(1-r^n)}{1-r}$ for the sum of the first n terms of a GP where $r \neq 1$, and use this formula to solve problems
- Examine the behaviour of a_n and S_n as $n \to \infty$ for a GP when $|r| < 1$
- Develop the formula $S_\infty = \frac{a}{1-r}$ for the limiting sum of a geometric series with $|r| < 1$, and use this formula to solve problems
- Apply the formulas for geometric sequences and series to model and solve growth and decay problems where a quantity is an exponential function of time

Differential calculus

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- selects and applies differentiation methods to solve problems **MAV-12-04**

Content

Differentiation with exponential functions

- Apply the rules of differentiation to find the derivative of $f(x)=k e^{ax}$, where k and a are constants
- Use the chain rule to prove $\frac{d}{dx}\left(e^{ax+b}\right)=a e^{ax+b}$, where a and b are constants and $a \neq 0$
- Prove and use the formula $\frac{d}{dx}\left(a^x\right)=(\ln a) a^x$, where a is a constant and $a>0$
- Use the chain rule to differentiate functions of the form $e^{f(x)}$

Differentiation with logarithmic functions

- Use chain rule on the identity $e^{\ln x}=x$ to prove the formula $\frac{d}{dx}(\ln x)=\frac{1}{x}$ for the derivative of the natural logarithm function where $x>0$
- Prove and use the formula $\frac{d}{dx}\left(\log_a x\right)=\frac{1}{x \ln a}$, where a is a constant and $a>0$
- Use the chain rule to differentiate functions of the form $\ln f(x)$

Differentiation with trigonometric functions

- Determine the formulas $\frac{d}{dx}(\sin x)=\cos x$ and $\frac{d}{dx}(\cos x)=-\sin x$ informally by using the graphs of $y=\sin x$ and $y=\cos x$ and verify with graphing applications
- Use the rules of differentiation to show that $\frac{d}{dx}(\tan x)=sec^2 x$
- Use the rules of differentiation to find the derivatives of $cosec\, x$, $sec\, x$ and $\cot x$
- Use the chain rule to differentiate functions of the form $\sin f(x)$, $\cos f(x)$ and $\tan f(x)$

Using derivatives

- Apply the product, quotient and chain rules to differentiate functions of the form $f(x) g(x)$, $\frac{f(x)}{g(x)}$ or $f(g(x))$, where $f(x)$ and $g(x)$ are any of the functions within the scope of the Mathematics Advanced course

Example(s):

Differentiate with respect to x: $x e^x$ and $\sin^2 x$.

- Solve problems involving equations of tangents and normals to curves involving any of the functions within the scope of the Mathematics Advanced course

Integral calculus

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- solves problems involving indefinite and definite integrals **MAV-12-05**

Content

Primitive functions

- Define a primitive of a function $f(x)$ as a function $F(x)$ whose derivative $F'(x)=f(x)$ and recognise the process of finding the primitive as the reverse of differentiation
- Recognise that a function whose derivative is everywhere zero is a constant function
- Prove by differentiation that a primitive of $f(x)=x^n$ is $F(x)=\frac{x^{n+1}}{n+1}$, for all real $n\neq -1$
- Prove by differentiation that if $F(x)$ and $G(x)$ are primitives of $f(x)$ and $g(x)$, and k is a constant, then $F(x)+G(x)$ is a primitive of $f(x)+g(x)$, and $kF(x)$ is a primitive of $kf(x)$
- Recognise that primitives of a function $f(x)$ are not unique, and that any two primitives of $f(x)$ differ by a constant, so that if $F(x)$ is a primitive of $f(x)$, the general primitive of $f(x)$ is $F(x)+C$, for some constant C
- Determine the primitive of a given function $f(x)$, where $f(x)$ is a sum of functions of the form kx^n for all real $n\neq -1$

Example(s):

Determine the primitive function of $f(x)=4x^3+x^2-5x+1$ and $f(p)=\frac{1}{\sqrt[\square]{p}}$.

- Determine the primitive function for functions of the form $f(x)=(ax+b)^n$, for all real $n\neq -1$, where a and b are constants
- Use algebraic manipulation to express given functions in forms suitable for determining primitive functions

Example(s):

Determine the primitive function of $g(r)=\frac{r^5-2r^3}{r^2}$.

- Determine $f(x)$, given $f'(x)$ and an initial condition $f(a)=b$ where a and b are constants

The definite integral

- Examine for a function $f(x)$, which indicates the rate of change of a quantity, the meaning of $\sum_a^b f(x)\Delta x$, where the interval $a \leq x \leq b$ is divided into subintervals of length Δx, and describe $\sum_a^b f(x)\Delta x$ as an estimate of the total change in that quantity over the interval $a \leq x \leq b$

Example(s):

Describe the area under a velocity–time graph as representing a distance travelled; describe the area under a graph of daily snowfalls over a month as the total amount of snow that has fallen in that month.

- Consider the definite integral as $\int_a^b f(x)\,dx = \lim_{\Delta x \to 0} \sum_a^b f(x)\Delta x$, noting that this implies that the result of a definite integral will be negative when $f(x) \leq 0$ throughout the interval $a \leq x \leq b$
- Define informally that a function is continuous on the interval $a \leq x \leq b$ if it can be drawn between the two endpoints of the interval without taking the pen off the paper
- Graph the region between the continuous function $y = f(x)$ and the x-axis, where $f(x) \geq 0$ on the interval $a \leq x \leq b$
- Use a graphing application to compare different methods of approximating the area, A, of the region between the continuous function $y = f(x)$ and the x-axis, where $f(x) \geq 0$ on the interval $a \leq x \leq b$, by summing the areas of trapezia or rectangles each of width $\Delta x = \frac{b-a}{n}$ and approximate height $f(x)$ for any x lying in its base, and observe the effect on the precision of the approximation of A as the number n of subintervals of $a \leq x \leq b$ increases, that is as $\Delta x \to 0$

Example(s):

Use a graphing application to approximate the area under the curve $y = x^2$ in the interval $[0,1]$ using sums of rectangles and compare the left-endpoint estimate, right-endpoint estimate and midpoint estimate.

- Evaluate the definite integral $\int_a^b f(x)\,dx$ by calculating areas using geometrical formulas, where the shape of $f(x)$ allows such calculations, in cases where $f(x) \geq 0$throughout $a \leq x \leq b$, $f(x) \leq 0$ throughout $a \leq x \leq b$ or where $f(x)$ changes sign in the interval $a \leq x \leq b$

Example(s):

Find the value of $\int_0^2 \sqrt{4 - x^2}\,dx$ using geometric ideas.

The Fundamental Theorem of Calculus

- Consider the function defined by $A(x)=\int_a^x f(t)\,dt$ and use a graphing application to recognise that $A(x)$ is a primitive of $f(x)$
- Recognise the Fundamental Theorem of Calculus as $\int_a^b f(x)\,dx=[F(x)]_a^b=F(b)-F(a)$ for a continuous function f on the interval $a\le x\le b$ where $F(x)$ is any primitive of $f(x)$

Indefinite integrals

- Use the notation $\int f(x)\,dx$ for the general primitive of $f(x)$, called the indefinite integral of $f(x)$, so that $\int f(x)\,dx=F(x)+C$, for some constant C, where $F(x)$ is any primitive of $f(x)$
- Recognise integration as the process of finding the indefinite integral of a function
- Use the formula $\int x^n\,dx=\frac{1}{n+1}x^{n+1}+C$ for real $n\neq-1$
- Use the identities $\int(f(x)+g(x))\,dx=\int f(x)\,dx+\int g(x)\,dx$ and $\int kf(x)\,dx=k\int f(x)\,dx$ for primitives
- Prove by differentiation, and apply $\int u^n\frac{du}{dx}\,dx=\frac{1}{n+1}u^{n+1}+C$, where u is a function of x, or $\int f'(x)[f(x)]^n\,dx=\frac{1}{n+1}[f(x)]^{n+1}+C$, for real $n\neq-1$

Example(s):

Find $\int(4x+8)(x^2+4x-3)^3\,dx$.

Integration with exponential functions

- Establish and use the formula $\int e^x\,dx=e^x+C$
- Establish and use the formula $\int e^{ax+b}\,dx=\frac{1}{a}e^{ax+b}+C$, where a and b are constants and $a\neq0$
- Establish and use the formula $\int a^x\,dx=\frac{a^x}{\ln a}+C$, where a is a constant and $a>0$
- Establish and use $\int e^u\frac{du}{dx}\,dx=e^u+C$, where u is a function of x, or $\int f'(x)e^{f(x)}\,dx=e^{f(x)}+C$
- Find primitives of functions involving exponential functions

Integration with logarithmic functions

- Derive and use the formula $\int\frac{1}{x}\,dx=\ln|x|+C$ where $x\neq0$
- Establish and use the formula $\int\frac{1}{ax+b}\,dx=\frac{1}{a}\ln|ax+b|+C$, where a and b are constants and $a\neq0$
- Establish and use $\int\frac{u'}{u}\,dx=\ln|u|+C$, where u is a function of x, or $\int\frac{f'(x)}{f(x)}\,dx=\ln|f(x)|+C$, on a domain where $f(x)\neq0$

Integration with trigonometric functions

- Establish and use the formulas $\int \sin x \, dx = -\cos x + C$, $\int \cos x \, dx = \sin x + C$ and $\int sec^2 x \, dx = \tan x + C$
- Establish and use indefinite integrals of the form $\int f(ax+b)\,dx$, where a and b are constants and $a \neq 0$, and $f(x) = \sin x$, $f(x) = \cos x$ and $f(x) = sec^2 x$
- Determine indefinite integrals of the form $\int f'(x) \sin f(x)\,dx$, $\int f'(x) \cos f(x)\,dx$ and $\int f'(x) sec^2 f(x)\,dx$

Areas and the definite integral

- Apply $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F(x)$ is a primitive of $f(x)$, to calculate definite integrals and solve related theoretical problems involving functions within the scope of the Mathematics Advanced course
- Describe, in the case where $f(x) \geq 0$ for all values of x in the interval $a \leq x \leq b$, the area bounded by the graph of the continuous function $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, as $\int_a^b f(x)\,dx$
- Recognise, in the case where $f(x) \leq 0$ for all values of x in the interval $a \leq x \leq b$, the area bounded by the graph of the continuous function $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, as $\left|\int_a^b f(x)\,dx\right|$ or $-\int_a^b f(x)\,dx$
- Conclude, for a continuous function $y = f(x)$ on the interval $a \leq x \leq b$, that $\int_a^b f(x)\,dx =$ (area of regions between curve and x-axis lying above the x-axis) - (area of regions between curve and the x-axis lying below the x-axis)

Example(s):

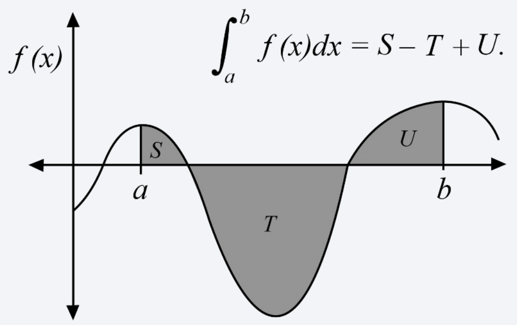


Image long description: A curve $y = f(x)$ starting at $x = 0$ and $y < 0$ and crossing the x-axis three times. The area between the curve and the x-axis from $x = a$ to $x = b$ is shaded. The area S lies above the x-axis between $x = a$ and the second x-intercept. The area T lies below the x-axis between the second and third x-intercepts. The area U lies above the x-axis between the third x-intercept and $x = b$. The caption shows $\int_a^b f(x)\,dx = S - T + U$.

- Use definite integrals to solve problems involving the areas of regions bounded by the graph of the continuous function $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$, in cases where

$f(x) \geq 0$ throughout $a \leq x \leq b$, $f(x) \leq 0$ throughout $a \leq x \leq b$ or where $f(x)$ changes sign in the interval $a \leq x \leq b$, with or without the graph provided

- Use definite integrals to solve problems involving the areas of regions bounded by the graph of the continuous function $y = f(x)$, the y-axis and the lines $y = a$ and $y = b$, in cases where $x \geq 0$ throughout $a \leq y \leq b$, $x \leq 0$ throughout $a \leq y \leq b$ or where x changes sign in the interval $a \leq y \leq b$ with or without the graph provided
- Use the fact that the graphs of $y = a^x$ and $y = \log_a x$ are reflections of each other in the line $y = x$ to solve problems involving areas between the x-axis or y-axis and a curve involving either an exponential or logarithmic function
- Recognise and use the result, where $f(x)$ is continuous on the interval $a \leq x \leq c$, $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$ for all c such that $a \leq b \leq c$
- Define and use the result $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$, where $f(x)$ is continuous on the interval $a \leq x \leq b$
- Recognise and use symmetry, particularly odd and even functions, to simplify and solve integration problems
- Use the Trapezoidal rule to approximate integrals

Example(s):

Use the Trapezoidal rule with four trapezia to find an approximation for $\int_1^3 \ln x\,dx$.

- Use an online computational application to evaluate definite and indefinite integrals involving functions within and beyond the scope of the Mathematics Advanced course
- Model and solve practical problems involving integrals and areas of regions bounded by a curve and the x-axis, or by a curve and the y-axis, involving functions within the scope of the Mathematics Advanced course

Example(s):

Find areas of shapes used in design; identify Lorenz curves, that is functions with the properties: $f(x)$ is differentiable on the interval $0 \leq x \leq 1$, $f(0) = 0$ and $f(1) = 1$, and $f(x) \leq x$ for $0 \leq x \leq 1$ and evaluate the Gini coefficient $G = 2\int_0^1 [x - f(x)]\,dx$ for functions whose graphs are Lorenz curves.

Applications of calculus

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- applies calculus to graph functions and model and solve problems involving optimisation, rates of change and motion in a line **MAV-12-06**

Content

Turning points, inflections and curve-sketching

- Define a function $f(x)$ to be differentiable at $x=a$ if $\frac{\lim_{h \to 0} f(a+h)-f(a)}{h}$ exists, that is if there is a non-vertical tangent to the curve at the point $P(a, f(a))$ and recognise that if $f(x)$ is differentiable at $x=a$ then it is continuous at $x=a$
- Identify any values of x where a function is continuous, but not differentiable, given either the equation of the function or its graph

Example(s):

The function $y=f(x)$ is given by $f(x)=\begin{cases} 2-x, x<1 \\ x^2, x \geq 1 \end{cases}$. Determine if $f(x)$ is continuous and differentiable when $x=1$.

- Use repeated differentiation to find second derivatives of a function $y=f(x)$, denoting them by $f''(x)$ or $\frac{d^2y}{dx^2}$ or y''
- Analyse the stationary points of a function by testing values for $f'(x)$, then classify the stationary points as local minima or local maxima where gradients change around the point, or horizontal points of inflection where gradients have the same sign on both sides of the point
- Interpret the second derivative y'' as the gradient function of the first derivative y', and deduce that if $y''>0$ the curve is concave up and if $y''<0$ the curve is concave down
- Define a point of inflection on a curve as a point where the concavity changes
- Analyse the value of $f''(x)$ either side of the roots of $f''(x)=0$, and use the resulting concavities to identify which zeroes of $f''(x)$ are points of inflection
- Use the second derivative to classify a stationary point as a local maximum, local minimum or a horizontal point of inflection

- Graph a function by determining local maxima and minima and points of inflection, horizontal and non-horizontal, considering any even or odd symmetry, the domain, any vertical asymptotes or other discontinuities, and where applicable, the behaviour of a function as $x \to \pm\infty$

Example(s):

Find the stationary points on the graph of $y = x^2 e^x$ and determine their nature. Find any point(s) of inflection and graph the function.

- Graph $y = f'(x)$ and $y = f''(x)$ for a function $y = f(x)$, given only a graph of $y = f(x)$

Optimisation

- Define a global maximum of a function $f(x)$ to be a point $P(a, f(a))$ on the graph where $f(x) \leq f(a)$, for all x in the domain, and define a global minimum similarly

Example(s):

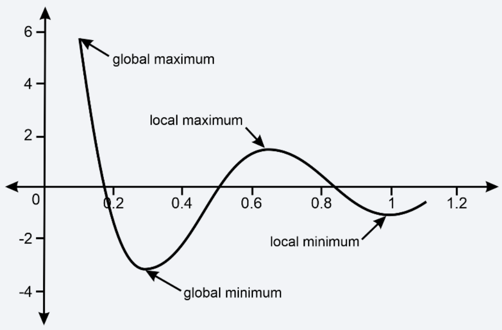


Image long description: The curve on the number plane shows an endpoint at $(0.1, 6)$ labelled global maximum. A minimum turning point lies below the x-axis at $(0.3, -3.0)$ and is labelled global minimum. A maximum turning point lies above the x-axis at $(0.7, 2)$ and is labelled local maximum. A minimum turning point lies below the x-axis at $(1.0, -1.0)$ and is labelled local minimum. The curve crosses the x-axis at approximately $x = 0.18$, 0.5 and 0.82.

- Examine whether any discontinuities or endpoints of the domain on which $f(x)$ is being considered are points of maxima or minima

Example(s):

Recognise that $y = |x|$ is not differentiable at $x = 0$, and that there is a local minimum at $x = 0$.

- Model optimisation problems in a variety of contexts by defining variables, noting domain restrictions if necessary, and establishing functions to represent the relationship between variables
- Solve optimisation problems by using calculus to find local and global maxima and minima of differentiable functions, checking discontinuities of $f'(x)$ and endpoints of the domain if applicable
- Formulate conclusions to optimisation problems by evaluating solutions given the constraints of the domain

Rates of change

- Use differentiation to find and interpret the first and second derivatives, $\frac{dQ}{dt}$ and $\frac{d^2Q}{dt^2}$, in practical problems where a quantity Q is a function of time t

Example(s):

A valve is slowly opened in a pipeline such that the volume flow rate R varies with time according to the relation $R = kt$, $(t > 0)$, where k is a constant. Calculate the total volume of water that flows through the valve in the first 10 seconds if k = 1.3 m^3s^{-2}.

- Use integration to solve practical problems on the rate of change of a quantity Q, where $\frac{dQ}{dt}$ or $\frac{d^2Q}{dt^2}$ is given as a function of time t, together with sufficient initial conditions
- Describe and examine the graphs of practical situations where the rate of change of a quantity is proportional to the quantity
- Explain that if the rate of change of a positive quantity Q over time t is proportional to the size of Q, then this may be represented by $\frac{dQ}{dt} = kQ$ where k is the constant of proportionality, and if $k > 0$ the quantity Q is increasing at a rate proportional to the value of Q at time t, while if $k < 0$ the quantity Q is decreasing at a rate proportional to the value of Q at time t
- Verify by substitution that the function $Q = Ae^{kt}$ satisfies the relationship $\frac{dQ}{dt} = kQ$ with A being the initial value of Q
- Recognise that the equation $\frac{dQ}{dt} = kQ$ where $Q > 0$, and its solution $Q = Ae^{kt}$ represent exponential growth when $k > 0$ and exponential decay when $k < 0$
- Graph the function $Q = Ae^{kt}$, where $k > 0$ and $k < 0$ and $A > 0$ and $A < 0$, with and without graphing applications, and identify any asymptotes
- Model and solve growth and decay problems in various contexts using $\frac{dQ}{dt} = kQ$, $Q = Ae^{kt}$ and the graph of $Q = Ae^{kt}$ for $t \geq 0$, and justify conclusions in the context of the problem
- Determine the velocity and acceleration of a particle moving in a straight line given its displacement from a point as a function of time, and use the notation $\frac{d^2x}{dt^2}$ or $\ddot{x}$ to represent acceleration
- Solve problems relating to the motion of a particle moving in a straight line, using both differentiation and integration to connect the concepts of displacement, velocity and acceleration

Random variables

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- solves problems involving discrete probability distributions, continuous random variables and the normal distribution **MAV-12-07**

Content

Discrete random variables

- Use the relative frequencies of discrete random variable datasets to estimate the probabilities that the random variable X takes each of its values, and explain why these estimated probabilities add to 1
- Denote the probability that the discrete random variable X takes the value x by $P(X=x)$, or by $P(x)$ if the discrete random variable X is understood
- Define a discrete probability distribution to be the set of values x taken by a discrete random variable X, together with the probabilities $P(x)$ that x is the outcome of the experiment
- Define a discrete random variable to be uniformly distributed if it has finitely many values, all with the same probability, and use it to model random phenomena with equally likely outcomes
- Recognise the mean, or expected value, μ or $\mathrm{E}(X)$, of a discrete random variable X as a measure of centre for its distribution
- Generate and use the formulas $\mathrm{E}(X)=\mu=\Sigma\, xP(x)$ and $Var(X)=\sigma^2=\Sigma(x-\mu)^2 P(x)=\Sigma\, x^2 P(x)-\mu^2$ for the random variable X, where $Var(X)$ is the variance and σ is the standard deviation of the distribution
- Recognise that the variance is an expected value because $\mathrm{Var}(X)=\mathrm{E}\left((X-\mu)^2\right)$

- Generate a probability distribution for a given discrete random variable and represent the probability distribution in graphical and tabular form

Example(s):

The probability distribution for the random variable X which takes on values 1, 2, 3, and 4, each of which are equally likely is represented using a table:

x	1	2	3	4
$P(x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Image long description: The first row of the table has the values of x, 1, 2, 3 and 4. The second row of the table has the values of $P(x)$, and $\frac{1}{4}$ repeated in the remaining cells.

- Solve problems involving probabilities, expectation and variance of discrete random variables

Continuous random variables

- Estimate the probability that a continuous random variable falls in some interval using relative frequencies and histograms obtained from data
- Recognise that the probability of a particular value for a continuous random variable is 0 and hence that $P(a<X<b)=P(a\leq X<b)=P(a<X\leq b)=P(a\leq X\leq b)$ since $P(X=a)=P(X=b)=0$ when X is a continuous random variable
- Define the cumulative distribution function (CDF), $F(x)$, as the probability of a random variable, X, having values less than or equal to x, so $F(x)=P(X\leq x)$ and $P(a\leq x\leq b)=F(b)-F(a)$
- Recognise that the cumulative distribution function, $F(x)$, is non-decreasing for all x in its domain, and graph cumulative distribution functions, given a formula for $F(x)$, with and without graphing applications
- Define a probability density function (PDF), $f(x)$, for a random variable X with cumulative distribution function $F(x)$ as $f(x)=F'(x)$ and recognise that $P(a<X<b)=\int_a^b f(x)\,dx$
- Recognise the properties of a probability density function: $f(x)\geq 0$ for all x in the domain of $f(x)$, and $\int_a^b f(x)\,dx=1$ if the domain of $f(x)$ is $[a,b]$, or $\int_{-\infty}^{\infty} f(x)\,dx=1$ if the domain of $f(x)$ is all real x
- Apply the properties of a probability density function to solve problems and justify conclusions
- Find the mode from a given probability density function
- Obtain the cumulative distribution function using the formula $F(x)=\int_a^x f(t)\,dt$ where $f(x)$ is a given probability density function defined on the interval $[a,b]$

- Determine and use the probability density function $f(x)=\frac{1}{b-a}$ for a continuous uniform distribution for a random variable X taking values in the interval $[a,b]$
- Use a cumulative distribution function to calculate the median and quartiles for a continuous random variable
- Find the probability density function from a given cumulative distribution function
- Generate the expression for the expected value of a continuous random variable, $E(X)=\mu=\int_a^b xf(x)\,dx$, where the probability density function $f(x)$ is defined on the interval $[a,b]$
- Generate the expression for the variance of a continuous random variable, $\mathrm{Var}(X)=E(X^2)-\mu^2=\int_a^b x^2 f(x)\,dx-\mu^2$, where the probability density function $f(x)$ is defined on the interval $[a,b]$
- Evaluate the expected value and the variance of a continuous random variable, where the probability density function $f(x)$ is defined on the interval $[a,b]$, that involve integration of functions within the scope of the Mathematics Advanced course
- Evaluate the expected value and the variance of a continuous random variable, where the probability density function $f(x)$ is defined on the interval $[a,b]$, that involve integration of functions beyond the scope of the Mathematics Advanced course using an online computational application

The normal distribution

- Identify the normal distribution as a continuous probability distribution that is used to model many naturally occurring phenomena
- Identify the graph of the probability density function of a normal distribution, the normal curve, as an 'ideal' bell-shaped curve, symmetrical about its mean which is equal to its mode and median, and as having most values concentrated about the mean
- Identify contexts that can be approximately modelled by a normal random variable

Example(s):

The heights of a group of students can be modelled by a normal random variable.

- Use the notation $X \sim N(\mu,\sigma^2)$ to represent a normally distributed random variable that has mean μ and standard deviation σ
- Represent probabilities associated with the normal distribution by areas of shaded regions under the normal curve, which may extend to $\pm\infty$
- Apply the empirical rule to make judgements and solve problems involving probabilities of normally distributed data: that for normal distributions, approximately 68% of data lie within one standard deviation of the mean, approximately 95% within two standard deviations of the mean and approximately 99.7% within three standard deviations of the mean
- Use graphing applications to explore the normal distribution, graph the probability density function $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, verify the empirical rule and graph the cumulative distribution function
- Recognise features of the normal curve, and identify the global maximum and points of inflection

- Distinguish between a standard normal distribution with mean 0 and standard deviation 1, and the non-standard normal distribution with mean μ and standard deviation σ
- Define the z-score, or standardised score, by the formula $z = \frac{x - \mu}{\sigma}$, where μ is the mean and σ is the standard deviation, and x is an observed value of a random variable
- Interpret the z-score as the number of standard deviations a score lies above or below the mean
- Use z-scores to compare scores from different sets of data and justify conclusions
- Use z-scores to identify probabilities of events less or more extreme than a given outcome and solve problems using tables for the standard normal distribution
- Solve problems involving finding the mean or standard deviation of a normal random variable given the probability of an event less or more extreme than a given outcome
- Use z-scores to make judgements related to probabilities of certain events or given sets of data assuming an underlying normal distribution

Financial mathematics

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- models and solves problems to make informed decisions about financial situations **MAV-12-08**

Content

Reducing balance loans

- Recognise a reducing balance loan algebraically as a compound interest loan with periodic repayments
- Examine the effect of varying the interest rate and repayment amount on the time taken to repay a loan, with or without digital tools or by using a given graph
- Solve problems that involve reducing balance loans by calculating the total amount paid, equal periodic repayments, the amount still owing and the time taken to repay the loan

Annuities

- Identify an annuity as either an investment account with regular, equal contributions and interest compounding at the end of each period, or as a single sum investment from which regular equal withdrawals are made
- Define and model the future value of an annuity as the sum of all the payments, together with the interest they have earned
- Examine the effect of varying the amount initially invested, the value of the periodic payment, the interest rate and the duration of the annuity on the total value of the investment, using digital tools
- Solve problems involving annuities in which payments are made at the end of each time period and the future value of an annuity is calculated at the end of one of these periods using the formula for the sum of the first n terms of a geometric sequence

Assessment

Course standards

Course standards are comprised of syllabus standards and performance standards. Syllabus outcomes and content form the syllabus standards that describe what students are expected to know, understand and do. Performance standards, including the Common Grade Scale for Preliminary courses, HSC Performance Band Descriptions and Achievement Level Descriptions (for HSC courses with grades only) describe how well students have achieved in relation to syllabus standards. Performance standards are used to report student achievement on NESA credentials.

Common Grade Scale for Preliminary courses

Stage 6 – Year 11

The Common Grade Scale for Preliminary courses provides generic, holistic descriptions for performance at each of the 5 grade levels. It is used to report student achievement in Year 11 courses in all NSW schools.

Performance Band Descriptions for Mathematics Advanced

Stage 6 – Year 12

HSC performance band descriptions summarise the knowledge, understanding and skills typically demonstrated by students at each band level in a particular course. NESA uses these holistic descriptions to report a student's overall level of achievement in an HSC course.

Band 6

A student who performs at this level typically:

- demonstrates extensive knowledge, understanding and skills appropriate to the course
- integrates and applies mathematical concepts, skills and techniques consistently and accurately in a range of familiar and unfamiliar situations
- selects and applies a wide range of strategies to solve a variety of multi-step mathematical problems
- constructs and analyses mathematical models to solve problems in a variety of situations
- communicates reasoning and justification effectively, using appropriate mathematical language, notation, diagrams and graphs.

Band 5

A student who performs at this level typically:

- demonstrates thorough knowledge, understanding and skills appropriate to the course
- integrates and applies mathematical concepts, skills and techniques accurately in familiar and unfamiliar situations
- selects and applies a range of strategies to solve a variety of multi-step mathematical problems
- constructs and analyses mathematical models to solve problems
- communicates reasoning and justification using appropriate mathematical language, notation, diagrams and graphs.

Band 4

A student who performs at this level typically:

- demonstrates sound knowledge, understanding and skills appropriate to the course
- uses mathematical concepts, skills and techniques in familiar and some unfamiliar situations
- selects and uses strategies to solve mathematical problems
- interprets mathematical models to solve problems
- communicates reasoning and justification using mathematical language, notation, diagrams and graphs.

Band 3

A student who performs at this level typically:

- demonstrates basic knowledge, understanding and skills appropriate to the course
- uses mathematical concepts, skills and techniques in familiar situations
- solves simple mathematical problems
- interprets some mathematical models
- communicates reasoning using mathematical language, notation, diagrams and graphs.

Band 2

A student who performs at this level typically:

- demonstrates elementary knowledge, understanding and skills appropriate to the course
- uses some mathematical concepts, skills and techniques to solve simple familiar problems with limited accuracy
- uses some mathematical language, notation, diagrams and graphs.

Band 1

Performance reported as Band 1 indicates that the minimum standard expected was not demonstrated.

School-based assessment

Schools are required to develop an assessment program for each Year 11 and Year 12 course. NESA provides information about the responsibilities of schools in developing assessment programs in course-specific assessment requirements and in the Assessment Certification Examination (ACE) rules and requirements.

Year 11 Mathematics Advanced school-based assessment

Stage 6 – Year 11

Schools are required to submit to NESA a grade for each student based on their achievement at the end of the course.

Teachers use professional, on-balance judgement to allocate grades based on the Common Grade Scale for Preliminary courses.

Teachers consider all available assessment information, including formal and informal assessment, to determine the grade that best matches each student’s achievement at the end of the course.

See Assessment Certification Examination (ACE) rules and requirements for further information.

Assessment programs must reflect course components and weightings

The course components and component weightings for Year 11 are mandatory.

Course component	Weighting %
Knowledge and understanding of course content	50
Skills in Working mathematically	50

Schools may determine specific elements of their assessment program

Schools have authority to determine the number, type of task and the weighting allocated to an assessment task. Schools may also follow the sample assessment programs provided by NESA.

Sample assessment program

NESA's sample Year 11 formal school-based assessment program for Mathematics Advanced is:

- 3 assessment tasks,
  including:
  - a formal written examination
  - a weighting for any individual task of 20% to 40%.

Some students with disability may require adjustments in order to access assessment opportunities and demonstrate achievement of outcomes.

Year 12 Mathematics Advanced school-based assessment

Stage 6 – Year 12

NESA requires schools to submit a school-based assessment mark for each Year 12 student who completes this course. Formal school-based assessment tasks should reflect the syllabus outcomes and content. The mark submitted by the school provides a summation of each student's achievement measured at several points throughout the course.

A school's program of school-based assessment includes both mandatory and non-mandatory elements.

See Assessment Certification Examination (ACE) rules and requirements for further information.

Assessment programs must reflect course components and weightings

The course components and component weightings for Year 12 are mandatory.

Course component	Weighting %
Knowledge and understanding of course content	50
Skills in Working mathematically	50

Schools may determine specific elements of their assessment program

Schools have authority to determine the number, type of task and the weighting allocated to an assessment task. Schools may also follow the sample assessment programs provided by NESA.

Sample assessment program

NESA's sample Year 12 formal school-based assessment program for Mathematics Advanced is:

- 4 assessment tasks,
  including:

  - a formal written examination
  - a weighting for any individual task of 10% to 40%.

Some students with disability may require adjustments in order to access assessment opportunities and demonstrate achievement of outcomes.

Formal school-based assessment in this course should focus on the Year 12 outcomes. The Year 11 course is assumed knowledge and may be assessed.

Sample assessment schedules

Stage 6

- Sample assessment schedule Year 11
- Sample assessment schedule Year 12: A
- Sample assessment schedule Year 12: B

Sample assessment task notifications

Stage 6

- Sample formal assessment task notification: Year 11 (Working with functions)
- Sample formal assessment task notification: Year 12 (Applications of probability)

HSC examinations

The external HSC examination measures student achievement in a range of syllabus outcomes.

The external examination and its marking relate to the syllabus by:

- providing clear links to syllabus outcomes
- enabling students to demonstrate the levels of achievement outlined in the Performance Band Descriptions
- applying marking guidelines based on criteria that relate to the quality of the response
- aligning performance in the examination each year to the standards established for the course.

Examination questions may require students to integrate knowledge, understanding and skills developed through studying the course.

The Year 11 course is assumed knowledge for the Year 12 course.

Some students with disability may be eligible for disability provisions when completing the HSC examination.

The examination will be based on the Mathematics Advanced Year 12 course and will focus on the Year 12 outcomes. The Mathematics Advanced Year 11 course will be assumed knowledge for this examination and may be examined.

Mathematics Advanced HSC examination specifications

Stage 6 – Year 12

The examination will consist of a written paper worth 100 marks.

Time allowed: 3 hours plus 10 minutes reading time.

A reference sheet will be provided.

NESA-approved calculators may be used.

The paper will consist of two sections.

Section I (10 marks)

- There will be objective-response questions to the value of 10 marks.

Section II (90 marks)

- Questions may contain parts.
- There will be 37 to 42 items.
- At least two items will be worth 4 or 5 marks.

HSC Mathematics Advanced – annotated sample examination materials

Stage 6 – Year 12

- HSC Mathematics Advanced – annotated sample examination materials
- Mathematics Advanced – HSC reference sheet

Further information

Stage 6 – Year 12

- Assessment in Stage 6
- Principles of effective assessment
- Assessment in practice
- School-based assessment requirements
- Adjustments to assessment for students with disability
- Life Skills
- Assessment Certification Examination (ACE) website

Mathematics Extension 1 11–12 (2024)

Implementation from 2026

The new Mathematics Extension 1 11–12 Syllabus (2024) is to be implemented from 2026 and will replace the **Mathematics Extension 1 Stage 6 Syllabus (2017)**.

2025

- Plan and prepare to teach the new syllabus

2026, Term 1

- Start teaching the new syllabus for Year 11
- Start implementing the new Year 11 school-based assessment requirements
- Continue to teach the **Mathematics Extension 1 Stage 6 Syllabus (2017)** for Year 12

2026, Term 4

- Start teaching the new syllabus for Year 12
- Start implementing the new Year 12 school-based assessment requirements

2027

- First HSC examination for the new syllabus

Overview

Syllabus overview

Organisation of Mathematics Extension 1 11–12

The organisation of outcomes and content for Mathematics Extension 1 11–12 highlights the important role Working mathematically plays across all areas of mathematics and reflects the strengthened connections between concepts. Working mathematically has been embedded in the outcomes and content of the syllabus.

Mathematics Extension 1 outcomes and their related content are organised into 7 areas of study:

- Functions
- Proof
- Vectors
- Trigonometric functions
- Combinatorics
- Calculus
- Statistical analysis.

Figure 1 gives a broad overview of the organisation of content, including the focus areas to be completed in each area of study.

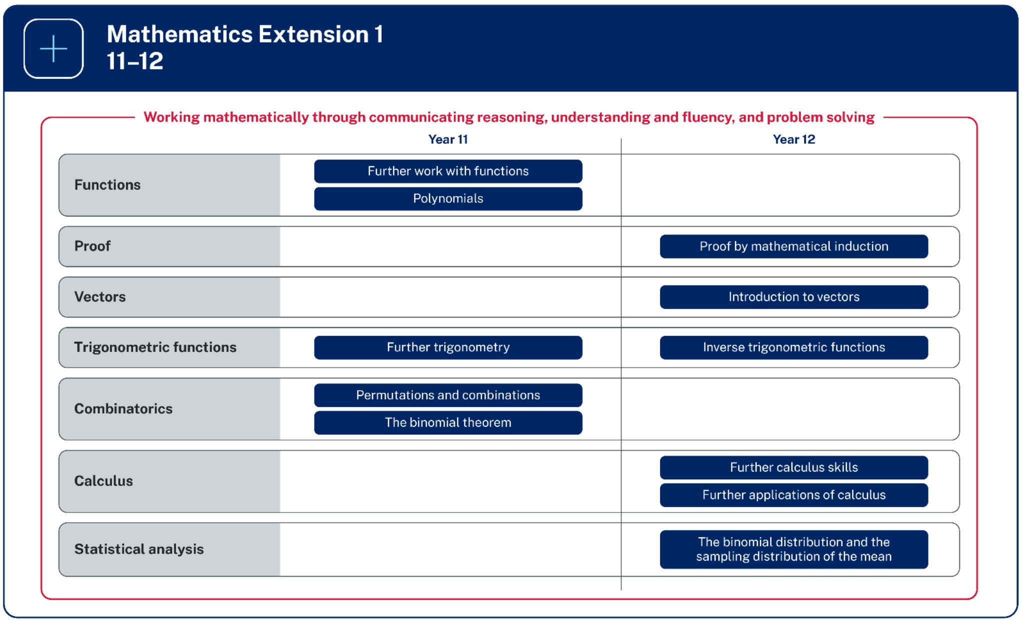


Figure 1: The organisation of Mathematics Extension 1 11–12

Image long description: Seven rows represent the areas of study for Mathematics Extension 1 11–12. Each row is divided in half, showing the focus areas for Year 11 and Year 12. The Functions area of study includes Year 11 focus areas Further work with functions and Polynomials. The Proof area of study includes the Year 12 focus area Proof by mathematical induction. The Vectors area of study includes the Year 12 focus area Introduction to vectors. The Trigonometric functions area of study includes the Year 11 focus area Further trigonometry, and the Year 12 focus area Inverse trigonometric functions. The Combinatorics area of study includes Year 11 focus areas Permutations and combinations, and The binomial theorem. The Calculus area of study includes the Year 12 focus areas Further calculus skills and Further applications of calculus. The Statistical analysis area of study includes the Year 12 focus area The binomial distribution and the sampling distribution of the mean. All content is surrounded by a box labelled with the phrase, ‘Working mathematically through communicating reasoning, understanding and fluency, and problem solving’.

Protocols for collaborating with Aboriginal and Torres Strait Islander Communities and engaging with Cultural works

NESA is committed to working in partnership with Aboriginal Communities and supporting teachers, schools and schooling sectors to improve educational outcomes for young people.

It is important to respect appropriate ways of interacting with Aboriginal Communities and Cultural material when teachers plan, program and implement learning experiences that focus on Aboriginal and Torres Strait Islander Priorities.

Indigenous Cultural and Intellectual Property (ICIP) protocols need to be followed. Aboriginal and Torres Strait Islander Peoples’ ICIP protocols include Cultural Knowledges, Cultural Expression and Cultural Property and documentation of Aboriginal and Torres Strait Islander Peoples’ Identities and lived experiences. It is important to recognise the diversity and complexity of different Cultural groups in NSW, as protocols may differ between local Aboriginal Communities.

Teachers should work in partnership with Elders, parents, Community members, Cultural Knowledge Holders, or a local, regional or state Aboriginal Education Consultative Group. It is important to respect Elders and the roles of men and women. Local Aboriginal Peoples should be invited to share their Cultural Knowledges with students and staff when engaging with Aboriginal histories and Cultural Practices.

Course description

Mathematics Extension 1 focuses on the development of students mathematical arguments and proofs, and use of mathematical models. The course allows students to develop a thorough knowledge and understanding of and competence in further aspects of mathematics as an extension of the Mathematics Advanced 11–12 course.

What students learn

Through the study of Mathematics Extension 1, students:

- develop thorough knowledge, understanding and skills in Working mathematically and in communicating concisely and precisely
- develop rigorous mathematical arguments and proofs, and use mathematical models extensively
- develop awareness of the interconnected nature of mathematics, its beauty and its functionality
- gain an appropriate mathematical background for future pathways that may involve mathematics and its applications.

Course structure and requirements

Mathematics Extension 1 consists of the courses Mathematics Extension 1 Year 11 and Mathematics Extension 1 Year 12. Students studying one or both Extension 1 and 2 courses must study both Mathematics Advanced Year 11 and Mathematics Extension 1 Year 11 courses before undertaking the study of Mathematics Extension 1 Year 12, or both Mathematics Extension 1 Year 12 and Mathematics Extension 2 Year 12. An alternative approach is for students to study both Mathematics Advanced Year 11 and Mathematics Advanced Year 12 before undertaking the study of Mathematics Extension 1 Year 11 and Mathematics Extension 1 Year 12, or both Mathematics Extension 1 Year 12 and Mathematics Extension 2 Year 12.

The course numbers and units for each year of study are set out below.

Course numbers:

- Mathematics Extension 1 (Year 11, 1 unit): TBA
- Mathematics Extension 1 (Year 12, 1 unit): TBA

Year 11 course structure

Area of study	Focus area
Functions	Further work with functions Polynomials
Trigonometric functions	Further trigonometry
Combinatorics	Permutations and combinations

	The binomial theorem

Year 12 course structure

Area of study	**Focus area**
Proof	Proof by mathematical induction
Vectors	Introduction to vectors
Trigonometric functions	Inverse trigonometric functions
Calculus	Further calculus skills Further applications of calculus
Statistical analysis	The binomial distribution and sampling distribution of the mean

Further information for Mathematics Extension 1 Year 11

- Course number: TBA
- Course hours: 60
- Course units: 1
- Enrolment type: Elective
- Endorsement type: Board developed
- Study via self-tuition: Yes

Exclusions

- Mathematics Standard (Year 11, 2 units): TBA
- Mathematics Life Skills (Year 11, 2 units): TBA

Further information for Mathematics Extension 1 Year 12

- Course number: TBA
- Course hours: 60
- Course units: 1
- Enrolment type: Elective
- Endorsement type: Board developed
- Study via self-tuition: Yes

Exclusions

- Mathematics Standard 1 (Year 12, 2 units): TBA
- Mathematics Standard 2 (Year 12, 2 units): TBA
- Mathematics Life Skills (Year 12, 2 units): TBA

Rationale

Mathematics in the Stage 6 Curriculum

Mathematical ideas have evolved and continue to develop across cultures and have been practised in Australia by Aboriginal and Torres Strait Islander Peoples for thousands of years.

The utility of mathematics extends from counting and measuring to explaining real-world phenomena and its inherent beauty. It has evolved in sophisticated ways to become the language used to describe and understand much of the modern world.

The creative application of mathematical concepts is integral to scientific and technological advancement. The symbolic nature of mathematics provides a precise means of communication.

The Mathematics Stage 6 syllabuses are designed to offer students opportunities to think mathematically. Mathematical thinking is supported through questioning, communicating, reasoning, reflecting and the use of appropriate technology, and is encouraged through providing students with opportunities to generalise, challenge, find connections and to think critically and creatively. Using mathematical thinking, students apply their knowledge and skills to deepen their understanding of the world.

Mathematics Extension 1

Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 form a continuum to provide students with opportunities to acquire knowledge, understanding and skills in mathematical concepts at progressively higher levels and with applications in an increasing number of contexts. Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 provide students with opportunities to learn about the interconnected nature of mathematics, its beauty and its functionality. The concepts and techniques of differential and integral calculus are the basis of the courses.

Mathematics Extension 1 enables students to develop a thorough knowledge and understanding of and competence in further aspects of mathematics as an extension of the Mathematics Advanced course. The course provides opportunities to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively.

Mathematics Extension 1 supports students in tertiary study in mathematics and related fields.

Aim

The aim of Mathematics Extension 1 is for students to extend their knowledge and understanding of Working mathematically from Mathematics Advanced and Stage 5, further their understanding of the relationship between real-world problems and mathematical models, make connections within mathematics, and enhance their skills in using the language of mathematics to communicate in a concise and systematic manner.

Table of outcomes

MAO-WM-01 Working mathematically

develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly

This outcome is aligned to all content in each Stage.

Year 11	Year 12
ME1-11-01 solves problems involving inequalities, functions and their inverses, graphical relationships between functions, and parametric equations	**ME1-12-01** uses mathematical induction to prove results involving sums and divisibility
ME1-11-02 applies the remainder and factor theorem and sums and products of zeroes to solve problems involving polynomials	**ME1-12-02** operates with 2D and 3D vectors and uses 2D vectors to solve problems involving motion in two dimensions
ME1-11-03 solves problems in three dimensions using trigonometry and simplifies expressions, proves results and solves problems involving compound angles using trigonometric identities	**ME1-12-03** solves problems involving inverse trigonometric functions
ME1-11-04 uses permutations and combinations to solve problems involving counting, ordering and probability	**ME1-12-04** selects and applies differentiation and integration techniques to solve problems
ME1-11-05 uses the binomial theorem to solve problems and prove identities	**ME1-12-05** applies calculus to solve problems involving polynomials, further rates of change, areas and volumes and differential equations
	ME1-12-06 solves problems involving binomial distributions, sampling distribution of the mean and the central limit theorem

Outcomes and content for Year 11

Further work with functions

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- solves problems involving inequalities, functions and their inverses, graphical relationships between functions, and parametric equations **ME1-11-01**

Content

Graphical relationships

- Examine the relationship between the graph of $y=f(x)$ and the graph of $y=\frac{1}{f(x)}$ using graphing applications, and graph $y=\frac{1}{f(x)}$ given $y=f(x)$ in algebraic or graphical form, identifying any vertical and horizontal asymptotes of $y=f(x)$ and $y=\frac{1}{f(x)}$
- Graph $y=sec\,x$, $y=cosec\,x$ and $y=\cot x$ in both radians and degrees, identifying key properties including asymptotes, period, domain, range and symmetry, and compare each graph with the graph of its reciprocal
- Examine the relationship between the graph of $y=f(x)$ and the graphs of $y=|f(x)|$ and $y=f(|x|)$ using graphing applications, and graph $y=|f(x)|$ and $y=f(|x|)$ given $y=f(x)$ in algebraic or graphical form
- Examine the relationship between the graphs of $y=f(x)$ and $y=g(x)$ and the graphs of $y=f(x)+g(x)$ and $y=f(x)-g(x)$ using graphing applications, and graph $y=f(x)+g(x)$ and $y=f(x)-g(x)$ given $y=f(x)$ and $y=g(x)$ in algebraic or graphical form
- Determine the domains and ranges of the sum and difference of functions where possible, and, if appropriate, verify them using a graphing application
- Apply knowledge of graphical relationships to solve problems involving graphs of functions, justifying conclusions in the context of the problem where appropriate

Inverse functions

- Describe a function as one-to-one if every element in the range of the function corresponds to exactly one element of the domain
- Establish that the reflection of a point in the line $y=x$ reverses the coordinates of the point
- Define an inverse function f^{-1} informally as a function that reverses or undoes the effect of the function f
- Recognise that inverse functions exist for one-to-one functions
- Determine the equation for the inverse function $f^{-1}(x)$ of a given one-to-one function $y=f(x)$ by interchanging the variables x and y in $y=f(x)$
- Compare the graphs of a function and its inverse function using graphing applications, and recognise that the two graphs are reflections of each other in the line $y=x$

- Establish that the domain of $f^{-1}(x)$ is the range of $f(x)$, and the range of $f^{-1}(x)$ is the domain of $f(x)$, and use this relationship to solve problems
- Graph the inverse function of a given one-to-one function
- Explain that the reflection in the line $y=x$ exchanges horizontal and vertical lines and recognise that the horizontal line test can therefore be applied to the graph of $y=f(x)$ to determine whether its reflection in the line $y=x$ is a function
- Apply restrictions to the domain of a function, if it is not one-to-one, to obtain an inverse function
- Define formally $f^{-1}(x)$ to be the inverse function of $f(x)$ if the relationships $f\left(f^{-1}(x)\right)=x$ and $f^{-1}\left(f(x)\right)=x$ hold, and use this definition to solve problems
- Solve problems based on the relationship between a function and its inverse function using algebraic and graphical techniques, including determining the points of intersection of a function and its inverse, where they exist

Parametric form of a function or relation

- Recognise that a curve may be represented by two parametric equations that give x and y as functions of a parameter and that the curve may also have a Cartesian equation in x and y
- Express linear and quadratic functions and circles in parametric form

Example(s):

$x=2\cos t$, $y=2\sin t$ are the parametric equations of the circle $x^2+y^2=4$.

- Convert linear and quadratic functions and circles from parametric form to Cartesian form
- Graph linear and quadratic functions and circles expressed in parametric form

Inequalities

- Solve cubic inequalities where the cubic is expressed as a product of linear factors
- Solve inequalities involving rational expressions with variables in the denominator

Example(s):

Find the value(s) of x for which $\dfrac{x-2}{x+3}>-2$.

- Solve absolute value inequalities of the form $|ax+b|\geq k$, $|ax+b|\leq k$, $|ax+b|<k$, $|ax+b|>k$, where a, b and k are constants, using algebraic and graphical methods or the characterisation of $|x|$ as the distance of x from the origin

Polynomials

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- applies the remainder and factor theorem and sums and products of zeroes to solve problems involving polynomials **ME1-11-02**

Content

Language and graphs of polynomials

- Define a polynomial function $P(x)$ of degree n, where n is a non-negative integer, to be a function that can be expressed in the form $P(x)=a_n x^n+a_{n-1}x^{n-1}+\ldots+a_2 x^2+a_1 x+a_0$, for real $a_n, \ldots, a_0$ and $a_n \neq 0$
- Define the leading term of $P(x)$ to be the term of highest degree and define the leading coefficient of $P(x)$ to be the coefficient of the leading term
- Define the constant term of $P(x)$ to be a_0
- Define a polynomial to be monic if its leading coefficient is 1
- Define the zero polynomial to be $P(x)=0$ to be the polynomial with all the coefficients equal to zero and recognise that the zero polynomial has no leading term, no leading coefficient, no degree and constant term 0
- Determine the degree of $P(x)+Q(x)$ when two non-zero polynomials $P(x)$ and $Q(x)$, of degrees n and m respectively, are added
- Explain how the leading coefficient and the degree determine whether $y \to \infty$ or $y \to -\infty$ as $x \to \infty$ and as $x \to -\infty$
- Define the zeroes of $P(x)$ to be the numbers α such that $P(\alpha)=0$, and define the roots of the polynomial equation $P(x)=0$ to be its solutions, and recognise that every real number is a zero of the zero polynomial
- Define α as a repeated zero or multiple zero of a non-zero polynomial $P(x)=(x-\alpha)Q(x)$ when $(x-\alpha)$ is a factor of $Q(x)$, and α as a single zero of $P(x)$ when $(x-\alpha)$ is not a factor of $Q(x)$, for $Q(x)\neq 0$

- Define α as a zero of $P(x)$ of multiplicity m if $P(x)=(x-\alpha)^m Q(x)$, where m is a positive integer and $Q(\alpha)\neq 0$
- State the multiplicity of each root of a polynomial equation given in factored form

Example(s):

The polynomial equation $P(x)=0$ is said to have one root of multiplicity 3 when $P(x)=(x-1)^3$.

- Find the zeroes of a polynomial that is expressed as a product of linear factors, determine their multiplicity and graph the polynomial

Remainder and factor theorems

- Examine the process of division of polynomials by comparing with the process of division with remainders for whole numbers, and use the terms dividend, divisor, quotient and remainder
- Express in the form $P(x)=A(x)Q(x)+R(x)$ the result of dividing $P(x)$ by a divisor $A(x)$, that is not the zero polynomial, with quotient $Q(x)$ and remainder $R(x)$, and explain why either $R(x)=0$ or $deg\,R(x)<deg\,A(x)$
- Express the result of the division also in the form $\frac{P(x)}{A(x)}=Q(x)+\frac{R(x)}{A(x)}$
- Explain why division by $x-\alpha$ yields $P(x)=(x-\alpha)Q(x)+r$, where r is a constant, and why $x-\alpha$ is a factor if and only if $r=0$
- Prove and apply the remainder theorem for polynomials: when $P(x)$ is divided by $x-\alpha$, the remainder is $P(\alpha)$, and solve related polynomial problems
- Prove and apply the factor theorem for polynomials: $P(\alpha)=0$ if and only if $x-\alpha$ is a factor of $P(x)$, and solve related polynomial problems
- Use the factor theorem to find all factors of $P(x)$ of the form $x-\alpha$, where α is an integer, and use division to find the remaining factor of the polynomial

Sums and products of zeroes of polynomials

- Prove that if a quadratic $P(x)=ax^2+bx+c$ has zeroes α and β then $\alpha+\beta=-\frac{b}{a}$, the sum of the zeroes, and $\alpha\beta=\frac{c}{a}$, the product of the zeroes
- Prove that if a cubic $P(x)=ax^3+bx^2+cx+d$ has three zeroes α, β, γ, then $\alpha+\beta+\gamma=-\frac{b}{a}$, $\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}$ and $\alpha\beta\gamma=-\frac{d}{a}$
- Prove that if a quartic $P(x)=ax^4+bx^3+cx^2+dx+e$ has four zeroes α, β, γ, δ then $\alpha+\beta+\gamma+\delta=-\frac{b}{a}$, $\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\alpha=\frac{c}{a}$, $\alpha\beta\gamma+\beta\gamma\delta+\gamma\delta\alpha+\delta\alpha\beta=-\frac{d}{a}$ and $\alpha\beta\gamma\delta=\frac{e}{a}$
- Use the formulas for the sums and products of zeroes to solve problems involving zeroes and coefficients of quadratic, cubic and quartic polynomials

Further trigonometry

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- solves problems in three dimensions using trigonometry and simplifies expressions, proves results and solves problems involving compound angles using trigonometric identities **ME1-11-03**

Content

Trigonometry in three dimensions

- Interpret information about a 3-dimensional context given in diagrammatical form
- Interpret information about a 3-dimensional context given in written form
- Apply trigonometry to solve problems in three dimensions where a diagram is given

Further trigonometric identities

- Derive the sum and difference expansions for the trigonometric functions $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$, $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ and $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- Use sum and difference expansion formulas to solve problems and prove results
- Derive the double angle formulas $\sin 2A = 2\sin A \cos A$, $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2sin^2 A$ and $\tan 2A = \dfrac{2\tan A}{1 - tan^2 A}$
- Use the double angle formulas to solve problems and prove results

Further trigonometric equations

- Solve trigonometric equations involving factorisation and/or substitution of trigonometric identities over restricted domains
- Examine the representations of $a\cos x + b\sin x$ as $R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$with and without graphing applications, where $a \neq 0$and $b \neq 0$
- Apply the representations of $a\cos x + b\sin x$ as $R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$ to graph functions, solve equations of the form $a\cos x + b\sin x = c$ over restricted domains, and model and solve related problems
- Interpret solutions of trigonometric equations graphically with and without graphing applications
- Model and solve practical problems using trigonometric equations and analyse their solutions in a variety of contexts

Example(s):

The current I (in amperes) in a certain circuit after t seconds is given by $I = 2\sin\left(t - \frac{\pi}{3}\right) - \cos\left(t + \frac{\pi}{2}\right)$. Find the maximum current and the earliest time that it occurs.

Permutations and combinations

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- uses permutations and combinations to solve problems involving counting, ordering and probability **ME1-11-04**

Content

- Use the notation $n!$ (read as n factorial), where $n! = n(n-1)(n-2) \times \ldots \times 3 \times 2 \times 1$ for positive integers n
- Use $n! = n \times (n-1)!$ and the convention $0! = 1$ in calculations and to simplify algebraic expressions involving factorials
- Establish and use the multiplication principle: if a selection can be made in two stages, where there are m choices for the first stage and n choices for the second stage then there are $m \times n$ choices for the selection
- Apply the multiplication principle to explain why the number of ways of ordering n distinct objects in a straight line is $n!$
- Define a permutation as an ordered selection of some or all objects from a set of distinct objects
- Use the notation ${}^{n}P_{r}$ to represent an ordered selection of r objects from n distinct objects and observe that ${}^{n}P_{n} = n!$ and ${}^{n}P_{0} = 1$
- Use the multiplication principle to establish that the number of ordered selections of r objects from n distinct objects is $n(n-1)(n-2) \times \ldots \times (n-r+1)$ and show that ${}^{n}P_{r} = n(n-1)(n-2) \times \ldots \times (n-r+1) = \dfrac{n!}{(n-r)!}$
- Solve problems involving permutations, including situations where the objects are not all distinct
- Solve problems involving permutations with restrictions on the placement of one or more objects
- Explain why the number of ways to arrange n distinct objects in a circle is $(n-1)!$
- Solve problems involving circular arrangements of distinct objects with or without restrictions on the placement of one or more objects
- Define a combination and use the notation ${}^{n}C_{r}$ or $\binom{n}{r}$ to represent the number of ways of selecting a subset of r objects from n distinct objects, where order is not important
- Establish and use the formula ${}^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$
- Show that ${}^{n}C_{n} = {}^{n}C_{0} = 1$ and ${}^{n}C_{1} = {}^{n}C_{n-1} = n$
- Show that ${}^{n}C_{r} = {}^{n}C_{n-r}$, for $0 \le r \le n$ by selecting r objects from n distinct objects for inclusions and $n-r$ objects from n distinct objects for exclusion
- Prove ${}^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-1}C_{r}$ for $1 \le r \le n-1$ algebraically and using combinatorial arguments
- Solve problems involving combinations with or without restrictions on the selection of one or more objects

- Solve problems involving both permutations and combinations, including problems which require consideration of cases
- Solve probability problems involving permutations and combinations

The binomial theorem

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- uses the binomial theorem to solve problems and prove identities **ME1-11-05**

Content

- Recognise that a binomial is an expression with two terms, and that a binomial expansion is an expansion of a power of a binomial
- Examine the symmetry formed by the coefficients of decreasing powers of x in the expansion of $(x+y)^n$ for $n=0,1,2,3,4,5$ and arrange the coefficients into Pascal's triangle
- Recognise the equivalence between the coefficient of $x^{n-r}y^r$ in the expansion of $(x+y)^n$ and ${}^nC_r$, when n is a positive integer
- Use patterns and symmetry in Pascal's triangle to confirm the identities ${}^nC_r={}^{n-1}C_{r-1}+{}^{n-1}C_r$ for $1\le r\le n-1$ and ${}^nC_r={}^nC_{n-r}$ for $0\le r\le n$
- Derive the binomial theorem: $(x+y)^n={}^nC_0x^n+{}^nC_1x^{n-1}y+{}^nC_2x^{n-2}y^2+\ldots+{}^nC_{n-1}xy^{n-1}+{}^nC_ny^n$, when n is a positive integer
- Apply the binomial theorem to expand and simplify expressions of the form $(x+y)^n$
- Use the binomial theorem to determine the coefficient of a term with a specific power or the constant term in a binomial expansion

Example(s):

Find the term independent of x and the coefficient of x^6 in the expansion of $\left(2x-\frac{1}{x^2}\right)^{12}$.

- Use the identities ${}^nC_0=1, {}^nC_n=1, {}^nC_r={}^{n-1}C_{r-1}+{}^{n-1}C_r$ for $1\le r\le n-1$ and ${}^nC_r={}^nC_{n-r}$ for $0\le r\le n$ to simplify expressions involving the binomial coefficients
- Prove identities involving binomial coefficients in binomial expansions by substituting values, comparing coefficients or applying a combinatorial argument to a specified context

Example(s):

Use the identity $(1+x)^4(1+x)^9=(1+x)^{13}$ to show that ${}^4C_0{}^9C_4+{}^4C_1{}^9C_3+{}^4C_2{}^9C_2+{}^4C_3{}^9C_1+{}^4C_4{}^9C_0={}^{13}C_4$. Prove the same result by considering the number of ways a committee of 4 members can be chosen from a group of 13 people comprising 4 men and 9 women.

- Apply given or proven identities involving binomial coefficients to prove further identities, without the use of calculus

Example(s):

Use the identity ${}^{n}C_{r} = {}^{n}C_{n-r}$ to show that ${}^{2n}C_{n} = \left({}^{n}C_{0}\right)^2 + \left({}^{n}C_{1}\right)^2 + \left({}^{n}C_{2}\right)^2 + \ldots + \left({}^{n}C_{n-1}\right)^2 + \left({}^{n}C_{n}\right)^2$.

Outcomes and content for Year 12

Proof by mathematical induction

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- uses mathematical induction to prove results involving sums and divisibility **ME1-12-01**

Content

- Examine the nature of inductive proof, including the statement to be proved, the base case and the inductive step
- Prove results for sums using mathematical induction

Example(s):

Prove $\sum_{k=1}^{n}(2k-1)=n^2$ for $n \geq 1$ and prove $1+2+2^2+\ldots+2^n=2^{n+1}-1$ for $n \geq 0$.

- Prove divisibility results using mathematical induction

Example(s):

Prove 3^n+7^n is divisible by 10 for all odd n.

- Identify errors in false 'proofs by induction', such as cases where only one of the two required steps of a proof by induction is true

Example(s):

Show that the inductive step can be proven for the false proposition that $1+2+3+\ldots+n=\frac{1}{2}(n-1)(n+2)$ for integers $n \geq 1$, but that the initial case does not hold true.

Introduction to vectors

Outcomes

A student:

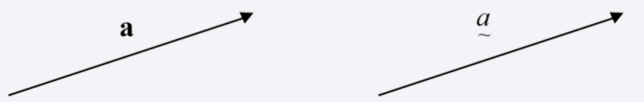
- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- operates with 2D and 3D vectors and uses 2D vectors to solve problems involving motion in two dimensions **ME1-12-02**

Content

Vector representation and notation

- Define a vector as a quantity having both magnitude and direction
- Associate vectors with directed line segments and recognise that a vector may have many directed line segments associated with it
- Identify and use notation for vectors in both two dimensions and three dimensions, including a, $\underset{\sim}{a}$ and $\overrightarrow{AB}$, where $\overrightarrow{AB}$ is the vector with magnitude and direction those of the directed line segment from A to B

Example(s):



(upright boldface "a" used in printed text)

(undertilde notation for handwritten vectors)

Image long description: Two identical arrows representing vectors, sloping upwards from left to right. The first is labelled with '**a**' and is captioned with 'upright boldface "a" used in printed text'. The second is labelled with '*a*' and has a tilde underneath with the caption 'undertilde notation for handwritten vectors'.

- Use notations $|a|$, $|\underset{\sim}{a}|$ and $|\overrightarrow{AB}|$ to represent the magnitude of a vector
- Describe a position vector as a vector with its tail at the origin
- Represent vectors graphically with and without graphing applications
- Recognise and use the fact that two vectors are equal if they have the same magnitude and direction to solve problems

Introduction to 2D and 3D vectors

- Use Cartesian coordinates to represent points in 2-dimensional (2D) and 3-dimensional (3D) space with and without graphing applications
- Use the midpoint and distance formulas in two dimensions and three dimensions

- Identify that, in three dimensions, all points on the xy-plane have a z-coordinate of 0, and deduce similar properties for points on the xz and yz-planes and thus the equations of the three coordinate planes
- Define the zero vector 0, written as $\underset{\sim}{0}$, as the vector with zero magnitude and no direction
- Define unit vectors as vectors of magnitude 1
- Recognise and use $\hat{a}$ and $\underset{\sim}{\hat{a}}$ as the notation for the unit vector in the direction of a
- Define the standard perpendicular unit vectors i and j in two dimensions and i, j and k in three dimensions
- Express 2D vectors in component form, $x i + y j$; as an ordered pair, (x, y); and in column vector notation $\begin{pmatrix} x \\ y \end{pmatrix}$
- Recognise $\overrightarrow{AB} = \begin{pmatrix} u - x \\ v - y \end{pmatrix}$ as the vector associated with the directed line segment from the point $A(x, y)$ to $B(u, v)$ in two dimensions
- Express 3D vectors in component form, $x i + y j + z k$; as an ordered triple, (x, y, z); and in column vector notation $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$
- Recognise $\overrightarrow{AB} = \begin{pmatrix} u - x \\ v - y \\ w - z \end{pmatrix}$ as the vector associated with the directed line segment from $A(x, y, z)$ to $B(u, v, w)$ in three dimensions

Operating with vectors

- Define a scalar as a real number that is used to multiply a vector
- Represent geometrically a scalar multiple of a vector in two dimensions and three dimensions with and without graphing applications
- Perform multiplication of a vector by a scalar algebraically in component form
- Establish and identify $a = k b$, for a non-zero scalar k, as a condition for two non-zero vectors a and b to be parallel to each other and determine with justification if two vectors are parallel to one another
- Identify $\begin{pmatrix} b \\ -a \end{pmatrix}$ and $\begin{pmatrix} -b \\ a \end{pmatrix}$ as vectors perpendicular to $\begin{pmatrix} a \\ b \end{pmatrix}$ and with equal magnitude
- Perform addition and subtraction of vectors algebraically in component form, and verify, with and without graphing applications, that geometrically these are obtained using the triangle law or the parallelogram law
- Establish and calculate the magnitude of a vector using $|x i + y j| = \sqrt{x^2 + y^2}$ for 2D vectors and $|x i + y j + z k| = \sqrt{x^2 + y^2 + z^2}$ for 3D vectors
- Use the magnitude of a vector to find the unit vector $\hat{a} = \frac{1}{|a|} a$ in two dimensions and three dimensions

Further operations with vectors

- Define $a \cdot b = x_1 x_2 + y_1 y_2$ as the scalar (dot) product of vectors $a = x_1 i + y_1 j$ and $b = x_2 i + y_2 j$ and use the scalar product to solve problems

- Define $a \cdot b = x_1 x_2 + y_1 y_2 + z_1 z_2$ as the scalar product of vectors $a = x_1 i + y_1 j + z_1 k$ and $b = x_2 i + y_2 j + z_2 k$ and use the scalar product to solve problems
- Use $a \cdot b = |a||b| \cos\theta$ as a geometric expression of the scalar product of non-zero vectors a and b in two dimensions and three dimensions, where θ is the angle between the vectors and $0° \le \theta \le 180°$.
- Verify the equivalence of $a \cdot b = |a||b| \cos\theta$ with the algebraic definition of the scalar product, $a \cdot b = x_1 x_2 + y_1 y_2$ for two dimensions and $a \cdot b = x_1 x_2 + y_1 y_2 + z_1 z_2$ for three dimensions

Example(s):

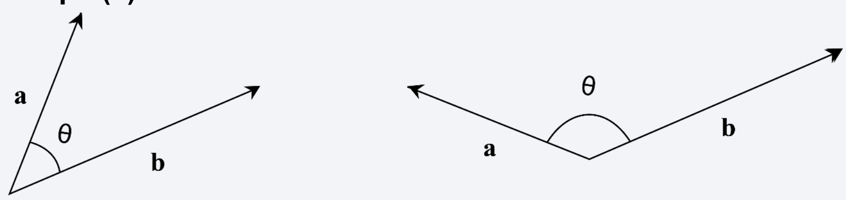


Image long description: Two vector diagrams, each represented by two arrows connected at a point. The first shows the arrows labelled a and b with an acute angle labelled theta between them. The second shows the arrows labelled a and b with an obtuse angle labelled theta between them.

- Derive and use the property $a \cdot a = |a|^2$ to establish the scalar product definition of the magnitude of a vector ($|a| = \sqrt{a \cdot a}$) in two dimensions and three dimensions
- Calculate the angle between two non-zero vectors a and b, in both two dimensions and three dimensions, using the scalar product by deriving and applying the relationship $\cos\theta = \dfrac{a \cdot b}{|a||b|} = (\hat{a} \cdot \hat{b})$
- Establish $a \cdot b = 0$ as a condition for two non-zero vectors a and b to be perpendicular to each other and use it to determine if two vectors are perpendicular
- Establish $a \cdot b = \pm|a||b|$ as another way to determine if two non-zero vectors a and b are parallel

- Define the projection of a vector a onto a vector b, denoted by $proj_b a$, to be the vector component of a in the direction of vector b

Example(s):

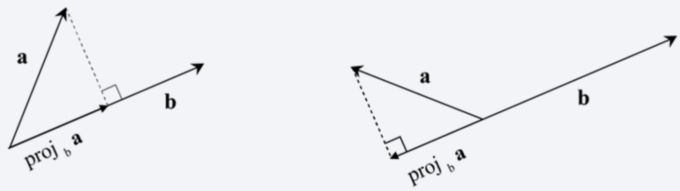


Image long description: Two vector diagrams both representing vectors, each showing two arrows connected at a point. In the first diagram, the angle between the arrows is acute. One arrow is labelled with boldface a, the other with boldface b, with the b arrow as the longer vector, pointing up and to the right. A dotted line perpendicular to arrow b joins the tip of arrow a. There is an arrow drawn on top of arrow b from the vertex, which is labelled proj b a, with a subscripted b and boldface a. In the second diagram the angle between the arrows is obtuse. One arrow is labelled with boldface a and the other with boldface b, with b as the longer vector, pointing up and to the right. A dotted line perpendicular to arrow b joins the tip of arrow a, pointing up and to the left, to an extension of arrow b, pointing down and to the left. The arrow drawn on top of arrow b from the vertex is labelled proj b a with a subscripted b and boldface a.

- Examine the proof of the formula $proj_b a = \left(\frac{a \cdot b}{|b|^2}\right) b = (a \cdot \hat{b})\hat{b}$ and use the formula to solve problems
- Determine that the component of a vector a perpendicular to another vector b is $a - proj_b a$

Example(s):

Define and find the vector component of **a** perpendicular to **b**.

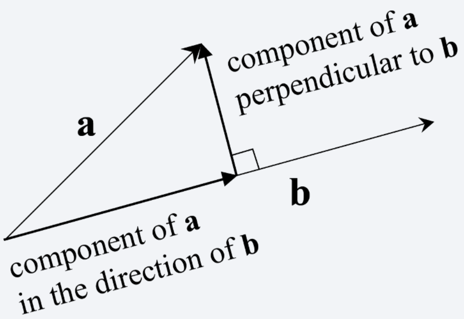


Image long description: Two arrows representing vectors, connected at a point. The angle between the arrows is acute. The arrows are labelled with boldface a and boldface b with b as the longer vector. An arrow, perpendicular to arrow b and starting from arrow b, joins the tip of arrow a and is labelled ‘component of (boldface) a perpendicular to (boldface) b’. Another arrow is drawn on top of arrow b from the vertex to the point where the perpendicular arrow starts and is labelled ‘component of (boldface) a in the direction of (boldface) b’.

Motion in vector form in two dimensions

- Describe the position of an object at a point in 2D space using a vector

- Describe the changing positions of an object by expressing its vector as a function of time using $r(t) = x(t)i + y(t)j$, or $r = xi + yj$ where x and y are functions of time t
- Recognise that $x(t)$ and $y(t)$ form a pair of parametric equations for the path of the object
- Find the Cartesian equation of the path of an object, where the path is a straight line, parabola or circle
- Express the change in an object's position between two points as a displacement vector and recognise the magnitude of the displacement vector as the distance between the two points
- Solve motion problems involving constant velocity using vectors
- Solve relative velocity problems involving constant crosswind/cross-current using vector diagrams, and describe the direction of a vector where required
- Find the velocity vector $v = \dot{x}i + \dot{y}j$ and acceleration vector $a = \ddot{x}i + \ddot{y}j$ of an object using differential calculus
- Find the position vector and the velocity vector of an object using integral calculus given its acceleration vector
- Solve motion problems involving non-constant velocity using vectors

Projectile motion

- Recognise that the gravitational force on a mass may be regarded as a constant acting in a downwards direction when the motion of the object is restricted to a small region near the Earth's surface
- Model and analyse a projectile's path where the projectile is a point and air resistance is negligible, subject to only acceleration due to gravity, assuming that the projectile is moving close to the Earth's surface
- Represent the motion of a projectile using vectors
- Recognise that the horizontal and vertical components of the motion of a projectile can be represented by horizontal and vertical vectors
- Derive and use the equations of motion of a projectile in vector form by splitting 2D motion into horizontal and vertical components to solve problems on projectiles
- Find the Cartesian equation of the path of a projectile using parametric equations for the horizontal and vertical components of the displacement vector
- Determine features of the flight of a projectile, including time of flight, maximum height, range, instantaneous velocity and impact velocity
- Solve problems relating to the path of a projectile in which the initial velocity and/or angle of projection may be unknown, in a variety of contexts

Inverse trigonometric functions

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- solves problems involving inverse trigonometric functions **ME1-12-03**

Content

Definitions of inverse trigonometric functions

- Graph $y=\sin x$, $y=\cos x$ and $y=\tan x$, for $-2\pi \le x \le 2\pi$ using graphing applications and recognise that these three functions fail the horizontal line test
- Examine possible domain restrictions of $y=\sin x$, $y=\cos x$ and $y=\tan x$ to obtain the corresponding inverse functions using graphing applications
- Define $\sin^{-1} x$ or $\arcsin x$ to be the inverse function of $y=\sin x$ restricted to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, and determine the domain and range of $y=\sin^{-1} x$
- Define $\cos^{-1} x$ or $\arccos x$ to be the inverse function of $y=\cos x$ restricted to $0 \le x \le \pi$, and determine the domain and range of $y=\cos^{-1} x$
- Define $\tan^{-1} x$ or $\arctan x$ to be the inverse function of $y=\tan x$ restricted to $-\frac{\pi}{2} < x < \frac{\pi}{2}$, and determine the domain and range of $y=\tan^{-1} x$
- Evaluate and simply expressions, and prove results using the definitions of $\sin^{-1} x$, $\cos^{-1} x$ and $\tan^{-1} x$

Example(s):

Find the exact value of $\cos\left(\sin^{-1}\left(-\frac{1}{4}\right)\right)$.

Graphs of inverse trigonometric functions

- Graph $y=\sin x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, then use a reflection in $y=x$ to graph $y=\sin^{-1} x$ with and without graphing applications
- Graph $y=\cos x$ for $0 \le x \le \pi$, then use a reflection in $y=x$ to graph $y=\cos^{-1} x$ with and without graphing applications
- Graph $y=\tan x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, then use a reflection in $y=x$ to graph $y=\tan^{-1} x$ with and without graphing applications
- Classify $y=\sin^{-1} x$, $y=\cos^{-1} x$ and $y=\tan^{-1} x$ as odd, even, or neither odd nor even
- Examine the properties $\sin^{-1}(-x)=-\sin^{-1} x$, $\cos^{-1}(-x)=\pi-\cos^{-1} x$, $\tan^{-1}(-x)=-\tan^{-1} x$ and $\cos^{-1} x+\sin^{-1} x=\frac{\pi}{2}$ using graphing applications and prove them algebraically

- Use the properties $\sin^{-1}(-x) = -\sin^{-1}x$, $\cos^{-1}(-x) = \pi - \cos^{-1}x$, $\tan^{-1}(-x) = -\tan^{-1}x$ and $\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$ to solve problems, simplify expressions and prove results

Example(s):

Solve $\cos^{-1}x = \sin^{-1}\left(-\frac{5}{13}\right)$ for x.

- Graph the composite functions $y = \sin(\sin^{-1}x)$, $y = \cos(\cos^{-1}x)$ and $y = \tan(\tan^{-1}x)$ by considering their domains and ranges
- Examine the graphs of the composite functions $y = \sin(\sin^{-1}x)$, $y = \cos(\cos^{-1}x)$ and $y = \tan(\tan^{-1}x)$, establish the values of x for which the formulas $\sin(\sin^{-1}x) = x$, $\cos(\cos^{-1}x) = x$ and $\tan(\tan^{-1}x) = x$ are true, and apply the formulas to solve problems, simplify expressions and prove results
- Graph the composite functions $y = \sin^{-1}(\sin x)$, $y = \cos^{-1}(\cos x)$ and $y = \tan^{-1}(\tan x)$ by considering their domains and ranges
- Examine the graphs of the composite functions $y = \sin^{-1}(\sin x)$, $y = \cos^{-1}(\cos x)$ and $y = \tan^{-1}(\tan x)$, establish the values of x for which the formulas $\sin^{-1}(\sin x) = x$, $\cos^{-1}(\cos x) = x$ and $\tan^{-1}(\tan x) = x$ are true, and apply the formulas to solve problems, simplify expressions and prove results

Example(s):

Given that $\cos\left(\frac{23\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$, find the exact value of $\cos^{-1}\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)$.

- Use graphing applications to explore reflections, translations and dilations of the functions $f(x) = \sin^{-1}x$, $f(x) = \cos^{-1}x$ and $f(x) = \tan^{-1}x$, and confirm that the principles of transformations hold for the inverse trigonometric functions
- Apply the principles of transformations to inverse trigonometric functions to determine the function rule and graph the function, finding the domain and range of the transformed graph and determining any intercepts and asymptotes where appropriate

Example(s):

Graph $y = -2\cos^{-1}(3x)$ and describe its graphical transformation from $y = \cos^{-1}x$.

Further calculus skills

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- selects and applies differentiation and integration techniques to solve problems **ME1-12-04**

Content

Further derivatives of functions

- Find the derivative of a function defined parametrically using the chain rule
- Solve problems involving derivatives of functions defined parametrically
- Verify using the chain rule that the derivative of the inverse function is the reciprocal of the derivative of the function, evaluated at the value of the inverse function, that is $\left(f^{-1}\right)'(x)=\dfrac{1}{f'\left(f^{-1}(x)\right)}$
- Solve problems involving derivatives of inverse functions

Example(s):

A particular function $y=f(x)$ has an inverse function $y=f^{-1}(x)$. The tangent l to $y=f(x)$ at the point $(2,3)$ has a gradient of 1. Show that the tangent to $y=f^{-1}(x)$ at the point $(3,2)$ is parallel to l.

- Examine the proofs of the derivatives of $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$
- Use the chain rule to show that $\dfrac{d}{dx}\left[\sin^{-1}f(x)\right]=\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$, $\dfrac{d}{dx}\left[\cos^{-1}f(x)\right]=-\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}=-\dfrac{d}{dx}\left[\sin^{-1}f(x)\right]$ and $\dfrac{d}{dx}\left[\tan^{-1}f(x)\right]=\dfrac{f'(x)}{1+\left[f(x)\right]^2}$ and apply the results to solve problems involving derivatives of $\sin^{-1}f(x)$, $\cos^{-1}f(x)$ and $\tan^{-1}f(x)$
- Apply the product, quotient and chain rules to find derivatives of functions of the form $f(x)g(x)$, $\dfrac{f(x)}{g(x)}$ and $f(g(x))$ where $f(x)$ and $g(x)$ are any of the functions covered in the scope of the *Mathematics Advanced 11–12 Syllabus* (2024) or inverse trigonometric functions and solve related problems

Example(s):

Find the equation of the tangent to the curve $y=\tan^{-1}\left(x^2\right)$ at the point where $x=1$.

Techniques of integration

- Find indefinite and definite integrals involving expressions of the form $\frac{1}{\sqrt{a^2-x^2}}$ or $\frac{a}{a^2+x^2}$

Example(s):

Find $\int \frac{x^2}{x^2+4}dx$ and $\int \frac{dx}{\sqrt{1-9x^2}}$.

- Use integration by substitution to evaluate definite and indefinite integrals given the substitution, where the substitution is expressed as a function of the variable of integration or where the variable of integration is the subject of the substitution

Example(s):

Find $\int \frac{e^{\sqrt{x}+1}}{\sqrt{x}}dx$ using the substitution $u=\sqrt{x}+1$. Find $\int_0^1 \frac{dx}{\left(1+x^2\right)^{\frac{3}{2}}}$ using the substitution $x=\tan\theta$.

- Prove and use the identities $\sin^2 nx=\frac{1}{2}(1-\cos 2nx)$ and $\cos^2 nx=\frac{1}{2}(1+\cos 2nx)$ to find integrals involving $\int \sin^2 nx \; dx$ and $\int \cos^2 nx \; dx$

Further applications of calculus

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- applies calculus to solve problems involving polynomials, further rates of change, areas and volumes and differential equations **ME1-12-05**

Content

Multiplicity of zeroes of polynomial functions

- Use the product rule to prove that if $P(x)$ has a zero, α, of multiplicity $m>1$, then α is a zero of $P'(x)$ of multiplicity $m-1$, and use the result to determine the multiplicity of a discovered zero of $P(x)$ and solve related polynomial problems

Example(s):

Let $P(x)=x^3+ax^2+bx-4$ where *a* and *b* are real numbers. If $x=2$ is a double root of $P(x)=0$, find the values of a and b.

- Prove that if α is a zero of multiplicity m, meeting the x-axis at $A(\alpha,0)$, then if $m=1$, the curve crosses the x-axis at A at an acute or obtuse angle; if $m>1$ is even, the curve is tangent to the x-axis at A and does not cross it; if $m>1$ is odd, the curve has a horizontal inflection at A
- Graph a polynomial function in factored form, identifying its turning points and points of inflection if possible, and explore its behaviour as $x\to\infty$ and $x\to-\infty$, verifying the shape of the graph using graphing applications

Example(s):

Graph the polynomials $P(x)=(x+1)^2(2-x)^3$ and $P(x)=(x-1)(x^2+3x+1)$

Further rates of change

- Develop models in contexts where a rate of change of a function can be expressed as a rate of change of a composition of two functions, so that the chain rule can be applied
- Solve problems involving related rates of change using the chain rule, given the required formulas for problems relating to area, surface area or volume
- Describe and examine the graphs of practical situations where the rate of change of a quantity is proportional to the amount $Q-P$ by which the quantity Q exceeds some fixed value P
- Explain that if the rate of change of a quantity Q over time t is proportional to the difference $Q-P$ at any instant, then this may be represented by the equation $\frac{dQ}{dt}=k(Q-P)$, where k is a constant

- Verify by substitution that the function $Q=P+Ae^{kt}$, where A is a constant, satisfies the relationship $\frac{dQ}{dt}=k(Q-P)$, and that $Q=P$ in the case where $A=0$
- Graph the function $Q=P+Ae^{kt}$, where $k>0$ and $k<0$ and $A>0$ and $A<0$, with and without graphing applications, and identify any asymptotes
- Use $\frac{dQ}{dt}=k(Q-P)$, $Q=P+Ae^{kt}$ and the graph of $Q=P+Ae^{kt}$ for $t\geq 0$, where $k>0$ or $k<0$ and $A>0$ or $A<0$, to model and solve problems where a limiting value of Q exists, including Newton's Law of Cooling and ecosystems with a natural carrying capacity, and justify conclusions in the context of the problem

Areas between curves and volumes of solids of revolution

- Calculate areas of regions between curves determined by functions in both real-life and abstract contexts
- Examine a solid of revolution whose boundary is formed by rotating an arc of a function about the x-axis or y-axis with and without graphing applications
- Calculate the volume of a solid of revolution formed by rotating a region in the plane about the x-axis or y-axis in both real-life and abstract contexts
- Calculate the volume of a solid of revolution formed by rotating the region between two curves about either the x-axis or y-axis in both real-life and abstract contexts

Differential equations

- Define a differential equation as an equation involving an unknown function and one or more of its derivatives
- Define and identify the order of a differential equation as the order of the highest derivative contained within the equation

Example(s):

$x^2\frac{dy}{dx}=\tan y$ is a first order differential equation and $\frac{d^2y}{dx^2}-2\frac{dy}{dx}=e^x$ is a second order differential equation.

- Recognise that a solution to a first order differential equations is a function, and that there may be infinitely many functions that are solutions to a given first order differential equation
- Recognise the solutions to differential equations in the context of slope fields, and that slope fields are useful in determining the behaviour of solutions when the differential equation cannot be easily solved
- Recognise that a unique solution of a differential equation can be determined when sufficient initial conditions are given, and refer to a problem involving a differential equation and initial conditions as an initial value problem (IVP)
- Graph solutions to first order differential equations given a slope field and identify the unique solution curve that satisfies a set of initial conditions
- Explore problems given a slope field representing a practical context and justify conclusions
- Form a slope field for a first order differential equation using graphing applications

- Recognise the features of a slope field corresponding to a first order differential equation and vice versa

Example(s):

Which of the following slope fields best represents the differential equation $\frac{dy}{dx} = x - y$?

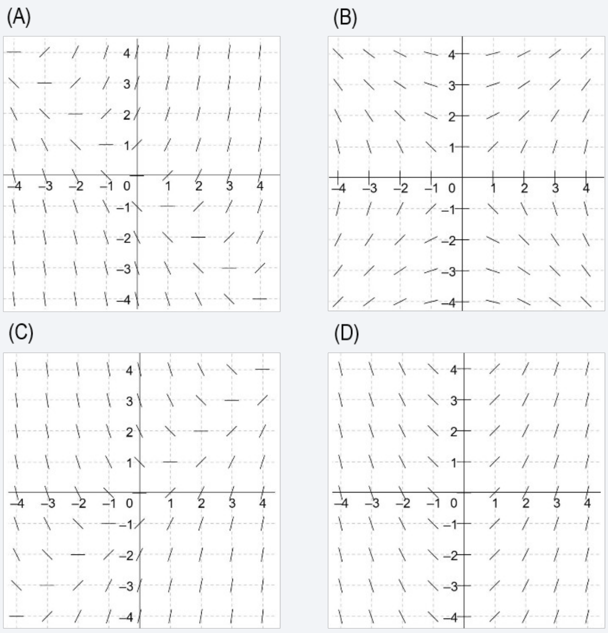


Image long description: Option A represents a slope field with all slope segments on the line $y = -x - 1$ having gradient -1. Option B represents a slope field with all slope segments on the line $y = x$ having gradient 1. Option C represents a slope field with all slope segments on the line $y = x - 1$ having gradient 1. Option D represents a slope field with all slope segments on the line $x = 0$ having 0 gradient. Image generated using Desmos.

- Solve first order differential equations of the form $\frac{dy}{dx} = f(x)$
- Solve first order differential equations of the form $\frac{dy}{dx} = g(y)$, where possible expressing the solution as a function with y as the subject

Example(s):

Solve the differential equations $\frac{dy}{dx} = 3y$ and $\frac{dy}{dx} = 20e^{-5y}$.

- Recognise and solve the first order differential equations for exponential growth and decay: $\frac{dQ}{dt} = kQ$ and $\frac{dQ}{dt} = k(Q - P)$

- Solve first order differential equations of the form $\frac{dy}{dx}=f(x)g(y)$ using separation of variables, where possible expressing the solution as a function with y as the subject

Example(s):

Given $\frac{dy}{dx}=-\frac{x}{y e^{x^2}}$, find the particular solution that goes through the point $(0,1)$.

- Graph solutions of first order differential equations using graphing applications and examine the behaviour of solutions for different values of the constant of integration and initial conditions
- Solve differential equations of the form $\frac{dP}{dt}=kP\left(1-\frac{P}{C}\right)$ for some constants k and C, given the appropriate decomposition into partial fractions, to obtain the logistic function
- Model and solve differential equations in practical scenarios including in chemistry, biology and economics

The binomial distribution and sampling distribution of the mean

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- solves problems involving binomial distributions, sampling distribution of the mean and the central limit theorem **ME1-12-06**

Content

Bernoulli distributions

- Define a Bernoulli random variable as a model for two-outcome situations, referred to as success and failure
- Model a Bernoulli trial using a random variable, X, where the probability of success, $X=1$, is p and the probability of failure, $X=0$, is $q=1-p$
- Define a Bernoulli distribution as the discrete probability distribution of a Bernoulli random variable
- Establish and use the formulae $\mathrm{E}(X)=\mu=p$ to find the mean, and $Var(X)=p(1-p)$ to find the variance of the Bernoulli distribution with parameter p
- Identify contexts suitable to be modelled by Bernoulli random variables
- Solve practical problems involving Bernoulli random variables

Binomial distributions

- Define a binomial random variable, X, as the number of 'successes' in n independent and identically distributed Bernoulli trials, with the same probability of success p in each trial, where n is a fixed finite number
- Identify contexts suitable to be modelled using binomial distributions

Example(s):

A scenario that can be modelled using a binomial distribution: 5 cards are randomly drawn from a standard deck of playing cards with replacement. A scenario that cannot be modelled using a binomial distribution: 5 cards are randomly drawn from a standard deck of playing cards without replacement.

- Define a binomial experiment as a fixed number of Bernoulli trials
- Use the notation $X \sim Bin(n, p)$ to represent a binomial random variable, X, where n is the number of trials and probability of success is p
- Recognise the significance of ${}^nC_r$ as the number of ways in which an outcome with r successes can occur in n trials
- Use the formula $P(X=r)={}^nC_r\, p^r(1-p)^{n-r}$ to find the probability of r successes in a binomial experiment involving n trials
- Calculate the expected frequencies of the various possible outcomes from a binomial experiment, where the expected frequency of an outcome happening r times in n trials is $n \times P(X=r)$

- Use the formulae $\mathrm{E}(X)=\mu=np$ to find the mean, and $Var(X)=np(1-p)$ to find the variance when $X\sim Bin(n,p)$
- Solve practical problems involving binomial distributions and binomial probabilities with and without online computational applications, excluding the normal approximation to the binomial distribution

Sampling distribution of the mean and the central limit theorem

- Define a statistical population as the entire group of people or objects about which information is sought
- Define a sample as a selection of people or objects drawn from a population
- Recognise that a statistic taken from a sample will be a good approximation if the sample is random and the sample size is large enough, typically $n\geq 30$
- Recognise that, in general, the distribution of a population and a statistic, such as its mean, are unknown
- Recognise that it is likely that the distribution of a random sample will resemble the distribution of the population when the sample size, n, is large enough, typically $n\geq 30$
- Define the sample mean for a sample $X_1, X_2, \ldots, X_n$ taken from a population as $\overline{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$
- Recognise that the sample means obtained from repeated sampling may be different, even if all the samples are of the same size
- Define, for a given sample size n, the sampling distribution of the mean to be the distribution of the sample means of all samples of size n
- Recognise that sample mean, $\overline{X}$, is a random variable that estimates the population mean, μ, when the size of the sample is large enough and consider its usefulness in making predictions in a variety of contexts including politics, finance, agriculture and biology

Example(s):

Assume the weight of pumpkins in a particular farm is normally distributed with a mean of 13.5 kg and a standard deviation of 3 kg. A sample of 15 pumpkins is randomly selected from the farm. Find the expected value, variance and the standard deviation of the distribution of the sample mean. Let $X_1, X_2, X_3, \ldots, X_{15}$ denote the weights of the pumpkins in the sample.

- State the central limit theorem as: for a population with mean μ and variance σ^2, provided the sample size n is large enough ($n\geq 30$), the sampling distribution is approximately normal with mean μ and variance $\frac{\sigma^2}{n}$; that is, $\overline{X}$ is approximately $N\left(\mu, \frac{\sigma^2}{n}\right)$
- Recognise the significance of the central limit theorem for populations that are not necessarily normally distributed; that, irrespective of the population distribution, for a sufficiently large sample size, the sampling distribution of the mean is approximately normal
- Use the formulas $E(\overline{X})=\mu$ and $\mathrm{Var}(\overline{X})=\frac{\sigma^2}{n}$ as the mean and the variance of the sampling distribution of the mean for samples of size n drawn from a population with mean μ and variance σ^2
- Examine the effect of the sample size on the variance of sample means with digital tools

- Apply the central limit theorem to estimate the probability that the sample mean lies within given bounds

Example(s):

A group of researchers conducted a random sampling study on the sleep time of Year 12 students. The population has an average of 7.4 hours of sleep time with a standard deviation of 2.42 hours. If a sample of 100 students is taken, what is the probability that their sample mean will be less than 7 hours?

Assessment

Course standards

Course standards are comprised of syllabus standards and performance standards. Syllabus outcomes and content form the syllabus standards that describe what students are expected to know, understand and do. Performance standards, including the Common Grade Scale for Preliminary courses, HSC Performance Band Descriptions and Achievement Level Descriptions (for HSC courses with grades only) describe how well students have achieved in relation to syllabus standards. Performance standards are used to report student achievement on NESA credentials.

Common Grade Scale for Preliminary courses

Stage 6 – Year 11

The Common Grade Scale for Preliminary courses provides generic, holistic descriptions for performance at each of the 5 grade levels. It is used to report student achievement in Year 11 courses in all NSW schools.

Performance Band Descriptions for Mathematics Extension 1

Stage 6

HSC performance band descriptions summarise the knowledge, understanding and skills typically demonstrated by students at each band level in a particular course. NESA uses these holistic descriptions to report a student's overall level of achievement in an HSC course.

Band E4

A student who performs at this level typically:

- demonstrates extensive knowledge, understanding and skills appropriate to the course
- synthesises mathematical concepts, techniques and results to efficiently solve problems
- demonstrates sophisticated multi-step logic to solve problems in familiar and unfamiliar situations
- constructs, analyses and applies mathematical models effectively to solve problems in a wide variety of situations
- communicates complex reasoning, justification and arguments effectively, using appropriate mathematical language, notation, diagrams and graphs.

Band E3

A student who performs at this level typically:

- demonstrates thorough knowledge, understanding and skills appropriate to the course
- synthesises mathematical concepts, techniques and results to solve problems
- demonstrates well-developed multi-step logic to solve problems in familiar and unfamiliar situations
- constructs, analyses and applies mathematical models to solve problems in a variety of situations
- communicates reasoning and justification effectively, using appropriate mathematical language, notation, diagrams and graphs.

Band E2

A student who performs at this level typically:

- demonstrates sound knowledge, understanding and skills appropriate to the course
- uses mathematical concepts, techniques and results to solve problems
- constructs and applies mathematical models to solve problems in a range of situations
- communicates reasoning and justification using mathematical language, notation, diagrams and graphs.

Band E1

Performance reported as Band E1 indicates that the minimum standard expected was not demonstrated.

School-based assessment

Schools are required to develop an assessment program for each Year 11 and Year 12 course. NESA provides information about the responsibilities of schools in developing assessment programs in course-specific assessment requirements and in the Assessment Certification Examination (ACE) rules and requirements.

Year 11 Mathematics Extension 1 school-based assessment

Stage 6 – Year 11

Schools are required to submit to NESA a grade for each student based on their achievement at the end of the course.

Teachers use professional, on-balance judgement to allocate grades based on the Common Grade Scale for Preliminary courses.

Teachers consider all available assessment information, including formal and informal assessment, to determine the grade that best matches each student's achievement at the end of the course.

See Assessment Certification Examination (ACE) rules and requirements for further information.

Assessment programs must reflect course components and weightings

The course components and component weightings for Year 11 are mandatory.

Course component	Weighting %
Knowledge and understanding of course content	50
Skills in Working mathematically	50

Schools may determine specific elements of their assessment program

Schools have authority to determine the number, type of task and the weighting allocated to an assessment task. Schools may also follow the sample assessment programs provided by NESA.

Sample assessment program

NESA's sample Year 11 formal school-based assessment program for Mathematics Extension 1 is:

- 3 assessment tasks, including:
  - a formal written examination
  - a weighting for any individual task of 20% to 40%.

Some students with disability may require adjustments in order to access assessment opportunities and demonstrate achievement of outcomes.

Year 12 Mathematics Extension 1 school-based assessment

Stage 6 – Year 12

NESA requires schools to submit a school-based assessment mark for each Year 12 student who completes this course. Formal school-based assessment tasks should reflect the syllabus outcomes and content. The mark submitted by the school provides a summation of each student's achievement measured at several points throughout the course.

A school's program of school-based assessment includes both mandatory and non-mandatory elements.

See Assessment Certification Examination (ACE) rules and requirements for further information.

Assessment programs must reflect course components and weightings

The course components and component weightings for Year 12 are mandatory.

Course component	Weighting %
Knowledge and understanding of course content	50
Skills in Working mathematically	50

Schools may determine specific elements of their assessment program

Schools have authority to determine the number, type of task and the weighting allocated to an assessment task. Schools may also follow the sample assessment programs provided by NESA.

Sample assessment program

NESA's sample Year 12 formal school-based assessment program for Mathematics Extension 1 is:

- 4 assessment tasks, including:
  - a formal written examination
  - a weighting for any individual task of 10% to 40%.

Some students with disability may require adjustments in order to access assessment opportunities and demonstrate achievement of outcomes.

Formal school-based assessment in this course should focus on the Year 12 outcomes. The Year 11 course is assumed knowledge and may be assessed.

Sample assessment schedules

Stage 6

- Sample assessment schedule Year 11
- Sample assessment schedule Year 12

Sample assessment task notifications

Stage 6

- Sample formal assessment task notification: Year 11 (Polynomials and Further trigonometry)
- Sample formal assessment task notification: Year 12 (Application and modelling with trigonometry, vectors and calculus)

HSC examinations

The external HSC examination measures student achievement in a range of syllabus outcomes.

The external examination and its marking relate to the syllabus by:

- providing clear links to syllabus outcomes
- enabling students to demonstrate the levels of achievement outlined in the Performance Band Descriptions
- applying marking guidelines based on criteria that relate to the quality of the response
- aligning performance in the examination each year to the standards established for the course.

Examination questions may require students to integrate knowledge, understanding and skills developed through studying the course.

The Year 11 course is assumed knowledge for the Year 12 course.

Some students with disability may be eligible for disability provisions when completing the HSC examination.

The examination will be based on the Mathematics Extension 1 Year 12 course and will focus on the Year 12 outcomes. The Mathematics Advanced course and the Mathematics Extension 1 Year 11 course will be assumed knowledge for this examination. The Mathematics Extension 1 Year 11 course may be examined.

Students will also be required to complete either the Mathematics Advanced examination paper or the Mathematics Extension 2 examination paper, in addition to the Mathematics Extension 1 paper, based on their course enrolment.

Mathematics Extension 1 HSC examination specifications

Stage 6 – Year 12

The examination will consist of a written paper worth 70 marks.

Time allowed: 2 hours plus 10 minutes reading time.

A reference sheet will be provided.

NESA-approved calculators may be used.

The paper will consist of two sections.

Section I (10 marks)

- There will be objective-response questions to the value of 10 marks.

Section II (60 marks)

- Questions may contain parts.

- There will be 23 to 28 items.
- At least one item will be worth 4 or 5 marks.

HSC Mathematics Extension 1 – annotated sample examination materials

Stage 6 – Year 12

- HSC Mathematics Extension 1 – annotated sample examination materials
- Mathematics Extension 1 – HSC reference sheet

Further information

Stage 6 – Year 12

- Assessment in Stage 6
- Principles of effective assessment
- Assessment in practice
- School-based assessment requirements
- Adjustments to assessment for students with disability
- Life Skills
- Assessment Certification Examination (ACE) website

Mathematics Extension 2 11–12 (2024)

Implementation from 2026

The new Mathematics Extension 2 11–12 Syllabus (2024) is to be implemented from 2026 and will replace the **Mathematics Extension 2 Stage 6 Syllabus (2017)**.

2025

- Plan and prepare to teach the new syllabus

2026, Term 4

- Start teaching the new syllabus for Year 12
- Start implementing the new Year 12 school-based assessment requirements

2027

- First HSC examination for the new syllabus

Overview

Syllabus overview

Organisation of Mathematics Extension 2 Year 12

The organisation of outcomes and content for Mathematics Extension 2 Year 12 highlights the important role Working mathematically plays across all areas of mathematics and reflects the strengthened connections between concepts. Working mathematically has been embedded in the outcomes and content of the syllabus.

Mathematics Extension 2 Year 12 outcomes and their related content are organised into 5 areas of study:

- Proof
- Vectors
- Complex numbers
- Calculus
- Mechanics.

Figure 1 shows the organisation of Mathematics Extension 2 Year 12.

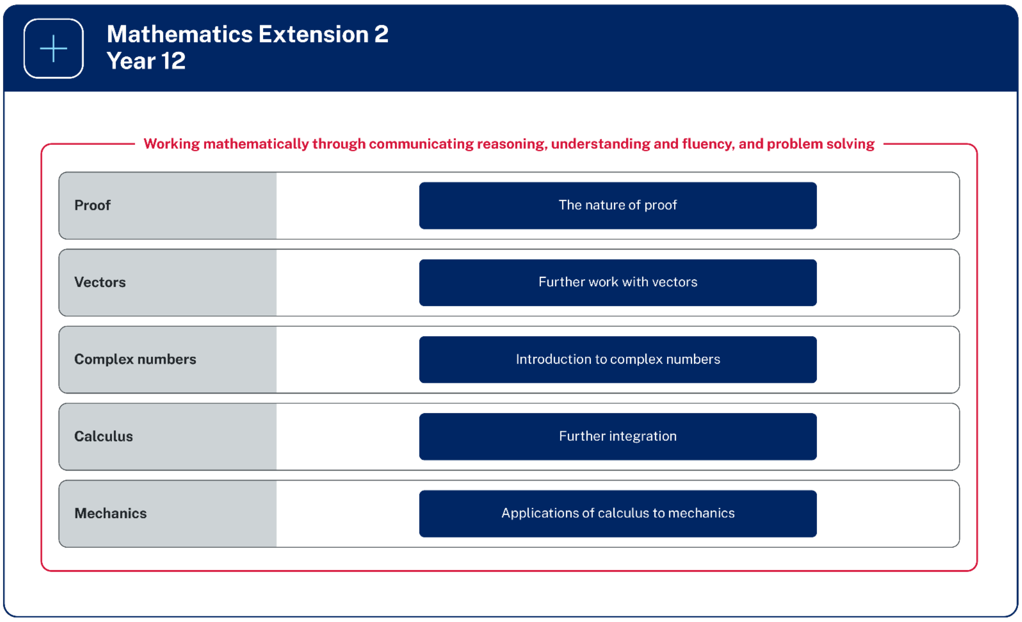


Figure 1: The organisation of Mathematics Extension 2 Year 12

Image long description: Five rows represent the areas of study for Mathematics Extension 2 Year 12. The Proof area of study includes the focus area The nature of proof. The Vectors area of study includes the focus area Further work with vectors. The Complex numbers area of study includes the focus area Introduction to complex numbers. The Calculus area of study includes the focus area Further integration. The Mechanics area of study includes the focus area Applications of calculus to mechanics. All content is surrounded by a box labelled with the phrase, 'Working mathematically through communicating reasoning, understanding and fluency, and problem solving'.

Protocols for collaborating with Aboriginal and Torres Strait Islander Communities and engaging with Cultural works

NESA is committed to working in partnership with Aboriginal Communities and supporting teachers, schools and schooling sectors to improve educational outcomes for young people.

It is important to respect appropriate ways of interacting with Aboriginal Communities and Cultural material when teachers plan, program and implement learning experiences that focus on Aboriginal and Torres Strait Islander Priorities.

Indigenous Cultural and Intellectual Property (ICIP) protocols need to be followed. Aboriginal and Torres Strait Islander Peoples' ICIP protocols include Cultural Knowledges, Cultural Expression and Cultural Property and documentation of Aboriginal and Torres Strait Islander Peoples' Identities and lived experiences. It is important to recognise the diversity and complexity of different Cultural groups in NSW, as protocols may differ between local Aboriginal Communities.

Teachers should work in partnership with Elders, parents, Community members, Cultural Knowledge Holders, or a local, regional or state Aboriginal Education Consultative Group. It is important to respect Elders and the roles of men and women. Local Aboriginal Peoples should be invited to share their Cultural Knowledges with students and staff when engaging with Aboriginal histories and Cultural Practices.

Course description

Mathematics Extension 2 focuses on key ideas of algebra and calculus and appreciation of mathematical invention, intuition and exploration. Mathematics Extension 2 extends students' conceptual knowledge and understanding through exploration of new areas of mathematics not covered in Mathematics Advanced and Mathematics Extension 1.

What students learn

Through the study of Mathematics Extension 2, students:

- develop strong knowledge, understanding and skills in Working mathematically and in communicating concisely and precisely
- acquire knowledge, understanding and skills in relation to mathematical concepts that have applications in an increasing number of contexts
- gain an appropriate mathematical background for future pathways which are founded in mathematics and its applications.

Course structure and requirements

Mathematics Extension 2 is a Year 12-only course. Students studying the Mathematics Extension 2 Year 12 course must:

- have studied the Mathematics Advanced and the Mathematics Extension 1 Year 11 courses
- study the Mathematics Advanced Year 12 and Mathematics Extension 1 Year 12 courses concurrently with Mathematics Extension 2 Year 12.

An alternative approach is for students to study Mathematics Advanced Year 11 and Mathematics Advanced Year 12 before studying Mathematics Extension 1 Years 11–12 and Mathematics Extension 2 Year 12.

The course numbers and units for each year of study are set out below.

Course numbers:

- Mathematics Extension 2 (Year 12, 1 unit): TBA

Year 12 course structure

Area of study	Focus area
Proof	The nature of proof
Vectors	Further work with vectors
Complex numbers	Introduction to complex numbers
Calculus	Further integration
Mechanics	Applications of calculus to mechanics

Further information for Mathematics Extension 2 Year 12

- Course number: TBA
- Course hours: 60

- Course units: 1
- Enrolment type: Elective
- Endorsement type: Board developed
- Study via self-tuition: Yes

Exclusions

- Mathematics Standard (Year 11, 2 units): TBA
- Mathematics Standard 1 (Year 12, 2 units): TBA
- Mathematics Standard 2 (Year 12, 2 units): TBA
- Mathematics Life Skills (Year 11, 2 units): TBA
- Mathematics Life Skills (Year 11, 2 units): TBA

Rationale

Mathematics in the Stage 6 Curriculum

Mathematical ideas have evolved and continue to develop across cultures and have been practised in Australia by Aboriginal and Torres Strait Islander Peoples for thousands of years.

The utility of mathematics extends from counting and measuring to explaining real-world phenomena and to the inherent beauty of mathematics. Mathematics has evolved in sophisticated ways to become the language used to describe and understand much of the modern world.

The creative application of mathematical concepts is integral to scientific and technological advancement. The symbolic nature of mathematics provides a precise means of communication.

The Mathematics Stage 6 syllabuses are designed to offer students opportunities to think mathematically. Mathematical thinking is supported through questioning, communicating, reasoning, reflecting and the use of appropriate technology, and is encouraged through providing students with opportunities to generalise, challenge, find connections and to think critically and creatively. Using mathematical thinking, students apply their knowledge and skills to deepen their understanding of the world.

Mathematics Extension 2

Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 form a continuum to provide students with opportunities to acquire knowledge, understanding and skills in mathematical concepts at progressively higher levels and with applications in an increasing number of contexts. Mathematics Advanced, Mathematics Extension 1 and Mathematics Extension 2 provide students with opportunities to learn about the interconnected nature of mathematics, its beauty and its functionality. The concepts and techniques of differential and integral calculus are the basis of the courses.

Mathematics Extension 2 provides students with the opportunity to develop strong mathematical manipulation skills and a deep understanding of the fundamental ideas of algebra and calculus. Students strengthen their appreciation of mathematics as an activity with its own intrinsic value, involving invention, intuition and exploration. Mathematics Extension 2 extends students' conceptual knowledge and understanding through the exploration of new areas of mathematics not seen in Mathematics Advanced and Mathematics Extension 1.

Mathematics Extension 2 provides a basis for a wide range of applications of mathematics as well as a strong foundation for further study of the subject at tertiary level.

Aim

The aim of Mathematics Extension 2 is for students to extend their knowledge from Mathematics Extension 1 and their understanding of Working mathematically. They enhance their skills to solve difficult problems, generalise, make connections in mathematics, and become fluent in using mathematical models and language to communicate in a concise and systematic manner.

Table of outcomes

MAO-WM-01 Working mathematically

develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly

This outcome is aligned to all content in each Stage.

Year 12
ME2-12-01 selects and applies the language, notation and methods of proof to prove results
ME2-12-02 uses vectors to model lines and curves, solve problems, and prove results involving geometry and coordinate geometry
ME2-12-03 uses algebraic and geometric representations of complex numbers to prove results and model and solve problems
ME2-12-04 selects and uses techniques of integration to solve problems
ME2-12-05 uses mechanics to model and solve practical problems

Outcomes and content for Year 12

The nature of proof

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- selects and applies the language, notation and methods of proof to prove results **ME2-12-01**

Content

The language and notation of proof

- Use the formal language of proof, including the terms 'statement', 'proposition', 'implication', 'converse', 'negation', 'contradiction', 'counterexample', 'equivalence' and 'contrapositive'
- Define a statement or proposition as a sentence that is either true or false, but not both
- Use the notation $P \wedge Q$ to represent the statement 'P and Q' and the notation $P \vee Q$ to represent the statement 'P or Q'
- Define and use the negation of P as 'not P', denoted $\neg P$ or $\sim P$
- Define an implication as an 'if–then' statement, where 'if P then Q' is denoted $P \Rightarrow Q$ or $P \rightarrow Q$, read as 'P implies Q'
- Use the quantifiers 'for all' $(\forall)$, and 'there exists' $(\exists)$ in formulating statements
- Negate statements including the negation of a negation $\neg(\neg P) = P$, the negation of an implication $\neg(P \Rightarrow Q) = (P \text{ and } \neg Q) = (P \wedge \neg Q)$, the negation $\neg(P \text{ and } Q) = (\neg P \text{ or } \neg Q)$, that is $\neg(P \wedge Q) = (\neg P \vee \neg Q)$, and the negation $\neg(P \text{ or } Q) = (\neg P \text{ and } \neg Q)$, that is $\neg(P \vee Q) = (\neg P \wedge \neg Q)$, noting that $P \Rightarrow Q = \neg(P \text{ and } \neg Q) = (\neg P \text{ or } Q)$, that is $P \Rightarrow Q = \neg(P \wedge \neg Q) = (\neg P \vee Q)$
- Define and use the converse of 'if P then Q' as 'if Q then P', denoted $Q \Rightarrow P$
- Recognise that the converse of a true implication may or may not be true
- Define equivalence of P and Q, as both $P \Rightarrow Q$ and $Q \Rightarrow P$, denoted $P \Longleftrightarrow Q$ or $P \leftrightarrow Q$, read as 'P if and only if Q', commonly abbreviated 'P iff Q'
- Define the contrapositive of 'if P then Q' as 'if not Q then not P', denoted $\neg Q \Rightarrow \neg P$
- Recognise that an implication is equivalent to its contrapositive, that is $(P \Rightarrow Q) \Longleftrightarrow (\neg Q \Rightarrow \neg P)$, and use this to prove results

Illustrations of proofs

- Use proof by contradiction to prove the truth of mathematical statements
- Use examples and counterexamples to test the truth of mathematical statements
- Prove results involving integers

Example(s):

Prove result involving odd and even integers or divisibility.

Proof of inequalities

- Prove results involving inequalities using the definition of $a>b$ for real a and b, that is $a>b$ if and only if $a - b>0$
- Prove results involving inequalities using the property that squares of real numbers are non-negative, in particular $(a\pm b)^2\geq 0$
- Prove and use results for numbers: if $a>b$ then $a\pm c>b\pm c$; if $a>b>0$ then $\frac{1}{b}>\frac{1}{a}>0$and vice versa; if $|a|>|b|$ then $a^2>b^2$ and vice versa; if $a>b$ and $b>c$ then $a>c$; if $a>b$ and $c>d$ then $a+c>b+d$; if $a>b$ and $c>0$ then $ac>bc$; if $a>b$ and $c<0$ then $ac<bc$
- Prove and use the triangle inequality $|a+b|\leq|a|+|b|$ and interpret the inequality geometrically
- Establish and use the relationship between the arithmetic mean and geometric mean for two non-negative numbers, that is $\forall\, a,b\geq 0, \frac{a+b}{2}\geq\sqrt{ab}$
- Prove results involving inequalities using previously obtained or known inequalities

Example(s):

Prove $x^2+y^2\geq 2xy$ and use the result to prove other inequalities.

- Prove inequalities involving geometry
- Prove results using the squeeze theorem: if $f(x)\leq g(x)\leq h(x)$ for all x that are near k, but not necessarily at k, and $\lim_{x\to k} f(x)=\lim_{x\to k} h(x)=L$, then $\lim_{x\to k} g(x)=L$
- Prove inequalities using graphical or calculus techniques or a combination of both

Example(s):

Prove results by considering where functions are increasing or decreasing.

Further proof by mathematical induction

- Prove results involving trigonometric, logarithmic, exponential, polynomial or other identities, including the binomial theorem, using mathematical induction
- Prove inequality results using mathematical induction
- Prove results in calculus using mathematical induction
- Explain that a recursive formula, or recurrence relation, is a formula that defines each term of a sequence using a preceding term
- Prove results involving first-order recursive formulas using mathematical induction
- Prove geometric results using mathematical induction

Further work with vectors

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- uses vectors to model lines and curves, solve problems, and prove results involving geometry and coordinate geometry **ME2-12-02**

Content

Vector equations of lines and curves

- Define the direction vector of a straight line and identify that a straight line through two points P and Q has $\overrightarrow{PQ}$ as a possible direction vector in both two dimensions and three dimensions
- Establish the relationship between the gradient, m, of a straight line in two dimensions and its direction vector
- Examine and use $r = a + \lambda b$ as the vector equation of a straight line in two dimensions and three dimensions to solve problems, where r is the position vector of a point on the line, a is the position vector of a particular point on the line, b is a direction vector of the line and λ is a scalar parameter
- Identify $r = p + \lambda(q - p)$ as one possible vector equation for the straight line through points P and Q, where p and q are the position vectors of P and Q respectively, noting its equivalence with the form $r = \lambda q + (1 - \lambda) p$, and establish the correspondence between the value of λ and the position of the point specified by it along the line
- Express a line in two dimensions given in gradient–intercept form ($y = mx + c$) as a vector equation, and vice versa
- Determine when two lines in vector form are parallel in two dimensions and three dimensions
- Determine whether a given point lies on a line in vector form
- Determine using vector methods whether three points are collinear in two dimensions and three dimensions
- Determine the point of intersection of two non-parallel lines expressed as vector equations in two dimensions
- Determine whether two distinct lines $r_1 = a_1 + \lambda_1 b_1$ and $r_2 = a_2 + \lambda_2 b_2$ in three dimensions intersect, and if so, find the unique value of either λ_1 or λ_2 corresponding to the point of intersection and determine its coordinates
- Define skew lines in three dimensions and apply vector methods to determine whether two lines in three dimensions are skew
- Recognise that a parametrically defined curve in two dimensions or three dimensions can be expressed as a vector equation: $r = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ or $r = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix}$, where t is a parameter
- Examine and identify curves given as vector equations in two dimensions and three dimensions using graphing applications
- Identify the vector equation of a curve given its graph
- Recognise $r = r\cos(\theta) i + r\sin(\theta) j$ and $|r| = r$ as vector equations of a circle in two dimensions with radius r centred at the origin

- Recognise $r = (r\cos(\theta) + x_C)i + (r\sin(\theta) + y_C)j$ and $|r - c| = r$ as vector equations of a circle in two dimensions with radius r and centre C with position vector $c = \begin{pmatrix} x_C \\ y_C \end{pmatrix}$
- Recognise $|r - c| = r$ as the vector equation of a sphere in three dimensions with radius r and centre C with position vector c
- Use the circle equations and sphere equations to solve problems

Example(s):

Let S be a sphere with equation $\left| r - \begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} \right| = 15$. Show that the point $P(1, -1, 2)$ lies inside the sphere S.

- Determine the Cartesian equation of a curve in two dimensions given its vector equation and graph the curve, where the curve is within the scope of this syllabus

Example(s):

A curve has vector equation $r = (2\sin t - 1)i + (2\cos t + 3)j$. Write down a pair of parametric equations for this curve. Determine the Cartesian equation for this curve. Sketch the curve on a Cartesian plane.

Vectors and geometry

- Examine and use properties of the scalar (dot) product, including commutativity ($a \cdot b = b \cdot a$), distributivity $(a \cdot (b + c) = a \cdot b + a \cdot c)$ and scalar multiplication ($k(a \cdot b) = (ka) \cdot b = a \cdot (kb)$)
- Establish and use the results $i \cdot i = j \cdot j = k \cdot k = 1$ and $i \cdot j = j \cdot k = k \cdot i = 0$
- Prove and use the Cauchy–Schwarz inequality for vectors: $|a \cdot b| \le |a||b|$
- Define the medians, altitudes, perpendicular bisectors and angle bisectors of a triangle and recognise these definitions in vector proofs
- Solve problems and prove geometric results in two dimensions and three dimensions using vectors

Introduction to complex numbers

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- uses algebraic and geometric representations of complex numbers to prove results and model and solve problems **ME2-12-03**

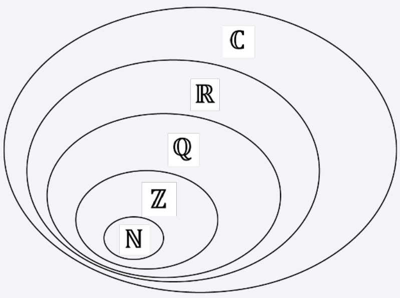
Content

Arithmetic of complex numbers

- Define the number i by $i^2 = -1$
- Use the number i to solve quadratic equations of the form $x^2 + k = 0$ where k is a positive real number
- Define the complex numbers ($\mathbb{C}$) as the set of numbers of the form $a + ib$, where a and b are real numbers
- Use complex numbers to express the roots of quadratic equations of the form $ax^2 + bx + c = 0$, where a, b and c are real numbers and the discriminant $\Delta = b^2 - 4ac < 0$
- Refer to a as 'the real part of the complex number $z = a + ib$', denoted by $\mathrm{Re}(z)$
- Refer to b as 'the imaginary part of the complex number $z = a + ib$', denoted by $\mathrm{Im}(z)$
- Classify numbers as belonging to the set of natural numbers ($\mathbb{N}$), integers ($\mathbb{Z}$), rational numbers ($\mathbb{Q}$), real numbers ($\mathbb{R}$) or complex numbers ($\mathbb{C}$), each of which is an extension of the previous

Example(s):

Classify each of the numbers 5, $\frac{18}{6}$, -8, $\sin\frac{\pi}{6}$, $\frac{22}{7}$, $\sqrt{5}$, $\sqrt{49}$ and $2+3i$ using the smallest possible subset presented in the number category Venn diagram below.



- Identify and use the condition for two complex numbers z_1 and z_2 to be equal, that is $z_1 = z_2$ if and only if $\mathrm{Re}(z_1) = \mathrm{Re}(z_2)$ and $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$
- Define and perform complex number addition, subtraction and multiplication, with and without digital tools
- Define the complex conjugate of a complex number $z = a + ib$ as $\bar{z} = a - ib$ and use it to solve problems
- Define and calculate the modulus of a complex number $z = a + ib$ as $|z| = \sqrt{z\bar{z}} = \sqrt{a^2 + b^2}$

- Establish relationships $\mathrm{Re}(z)=\frac{z+\bar{z}}{2}$ and $\mathrm{Im}(z)=\frac{z-\bar{z}}{2i}$ and $\frac{1}{z}=\frac{\bar{z}}{|z|^2}$ and use them to solve problems
- Divide one complex number by another non-zero complex number, with and without digital tools, and give the result in the form $a+ib$
- Find the two square roots of a complex number $z=a+ib$

Geometric representation of complex numbers

- Plot the complex number $z=a+ib$ as a point on the complex plane with and without graphing applications
- Define and calculate the argument of a non-zero complex number $z=a+ib$ as $arg(z)=\theta$, where θsatisfies $\sin\theta=\frac{b}{|z|}$ and $\cos\theta=\frac{a}{|z|}$, noting that the argument has multiple values that differ by multiples of 2π
- Define and use the principal argument $Arg(z)$ of a non-zero complex number z as the unique value of the argument in the interval $(-\pi,\pi]$
- Define and use complex numbers in polar or modulus–argument form that expresses a complex number in terms of its modulus and argument, $z=r(\cos\theta+i\sin\theta)$, where r is its modulus and θ is an argument of z, and represent complex numbers in this form on the complex plane
- Use multiplication, division and powers of complex numbers in polar form and interpret these geometrically
- Convert between complex numbers in Cartesian form and polar form and use complex numbers in Cartesian form and polar form to solve problems
- Prove and use identities involving the modulus of complex numbers: $|z_1 z_2|=|z_1||z_2|$, $\left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|}$ and $|z^n|=|z|^n$, where n is an integer
- Prove and use identities involving the argument of complex numbers: $arg(z_1 z_2)=arg(z_1)+arg(z_2)$, $arg\left(\frac{z_1}{z_2}\right)=arg(z_1)-arg(z_2)$ and $arg(z^n)=n\,arg(z)$ where n is an integer
- Prove and use identities involving the complex conjugate of complex numbers: $\overline{z_1}+\overline{z_2}=\overline{z_1+z_2}$, $\overline{z_1}\,\overline{z_2}=\overline{z_1 z_2}$
- Prove and use the triangle inequality $|z_1+z_2|\le|z_1|+|z_2|$ for complex numbers z_1 and z_2

Solving equations with complex numbers

- Solve quadratic equations of the form $ax^2+bx+c=0$, where a, b and c are complex numbers
- Recognise that solutions to quadratic equations with real coefficients are complex conjugates of each other and use this to solve problems
- Prove the complex conjugate root theorem: if the complex number $z=a+ib$ is a root of the polynomial equation $P(x)=0$ with real coefficients, then the complex conjugate $\bar{z}=a-ib$ is also a root of $P(x)=0$
- Solve problems involving complex conjugate roots of polynomial equations with real coefficients

Powers and roots of complex numbers

- Using proof by mathematical induction prove de Moivre's theorem for positive integer powers: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$
- Prove that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ for negative integers n
- Use de Moivre's theorem to find any integer power of a given complex number
- Use de Moivre's theorem to derive trigonometric identities
- Determine the n^{th} roots of ± 1 in polar form and their location on the unit circle
- Illustrate the geometrical relationship connecting the n^{th} roots of ± 1
- Determine the n^{th} roots of complex numbers and their location on the complex plane
- Recognise that a complex number can be represented as a vector, where the magnitude and direction of the vector are determined by the modulus and argument of the complex number respectively
- Examine and use addition and subtraction of complex numbers as vectors on the complex plane
- Examine and use the geometric interpretation of multiplying complex numbers, including rotation and dilation on the complex plane, with and without graphing applications
- Prove geometric results using complex numbers as vectors
- Solve problems and prove results using the n^{th} roots of complex numbers

Describing lines, curves and regions

- Graph vertical and horizontal lines of the form $\mathrm{Re}(z)=c$ or $\mathrm{Im}(z)=k$ where c and k are real constants
- Graph the line corresponding to the equation $|z-z_1|=|z-z_2|$, where z_1 and z_2 are complex numbers, and give a geometrical description of the line
- Graph the circles corresponding to the equations $|z|=r$ and $|z-z_1|=r$, where z_1 is a complex number and r is a positive real number
- Graph rays corresponding to the equations $arg(z)=\theta$ and $arg(z-z_1)=\theta$, where z_1 is a complex number
- Graph regions associated with lines, rays and circles defined using complex numbers, giving a geometrical description of any such curves or regions, and using circle geometry theorems where necessary

Further integration

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- selects and uses techniques of integration to solve problems **ME2-12-04**

Content

- Derive the identities for trigonometric products as sums and differences for $\cos A\cos B=\frac{1}{2}[\cos(A-B)+\cos(A+B)]$, $\sin A\sin B=\frac{1}{2}[\cos(A-B)-\cos(A+B)]$, $\sin A\cos B=\frac{1}{2}[\sin(A+B)+\sin(A-B)]$ and $\cos A\sin B=\frac{1}{2}[\sin(A+B)-\sin(A-B)]$
- Use the identities for trigonometric products as sums and differences to solve problems and prove results
- Solve trigonometric equations by applying the formulas for trigonometric products as sums and differences for $\cos A\cos B$, $\sin A\sin B$ and $\sin A\cos B$ over restricted domains

Example(s):

Solve $\cos 3x+\cos x=0$ for $0\le x\le \pi$.

- Use identities relating the trigonometric products as sums and differences to solve problems involving integrals of the form $\int \sin(mx)\cos(nx)\,dx$, $\int \sin(mx)\sin(nx)\,dx$ or $\int \cos(mx)\cos(nx)\,dx$
- Derive the expressions $\sin A=\frac{2t}{1+t^2}$, $\cos A=\frac{1-t^2}{1+t^2}$ and $\tan A=\frac{2t}{1-t^2}$ where $t=\tan\frac{A}{2}$ (the t-formulas) and use them to solve trigonometric equations over restricted domains

Example(s):

Solve $3\sin\theta-2\cos\theta=2$ for $0\le\theta\le 2\pi$ using the substitution $t=\tan\frac{\theta}{2}$.

- Find indefinite integrals and evaluate definite integrals using the method of integration by substitution, where the substitution may or may not be given
- Decompose rational functions whose denominators can be expressed as a product of distinct linear factors, distinct irreducible quadratic factors and perfect square factors into partial fractions

- Integrate rational functions whose denominators can be expressed as a product of distinct linear factors, distinct irreducible quadratic factors and perfect square factors, using partial fraction decomposition

Example(s):

Find $\int \frac{3\,dx}{x^2-2x}$, $\int \frac{3x+6}{x^2+4x+5}dx$ and $\int \frac{2x-5}{x^2-6x-1}dx$.

- Integrate rational functions by completing the square on a quadratic denominator

Example(s):

Find $\int \frac{dx}{x^2-2x+3}$.

- Integrate rational functions where the degree of the numerator is not less than the degree of the denominator

Example(s):

Find $\int \frac{x^2+1}{x+1}dx$.

- Integrate functions by changing an integrand into an appropriate form using algebraic manipulation

Example(s):

Find $\int \frac{3x+6}{x^2+4x+5}dx$ and $\int \frac{2x-5}{x^2-6x-1}dx$.

- Derive the method for integration by parts
- Find indefinite integrals and evaluate definite integrals using the method of integration by parts, including problems where more than one application is required
- Derive and use recurrence relations involving integration by parts

Example(s):

Let $I_n = \int_1^2 x^n \sqrt{x-1}\,dx$ for integers $n \geq 0$. Show that $I_n = \frac{2^{n+1}+2nI_{n-1}}{2n+3}$ for $n \geq 1$. Hence, find $\int_1^2 x^2 \sqrt{x-1}\,dx$.

- Solve theoretical problems involving multiple techniques of integration

Example(s):

Find $\int sec^3 x \tan^3 x \, d x$.

- Solve practical problems involving multiple techniques of integration

Applications of calculus to mechanics

Outcomes

A student:

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- uses mechanics to model and solve practical problems **ME2-12-05**

Content

Forces and further motion in a straight line

- Derive expressions for acceleration: $\frac{dv}{dt}$, where v is a function of t, as well as $v\frac{dv}{dx}$ and $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$, where v is a function of x
- Solve problems involving velocity and acceleration expressed in terms of displacement, and acceleration expressed in terms of velocity
- Examine Newton's three laws of motion, including force, acceleration, action and reaction under a constant and non-constant force
- Find acceleration, $\ddot{x}$, using the formula $F=m\ddot{x}$, where F is the force acting on a mass, m
- Recognise that forces are vector quantities, analyse concurrent forces on a body by resolving them into perpendicular components and use vector projections to determine how much of a given force acts in a given direction in both 2D and 3D contexts

Simple harmonic motion

- Define simple harmonic motion as motion in which acceleration is proportional to, and in the opposite direction to, displacement, that is $\ddot{x}=-n^2(x-c)$, where x is the displacement of a particle from the centre of motion at $x=c$ and $\ddot{x}$ is the acceleration
- Recognise that the force $F(x)$ acting on a body of mass, m, executing simple harmonic motion according to the equation $\ddot{x}=-n^2(x-c)$, is a restoring force which acts towards the centre of motion causing the body to oscillate around the centre of motion
- Verify that $x=A\sin(nt+\alpha)+c$, $x=A\cos(nt+\alpha)+c$, or $x=A\cos(nt+\alpha)+B\sin(nt+\beta)$ are solutions to the differential equation defining simple harmonic motion
- Describe simple harmonic motion using displacement, velocity, acceleration, force, amplitude and period
- Prove that motion is simple harmonic by obtaining an equation of the form $\ddot{x}=-n^2(x-c)$, when given an equation for acceleration, velocity or displacement
- Graph x, $\dot{x}$ and $\ddot{x}$ as functions of t with and without graphing applications for a particle moving in simple harmonic motion where x is of the form $x=A\cos(nt+\alpha)+c$ or $x=A\sin(nt+\alpha)+c$
- Determine equations for simple harmonic motion when given graphs of acceleration, velocity or displacement in terms of time
- Derive $v^2=n^2\left[A^2-(x-c)^2\right]$ for a particle moving in simple harmonic motion according to $\ddot{x}=-n^2(x-c)$, where A is the amplitude
- Determine equations for the displacement, x, and velocity, v, in terms of time for an object executing simple harmonic motion according to a given equation $\ddot{x}=-n^2(x-c)$ and satisfying given initial conditions

- Model and solve problems involving simple harmonic motion using relevant formulas and graphs

Modelling motion without resistance

- Derive the equations of motion for a particle travelling, without resistance, in a straight line with constant and variable acceleration and use the equations of motion to solve problems
- Analyse and solve problems relating to motion on a smooth inclined plane by resolving forces into components parallel and perpendicular to the inclined plane
- Solve motion problems involving a single smooth pulley and a smooth inclined plane where a body hangs vertically or lies on a smooth horizontal or inclined plane

Rectilinear resisted motion

- Derive, from Newton's laws of motion, $\sum F = m\ddot{x}$, the equation for acceleration of a particle moving in a straight line and in the absence of external forces, except for a resistance oppositely directed to the motion and with a magnitude proportional to a power of the speed
- Derive an expression for velocity as a function of time of a particle moving in a straight line and in the absence of external forces, except for a resistance oppositely directed to the motion and with a magnitude proportional to a power of the speed
- Derive an expression for velocity as a function of displacement of a particle moving in a straight line and in the absence of external forces, except for a resistance oppositely directed to the motion and with a magnitude proportional to a power of the speed
- Derive an expression for displacement as a function of time of a particle moving in a straight line and in the absence of external forces, except for a resistance oppositely directed to the motion and with a magnitude proportional to a power of the speed
- Solve problems, excluding those with pulley systems, involving a particle moving in a straight line subject to a resistance oppositely directed to the motion and with a magnitude proportional to a power of the speed

Vertical resisted motion

- Derive, from Newton's laws of motion, $\sum F = m\ddot{x}$, the equation for acceleration of a particle moving vertically (upwards or downwards) in a resistive medium and under the influence of constant gravity, where the particle experiences a resistance, R, oppositely directed to the motion, whose magnitude is proportional to the first or second power of the speed
- Derive expressions for velocity both as a function of time and of displacement for vertical resisted motion under the influence of constant gravity, with a resistance, R, oppositely directed to the motion, whose magnitude is proportional to the first or second power of the speed
- Derive an expression for displacement as a function of time for vertical resisted motion under the influence of constant gravity, with a resistance, R, oppositely directed to the motion, whose magnitude is proportional to the first or second power of the speed
- Define the terminal velocity of a particle falling through a medium as the constant velocity the particle reaches when the resistance of the medium prevents further acceleration
- Determine the terminal velocity of a falling particle from its equation of motion for vertical resisted motion under the influence of constant gravity, with a resistance, R, oppositely directed to the motion, and whose magnitude is proportional to the first or second power of the speed
- Solve vertical resisted motion problems using the expressions derived for acceleration, velocity and displacement, including finding the maximum height reached by a particle projected vertically upwards and the time taken to reach this maximum height, and finding the time taken for a particle to return to the level from which it was projected and its terminal velocity

Projectiles and resisted motion

- Distinguish between the shape of the trajectory of a projectile moving under the influence of gravity and with negligible resistance and the shape of the trajectory of a projectile moving under the influence of gravity subject to resistance whose magnitude is proportional to the speed
- Establish and use the equations for acceleration for a projectile moving under the influence of gravity, projected at an angle to the horizontal, and subject to a resistance whose magnitude is proportional to the speed, to solve problems

Example(s):

Derive equations for horizontal and vertical displacement in terms of time for a projectile. The projectile is projected at an angle of θ to the horizontal and moving in a resisting medium. The magnitude of resistance is proportional to the speed and under the influence of gravity.

Assessment

Course standards

Course standards are comprised of syllabus standards and performance standards. Syllabus outcomes and content form the syllabus standards that describe what students are expected to know, understand and do. Performance standards, including the Common Grade Scale for Preliminary courses, HSC Performance Band Descriptions and Achievement Level Descriptions (for HSC courses with grades only) describe how well students have achieved in relation to syllabus standards. Performance standards are used to report student achievement on NESA credentials.

Performance Band Descriptions for Mathematics Extension 2

Stage 6 – Year 12

HSC performance band descriptions summarise the knowledge, understanding and skills typically demonstrated by students at each band level in a particular course. NESA uses these holistic descriptions to report a student's overall level of achievement in an HSC course.

Band E4

A student who performs at this level typically:

- demonstrates extensive knowledge, understanding and skills appropriate to the course
- creatively synthesises mathematical concepts, techniques and results to efficiently solve problems
- demonstrates sophisticated multi-step logic and mathematical insight to solve problems and prove results in familiar and unfamiliar situations
- constructs, analyses and applies mathematical models effectively to solve problems in a wide variety of situations
- communicates complex reasoning, justification and arguments effectively, using appropriate mathematical language, concise notation, diagrams, and algebraic and graphical techniques.

Band E3

A student who performs at this level typically:

- demonstrates thorough knowledge, understanding and skills appropriate to the course
- synthesises mathematical concepts, techniques and results to solve problems
- demonstrates complex multi-step logic to solve problems and prove results in familiar and unfamiliar situations
- constructs, analyses and applies mathematical models to solve problems in a variety of situations
- communicates reasoning and justification effectively, using appropriate mathematical language, notation, diagrams, and algebraic and graphical techniques.

Band E2

A student who performs at this level typically:

- demonstrates sound knowledge, understanding and skills appropriate to the course
- uses mathematical concepts, techniques and results to solve problems
- constructs and applies mathematical models to solve problems in a range of situations
- communicates reasoning and justification effectively, using mathematical language, notation, diagrams and graphs.

Band E1

Performance reported as Band E1 indicates that the minimum standard expected was not demonstrated.

School-based assessment

Schools are required to develop an assessment program for each Year 11 and Year 12 course. NESA provides information about the responsibilities of schools in developing assessment programs in course-specific assessment requirements and in the Assessment Certification Examination (ACE) rules and requirements.

Year 12 Mathematics Extension 2 school-based assessment

Stage 6 – Year 12

NESA requires schools to submit a school-based assessment mark for each Year 12 student who completes this course. Formal school-based assessment tasks should reflect the syllabus outcomes and content. The mark submitted by the school provides a summation of each student's achievement measured at several points throughout the course.

A school's program of school-based assessment includes both mandatory and non-mandatory elements.

See Assessment Certification Examination (ACE) rules and requirements for further information.

Assessment programs must reflect course components and weightings

The course components and component weightings for Year 12 are mandatory.

Course component	Weighting %
Knowledge and understanding of course content	50
Skills in Working mathematically	50

Schools may determine specific elements of their assessment program

Schools have authority to determine the number, type of task and the weighting allocated to an assessment task. Schools may also follow the sample assessment programs provided by NESA.

Sample assessment program

NESA's sample Year 12 formal school-based assessment program for Mathematics Extension 2 is:

- 4 assessment tasks, including:
  - a formal written examination
  - a weighting for any individual task of 10% to 40%.

Some students with disability may require adjustments in order to access assessment opportunities and demonstrate achievement of outcomes.

Formal school-based assessment in this course should focus on the Year 12 outcomes. The Year 11 course is assumed knowledge and may be assessed.

Sample assessment schedules

Stage 6 – Year 12

- Sample assessment schedule Year 12

Sample assessment task notifications

Stage 6

- Sample formal assessment task notification: Year 12 (Modelling with integration and geometrical applications of vectors)

HSC examinations

The external HSC examination measures student achievement in a range of syllabus outcomes.

The external examination and its marking relate to the syllabus by:

- providing clear links to syllabus outcomes
- enabling students to demonstrate the levels of achievement outlined in the Performance Band Descriptions
- applying marking guidelines based on criteria that relate to the quality of the response
- aligning performance in the examination each year to the standards established for the course.

Examination questions may require students to integrate knowledge, understanding and skills developed through studying the course.

Some students with disability may be eligible for disability provisions when completing the HSC examination.

The examination will be based on the Mathematics Extension 2 Year 12 course and will focus on the course outcomes. The Mathematics Advanced and Mathematics Extension 1 courses will be assumed knowledge for this examination.

Students will also be required to complete the Mathematics Extension 1 paper in addition to the Mathematics Extension 2 paper.

Mathematics Extension 2 HSC examination specifications

Stage 6 – Year 12

The examination will consist of a written paper worth 100 marks.

Time allowed: 3 hours plus 10 minutes reading time.

A reference sheet will be provided.

NESA-approved calculators may be used.

The paper will consist of two sections.

Section I (10 marks)

- There will be objective-response questions to the value of 10 marks.

Section II (90 marks)

- Questions may contain parts.
- There will be 37 to 42 items.
- At least two items will be worth 4 or 5 marks.

HSC Mathematics Extension 2 – annotated sample examination materials

Stage 6 – Year 12

- HSC Mathematics Extension 2 – annotated sample examination materials
- Mathematics Extension 2 – HSC reference sheet

Further information

Stage 6 – Year 12

- Assessment in Stage 6
- Principles of effective assessment
- Assessment in practice
- School-based assessment requirements
- Adjustments to assessment for students with disability
- Life Skills
- Assessment Certification Examination (ACE) website